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## A control theory approach to determining premiums and coverage in relation to externalities associated with pension benefit guarantee insurance

### Abstract

This paper analyzes premium rates, and externalities associated with providing Pension Benefit Guarantee Corporation (PBGC) insurance coverage for single employer, defined benefit pension plans in the U.S. The first section of this investigation examines micro and macro-economic factors impacting financial stability and actuarial viability of PBGC. The second section discusses externalities that make adequate rate setting difficult from a public policy perspective. The paper concludes with a linear control model that may assist policy makers in determining the extent of PBGC coverage as primary insurer, the Treasury Department's role as reinsurer for catastrophic loss, and premium rates that adequately covers losses.

**Keywords:** control theory, pension guarantee, public policy.

**JEL Classification:** C73, D62, G22, G23, G28, H11, J32.

### Introduction

**Micro and macro financial and economic factors impacting PBGC.** The Pension Benefit Guarantee Corporation (PBGC), created under the 1974 Employee Retirement Income Security Act, is a quasi-governmental insurer offering coverage to participants in defined benefit pension plans. While PBGC is considered a federal agency under the U.S. Department of Labor, its primary funding is based on income generated from premiums charged to employer sponsors of defined benefit plans. Presently, PBGC provides insurance to 44 million workers and retirees in 29,100 private defined benefit plans. The largest number of workers and retirees, 33.6 million, are members of single-employer plans<sup>1</sup>. In the PBGC's most recent 2011 annual report, this insurance agency's deficit for covering single-employer plans increased to \$23.266 billion, while its estimate of contingent loss reserves grew from \$170 to \$227 billion, while maintaining a flat premium structure at \$35 per participant<sup>2</sup>. These findings show that the current PBGC premium structure may be woefully inadequate to meet current and future insurance obligations. PBGC deficits are being financed by the U.S. Treasury through an open line of credit. The present article examines the externalities associated with PBGC premiums rates and reserving in relation to this Treasury financing of reserve deficits to determine public policy in setting of rates on PBGC coverage.

Table 1 giving PBGC's net financial position for the past 10 years, demonstrates how difficult, from a public policy standpoint, it may be to determine adequate rates for PBGC insurance losses. Over the last decade PBGC financial assets have been declining to the point where the insurer is running a negative tangible net worth position of \$23.266 billion. The steady decline in net worth position occurred during a time when PBGC premiums went from \$19 to \$35 participant (U.S. Department of Labor, 2010; U.S. Department of Labor, 2004; U.S. Department of Labor, 2005). Consequently, PBGC premium policies have not set flat or variable rates at the level necessary to reduce or eliminate reserve deficits in order to avoid extensive use of the U.S. Treasury line of credit.

Table 1. Net financial position of PBGC's single-employer program from 2001 to 2011

Fiscal year	Assets in millions	Liabilities in millions	Net financial position in millions
2011	\$78,960	\$102,226	-\$23,266
2010	\$77,827	\$99,421	-\$21,594
2009	\$68,736	\$89,813	-\$21,077
2008	\$64,612	\$75,290	-\$10,678
2007	\$67,241	\$80,352	-\$13,111
2006	\$59,972	\$78,114	-\$18,142
2005	\$56,470	\$79,246	-\$22,776
2004	\$38,993	\$62,298	-\$23,305
2003	\$34,016	\$45,254	-\$11,238
2002	\$25,430	\$29,068	-\$ 3,638
2001	\$21,768	\$14,036	\$ 7,732

Source: U.S. Department of Labor (2004) Pension Data Book; U.S. Department of Labor (2009) Pension Data Book; U.S. Department of Labor (2010) PBGC Annual Report; U.S. Department of Labor (2011) Pension Data Book.

An actuarial and financial approach to adequately funding PBGC insurance coverage, seeking to include the external relationships between insurer and policyholder, requires the present value of future

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<sup>1</sup> 33.6 million workers in 27,600 single employer plans, and 10.6 million workers in 1,500 multi-employer plans, U.S. Department of Labor (2009), PBGC Annual Report.

<sup>2</sup> U.S. Department of Labor (2011). PBGC Annual Report and <http://www.pbgc.gov/prac/prem/premium-rates.html> (accessed January 16, 2012).

premiums be sufficient to meet the present value of future obligations to workers and retirees in defined benefit pension plans covered by PBGC. Two critical variables in maintaining the viability of an insurer is the ability to (1) adjust premiums to fully reflect the risks assumed with coverage; and (2) alter underwriting and benefit structures to avoid adverse selection. PBGC deficits for the past 10 years would appear to support two underwriting positions: (1) the flat premium may have to increase to allow for healthier pension plans to support insurance reserves; and (2) variable rates need to be revised upward to fully reflect added risk on those plans with underfunded benefits. From a public policy standpoint, premium rates should be driven more on the basis of actuarial considerations in terms of meeting current and future losses, than the desires of the premium payers for modest rates.

From a microeconomic, insurance perspective, PBGC offers a unique form of coverage. Although the policy insures pension benefits for workers participating in defined benefit plans, the premiums are paid by firms sponsoring the pensions. The insured party is not the policyholder or premium payer for the coverage. The insurance is owned by the corporation, on behalf of the insured workers covered by PBGC. Consequently, the managers of the firm decide whether to continue their defined benefit plan and pay premiums, or terminate the pension altogether. Insured workers have little control over these decisions. This unique aspect to PBGC insurance creates externalities between PBGC, the employer offering coverage and the employee who is to receive the benefits of PBGC insurance in case a firm goes bankrupt. Plan terminations may occur as the result of bankruptcy or the firm's managerial decision to convert the pension into a defined contribution plan. When PBGC receives a plan either through voluntary termination or bankruptcy, they must provide a fixed level of benefits to workers under the plans. In most cases, plan assets are not sufficient to carry benefits at the levels originally provided by the employer. Consequently, active and retired employees will obtain less in retirement benefits under such circumstances (Jupiter, 1982)<sup>1</sup>. PBGC when taking over a plan will acquire control of pension assets which may have value sufficient to fund increased benefits to retired employees above PBGC's maximum benefit amount. However, diffi-

culties with handling assets acquired from terminated plans are significant including, but not limited to: (1) the cost of managing physical assets until a sale can be made (2) determining a fair value of the assets quickly (3) maintaining the value of the assets while an appraisal is being made of whether to dispose or retain the property, and (4) dealing with litigation costs associated with enforcing PBGC's right to the property. These problems create another external relationship between PBGC and retired and active worker insurance beneficiaries. If PBGC has limited funds due to deficits to manage acquired pension plan assets, there may be little value obtained when liquidating them. Due to the relationship between PBGC's financial resources and its ability to maximize the value of acquired assets, it would be important for PBGC to eliminate fund deficits and build a net financial position. Development of a model of these externalities may provide PBGC policy makers the ability to determine premium rates that will protect the interests of retirees and workers, as policyholders, in the long term.

A recent example of the reduction in pension payments to PBGC retirees, from the diminishing value of pension assets, is the United Airlines bankruptcy and pension termination. PBGC became an unsecured creditor in United Airlines when the company shifted \$10.2 billion in unfunded pension liabilities to the agency in December 2002. PBGC reached an agreement, during the United Airlines bankruptcy proceedings, to receive a \$5.6 billion claim on the new United Airlines. In February 2006, PBGC sold \$2.5 billion of this claim to hedge fund investors and banks for \$450 million or \$.18 on the dollar. Under PBGC's maximum benefit cap, some of the 120,000 United workers saw large cuts in their retirement income due to the insufficiency of the value of the pension assets received from the United Airlines bankruptcy (Schroeder, 2006). In past bankruptcies, PBGC has received such diverse assets as: "diamonds, a hog slaughtering facility, oil wells, a restaurant, interest in a nuclear fuel reconditioning partnership, and water rights in the Mojave Valley." The lack of PBGC financial resources constrained its ability to secure the highest possible price for these diverse assets (Schroeder, 2006).

### **1. Examination of the externalities associated with the provision of PBGC insurance**

Economic externalities derived from PBGC insurance is an outgrowth of its social welfare function, to provide retirement income to those who find themselves in bankrupt pension plans. While PBGC has been created to operate as a private insurer, the underwriting, sources of premium income, benefit levels, and reinsurance arrangements may be more subject to political process more than actuarial considerations.

<sup>1</sup> Under PBGC rules, an employer is liable to the agency for any plan asset insufficiency up to 30% of the employer's net worth. Net worth at the time of termination may include reorganization value, liquidating value of the employer's tangible and intangible property, the value of equity assumed in a plan of reorganization, or any other factor relevant in determining net worth, Part 4062 – Liability for termination of single-employer plans, PBGC regulations, 29 U.S.C. 1302(b)(3) 1362-1364, 1367, 1368, available at: <http://www.pbgc.gov/practitioners/law-regulations-informal-guidance/content/page14767.html>.

PBGC insurance is one segment of the social insurance safety net used to provide a floor of income to US retirees. Benefit income provided by PBGC, along with social security, helps meet retirement expenses for those retirees in bankrupt plans (Bodie, 1996). Although PBGC receives premium income to help defray part of its claim costs, the ultimate responsibility for paying claims may rest with the federal government. Currently PBGC has a \$100 million line of credit with the Department of the Treasury; however, that amount may be increased in case the claim costs rise beyond PBGC assets in reserve to meet retirement obligations<sup>1</sup>. Consequently, the federal government is the insurer of last resort and reinsurer to PBGC's insurance program. Any excess loss above PBGC's reserves to cover pension claims may require federal funding. Unlike other reinsurance agreements in the private market, PBGC's arrangement is unique in that the reinsurer [federal government] may have unlimited liability. Private reinsurers will cap the amount of insured losses they accept on the basis of their own insurance capacity. A significant reason for this relationship rests with the notion of the "too big to fail" theory of government assistance to failing corporations (Brigham and Ehrhardt, 2005).

The federal government's role as a catastrophic reinsurer to PBGC coverage creates externalities between defined benefit pension participants. PBGC's role taking premiums and laying off large loss exposure to the Treasury, allows for moral hazard relationships between PBGC, the Department of the Treasury and corporate sponsors of defined benefit plans. Corporate sponsors want to reduce the cost of coverage, PBGC needs to have premiums sufficient to cover loss experience, and employees want retirement insurance to cover benefit loss when a company goes bankrupt. PBGC, as a quasi-federal government entity, needs to have a method for determining premium rates in advance of coverage, to handle loss experience that permits them to get Congressional approval for rate changes. PBGC's past funding history shows how the rate setting process may be tied to financial results. From 1996 to 2001, PBGC enjoyed a positive financial position coinciding with its public policy objectives of increasing premium rate increases from Congress on covered plans. These rate changes allowed PBGC to cover its losses without reliance on the Treasury line of credit for that 5 year period (U.S. Department of Labor (2004, 2005, 2009, 2010, 2011) Annual Reports and Pension Insurance Data Books). One

method of determining PBGC premiums within this public policy setting is to develop a linear control model that seeks to minimize Treasury borrowing while covering current and future insured loss experience through premium adjustment. The next section provides a linear control model to help PBGC set premium rates sufficient to cover current and future claim experience subject to ceding large loss exposure to the Treasury line of credit up to a set retention limit.

## 2. A linear control model for determining PBGC's retention limit on insuring defined benefit pension plans<sup>2</sup>

In order for PBGC to remain viable as an insurer, premiums must be sufficient to cover: (1) current loss experience; and (2) claims that can reasonably be expected in the future. A pure risk premium representing the expected value of future losses, funds current claims while the loading is used to provide for administrative expenses and reserving for incurred but not yet reported loss. Before PBGC can determine an adequate premium rate, it must first decide on how much coverage to retain. This retention limit represents a maximum amount of coverage PBGC can reasonably reserve, based on premiums and investment return. Insuring defined benefit pensions involves two risks: (1) investment risk associated with the return on pension securities and (2) management risk which relates to the ability to adequately set aside funds to meet pension obligations. Each of these risks may depend upon exogenous economic conditions over time. The dynamic and fundamental nature of these risks are such that PBGC may be unable to retain responsibility for catastrophic coverage should large pension losses to entire companies or industries occur. Currently, the PBGC has a \$100 million statutory line of credit with the U.S. Treasury Department which may be used to maintain liquidity should there be massive withdrawals to the corporation (Federal Reserve Board of San Francisco, 2003). To remain viable, PBGC needs to price its insurance in such a way that the agency's capacity to retain insurance will increase with the size of its reserves. A dynamic control model will be used to identify the relationships involved in determining PBGC's retention limit on insuring defined benefit plans.

One way of examining the impact operating capacity has on insurance retention is to view PBGC as a primary insurer seeking to cede [transfer] excess loss insurance to an outside reinsurer [U.S. Treasury Department]. In addition to quantifying the connection between retention limit and PBGC reserve size, such a

<sup>1</sup> A similar situation played out in the late 1980's when the FSLIC had a \$750 million line of credit on its deposit insurance coverage with the Treasury, and that amount was later raised to accommodate the large losses experienced with failing S&Ls. See, Thompson (1981), Thompson, Medury, Ramjee and Ramjee (1990), Thompson and Zaman (1993).

<sup>2</sup> This material is based on results first reported in, Thompson, A. Ramjee, B. Ramjee (1984), Martin, Ifflander (1994), with the control model being an extension of one first presented in Bensoussan, Hurst and Naslund (1974).

model may prove useful in determining a suitable credit line for excess coverage. PGBC's investment funds can be divided into two parts. One consisting of technical reserves  $R'$  to meet current claim experience. The other fund represents longer term, free reserves  $R''$  used to meet unexpected adverse loss away from expected results<sup>1</sup>.  $R'$  directly relates to the amount of insurance PBGC can absorb in its risk portfolio.

Let,  $K_1(t)$  be the amount in  $R'$  at time  $t$ ;  $K_2(t)$  be the amount in  $R''$  at time  $t$ .

These two reserve funds will earn individual interest rates of return  $\delta_1$  and  $\delta_2$  over time.

Thus,  $\delta_1(t)$  is the short-term interest rate on funds held in  $K_1$ ;  $\delta_2(t)$  is the long-term rate on fixed income investments in  $K_2$ ,  $\delta_3(t)$  is the long-term rate capital appreciation rate on funds in  $K_2$ .

If  $F[x(t)]$  represents the cumulative claims distribution function, then the technical reserve for meeting expected claims is:

$$d(t) = \int_0^\infty [x(t)]F'[x(t)]dt,$$

where  $x(t) = K_1(t) + K_2(t) + P(t)$  and  $P(t) =$  PBGC premium income in period  $t$  while  $d(t)$  represents PBGC's demand for cash to meet claims in period  $t$ , [ $d(t) \in [0, \infty)$ ]. The controller is  $u(t)$ , the amount of insurance coverage PBGC cedes to the U.S. Department of the Treasury through credit line in time  $t$ .  $|u(t)| < M$ , where  $M$  is an upper bound on the amount of credit that PBGC can draw upon in any time period. In this case  $M$  would be equal to \$100 million. The set of state equations area as follows:

$$K_1(t) = \delta_1(t)K_1(t) - d(t) + u(t) - \alpha|u(t)| + \delta_2(t)K_2(t), \tag{1}$$

$$K_2(t) = \delta_3(t)K_2(t) - u(t). \tag{2}$$

According to equation (1), a change in the technical reserve,  $K_1(t)$  results from:

- ◆ a short-term interest return  $\delta_1(t) K_1(t)$ ;
- ◆ a decrement  $d(t)$  based on the payment of claims at time  $t$ ;
- ◆ an increase (or decrease) by using the credit line  $u(t)$ ;
- ◆ a payment of transactions costs for the use of the credit line  $\alpha |u(t)|$ ;
- ◆ a short-term return on fixed income assets  $\delta_2(t) K_2(t)$ .

Differential equation (2) represents changes to the long-term safety reserve,  $K_2(t)$  from:

- ◆ an increase (or decrease) in the long-term value of safety reserve assets  $\delta_3(t) K_2(t)$ ; and
- ◆ a decrease caused by the use of the PBGC credit line with the U.S. Department of the Treasury [ $-u(t)$ ].

PBGC's objective is to maximize the amount of money in reserve to meet its claim experience, therefore the goal is to maximize the functional [ $K_1(t) + K_2(t)$ ] with respect to the controller  $u(t)$  over time subject to the state equations (1) and (2). The canonical representation of this system is given as:

$$\text{Max}\{[1,1]K(t)\}$$

subject to:

$$\dot{\bar{K}}(t) = A \bar{K}(t) + b u(t) + c, \tag{3}$$

where:

$$\bar{K}(t) = [K_1(t), K_2(t)]',$$

$$A = \begin{bmatrix} \delta_1(t) & \delta_2(t) & 0 \\ 0 & 0 & \delta_3(t) \end{bmatrix}$$

$$B = [1 - \alpha, -1]', \text{ and } C = [-d(t), 0]',$$

with claims experience  $d(t)$  exogenously defined. PBGC wants to find the optimal decision rule  $u^*(t)$  from a set of rules  $\{u_i, \text{ for all } i = 1, 2, 3, \dots, n\}$ , which leads to a maximum value for [ $K_1(t) + K_2(t)$ ] at terminal time  $T$  [the length of PBGC's planning horizon]. Selecting any  $u_i(t)$  leads to some terminal value for [ $K_1(t) + K_2(t)$ ]. The set of admissible controls  $u_i(t)$  is bounded by  $M$ , and the state system is linear in  $u(t)$  and  $\bar{K}(t)$  which guarantees a unique solution. The Hamiltonian can be defined as:

$$H[K_1(t), K_2(t), p_1(t), p_2(t), u(t), t] = p_1(t)\{\delta_1(t) \times K_1(t) - d(t) + u(t) - \alpha|u(t)| + \delta_2(t) \times K_2(t) + p_2(t)\{\delta_3(t) \cdot K_2(t) - u(t)\}, \tag{4}$$

where  $p_1(t)$ , and  $p_2(t)$  are co-state or shadow price variables. Applying Pontryagin's maximum principle, an optimal policy can be obtained from the linear system of equations defined by:

$$\frac{\partial H}{\partial u}[K_1(t), K_2(t), p_1(t), p_2(t), u(t), t] = 0.$$

The controller  $u(t)$  can take on both positive or negative values depending on whether PBGC is using or restoring its \$100 million credit line in time  $t$ . Since  $|u(t)|$  is a discontinuous function, it is not possible to directly differentiate  $H$ . However, by suitably defining  $u(t)$  a derivative can be obtained over a finite interval. Let:

<sup>1</sup> Let  $F(x) =$  the cumulative claims distribution function which represents the probability that the amount of claims paid under PBGC insurance will not exceed the random variable  $x$ .  $R' = \int_0^\infty x F'(x) dx$  the technical reserve and  $R''$  is a safety reserve, an amount held in addition to  $R'$  used to meet contingent losses greater than expected claims.

$$u^+(t) - u^-(t),$$

$$u(t) = \text{for all } u^+(t) \geq 0, u^-(t) \geq 0,$$

$$u^+(t) \times u^-(t) = 0.$$

The decomposed controller  $u(t)$  has the following graphical configuration.

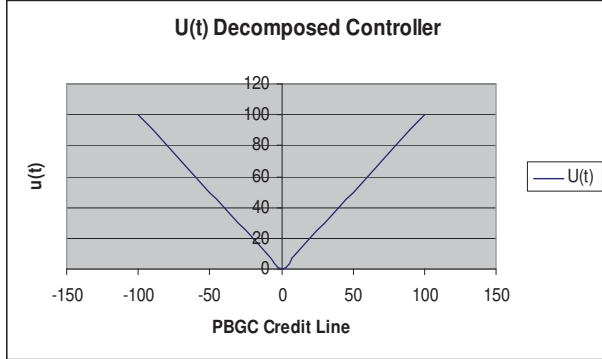


Fig. 1. PBGC credit line bang-bang controller

$u(t) = u^+(t)$ , whenever PBGC is using their credit line with the Treasury,  $u(t) = u^-(t)$ , whenever PBGC is restoring its line of credit with the Treasury,  $u(t) = 0$ , whenever PBGC is able to meet all its claim experience through premium income and is not using the line of credit with the Treasury. Under this new formulation the Hamiltonian is redefined as:

$$H[K_1(t), K_2(t), p_1(t), p_2(t), u(t), t] = p_1(t) \{ \delta_1(t) \times K_1(t) - d(t) + [u^+(t) - u^-(t)] - \alpha [u^+(t) - u^-(t)] + \delta_2(t) \times K_2(t) \}. \quad (5)$$

Maximizing the functional  $H$  with respect to  $u^+(t)$  and  $u^-(t)$ :

$$\frac{\partial H}{\partial u^+(t)} = [1 - \alpha] p_1(t) - p_2(t), \quad (6)$$

$$\frac{\partial H}{\partial u^-(t)} = -[1 - \alpha] p_1(t) - p_2(t). \quad (7)$$

Since  $H$  is linear in  $u^+(t)$  and  $u^-(t)$  the solution defines an on-off or what is known as a bang-bang switching policy where:

$$u^+(t) = \begin{cases} M & \text{if } \frac{\partial H}{\partial u^+(t)} > 0, [1 - \alpha] p_1(t) - p_2(t) > 0, \\ 0 & \text{if } [1 - \alpha] p_1(t) - p_2(t) \leq 0. \end{cases}$$

$$u^-(t) = \begin{cases} -M & \text{if } \frac{\partial H}{\partial u^-(t)} > 0, -[1 - \alpha] p_1(t) - p_2(t) > 0, \\ 0 & \text{if } -[1 - \alpha] p_1(t) - p_2(t) \leq 0. \end{cases}$$

PBGC will utilize the credit line whenever  $[1 - \alpha] p_1(t) > p_2(t)$  and will attain more insurance capacity by restoring the line when  $p_2(t) > [1 - \alpha] p_1(t)$ . The values of the co-state or shadow variables  $p_1(t)$  and  $p_2(t)$  may be found by examining the system of adjoint equations defined to be:

$$\dot{p}_1(t) = \frac{-\partial H}{\partial K_1(t)}, \quad (8)$$

$$\dot{p}_2(t) = \frac{-\partial H}{\partial K_2(t)}, \quad (9)$$

$$\dot{p}_1(t) = -[\delta_1(t) p_1(t)], \quad (10)$$

$$\dot{p}_2(t) = -[\delta_2(t) p_1(t) + \delta_3(t) p_2(t)]. \quad (11)$$

Solving (10) for  $p_1(t)$ :

$\dot{p}_1(t) = -\delta_1 t$  and integrating both sides over the closed interval  $[t, T]$  for  $p_1(t)$ , then

$$\int_t^T [p_1(t) / p_1(t)] dt = -\int_t^T \delta_1(t) dt, \text{ which implies that}$$

$$\ln[p_1(t)]_t^T = -\int_t^T \delta_1(t) dt, \text{ and}$$

$$p_1(t) = e^{-\int_t^T \delta_1(t) dt} \text{ given that } p_1(T) = 1. \quad (12)$$

Substituting equation (12) into (11):

$$\dot{p}_2(t) = -\delta_2(t) e^{-\int_t^T \delta_1(t) dt} - \delta_3(t) p_2(t). \quad (13)$$

Using the boundary condition that  $p_2(T) = 1$  and an application of the variation of parameters formula on this nonhomogeneous linear differential equation<sup>1</sup>:

$$p_2(t) = e^{\int_t^T \delta_3(t) dt} - \int_t^T \delta_2(t) e^{\int_t^s \delta_1(s) ds + \int_t^s \delta_3(u) du}. \quad (14)$$

This model describes a risk retention policy for PBGC consistent with the goal of optimizing insurance capacity over time. Given PBGC's retention limit  $M$ , claims experience  $d(t)$ , a time optimal policy for using the Treasury line of credit on an excess loss basis is defined by equations (6), (7), (12), and (14). Since  $M$  and  $d(t)$  are exogenous variables, PBGC can perform sensitivity analysis on the optimal solution by varying these two parameters to determine their impact on  $K(t)$ . Testing of a solution in this way will indicate how dramatically PBGC's retention limit ( $M$ ) may change as a result of increasing or decreasing claim experience [ $d(t)$ ]. Such analysis may provide an indication of the adequacy of the size of the retention limit based on number and size of recent claims.

### Conclusions

PBGC as sole insurer for American defined benefit pension plans faces a number of public policy issues related to premium setting and use of the Treasury

<sup>1</sup> See: Athans and Falb (1966), Hadley and Kemp (1971), as well as, [www.cbu.edu/~wschrein/media/DE/Errata04.pdf#search=variation%20of%20parameters%20formula](http://www.cbu.edu/~wschrein/media/DE/Errata04.pdf#search=variation%20of%20parameters%20formula). The theorem is as follows: The solution of:  $x + a(t)x = q(t)$ ,  $x(t_0) = x_0$  is given by the variation of parameters formula:  $-\int_t^T a(u) du$ ,  $x(t) = z(t, t_0)x_0 + \int_t^T z(ts) ds$ , where  $z(t,s) = e$ .

Department's line of credit for covering large loss claims. Rate setting may be proscribed by the external relationships between PBGC as insurer, corporate sponsors as insurance purchasers, and employees as insured parties to coverage. PBGC rates over the past 10 years have been inadequate to meet current and expected loss resulting in a \$23 billion financial deficit to insurance operations and an increasing need to access the US Treasury line of credit. In the past period from 1996 to 2001, PBGC was able to build reserves and cover current loss experience by raising premium rates. The linear control model presented here may be useful in helping PBGC determine rates on the basis of minimizing over reliance on the U.S. Treasury while maintaining reserves sufficient to handle retained insurance coverage. This linear control model would permit PBGC to base premium rates on its actual experience. If in a given year, actual loss experience against investment results on reserve assets were such as to require U.S. Treasury borrowing, premium rates increase would respond to such a change. On the other hand, if PBGC loss experience along with investment results produced a fund surplus, premium rates could be revised downward as long as the U.S. Treasury line limit was not exceeded. Major advantages to this approach are: (1) PBGC would be able to determine on a consistent and independent basis pre-

mium rates that would be linked to actual experience; (2) the model could assist policy makers in addressing how much PBGC retains in normal loss experience with catastrophic loss ceded to the US Treasury as reinsurer; (3) it would be possible to determine how fund investment returns tied to the economy might alter the need for premium rate changes. Limitations for this control model configuration might be: (1) the difficulty in determining the time horizon for setting premium rates based on say a 1, 2 or 5 year planning period; (2) calculating long-term loss reserves when PBGC claim experience has been erratic over a number of years; and (3) obtaining Congressional approval for premium rates to be tied to actual experience more than political forces. Despite these constraints, the model would move premium rate setting for PBGC towards adequately covering reasonable claims experience while directly recognizing the Treasury Department's role for handling catastrophic loss. A major motivation in this regard would be the potential for reducing over reliance on Treasury subsidies for losses other than catastrophic claims. One area for further research would be to review past PBGC claims to identify the characteristics of catastrophic versus normal loss experience in order to estimate PBGC's retention amount and what might be a reasonable ceded coverage for the U.S. Treasury.

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