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Bank loans pricing and Basel II: a multi-period risk-adjusted methodology under the new regulatory constraints

Abstract

Under the new Basel II regulatory framework, the need for an effective risk-adjusted pricing mechanism has become even more central in banking than in the past: banks are spurred to develop risk-adjusted measures, to avoid wasteful customers' cross-subsidization and support the value creation process for their shareholders. The paper aims at detecting how the Internal Ratings-Based approach affects the bank loan pricing mechanism, by developing a multi-period risk-adjusted pricing methodology, which allows us to separate the contribution of the two components of credit losses (the expected loss and the unexpected loss), under the prevalent repayment schemes. Following Hasan and Zazzara (2006), risk-adjusted pricing can be split into two main parts: a "technical" one, which is based on Basel II-consistent risk factors (probability of default, loss in case of default, exposure at default and maturity); the second part, not analyzed in this paper, is defined as "commercial" and includes commissions, operational costs, and other subjectively allocated costs. In this research we focus on the remuneration for both the expected and unexpected losses. The main inputs we need in our pricing formula can simply be drawn from an internal rating model and from easy-to-find market data (risk-free interest rates and shareholders' target return). The pricing formula we propose is consistent with the new Basel II regulatory approach to credit risk management and provides an immediate support for bank managers in making a loan price-related decision.

Keywords: asset pricing, banks, Basel II, risk management.
JEL Classification: G12, G21, G28, G32.

Introduction

Measuring and pricing credit risk are crucial in banking. Banks are influenced by different factors in their loan pricing decisions: counterparties' characteristics, summarized by their probability of default; facility characteristics, such as the presence of guarantees, the loan maturity, etc.; bank internal factors, such as the diversification degree of its credit portfolio or the cost of its funding structure; institutional (external) elements, partly related to the market (the availability of hedging instruments or the existence of an active secondary market), and partly concerning the bank regulatory framework.

The paradigm of value creation for bank shareholders, together with the new Basel II regime of capital requirements (the Accord), drove bank managers to develop effective risk-adjusted performance measures (RAPMs) during the past decades.

Basel II Pillar 1 defines the methodologies to calculate capital requirements for credit, market and operational risks. With regard to credit risk, the Accord allows banks to choose between two approaches: the standardized approach, which basically refines the old set of risk weights proposed in the 1988 Accord; the Internal Ratings-Based (IRB) approach, which allows banks to use their internal estimates of the credit risk components: their counterparties' probability of default (PD), the loss given default (LGD), i.e. the loss that the bank would face for a specific loan facility in case it defaults, the exposure at default (EAD) and the loan maturity (M). Specifically, there are two variants of the IRB approach: the IRB-Foundation, where banks only provide estimates of each borrower's PD, and the IRB-Advanced, where banks estimate all the credit risk components previously mentioned[1].

Credit risk can generate two types of losses, known as expected loss (EL) and unexpected loss (UL). EL depends on the borrower's PD and the LGD. Assuming the independence between PD and LGD, the expected loss rate (ELR) for a single loan/borrower j is simply given by the following product: $PD_j \times LGD_j$, whereas, for a whole credit portfolio, it is the sum of each loan's ELR. Since they are expected, these losses must be hedged by adequate accounting loan-loss provisions and represent a physiological cost of bank lending activity[2]. UL is function of the PD variability and the correlation between the portfolio assets and must be covered by an appropriate amount of economic capital. Ex post, UL equals the difference between the actual loss and EL. Ex ante, the unexpected loss can be measured through a portfolio model based on a Value-at-Risk (VaR) methodology[3]. Within the new regulatory framework, banks have to set aside an amount of regulatory capital to face the risk of unexpected losses, deviating these resources from their lending activity and suffering from the consequent opportunity cost.

[1] For further details on the Accord see BCBS (2006), and for an interesting perspective on the capital adequacy regime, see Hasan et al. (2009).
[2] See Saita (2003).
[3] See Hasan and Zazzara (2006) and Resti and Sironi (2007) for further details on the models used to measure UL.

By implementing one of the IRB approaches, banks are supposed to achieve capital savings when compared to the standardized approach, taking advantage of a higher risk-sensitivity, to implement more selective lending policies and to take more risk-sensitive pricing decisions. In the end, since the regulatory capital requirement affects bank loan pricing, IRB credit institutions would be able to better quantify and transfer both the expected and the unexpected losses.

This paper detects the bank loan pricing mechanism under the IRB framework, showing how the capital absorbed by a single loan should be taken into account in determining its price. We propose a formula to calculate risk-adjusted price measures for loans repaid under different amortization schemes. We develop prior researches' intuitions about the "technical" pricing methodology and go further by extending the generally adopted 1-year perspective, to get a more accurate and risk-sensitive price measure.

The rest of the paper is organized as follows: section 1 reviews the studies that have already analyzed loan pricing issues under the new regulatory regime; section 2 describes the adopted loan price methodology; sections 3 and 4 show the application of our pricing formula to calculate risk-adjusted interest rates and spreads for loans with different repayment schemes; the last section concludes and provides suggestions for further research developments.

1. Loan pricing under the new Basel II framework: a literature review

The implications of Basel II on loan pricing have already been investigated by previous literature, even if not extensively, due to the recent publication of the Accord's final version. In a paper devoted to credit risk modelling of small commercial loan portfolios, Dietsch and Petey (2002) assume that a bank has to maximize its expected portfolio return under the constraint that the economic capital requirement must be equal to an exogenous, certain amount. Given an expected Return on Equity (RoE), a 1-year maturity, a fixed recovery rate, and neglecting taxes and operating costs, they determine the risk-adjusted price consistent with the expected RoE. They show that the price of loans granted to SMEs depends on their classification as retail or corporate exposures.

Repullo and Suarez (2004) analyze the impact of the new capital requirements on the loan pricing in a perfectly competitive market for business loans, where the correlation in defaults across firms is driven by a single systematic risk factor. Furthermore, banks have zero intermediation costs, are funded with fully insured deposits and equity capital, remunerating the latter more than the former, though bank shareholders are supposed to be risk-neutral, and supply loans to a huge number of unrated firms to fund risky investment projects. They find that, under perfect competition and a 1-year planning horizon, the rates which equate the expected payments of a loan to its weighted marginal funding cost, are calculated by maximizing the expected discounted value of its net worth (gross loan returns minus gross deposit liabilities), holding the minimum possible amount of regulatory capital. Considering two groups of banks, lending to high-risk firms and to low-risk firms, respectively, due to the advantageous treatment for low-risk lending in the IRB method relative to Basel I, the rates of low-risk loans will be determined by the capital charges of the IRB approach and will be lower than under Basel I, while the rates of high-risk loans will be determined by the capital charges of the standardized approach. From a quantitative point of view, they show that the IRB approach may imply a reduction or an increase in loan rates, relative to Basel I, depending on the borrowers' creditworthiness. Based on their results, banks lending to high-risk loans will adopt the standardized approach, leaving their rates the same as under Basel I.

Hasan and Zazzara (2006) propose a methodology to estimate risk-adjusted spreads for bank corporate loans. They price bank loans through a formula fed by the same inputs needed to calculate the Basel II capital requirements. Following their approach, the loan spread can be split into two portions: the "technical" spread, which is directly and fully derivable from an internal rating model, and the "commercial" spread, which accounts for operational costs, commissions and other subjectively allocated costs. They focus on the former, explaining its link to some performance indicators, such as the EVA™ and the RAROC, and finding evidence of a significant relationship between risk and loan spread.

Based on the model of a risk-neutral bank operating under uncertainty in an imperfectly competitive loan market, Ruthenberg and Landskroner (2008) detect the impact of the two new regulatory approaches (IRB and standardized) using the PD distribution of a leading Israeli bank's customers. They show that big banks will attract high-quality borrowers, due to the reduction in loan rates stemming from the adoption of the IRB approach; low-quality firms will be funded by small intermediaries, which are more likely to adopt the standardized approach; retail customers will enjoy a reduction in loan rates if they borrow from IRB banks.

2. The loan pricing methodology

In this section we describe the loan pricing methodology adopted here in estimating risk-adjusted rates and spreads for bank credit exposures. Following Hasan and Zazzara (2006), the risk-adjusted price for bank loans can be split into two main components: the "technical" part, which takes into account both expected and unexpected losses and the opportunity cost for providing committed credit lines; the "commercial" component, which includes commissions, operational costs, and other subjectively allocated costs. We don't take care of these latter elements since their allocation doesn't have any relevance in terms of credit risk management.

We focus on the two main components of the "technical" price: the remuneration for EL and UL for loans with fixed exposure[1]. The main inputs we need to take both expected and unexpected losses into account in our pricing formula can simply be drawn from an internal rating model (PD, LGD, EAD and M) and from easy-to-find market data (risk-free interest rates and shareholders' target return). Our formula is consistent with the logic underlying the new Basel II regulatory approach to credit risk management and provides an immediate support for bank managers. In the next paragraphs we develop the pricing methodology for zero-coupon loans (ZCLs), where both interests and principal are repaid in a single sum on a set maturity. Then, we extend the analysis to the other prevalent repayment schemes.

2.1. The cost of the expected loss for zero-coupon loans. Bank remuneration to cover expected losses is calculated within a risk-neutral framework: let's assume a bank issuing a 1-year maturity loan of €1 to a borrower classified in the *i*-th rating class. The expected loan value must be equal to the future value of a risk-free investment:

$$(1+r_1+s_1^{EL,i})\left[(1-p_1^i)+Rp_1^i\right]=(1+r_1) \quad , \qquad (1)$$

where: r_1 is the risk-free rate for a 1-year horizon; $s_1^{EL,i}$ is the spread to remunerate expected losses for a 1-year maturity loan; p_1^i is the probability of default within 1 year; R is the recovery rate in the event of default, set flat for each rating class.

Expected losses must be covered by adding a spread to the risk-free rate. The left-hand side of equation (1) is the loan expected value, equal to the sum of the loan future value in case of survival, with probability $(1-p_1^i)$, and the loan recovered amount in case of default, with probability p_1^i. After some algebraic manipulations, we can get the 1-year risk-neutral interest rate $r_{1,neutral}^i$, expressed by the 1-year risk-free interest rate plus the 1-year spread for expected losses, and the corresponding spread $s_1^{EL,i}$:

$$r_{1,neutral}^i=r_1+s_1^{EL,i}=\frac{r_1+p_1^i(1-R)}{1\text{-}p_1^i(1-R)}, \qquad (2)$$

$$s_1^{EL,i}=\frac{r_1+p_1^i(1-R)}{1\text{-}p_1^i(1-R)}-r_1 \ . \qquad (2.1)$$

Extending this 1-year analysis to an *n*-year horizon, formula (1) becomes:

$$(1+r_n+s_n^{EL,i})^n\left[(1-p_n^i)+Rp_n^i\right]=(1+r_n)^n \quad , \qquad (3)$$

where the subscript *n* reflects the *n*-year perspective, p_n^i is the cumulative probability of default within *n* year, and R is set constant over time.

From equation (3) we can get the *n*-year risk-neutral interest rate $r_{n,neutral}^i$, and the corresponding spread $s_n^{EL,i}$, both calculated on an annual basis:

$$r_{n,neutral}^i=r_n+s_n^{EL,i}=\frac{1+r_n}{\sqrt[n]{1-p_n^i(1-R_n)}}-1, \qquad (4)$$

$$s_n^{EL,i}=\frac{1+r_n}{\sqrt[n]{1-p_n^i(1-R_n)}}-(1+r_n) \ . \qquad (4.1)$$

2.2. The cost of the unexpected loss for zero-coupon loans. Since the risk-neutrality assumption is unrealistic for banks, we include their risk aversion into our pricing formula: for each loan, the final interest rate must remunerate not only the cost of the expected loss, but also that of the unexpected loss. The resulting interest rate is generally defined as "risk-adjusted" just because it accounts for the burden of unexpected losses too, the actual risk for credit institutions.

UL is function of the correlation between bank loans and can be estimated through portfolio models, whose final objective is to calculate a VaR measure for both the single loan and the overall portfolio: this VaR, also named CaR (Capital at Risk), since it is referred to the bank capital, represents the amount of risk that must be covered by equity. Following a VaR

[1] With regard to loans with a variable exposure, we have to consider the opportunity cost that banks bear to grant to some borrowers the possibility to draw money up to a certain amount in a totally discretionary way. In this case, Hasan and Zazzara (2006) assume that banks apply the risk-adjusted interest rate (spread) of a loan with fixed exposure on the drawn portion, and charge the undrawn portion with the difference between the risk-adjusted rate and the return they would get if invested it at the risk-free interest rate.

approach, the difference between the maximum potential loss, calculated within a certain time interval and for a given confidence level, and the expected loss is a measure of the unexpected loss, and also represents the bank capital at risk.

We measure the bank economic capital needed to face the unexpected loss through the regulatory capital, which is calculated through the IRB closed formula[1], and is made up of Tier 1, or core capital, and Tier 2, or supplementary capital[2]. The last input we need to feed our pricing formula is the cost of the economic capital, which is function of the return expected by the capital providers.

Finally, in our model each loan is funded by both debt capital and equity capital, unlike some other frameworks where the equity capital has only a collateral function[3]. Including the above mentioned factors, equation (1) can be modified as follows:

$$(1+r_1+s_1^{EL,i}+s_1^{UL,i})\left[(1-p_1^i)+Rp_1^i\right]=RC_1^{B,i}(1+r_1+s_B)+RC_1^{S,i}(1+r_1+s_s)+\left[1-\left(RC_1^{B,i}+RC_1^{S,i}\right)\right](1+r_1), \quad (5)$$

where, apart from the previously defined variables, s_C and s_S are the constant spreads over the risk-free interest rate, required by the core capital holders and the supplementary capital holders, respectively; $RC_1^{C,i}$ and $RC_1^{S,i}$ are the amounts of core capital requirements and supplementary capital requirements, calculated under the Basel II rules for a 1-year maturity loan, respectively;

From equation (5), after some algebraic manipulations, we can derive the risk-adjusted interest rate remunerating both EL and UL for this 1-year ZCL:

$$r_1^{i,adj.}=r_1+s_1^{EL,i}+s_1^{UL,i}=\frac{RC_1^{C,i}(1+r_1+s_B)+RC_1^{S,i}(1+r_1+s_S)+\left[1-\left(RC_1^{C,i}+RC_1^{S,i}\right)\right](1+r_1)}{1-p_1^i(1-R)}-1\,. \quad (6)$$

The spread to compensate for the unexpected losses can be simply written as follows:

$$s_1^{EL,i}+s_1^{UL,i}=\frac{RC_1^{C,i}(1+r_B)+RC_1^{S,i}(1+r_S)+\left[1-\left(RC_1^{C,i}+RC_1^{S,i}\right)\right](1+r_1)}{1-p_1^i(1-R)}-\left(1+r_1\right). \quad (6.1)$$

If we take into account an n-year ZCL, equation (5) becomes:

$$(1+r_n+s_n^{EL,i}+s_n^{UL,i})^n\left[(1-p_n^i)+Rp_n^i\right]=RC_n^{C,i}(1+r_n+s_B)^n+RC_n^{S,i}(1+r_n+s_S)^n++\left[1-\left(RC_n^{C,i}+RC_n^{S,i}\right)\right](1+r_n)^n, \quad (7)$$

where the subscript n reflects the n-year perspective.

We have to point out that using the annualized probabilities of default as inputs of the formula to assess regulatory capital, we move away from what proposed by the Basel Committee since to feed the risk-weight functions, they use the 1-year probability of default, regardless of loan maturity.

In equations (8) and (8.1) we show how to calculate the n-year risk-adjusted interest rate and the spread to remunerate the two "technical" components that we take into account in our pricing mechanism. Both are calculated on an annual basis:

$$r_n^{i,adj.}=r_n+s_n^{EL}+s_n^{UL}==\sqrt[n]{\frac{RC_n^{C,i}(1+r_n+s_C)^n+RC_n^{S,i}(1+r_n+s_S)^n+\left[1-\left(RC_n^{C,i}+RC_n^{S,i}\right)\right](1+r_n)^n}{1-p_n^i(1-R)}}-1, \quad (8)$$

$$s_n^{EL}+s_n^{UL}=\sqrt[n]{\frac{RC_n^{B,i}(1+r_n+s_B)^n+RC_n^{S,i}(1+r_n+s_S)^n+\left[1-\left(RC_n^{B,i}+RC_n^{S,i}\right)\right](1+r_n)^n}{1-p_n^i(1-R)}}-\left(1+r_n\right). \quad (8.1)$$

[1] The Basel II IRB risk-weight functions, used to assess the regulatory capital for unexpected losses, are based on a specific model described in Gordy (2003). For further details on the regulatory formulas, see BCBS (2006).
[2] For details concerning the components of the two tiers, see BCBS (2006).
[3] See Resti and Sironi (2007) for an example of the first approach, and Hasan and Zazzara (2006) for an application of the second one.

3. Estimating risk-adjusted price measures for zero-coupon loans with fixed exposures: risk-adjusted spread break-down

In this paragraph we use the above described methodology to estimate the term structure of the "technical" risk-adjusted rates and spreads for ZCLs with fixed exposures, with regard to the Basel II corporate segment[1]. In our pricing simulations for the year 2009:

- we use a multi-period rating master scale as a source of cumulative probabilities of default for a 10-year time horizon (Table 1)[2], from which we calculate the annualized PDs to feed the regulatory formula for *n*-year maturity loans;
- we adopt the term structure of swap interest rates, as of January 1st 2009, as a proxy for the term structure of risk-free interest rates (see the bottom row of Table 1);

Table 1. The multi-period rating master scale and the term structure of swap interest rates

Average cumulative issuer-weighted global default (1983-2008)										
	Maturity year									
	1	2	3	4	5	6	7	8	9	10
Aaa	0.01%	0.02%	0.02%	0.05%	0.09%	0.14%	0.19%	0.19%	0.19%	0.19%
Aa	0.02%	0.06%	0.10%	0.17%	0.25%	0.29%	0.32%	0.35%	0.37%	0.41%
A	0.03%	0.13%	0.31%	0.48%	0.68%	0.89%	1.11%	1.34%	1.55%	1.71%
Baa	0.18%	0.52%	0.93%	1.41%	1.89%	2.36%	2.82%	3.24%	3.65%	4.14%
Ba	1.15%	3.17%	5.69%	8.29%	10.48%	12.47%	14.22%	15.85%	17.32%	18.74%
B	4.33%	9.83%	15.27%	20.09%	24.47%	28.67%	32.67%	36.00%	38.93%	41.45%
Caa	13.73%	23.51%	31.70%	38.41%	43.75%	47.62%	50.36%	53.52%	58.37%	64.78%
Ca-C	32.95%	44.30%	53.26%	58.41%	63.93%	66.49%	70.34%	74.99%	74.99%	74.99%
Investment grade	0.07%	0.23%	0.44%	0.67%	0.92%	1.15%	1.38%	1.60%	1.80%	2.01%
Speculative grade	4.35%	8.92%	13.37%	17.32%	20.69%	23.70%	26.39%	28.69%	30.71%	32.52%
Term structure of swap interest rates (as of January 1st 2009)										
Swap interest rates	2.68%	2.76%	2.96%	3.12%	3.36%	3.24%	3.57%	3.46%	3.66%	3.74%

Source: Moody's (2009) and Datastream.

- we set the recovery rate constant and equal to 55% of the credit exposure, consistently with the 45% LGD of the IRB-Foundation approach for senior unsecured claims on corporates, sovereigns and banks;
- we assume that the economic capital absorbed by each loan coincides with the regulatory capital. Based on the evidence referred to the Italian banking system[3], we hypothesize that 70% of the regulatory capital is core capital and the remaining 30% is supplementary capital;
- regardless of loan maturity, we hypothesize a constant risk-premium for both core capital holders and supplementary capital holders, which must be added to the risk-free interest rate to calculate their respective target remuneration. Assuming that the supplementary capital is made up of only subordinated debt, and following some suggestions from bank managers, we set a risk-premium of 800 bps. and 200 bps. for core capital and subordinated debt, respectively.

Based on formulas (8) and (8.1), fed with the above listed inputs, we get the term structure of the risk-adjusted price measures (Table 2). On average, risk-adjusted spreads for investment grades increase with maturity, whereas they move downward for speculative grades.

Spread break-down: EL vs. UL

Here we calculate the contribution of the two components (EL and UL) to the total spread, narrowing our analysis to some maturities, by estimating the share of the total spread explained by EL and UL, respectively. We do that by calculating the ratios of the spread to cover EL to the total spread, on the one hand, and the spread to remunerate UL to the total spread, on the other hand (see Table 3). As expected, for each maturity spreads of better rating classes are characterized by a lower incidence of expected losses, relative to unexpected ones. The EL weight increases with the decline of the counterparties' creditworthiness, and becomes larger than the UL one from rating Ba. On average, the incidence of the unexpected loss raises with the loan maturity for speculative grades, whereas it diminishes for the investment grades, even if at a slower pace.

[1] For details about the different segments of bank exposures within the Accord, see BCBS (2006).
[2] See Moody's (2009).
[3] See Bank of Italy (2009).

Table 2. The term structure of risk-adjusted rates (R) and spreads (S) for the corporate segment – zero-coupon loan

	Maturity year									
	1		3		5		7		10	
	R	S	R	S	R	S	R	S	R	S
Aaa	2.73%	0.04%	2.99%	0.03%	3.42%	0.07%	3.66%	0.10%	3.82%	0.09%
Aa	2.75%	0.06%	3.06%	0.10%	3.49%	0.14%	3.70%	0.14%	3.88%	0.14%
A	2.76%	0.08%	3.17%	0.21%	3.62%	0.27%	3.88%	0.31%	4.10%	0.36%
Baa	2.98%	0.29%	3.40%	0.44%	3.89%	0.53%	4.15%	0.58%	4.38%	0.64%
Ba	3.70%	1.02%	4.47%	1.51%	5.04%	1.69%	5.28%	1.71%	5.45%	1.71%
B	5.45%	2.77%	6.28%	3.31%	6.70%	3.34%	6.91%	3.34%	6.95%	3.21%
Caa	10.60%	7.91%	9.54%	6.57%	9.17%	5.81%	8.63%	5.06%	8.70%	4.97%
Ca-C	22.02%	19.33%	14.23%	11.26%	12.03%	8.67%	10.79%	7.22%	9.56%	5.82%
Investment grade	2.84%	0.16%	3.23%	0.27%	3.68%	0.33%	3.93%	0.36%	4.14%	0.40%
Speculative grade	5.47%	2.78%	5.91%	2.95%	6.24%	2.88%	6.34%	2.77%	6.34%	2.60%

Source: Our elaborations on data from Moody's (2009) and Datastream[TM].

Table 3. Spread break-down: EL* vs. UL*

	Maturity year									
	1		3		5		7		10	
	UL	EL	UL	EL	UL	EL	UL	EL	UL	EL
Aaa	89.08%	10.92%	92.05%	7.95%	88.13%	11.87%	87.09%	12.91%	89.63%	10.37%
Aa	86.22%	13.78%	84.33%	15.67%	83.10%	16.90%	84.50%	15.50%	86.44%	13.56%
A	84.66%	15.34%	77.34%	22.66%	76.40%	23.60%	76.52%	23.48%	77.86%	22.14%
Baa	71.26%	28.74%	67.42%	32.58%	66.66%	33.34%	67.52%	32.48%	69.54%	30.46%
Ba	47.79%	52.21%	40.72%	59.28%	40.43%	59.57%	42.52%	57.48%	46.43%	53.57%
B	26.34%	73.66%	25.36%	74.64%	26.98%	73.02%	28.85%	71.15%	32.59%	67.41%
Caa	14.53%	85.47%	17.55%	82.45%	20.31%	79.69%	23.53%	76.47%	26.75%	73.25%
Ca-C	7.53%	92.47%	12.58%	87.42%	16.34%	83.66%	19.86%	80.14%	25.11%	74.89%
Investment grade	78.61%	21.39%	74.58%	25.42%	73.93%	26.07%	74.66%	25.34%	76.57%	23.43%
Speculative grade	26.28%	73.72%	27.08%	72.92%	29.29%	70.71%	31.96%	68.04%	36.47%	63.53%

Note: * in percentage of the total spread.
Source: Our elaborations on data from Moody's (2009) and DataStream[TM].

4. Estimating risk-adjusted price measures for different repayment plans

The pricing model presented above refers to a zero-coupon loan but, in practice, bank loans are issued under different amortization plans. In this section we consider three different schemes: bullet loan (BL), where interests are paid at regular intervals and capital is repaid on the final maturity; constant capital repayment (CCR), where the capital component of the installment is taken constant for each maturity; and straight-line amortization (SLA), where the installment is constant over time.

For each of the three amortization schemes, we derive the flat term structure of the annualized risk-adjusted interest rates for a €1 loan, which is equivalent to the term structure of risk-adjusted interest rates referred to the ZCLs case: we decompose each amortization plan into a series of ZCLs whose amount equals the single installment value, and use the risk-adjusted interest rates previously derived for the ZCLs to calculate the constant risk-adjusted interest rate for any rating class.

In each of the cases described below, our analysis grounds on the following equilibrium condition at time t_0, when the loan is issued:

$$1 = \sum_{t=1}^{n} \frac{CF_t}{(1 + r^i_{t,adj})^t} \tag{9}$$

from which its value (€1) has to be equal to the sum of the loan cash-flows' present values (CF_t), discounted at the corresponding risk-adjusted interest rates ($r^i_{t,adj}$), calculated, for each rating class, using formula (8), and reported in Table 2.

Bullet loan (BL)

Let's suppose a €1 loan to a borrower ranked in the *i*-th rating class, with an *n*-year maturity and interest repayment at the end of each year. According to the equilibrium condition, we can write:

$$1 = \frac{r^i_{BL,adj}}{(1+r^i_{1,adj})} + \frac{r^i_{BL,adj}}{(1+r^i_{2,adj})^2} + \ldots + \frac{r^i_{BL,adj} + 1}{(1+r^i_{n,adj})^n}, \tag{10}$$

where $r^i{}_{BL,adj}$ is the annualized risk-adjusted rate, which is constant over time. The above formula can be rewritten as follows:

$$1 = r^i_{BL,adj} \cdot \sum_{t=1}^{n} \frac{1}{(1+r^i_{t,adj})} + \frac{1}{(1+r^i_{n,adj})^n}. \quad (11)$$

Since we calculated the risk-adjusted interest rates at the denominators of (11) through the methodology described in paragraphs 2 and 3, we derive the annualized risk-adjusted interest rate as follows:

$$r^i{}_{BL,adj} = \frac{1 - \frac{1}{(1+r^i_{n,adj})^n}}{\sum_{t=1}^{n} \frac{1}{(1+r^i_{t,adj})^t}}. \quad (12)$$

Constant capital repayment (CCR)

Let's consider a €1 loan to a borrower ranked in the *i*-th rating class, with an *n*-year maturity and installment repayment at the end of each year, with a constant principal repayment (*C*). The "equilibrium condition" here becomes:

$$1 = \frac{r^i_{CCR,adj} \cdot D_o + C}{(1+r^i_{1,adj})} + \frac{r^i_{CCR,adj} \cdot D_1 + C}{(1+r^i_{2,adj})^2} + ... + \frac{r^i_{CCR,adj} \cdot D_{n-1} + C}{(1+r^i_{n,adj})^n}, \quad (13)$$

where D_t is the outstanding debt used to calculate the interest repayment at time t+1 and $r^i_{CCR,adj}$ is the annualized risk-adjusted interest rate, which is constant over time. Equation (13) can be rewritten as:

$$1 = r^i_{CCR,adj} \cdot \left[\frac{D_o}{(1+r^i{}_{1,adj})} + \frac{D_1}{(1+r^i{}_{2,adj})^2} + ... + \frac{D_{n-1}}{(1+r^i{}_{n,adj})^n} \right] + \mathrm{C} \cdot \sum_{\mathrm{t}=1}^{n} \frac{1}{(1+r^i{}_{t,adj})^t}. \quad (14)$$

From equation (14) we derive the constant annualized risk-adjusted interest rate for this amortization scheme:

$$r^i_{CCR,adj} = \frac{1 - \mathrm{C} \cdot \sum_{\mathrm{t}=1}^{n} \frac{1}{(1+r^i{}_{t,adj})^t}}{\left[\frac{D_o}{(1+r^i{}_{1,adj})} + \frac{D_1}{(1+r^i{}_{2,adj})^2} + ... + \frac{D_{n-1}}{(1+r^i{}_{n,adj})^n} \right]}. \quad (15)$$

Straight-line amortization (SLA)

Let's suppose a €1 loan to a borrower within the *i*-th rating class, with an *n*-year maturity, whose constant installments (*I*) are paid at the end of each year. In this case, the "equilibrium condition" can be written as follows:

$$1 = \frac{I}{(1+r^i{}_{1,adj})} + \frac{I}{(1+r^i{}_{2,adj})^2} + ... + \frac{I}{(1+r^i{}_{n,adj})^n}. \quad (16)$$

Equation (16) can also be rewritten in the following way:

$$1 = I \cdot \sum_{t=1}^{n} \frac{1}{(1+r^i{}_{1,adj})^t} \quad (17)$$

from which, by using the risk-adjusted rates derived for the ZCL case, we can calculate the corresponding installment through the following formula:

$$I = \frac{1}{\sum_{t=1}^{n} \frac{1}{(1+r^i_{1,adj})^t}}. \quad (18)$$

Besides, since for this particular amortization plan, the relationship between the loan value and the installment can be formalized as follows:

$$1 = I \cdot a_{n\rangle r^i_{SLA,adj}}, \quad (19)$$

where

$$a_{n\rangle r^i_{SLA,adj}} = \frac{1 - \frac{1}{(1+r^i_{SLA,adj})^n}}{r^i_{SLA,adj}} \quad (20)$$

and $r^i_{SLA,adj}$ is the annualized risk-adjusted interest rate, which is constant for each maturity, we can replace equation (20) into equation (19) and obtain the annualized risk-adjusted interest rate, by solving the following equation using numerical methods:

$$1 = I \cdot \frac{1 - \frac{1}{(1+r^i_{SL,adj})^n}}{r^i_{SL,adj}}. \quad (21)$$

In Table 4 we report the risk-adjusted interest rates for the three repayment plans for five maturities. When the loan expires after 1 year we get the same interest rates for the three amortization schedules, and these rates are also equal to those calculated for the ZCL case. For maturities beyond 1 year, results are different, depending on the borrower rating class: regardless of the maturity, for rating classes ranging from Aaa to B, the bullet plan shows higher interest rates than the equal installment one, whereas the constant capital repayment is characterized by the lower values; vice versa for rating classes Caa and Ca-C. Obviously, these differences are due to the different distribution of interest and capital repayments during the loan time horizon and to the consequent impact on the perceived loan riskiness.

With regard to the risk-adjusted spreads, since it is not possible to derive them directly, we adopt the following three-step procedure:

1) we calculate the term structure of the risk-neutral interest rates for the zero-coupon loans, taking only the expected loss into account, via formula (4);
2) for each amortization plan, we estimate the constant annualized risk-neutral interest rates using the term structure derived above at point 1, to feed formulas (12), (15) and (21), respectively;
3) for each amortization plan, we derive the constant annualized risk-free interest rate using the term structure of the risk-free interest rates reported at the bottom of Table 1, to feed formulas (12), (15) and (21), respectively.

Consequently, the spread to cover EL is the difference from what we get at the second step of the procedure (the annualized risk-neutral interest rates) and what we derive at the third step (the annualized risk-free interest rate). The spread to remunerate UL is the difference between the risk-adjusted interest rates (Table 4 below) and the risk-neutral interest rates calculated at point 2. These spreads are shown in Table 5.

Table 4. The term structure of risk-adjusted rates – bullet loan (BL), constant capital repayment (CCR) and straight-line amortization (SLA)

	Maturity year														
	1			3			5			7			10		
	BL	CCR	SLA	BL	CCR	SLA	BL	CCR	SLA	BL	CCR	SLA	BL	CCR	SLA
Aaa	2.73%	2.73%	2.73%	2.99%	2.88%	2.88%	3.40%	3.12%	3.13%	3.61%	3.28%	3.29%	3.76%	3.46%	3.48%
Aa	2.75%	2.75%	2.75%	3.05%	2.93%	2.93%	3.46%	3.18%	3.19%	3.66%	3.33%	3.35%	3.81%	3.51%	3.54%
A	2.76%	2.76%	2.76%	3.16%	3.01%	3.01%	3.59%	3.29%	3.30%	3.82%	3.46%	3.48%	4.02%	3.67%	3.70%
Baa	2.98%	2.98%	2.98%	3.39%	3.24%	3.24%	3.85%	3.53%	3.54%	4.08%	3.71%	3.73%	4.29%	3.93%	3.96%
Ba	3.70%	3.70%	3.70%	4.45%	4.19%	4.19%	4.98%	4.58%	4.61%	5.19%	4.79%	4.82%	5.34%	5.01%	5.05%
B	5.45%	5.45%	5.45%	6.24%	5.99%	6.00%	6.63%	6.31%	6.33%	6.81%	6.47%	6.51%	6.85%	6.62%	6.66%
Caa	10.60%	10.60%	10.60%	9.59%	9.85%	9.83%	9.25%	9.53%	9.50%	8.78%	9.21%	9.15%	8.77%	8.96%	8.88%
Ca-C	22.02%	22.02%	22.02%	14.75%	16.70%	16.42%	12.68%	14.55%	14.19%	11.52%	13.28%	12.86%	10.46%	12.15%	11.70%
Investment grade	2.84%	2.84%	2.84%	3.22%	3.07%	3.08%	3.65%	3.35%	3.36%	3.87%	3.52%	3.54%	4.06%	3.72%	3.75%
Speculative grade	5.47%	5.47%	5.47%	5.90%	5.74%	5.74%	6.20%	5.96%	5.98%	6.28%	6.05%	6.07%	6.29%	6.14%	6.16%

Source: Our elaborations on data from Moody's (2009) and Datastream^{TM}.

Table 5. The term structure of risk-adjusted spreads – bullet loan (BL), constant capital repayment (CCR) and straight-line amortization (SLA)

	Maturity year														
	1			3			5			7			10		
	BL	CCR	SLA	BL	CCR	SLA	BL	CCR	SLA	BL	CCR	SLA	BL	CCR	SLA
Aaa	0.04%	0.04%	0.04%	0.03%	0.04%	0.04%	0.07%	0.05%	0.05%	0.09%	0.07%	0.07%	0.08%	0.08%	0.08%
Aa	0.06%	0.06%	0.06%	0.10%	0.09%	0.09%	0.13%	0.11%	0.11%	0.13%	0.12%	0.12%	0.14%	0.13%	0.13%
A	0.08%	0.08%	0.08%	0.21%	0.16%	0.17%	0.26%	0.21%	0.22%	0.30%	0.25%	0.25%	0.34%	0.29%	0.30%
Baa	0.29%	0.29%	0.29%	0.44%	0.39%	0.39%	0.52%	0.46%	0.46%	0.56%	0.50%	0.51%	0.61%	0.55%	0.55%
Ba	1.02%	1.02%	1.02%	1.49%	1.34%	1.35%	1.65%	1.51%	1.53%	1.67%	1.58%	1.60%	1.67%	1.62%	1.64%
B	2.77%	2.77%	2.77%	3.29%	3.14%	3.15%	3.30%	3.24%	3.25%	3.29%	3.26%	3.28%	3.18%	3.24%	3.26%
Caa	7.91%	7.91%	7.91%	6.63%	7.01%	6.98%	5.92%	6.46%	6.42%	5.26%	6.00%	5.92%	5.10%	5.57%	5.48%
Ca-C	19.33%	19.33%	19.33%	11.79%	13.85%	13.58%	9.35%	11.48%	11.11%	8.00%	10.06%	9.64%	6.79%	8.77%	8.29%
Investment grade	0.16%	0.16%	0.16%	0.26%	0.23%	0.23%	0.32%	0.28%	0.28%	0.35%	0.31%	0.31%	0.38%	0.34%	0.35%
Speculative grade	2.78%	2.78%	2.78%	2.94%	2.89%	2.90%	2.87%	2.89%	2.90%	2.76%	2.84%	2.85%	2.61%	2.75%	2.75%

Source: Our elaborations on data from Moody's (2009) and Datastream^{TM}.

As done before for the ZCL case, for each amortization plan we calculated the spread break-down in order to investigate the contribution of EL and UL to the whole spread: our evidence supports the results that we found in the zero-coupon loan case since spreads of better rating classes show a lower incidence of the expected loss, relative to the unexpected one. The weight of the expected loss increases with the rise of the counterparties' riskiness and becomes larger than that of the unexpected losses from rating Ba. On average, the incidence of the unexpected loss raises with the loan maturity for speculative grades, whereas it diminishes for the investment grades, even if at a slower pace (see Tables 6, 7 and 8 below).

Table 6. Spread break-down: EL* vs. UL* – bullet loan

	Maturity year									
	1		3		5		7		10	
	UL	EL	UL	EL	UL	EL	UL	EL	UL	EL
Aaa	89.08%	10.92%	91.97%	8.03%	88.21%	11.79%	87.18%	12.82%	89.43%	10.57%
Aa	86.22%	13.78%	84.34%	15.66%	83.14%	16.86%	84.42%	15.58%	86.16%	13.84%
A	84.66%	15.34%	77.39%	22.61%	76.44%	23.56%	76.51%	23.49%	77.67%	22.33%
Baa	71.26%	28.74%	67.45%	32.55%	66.67%	33.33%	67.43%	32.57%	69.21%	30.79%
Ba	47.79%	52.21%	40.77%	59.23%	40.39%	59.61%	42.27%	57.73%	45.63%	54.37%
B	26.34%	73.66%	25.32%	74.68%	26.80%	73.20%	28.45%	71.55%	31.60%	68.40%
Caa	14.53%	85.47%	17.48%	82.52%	20.06%	79.94%	22.95%	77.05%	25.55%	74.45%
Ca-C	7.53%	92.47%	12.45%	87.55%	15.96%	84.04%	19.09%	80.91%	23.45%	76.55%
Investment grade	78.61%	21.39%	74.61%	25.39%	73.95%	26.05%	74.61%	25.39%	76.29%	23.71%
Speculative grade	26.28%	73.72%	27.04%	72.96%	29.09%	70.91%	31.50%	68.50%	35.34%	64.66%

Note: * in percentage of the total spread.
Source: Our elaborations on data from Moody's (2009) and Datastream^{TM}.

Table 7. Spread break-down: EL* vs. UL* – constant capital repayment

	Maturity year									
	1		3		5		7		10	
	UL	EL	UL	EL	UL	EL	UL	EL	UL	EL
Aaa	89.08%	10.92%	90.77%	9.23%	89.26%	10.74%	88.12%	11.88%	88.46%	11.54%
Aa	86.22%	13.78%	84.56%	15.44%	83.68%	16.32%	83.89%	16.11%	84.81%	15.19%
A	84.66%	15.34%	78.72%	21.28%	77.24%	22.76%	76.83%	23.17%	77.02%	22.98%
Baa	71.26%	28.74%	68.31%	31.69%	67.26%	32.74%	67.24%	32.76%	68.00%	32.00%
Ba	47.79%	52.21%	42.42%	57.58%	40.99%	59.01%	41.36%	58.64%	42.90%	57.10%
B	26.34%	73.66%	25.44%	74.56%	26.03%	73.97%	26.95%	73.05%	28.62%	71.38%
Caa	14.53%	85.47%	16.47%	83.53%	18.13%	81.87%	19.83%	80.17%	22.04%	77.96%
Ca-C	7.53%	92.47%	10.64%	89.36%	13.01%	86.99%	15.08%	84.92%	17.81%	82.19%
Investment grade	78.61%	21.39%	75.48%	24.52%	74.49%	25.51%	74.46%	25.54%	75.13%	24.87%
Speculative grade	26.28%	73.72%	26.73%	73.27%	27.81%	72.19%	29.19%	70.81%	31.42%	68.58%

Note: * in percentage of the total spread.
Source: Our elaborations on data from Moody's (2009) and Datastream^{TM}.

Table 8. Spread break-down EL* vs. UL* – straight-line amortization

	Maturity year									
	1		3		5		7		10	
	UL	EL	UL	EL	UL	EL	UL	EL	UL	EL
Aaa	89.08%	10.92%	90.80%	9.20%	89.23%	10.77%	88.07%	11.93%	88.47%	11.53%
Aa	86.23%	13.77%	84.59%	15.41%	83.64%	16.36%	83.89%	16.11%	84.88%	15.12%
A	84.67%	15.33%	78.68%	21.32%	77.20%	22.80%	76.80%	23.20%	77.04%	22.96%
Baa	71.26%	28.74%	68.29%	31.71%	67.23%	32.77%	67.25%	32.75%	68.08%	31.92%
Ba	47.79%	52.21%	42.40%	57.60%	40.98%	59.02%	41.41%	58.59%	43.10%	56.90%
B	26.34%	73.66%	25.45%	74.55%	26.08%	73.92%	27.07%	72.93%	28.89%	71.11%
Caa	14.53%	85.47%	16.50%	83.50%	18.21%	81.79%	19.99%	80.01%	22.39%	77.61%
Ca-C	7.53%	92.47%	10.70%	89.30%	13.14%	86.86%	15.33%	84.67%	18.29%	81.71%
Investment grade	78.64%	21.36%	75.45%	24.55%	74.46%	25.54%	74.45%	25.55%	75.20%	24.80%
Speculative grade	26.28%	73.72%	26.75%	73.25%	27.87%	72.13%	29.33%	70.67%	31.73%	68.27%

Note: * in percentage of the total spread.
Source: Our elaborations on data from Moody's (2009) and Datastream^{TM}.

Concluding remarks

This paper detects how the Basel II IRB-Foundation approach affects the bank loan pricing, by developing a multi-period pricing methodology to estimate risk-adjusted rates and spreads for credit exposures with different repayment schemes. The main inputs we need to feed our pricing formula can be drawn from an internal rating model and from easy-to-find market data. Our model consistency with the new regulatory framework to credit risk measurement provides an immediate support for bank managers in making a loan price-related decision and allows us to find evidence of a significant relationship between the risk measures and the "technical" spreads charged on corporate loans.

Nevertheless, though very simple, our model needs to be refined spurring further studies aiming at:

i) removing the unrealistic assumption of a flat recovery rate for all corporate segments, as required in the IRB-Foundation approach. This condition doesn't consider that large corporations are more likely to offer appropriate guarantees or collaterals than small firms;

ii) relaxing the hypothesis of independence between PD, LGD and EAD, which allows to calculate the expected loss as the simple product of the three variables. To do that, we need to estimate the dependencies between the factors determining loan losses for both the single borrower and the whole credit portfolio;

iii) including into the analysis a way to account for portfolio effects: since we measure unexpected losses through the regulatory risk-weight formulas, asset correlations cannot be directly estimated because they are automatically derived through an algorithm which is based on the inverse relationship between PD and asset correlation supposed by Basel II. How well do the correlation values calibrated by the Committee reflect the risk profile and the actual loss experience of credit portfolios, and what are the consequences of their adoption in terms of loan pricing are relevant issues that must be addressed, given the strong impact of correlations on the IRB capital requirement and, finally, on the loan price.

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