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The fair value of pension liabilities: the case of embedded option in scenario analysis

Abstract

Pension funds evolved over time towards the adoption of more complex risk-sharing schemes in order to keep up with the financial market complexities and volatility. Among these, the adoption of an indexation policy is widespread and it is now conditional to the solvability of the fund. Pension funds recognizing conditional inflation indexation targets are obliged to pay an additional payoff that is linked to the inflation rate through some specific rule. The additional payoff normally takes the form of a contingent claim conditional to a "measure" of sustainability of the payoff itself. This contingent claim can be valued with the same techniques that are used to value options. This valuation technique is an indispensable tool for improving pension fund risk management and correlated fair valuation issues. The paper provides a valuation methodology for the inflation indexation as embedded option by means of scenario-based analysis. Results derive from a simulation procedure applied to an exemplar case and give the opportunity to state the nature and the value of the indexation option.

Keywords: conditional indexation, barrier option, liabilities valuation, pension fund.
JEL Classification: C3, G13, G23.

Introduction

In the last years many of the Defined Benefit (DB) pension funds in OECD countries reported lower funding levels and in some cases large funding gaps (OECD, 2009). Whereas the impact of the financial crisis is not such to harm the solvability of DB pension plans, the reduction of the funding levels resulted mainly in a reduction in the indexation granted to pension fund participants. These pension funds are expected to react to lower funding ratio by stopping the indexation of benefits to wage or price inflation until funding level recovers. The indexation represents a correction of the pension rights aimed at compensating the loss in terms of purchasing power due to inflation rate increases and therefore offers a hedge against the purchasing power risk faced by pension participants. The full indexation of the liabilities has been for last decades an undisputed guarantee offered to the participants of a pension fund, but it has become less sustainable for many DB pension funds since the 2000-2003 stock market collapse. Most of them opted to voluntary and conditional/limited indexation policy, depending on the financial position of the fund. It means that the compensation can also be null or only partial when the funding ratio falls below required level. In the UK, for example, indexation is typically restricted to the range of 0%-5% per year (limited indexation). In the Netherlands, pension funds mostly selected a solution consisting in a conditional indexation: the decision to grant indexation depends on the nominal funding ratio defined as the ratio of assets to liabilities. If the funding ratio falls below a threshold level, indexation is limited or skipped altogether assuming the features of an option (de Jong, 2008). From a participant's perspective, the conditional indexation implies that the "indexation risk" (or purchasing power risk) partly translates from the pension fund to its participants. From the pension fund management perspective, the solution to offer only conditional indexation has been seen as a good compromise given the adverse financial market conditions. The recent evolution of their full indexation policy towards a conditional indexation policy arises the need for a quantification of the risks arising from this. At the same time, several criticisms have been raised against pension fund management because it has under-estimated the implied effect of such a policy. The prospected payoff can be assimilated to an option scheme and should be accurately valuated in the definition of the pension fund's obligation towards its participants and should lead to an appropriate Asset & Liability Management (ALM) strategy. Different kinds of embedded option exist related to indexation. Our analysis relates to indexation conditional to the level of the funding ratio, as applied in the Netherlands, and is under consideration for introduction also in other pension systems. Within this context, the valuation of the embedded option concerning the inflation becomes relevant even in time. The main objective is twofold: the identification of the more appropriate option scheme to adopt as an efficient replication of the pension fund flows and the selection of an evaluation procedure consistent with the internal management approach. The paper investigates the opportunity to apply barrier option scheme to the case of a pension fund, whose indexation target is conditional to a specific value of the funding ratio, in order to provide a full valuation of the obligation towards participants. The prime result is to provide a valuation for the inflation indexation as

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embedded option consistent with the scenario analysis driving asset allocation policy. Numerical results derive from a simulation procedure applied to an exemplar case by means of scenario-based analysis. The dataset and the indexation rule correspond to a Dutch based DB pension funds (Bikker, 2007). Evidences give the opportunity to state the absolute value of the "inflation option" and the relative value with respect to the fund's liability. This valuation technique is an indispensable tool for improving pension fund risk management redesigning pension contracts and for supporting decision making processes (Ziemba, 1998).

The literature on pension funds focuses on the risks that various stakeholders assume in a pension funds in terms of embedded option approach. Seminal paper by Blake (1998) shows, for example, that a DB pension funds can be replicated by an investment in a portfolio containing the underlying asset (market value of the asset) plus a put minus a call option on this asset, by adopting a Black and Scholes (1973) pricing. As the whole fund can be replicated by an appropriate portfolio, also specific (innovative) feature can be treated as embedded option, such as the option to increase contributions in case of a low funding ratio. Even longevity options are written by active employees to the pension fund, allowing the fund to reduce pension entitlements in case of an unanticipated rise in longevity. The embedded options described above can be explicitly calculated using market-consistent valuation. The values of the embedded options are measured using arbitrage-free option pricing techniques and assuming complete markets and by means of simulation techniques.

Even the conditional indexation agreement depending on the funding ratio can be modelled as a structured product. In particular, it can be regarded as a barrier option embedded in the pension contract that the pension fund sells to its participants as suggested by de Jong (2008). Among different types of barrier option, we originally evaluate this Indexation Option (IO) as an outside barrier option call down-and-out. Next section describes the general functioning of the indexation rule adopted in a Dutch based pension funds and the computation of the asset and liabilities market value which composes the funding ratio. Successively the barrier options are presented together with the payoffs of the outside barrier options chosen to describe the indexation option. The following paragraph evaluates this option by means of scenario analysis in ALM context.

1. Dynamics of the pension fund

The indexation policy depends on the financial status of the fund expressed by the funding ratio at the end of the year *t* (FR). It is computed using the annual market values for both assets (A_t^U) and liabilities (L_t^U):

$$FR_t^U = \frac{A_t^U}{L_t^U}, \tag{1}$$

where FR_t^U – ultimate funding ratio – expresses the financial status of the fund as the capability of the amount of the resources available to cover the related nominal liabilities at the end of the year. It is usually expressed in percentage terms, so that a funding ratio of 105 corresponds to a 5% surplus of assets over liabilities.

In most of the DB pension funds, the indexation rule is defined as follows: if the funding ratio is greater than the required funding ratio, full indexation is granted.

According to the actual Dutch regulation, the required funding ratio is defined by the Pension Law and depends on both the Strategic Asset Allocation (SAA) of the fund and the duration mismatch between pension assets and liabilities. Let us assume that the required funding ratio has to be equal to two exemplar cases: 105 corresponding to the minimum solvency requirement and 115 as the average indexation requirement.

Therefore, if the funding ratio is lower than the threshold values (105; 115) the nominal liabilities at time $t+1$ correspond to the nominal liabilities at time t, without any indexation.

Hence, only if the nominal liabilities are counterbalanced in terms of assets, the pension fund will proceed to consider an update of the nominal liabilities to the inflation rate, granting indexation.

To compute the funding ratio, the market value of the assets and liabilities must be computed. At time 0 (evaluation time), the pension fund has a certain current value of the assets ($A_{t=0}$) and liabilities ($L_{t=0}$). The initial funding ratio is defined as:

$$FR_{t=0} = \frac{A_{t=0}}{L_{t=0}}, \tag{2}$$

where $A_{t=0}$ corresponds to the market value of the invested assets and $L_{t=0}$ to the present value of all the future obligations of the fund towards the participants as a whole. For each time t, according to the Liability Driven Investment (LDI) paradigm, the asset portfolio (A_t) is divided into two sections: the Matching portfolio ($A_{M,t}$) and the Risk-Return portfolio ($A_{RR,t}$). The Matching portfolio is assumed to earn exactly the liability return to match nominal liabilities as a result of a perfect immunization strategy. The Risk-return portfolio consists of different asset classes as equity and alternative assets. It is meant to provide enough resources to grant indexation. The amount invested in each portfolio is defined according to the ratio of the matching portfolio to the total value ($w_M = A_{M,t}/A_t$) and of the risk-return portfolio to the total value ($w_{RR,t} = A_{RR,t}/A_t$) and the portfolio is rebalanced to these pre-defined weights each year. Let us assume, using average percentage concerning the Dutch pension fund, that the percentage of assets invested in the Matching portfolio is 37%, while the remaining 63% is invested in the Risk-Return portfolio.

1.1. Market value of asset and liabilities. To compute the critical funding ratio conditioning indexation, we need to define the market value of asset and liabilities. On the liability side, the value of the liabilities is computed under the hypothesis of the run-off of the pension fund. We set the time t as the moment from which the pension fund is formally closed to new participants and the old ones do not pay any contribution (evaluation time). The pension fund only has annual nominal cash flows (CF) to be paid to the participants at the end of each subsequent year until the definitive closing date (n). The present value of all these future nominal obligations is computed market-to-market as:

$$L_t^U\left(i_{k,t}\right)=\sum_{k=0}^{n}\frac{CF_{t+k}}{(1+i_k)^k}, \tag{3}$$

where k is the maturity of each residual cash flow and i_k is the spot rate associated to the corresponding node on the interest rate yield curve. The notation $L_t^U\left(i_{k,t}\right)$ accounts for the fact that the present value is calculated on the basis of a yield curve estimated at time t. The cash flows are computed under usual assumptions about the life expectation of the participants, the expected retirement date and other variables according to a defined actuarial model that takes into account actuarial and longevity risk. We will not investigate these aspects, since we concentrate on the interest rate risk arising from the fair valuation and we define the value in (3) as the present value of an anticipated rent.

The interest rate yield curve is generated by the Nelson and Siegel (1987) model, that has the advantages that it is well-behaved at long maturities, and that its parameters can be set to model virtually any yield curve. The corresponding term structure of interest rate in each year (and next in each scenario) will be determined by combining the values of the three main parameters according to the following relationship:

$$i_k=\beta_0+\left(\beta_1+\beta_2\right)\frac{1-e^{-k/\tau}}{k/\tau}-\beta_2 e^{-k/\tau}, \tag{4}$$

where k is the relevant node; β_0 is an estimate of the long-run levels of interest rates; β_1 is the short-term component; β_2 is an estimate of the medium-term component; and τ is the decay factor. Parameters were fitted via a least-squares according to a standard procedure defined by Diebold and Li (2006). The yield curve is simulated on the basis of formula (4) and it is used to discount all the future cash flows according to the value of k. We want to remark that the ultimate value of the liabilities at time t is computed as the present value of all the future nominal obligations including the cash flow to be paid at the end of year t (anticipated rent), discounted at the interest rate yield curve estimated according to formula (4) at time t. Therefore, this value only takes into account the nominal obligation as defined at time t, excluding the eventual increase of the nominal liabilities due to the indexation decision.

From the ultimate value, we derive the corresponding primary value of the liabilities at time t, by subtracting the nominal cash flow to be paid at time t, in order to regard the primary value as the present value of the posticipated rent corresponding to the anticipated one as defined by (3). That is:

$$L_t^P\left(i_{k,t}\right)=L_t^U\left(i_{k,t}\right)-CF_t. \tag{5}$$

The primary value of the liabilities $L_t^P\left(i_{k,t}\right)$ represents the "end of the year" value evaluated on the basis of the yield curve as estimated at time t, and hereafter the initial value of the liabilities at the beginning of the next year filtered by the information available at time t and synthesized in the yield curve. Given these definitions, the "nominal" rate of growth of liabilities is given by:

$$r_{L,t+1}=\frac{L_{t+1}^U\left(i_{k,t+1}\right)}{L_t^P\left(i_{k,t}\right)}-1. \tag{6}$$

This value gives the increase in the value of the nominal liabilities from their initial value (primary) at the beginning of the year to the end of the same year, only due to the dynamics of cash flows and changes in the interest yield curve from one year to another.

Once the nominal growth of liabilities is computed, every year the primary value of the liabilities at time

t, that is to say the initial value of the liabilities at time $t+1$, is updated by the nominal rate of growth as in formula (6), to obtain the nominal ultimate value at time $t+1$ as below:

$$L_{t+1}^{U}\left(i_{k,t+1}\right)=L_{t}^{P}\left(i_{k,t}\right)\left(1+r_{L,t+1}\right). \quad (7)$$

Then, depending on the value of the funding ratio at time $t+1$, the indexation decision is taken and applied to the ultimate value in formula (7), to obtain the indexed ultimate value of the liabilities, as follows:

$$L_{t+1}^{Uindex}=L_{t+1}^{U}\left(i_{k,t+1}\right)\cdot\left(1+\pi_{t+1}\right), \quad (8)$$

where π_{t+1} is the inflation rate as recorded at time t+1. By subtracting the t+1 maturing cash flow (also updated by indexation), we compute a new primary value for the liabilities which also takes into account the indexation:

$$L_{t+1}^{Pindex}=L_{t+1}^{Uindex}-\left(CF_{t+1}\cdot\left(1+\pi_{t+1}\right)\right). \quad (9)$$

This value represents the initial value of the liabilities for the next year that will be accordingly updated by the nominal growth estimated in formula (7) and eventually by the indexation decision (8). It is denominated "Pindex" to be distinguished by the previously defined primary value, which does not include indexation. However, once the indexation is recognized, it is acquired and guaranteed: it becomes the "nominal" value for the next year.

Therefore formula (8) can be timely extended as:

$$L_{t+2}^{U}\left(i_{k,t+2}\right)=L_{t+1}^{Pindex}\left(1+r_{L,t+2}\right). \quad (10)$$

On the other side of the intermediation portfolio, the initial amount of assets at time 0 is invested every year, and therefore A_t represents the market value of portfolio of the pension fund. The value of the portfolio is the sum of the two parts described in the previous section:

$$A_t=A_{M,t}+A_{RR,t}. \quad (11)$$

The Matching portfolio $A_{M,t}$ is composed of fixed-income assets with duration equal to the duration of the liabilities and that it earns every year a return equal to the nominal rate of growth of the nominal liabilities as defined earlier (formulation 6).

$$r_{L,t}=r_{M,t}, \quad (12)$$

where $r_{M,t}$ is the rate of return of the Matching Portfolio at time t. By means of this position, the interest rate risk is partially offset. Due to the fact that the immunization is only in terms of duration, it only hedges from a parallel shift of the interest rate yield curve. The remaining interest rate risk (convexity risk) and the inflation risk should be hedged by the dynamics of the returns of the other part, the Risk-return portfolio $A_{RR,t}$. This portfolio is composed of: Property, Commodity, Equity Value, Equity Passive, Equity Emerging Market and Equity Growth. It should earn enough to complete the hedging of the nominal liabilities and also provide with extra-return to allow for indexation. The return on the risk-return portfolio of the pension fund is given by:

$$r_{RR,t}=\sum_{j=1}^{z}r_{j,t}\cdot\frac{A_{j,t}}{A_{RR,t}} \ \ with \ \ A_{RR,t}=\sum_{j=1}^{z}A_{j,t}, \quad (13)$$

where $r_{j,t}$ is the rate of return – at time t – of the j-th asset in the risk-return portfolio weighted by the percentage contribution of the j-th asset to the portfolio and where z is the total number of assets or securities in the portfolio itself.

Consistently with the liabilities framework, we define two different values of the assets. The first one, defined as ultimate asset value (A_{t+1}^{U}) is the reference value for the computation of nominal funding ratio on which the indexation will depend. It is calculated as:

$$A_{t+1}^{U}=A_{M,t}^{P}\left(1+r_{L,t}\right)+A_{RR,t}^{P}\left(1+r_{RR,t}\right). \quad (14)$$

It expresses the value of the invested assets before the indexation and the payment of the cash flow for the corresponding year, where A_t^P is the primary value for each portfolio. Similarly to the primary value of the liabilities, it is computed as:

$$A_{t+1}^{P}=A_{t+1}^{U}-\left(CF_{t+1}\cdot\left(1+\pi_{t+1}\right)\right). \quad (15)$$

2. The dynamics of the embedded option

2.1. Outside barrier options. Barrier options are contingent claims that either are born (in barrier or knock in) or expire (out barrier or knock out) when the underlying asset price reaches a specified value h defined as "barrier". Given the presence of the barrier, these options typically exhibit a lower value than corresponding plain vanilla options, with higher prospective expected return. There are put and call, as well as European and American varieties. The common feature is that they become activated or, on the contrary, null and void only if the underlying asset reaches a predetermined level (barrier) and, specifically, "in" options start their lives worthless and only become active in the event a predetermined knock-in barrier price is breached, while "out" options start their lives active and become null and void in the event a certain knock-out barrier price is breached.

Outside barrier option are two-asset options where the payoff is defined on one asset (the so called payoff asset) and the barrier is defined on another asset (the so called measurement asset). Several types of barrier option (put and call) can be formulized, but for the case under investigation we will refer to the *down-and-out option*, where the contract expires if the measurement asset price falls below the value barrier at the expiration date.

In order to configure the scheme of the conditional indexation policy we will refer to a barrier down-and-out option, characterized by the presence of two underlying assets, since the option payoff (the indexed addendum) is conditional to a special event: the funding ratio has not to fall below a defined minimum level (see section 2).

Therefore, recalling the scheme of the down-and-out outside barrier option, the funding ratio takes the place of the "measurement asset" and sets the condition which eliminates any positive payoff, given a decrease in the value of the measurement itself. According to these scheme, if the barrier is hit, there is no additional payoff and the option expires. The indexed addendum is the proper "payoff asset", which ultimately defines the positive payoff of the option. This framework, here originally applied to pension funds, exactly portrays the case of the minimum requirement for the funding ratio. In the majority of cases, the funding ratio is higher than the minimum requirement (both institutional and internal) and only if it goes down the minimum, the indexation will not be paid. Consistent with the dynamic of the pension fund (section 2), the possibility of knocking out depends solely on the fact that the measurement – that is to say the funding ratio – reaches the barrier level at certain times. If the option does not expire, that is to say if the funding ratio at time $t+1$ does not fall below the required ratio (the barrier h), the pension fund will recognize the indexation as stated by (8) with an optional positive payoff equal to $L_{t+1}^{U}\left(i_{k,t+1}\right)\cdot \pi_{t+1}$.

To evaluate an outside barrier option analytical solution has been developed (Zhang, 1995). The evaluation of the outside barrier option requires that the density function contains the lognormal distribution of the asset price payoff which is conditional upon the achievement or failure to achieve (depending on whether it is knock in or knock out) the barrier level by the price of the measurement asset *during the life of the option*. The crux is that in this pricing approach the barrier is modelled in a continuous framework. This assumption implies a density function even for the barrier since the option price relies on two defined stochastic processes put in a consistent Black and Scholes framework, that is to say respectively for the payoff asset and the measurement asset:

$$d\ln\left(S_t/S_0\right)=\mu_1 dt+\sigma_1 dW_t^1,$$

$$d\ln\left(R_t/R_0\right)=\mu_2 dt+\sigma_2 dW_t^2.$$

In other words, the price is based on a bi-variate density function, deriving from a lognormal distribution for both the measurement and the payoff assets. The two lognormal distributions are modelled in a stochastic environment by the application of known drift and diffusion coefficients (μ,σ), as well as on the base of a known correlation between the two relevant disturbance dynamics ($dW_t^1 \cdot dW_t^2 = \rho$).

2.2. Evaluating the indexation option. For the application of the outside barrier option to the indexation case, the recalled Black and Scholes approach above can not be appropriately used. This is due to the fact that it assumes a continue barrier over the life of the option and a lognormal distribution for both the measurement and payoff asset. In the pension fund case, the barrier is represented by a specified level of the funding ratio and is not observed continuously, but in a discrete time and on a specific date. Therefore, we will define the indexation option (IO) as an outside barrier option (down-and-out) having a discrete barrier. The observation time is set equal to the last day of each year, when the market value of the assets and liabilities are computed and the inflation rate is observed. For this reason, the lognormal distribution cannot be regarded as an accurate description of the relevant dynamic. At the same time, the payoff asset is more similar to an interest rate option. As a consequence, a numerical approach to the evaluation of the embedded option emerges as an obliged choice. We proceed on by using a scenario-based approach.

The simulation approach gives the opportunity to state simultaneously the value of the barrier and the value of the payoff. The implementation of this methodology consents the modelling of the relevant values according to correlation factors of the primary risk and value drivers, since these correlations are included in the scenario generation by means of the scenario generation scheme (see *infra*).

Since we concentrate on the "additional" amount paid if the relevant condition holds, we define the option payoff as "$L_{t+1}^{U}\left(i_{k,t+1}\right)\left(\pi_{t+1}\right)$ or nothing". In practice, if the funding ratio at time t+1 falls below the minimum requirement (barrier), the pension fund will recognize only the "nominal" liability value $L_{t+1}^{U}\left(i_{k,t+1}\right)$. On the other hand, if the funding ratio is equal or higher than the barrier, the pension fund will recognize the indexed value of the liability $L_{t+1}^{UIndex}\left(i_{k,t+1}\right)=L_{t+1}^{U}\left(i_{k,t+1}\right)\left(1+\pi_{t+1}\right)$, that is:

$$\begin{cases} L^U_{t+1}(i_{k,t+1}) & \forall \quad FR^U_{t+1t+1} < h \\ L^U_{t+1}(i_{k,t+1})(1+\pi_{t+1}) & \forall \quad FR^U_{t+1} \geq h \end{cases} = L^U_t(i_{k,t}) + \underbrace{\begin{cases} 0 & \forall \quad FR^U_{t+1} \leq h \\ L^U_{t+1}(i_{k,t+1})(\pi_{t+1}) & \forall \quad FR^U_{t+1} > h \end{cases}}_{indexation \quad option}, \tag{16}$$

where the last addendum is the payoff of the indexation option payoff (IOP) as:

$$IOP_{t+1} = \max\left[L^U_{t+1}(i_{k,t+1})(\pi_{t+1}),\ 0\right] \ \forall \ FR^U_{t+1} > h. \tag{17}$$

The previous formulation gives the payoff referred to time t+1. The present value at time t of the IOP_{t+1} calculated using the spot rate referring to the first node of the yield curve and observed in t ($i_{1,t}$) gives the price of the IO. And so on for the residual duration of the pension funds. Therefore, the present value of the whole indexation option payoff ($WIOP_t$) at time t is the sum of n indexation option payoffs differing for the time to maturity and discounting for the appropriate spot rate as observe in time t. Formally:

$$WIOP_t = \frac{IOP_{t+1}}{1+i_{1,t}} + \frac{IOP_{t+2}}{(1+i_{2,t})^2} + \cdots + \frac{IOP_{t+n}}{(1+i_{n,t})^n} = \sum_{k=1}^{n} \frac{IOP_{t+k}}{(1+i_{k,t})^k}. \tag{18}$$

Given the discretization of the barrier, the present value of IOP, that is to say the price/value of the option, is estimated by numerical methods, based on scenario analysis as far as the asset and liability values are concerned. More specifically, since each scenario s (with s = 1, 2, …, q) gives rise to a different yield curve the expected value of $WIOP_t$ is the present value of n option payoff in q states of the world, as follows:

$$E[WIOP_t] = \frac{1}{q}\sum_{s=1}^{q}\sum_{k=1}^{n} \frac{IOP_{t+k,s}}{(1+i_{k,t,s})^k}, \tag{19}$$

where $i_{k,t,s}$ is the spot rate observed in t referring to period t-(t+k) and to scenario s and $IOP_{t+k,s}$ refers to the IOP as it is at time t+k and scenario s.

3. Numerical evaluation and scenario analysis

As in most ALM studies, the scenarios for the economic relevant variables are generated by a statistical model called Vector Auto Regressive Model (VAR), introduced by Sims (1980). The model is formalized as follows:

$$x_{t+1} = a + Dx_t + \varepsilon_{t+1}, \tag{20}$$

where a denotes a vector of the intercepts, D denotes the matrix of coefficients, x_t is the state vector composed by the economic variables and ε_t is the vector of shocks to the system which is assumed to be normally distributed with zero mean and variance-covariance matrix $\sum_\varepsilon$: $\varepsilon_t \sim$ N(0, $\sum_\varepsilon$).

This model is preferred to others because it is able to create scenarios that are "in accordance with the past" (Boender, 1997). In particular, if the parameters of the VAR are estimated by Ordinary Least Square (OLS) procedure on a sufficiently long historical period, the long-term averages, standard deviations and (auto-) correlations of the scenarios generated are identical to the observations in the historical period used for the model estimation.

After the estimation of the coefficients D of the VAR model, the scenarios are generated by simulating recursively from the VAR model. For this, the estimated covariance matrix of the residuals $\sum_\varepsilon$. is decomposed by means of the Cholesky matrix (Gentle, 1998) (C), such that $CC' = \sum_\varepsilon$. The decomposition is used to estimate values of ε_t. This is done by sampling a vector u from a standard normal distribution N(0,1) so that $u \sim N(0,1)$ of which $Cu \sim N(0, CC')$ is derived. By multiplying the Cholesky decomposition with a vector of random numbers from a standard normal distribution, new shocks to the system are generated which give simulations of $\varepsilon = Cu$. The Cholesky matrix permits us to impose the historical covariance structure on the future scenarios. These values are used in the equation (20) in order to generate a fan of scenarios according to the formula:

$$x_{t+1} = a + Dx_t + C\varepsilon_{t+1}, \tag{21}$$

where x_{t+1} is a vector of future values for the variables, D is a matrix with the estimated coefficients, x_t is a vector of values for the variables in the previous node, C is the Cholesky matrix, and ε_{t+1} is a vector of random standard normally distributed innovations.

This methodology is applied to our dataset to generate a total number of q scenarios equal to 2500 for the relevant economic time series and the asset classes (j) for the period 2009-2022 on an annual basis. We use annual data of these series for the period from 1970 to 2006 as the inputs for the estimation of an unrestricted first order VAR model including assets returns, interest rates, and price inflation as endogenous variables. In particular, as inflation rate we consider the annual realized Dutch inflation since the Netherlands is the country where the conditional indexation is mostly adopted. As far as interest rate time series are concerned, starting from the initial estimated parameters of the Nelson & Siegel model as described in the previous section, we generate the three main parameters (β_0, β_1, β_2) in each node (s, t) to construct a yield curve for each scenario (s) and each time node in each node (t) to discount the liabilities' cash flow.

On the asset side, the asset returns for Property, Commodity, Equity Value, Equity Passive, Equity Emerging Market and Equity Growth are generated. Commodity dataset is represented by Goldman Sachs Commodity Index (GSCI), a composite index of Commodity sector returns which represents a broadly diversified, unleveraged, long-only position in Commodity futures. Property data is represented by ROZ/IPD Dutch Property Index. This index measures the total returns on directly held real estate investments belonging to institutional investors and real estate funds in the Netherlands. Concerning the investment in equities, Equity Growth is represented by worldwide used Morgan Stanley Capital International World Index (MSCIWI). Equity Value category is represented by MSCISWI hedged, which gives the performance of an index of securities where currency exposures affecting index principal are hedged against a specified currency. Finally, Emerging Markets Equity category is represented by MSCI Emerging Markets Index, which is a float-adjusted market capitalization index investing in 26 emerging economies.

On the liability side, we make use of an original dataset provided by a Dutch pension funds composed by all the residual cash flows from 2008 to 2022 in the hypothesis of the closing of the fund in 2022 – that is to say – it is closed to the entry of new participants. It is important to underline that these cash flows are estimated by actuarial simulation that are properly linked to the other simulated economic times series.

The option value at time 0 gives the value of the option written by the pension fund to the participants on the inflation rate. The valuation of the IO is applied to the dataset assuming that the investment horizon (n) is set equal to 14 years, the liabilities are conditionally (only) fully indexed to inflation rate and the barrier (h) is set equal to two exemplar levels: 105 (as minimum solvency requirement) and 115 (as a proxy of the required funding ratio according to the Dutch law).

4. Results

The methodology is applied to the dataset by means of MATLAB. An original script was devoted to the evaluation of the embedded option. The figure below shows the option payoff (OIP) for each scenario at the evaluation time (in our case 1/1/2009), as a function of the payoff asset, that is to say as a function of the inflation rate (formulation 17). The option payoff has value equal to zero when the option expires because the option in that scenario is knocked out or the payoff asset is not positive (as the case of a negative inflation). On the y-axis there is the histogram of the frequencies associated with each payoff, while on the x-axis there is the histogram representing the distribution frequency of the payoff assets across scenarios.

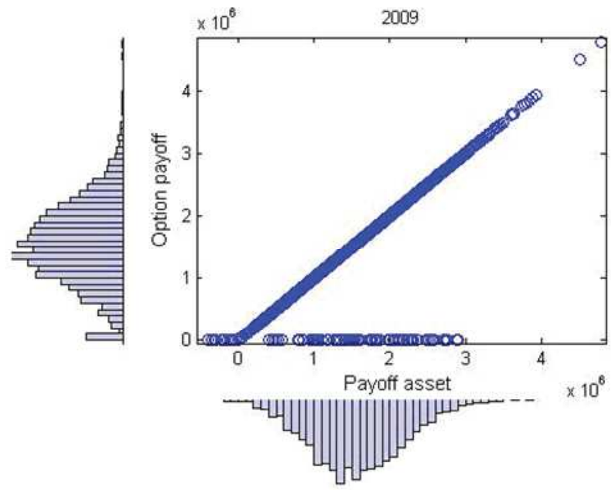

Fig. 1. Option payoff and payoff asset

The graph below relates the option payoff (and the relative frequency distribution) to the funding ratio dynamics at the evaluation time (1/1/2009).

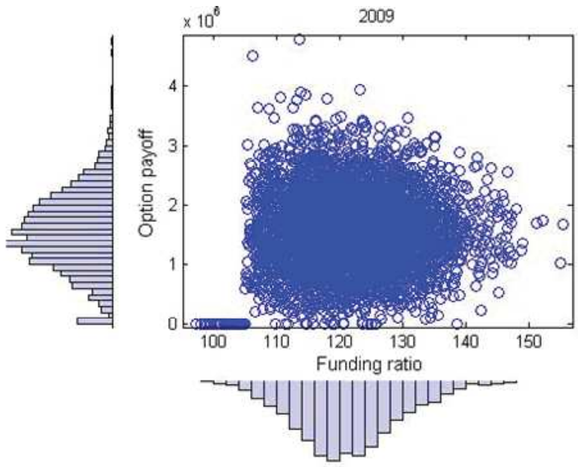

Fig. 2. Option payoff and funding ratio in 2009

Figure 3 shows the distribution of the option payoff (IOP) for each year as a stochastic process. Therefore, for each time node, we can observe the distribution of the annual payoff across scenarios (formulation 17). We notice that the means and the standard deviations of the payoff increase over time according to the increasing volatility of the underlying scenario over time. We can also notice that because of the higher volatility of the funding ratio, the frequency associated with the case where the option is knocked out increases over time. The application of formulation 19 gives us the value of the option. Starting from the monetary value, we can deduce the relative value to the nominal liabilities. In this case, the option value at evaluation time (1/1/2009) for the residual 14 years accounts for approximately 27% of the nominal liabilities, that is to say more than 1/4 of the nominal liabilities. It is not an irrelevant percentage of the value of the liabilities and cannot be neglected in a fair valuation.

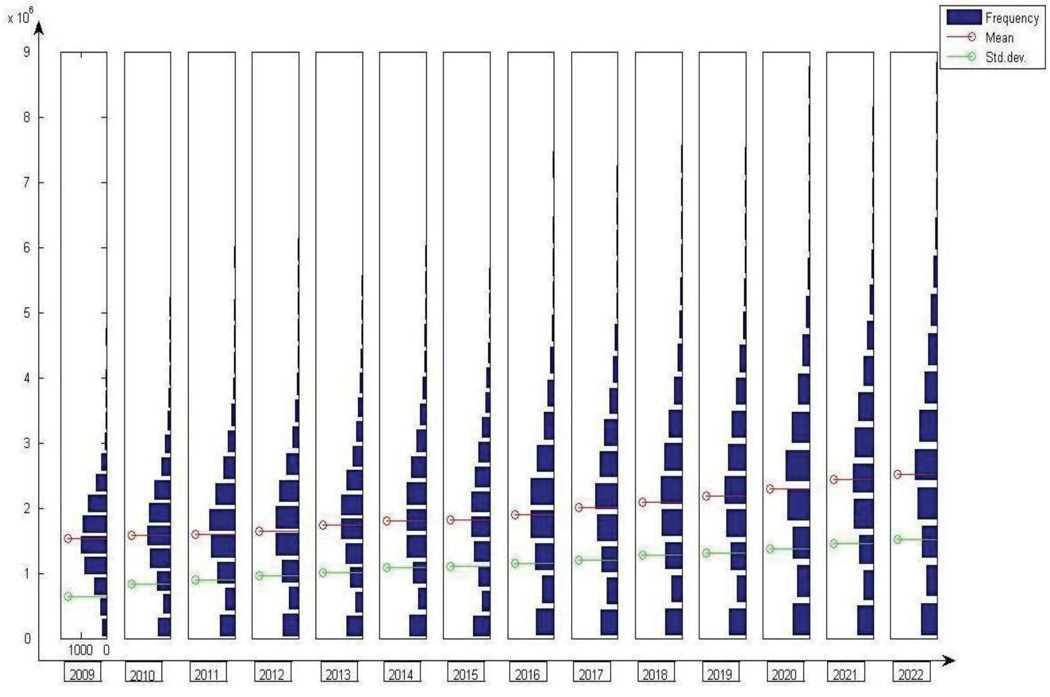

Fig. 3. The distribution frequency of the OIP over time with barrier set at 105

We also develop the same calculus setting the barrier level at 115. As we expected, the option value reaches the value of 22.38% of the liabilities. This is due to the higher level barrier that leads to a higher number of knock-out. As in the preceding case, the graph shows the distribution of the option payoff for each year under consideration as a stochastic process with barrier set at 115. We notice the higher frequency associated with the case where the option is knocked out and a lower means than in the case with barrier set at 105.

As expected, the selection of a higher barrier reduces the value (both absolute and relative) of the option, which accounts for more than 1/5 of the nominal value of the liabilities.

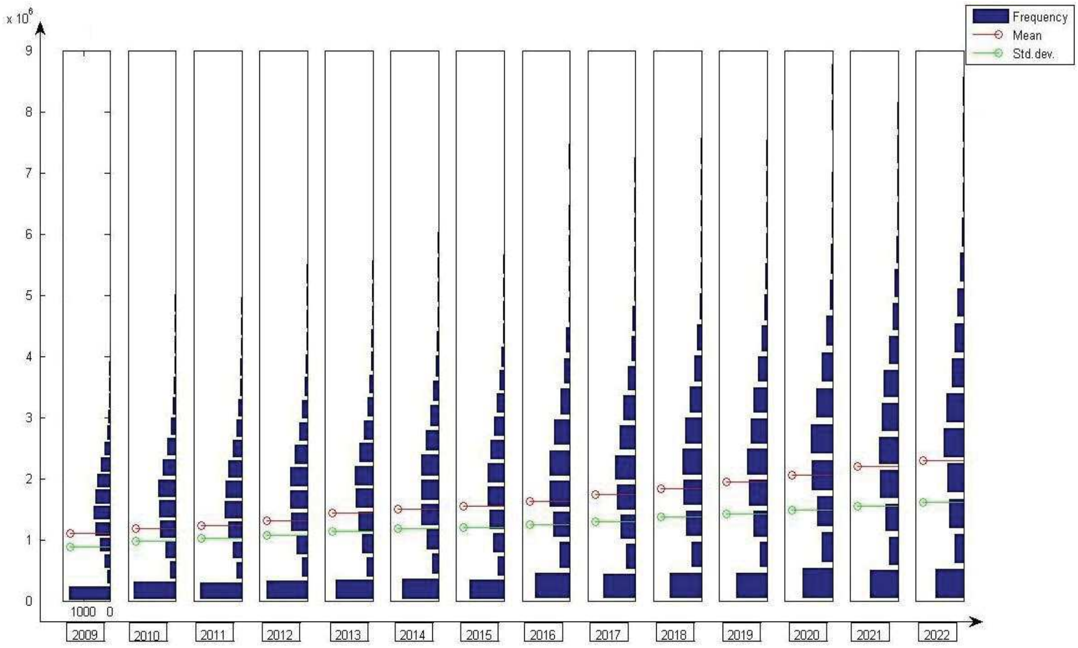

Fig. 4. The distribution frequency of the OIP over time with barrier set at 115

Conclusions

Conditional indexation is an important issue to be taken into account in the valuation of the liabilities. It is an embedded option written by the fund to the participants in the indexation agreements. As stated in the introduction, the paper was aimed at identifing an appropriate option scheme and at adopting a valuation procedure consistent with the ALM features.

With respect to these, the outside barrier option scheme – originally applied to replicate the conditional indexation policy – is able to depict the full cash flows dynamic and the adoption of a scenario based analysis allows for a valuation that can be immediately implemented for both managerial targets and accounting reports. This inner coherence gives the opportunity to calibrate performance measurement and improve risk management to assess both the suitability of the funding level and the effectiveness of the asset allocation.

Moreover, the results obtained for the indexation rule adopted by the Dutch pension funds consent to investigate what is the impact of this option on the fair value of the liabilities. We show that a knock-out call barrier option (with two reference assets) offers a good framework for this valuation. The option value in 2009 for the following 14 years amounts to 27% of the liability value when the barrier is 105 and 22% when the barrier is 115.

Further investigations should try to remove several assumptions we impose as the static asset allocation or also allow for partial and recovering indexation. Also, the definition of an optimal level for the barrier can be considered. This last point is of special interest for regulation and supervision application. The barrier level could be in fact selected to keep the solvency probability within a certain predefined level, in order to assure the survival of the fund. Accounting implementations are even possible, with special reference to those practices where the marking to market require a full unbundling of the basic components of the relevant obligations.

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