“An economic model of simultaneous purchasing of both an insurable asset and insurance coverage”

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An economic model of simultaneous purchasing of both an insurable asset and insurance coverage

Abstract

This study investigates the situation in which a consumer simultaneously purchases both an insurable asset and insurance coverage. The main results of this study are as follows. First, the optimal amount of the insurable asset in a risky situation is smaller than it is in a non-risky situation. Second, when the insurance premium rate is actuarially fair, the accident probability distribution does not affect the optimal amount of the insurable asset because the consumer purchases full insurance coverage. Third, when the insurance premium rate is actuarially unfair, it is ambiguous whether the optimal amount of the insurable asset is larger than it is in the actuarially fair case.

Keywords: insurable asset, insurance coverage, economic model.

Introduction

There is a substantial literature studying the amount of insurance coverage in many situations. Studies on the amount of insurance coverage are very important and they are closely related to many research topics on insurance markets. For example, discussions of the amount of insurance coverage lead to the issue of risk sharing problems among parties. Ehrlich and Becker (1972) argued that full insurance coverage is the equilibrium if the insurance premium is actuarially fair. As another example, discussions of the amount of insurance coverage are connected to asymmetric information problems such as adverse selection and moral hazard problems. Rothschild and Stiglitz (1976) showed that the equilibrium insurance coverage of low risk consumers is partial insurance coverage when accident probabilities are the private information of the consumers because of the existence of an adverse selection problem. Pauly (1968) argued that deductibles and coinsurance, which prevent the provision of full insurance coverage, are devices to alleviate the moral hazard problem. These studies have been revised and extended by many researchers over many years. For example, in the case of adverse selection problems, Dionne and Lasserre (1985), Cooper and Hayes (1987), Vázquez and Watt (1999), de Garidel-Thoron (2005), and Janssen and Karamychev (2005) investigated the optimal insurance coverage in the case of multiperiod insurance contracts. Doherty and Thistle (1996), Doherty and Posey (1998), Hoy and Polborn (2000), Hoel et al. (2006), and Barigozzi and Henriet (2011) discussed the optimal insurance coverage in the situation where both consumers and insurers initially do not know consumers' accident probabilities, which are subsequently identified through medical checkups and genetic testing.

The discussions of the amount of insurance coverage are wide-ranging and have derived many meaningful results. However, they have implicitly or explicitly assumed that the amount of the insurable asset is given because the consumers in these studies have an endowed income that may be lowered by an accident, meaning that they may want to purchase insurance coverage to protect their income. Thus, these studies did not investigate the situation in which the consumers endogenously decide the amounts of the insurable asset and the insurance coverage simultaneously. In the real world, for example, a consumer who wants to purchase an automobile considers not only the amount of an automobile but also the appropriate amount of automobile insurance coverage.

From that viewpoint, the purpose of this study is to investigate for how much an insurable asset is purchased when a consumer simultaneously chooses the amounts of both the insurable asset and the insurance coverage. In particular, this study compares the amounts of optimal insurable assets when insurance premium rates are actuarially fair and unfair.

1. The model

Suppose that the consumer has initial wealth but does not have an insurable asset. This initial wealth is denoted by $w > 0$. This consumer simultaneously chooses the amounts of an insurable asset and the insurance coverage. $v \equiv v(p)$ represents the consumer's revenue by using the insurable asset, the amount of which is $p > 0$, and we assume that $v_p \equiv \partial v/\partial p > 0, v_{pp} \equiv \partial^2 v/\partial p^2 \le 0$. This assumption implies that the consumer's utility from purchasing the insurable asset is increasing but the marginal utility is decreasing. Suppose that this insurable asset involves the risk of an accident, which will lower its value. $\tilde{p}$ represents the amount of damage in the case of the accident. Because nobody knows

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the actual amount of damage when the consumer purchases both the insurable asset and the insurance coverage, $\tilde{p}$ is shown as a random variable. For simplicity, the random variable $\tilde{p}$ is distributed along the normal distribution that is denoted by $N(\mu, \sigma^2)$. $\mu \equiv \mu(p)$ and $\sigma^2 \equiv \sigma^2(p)$ represent the mean and variance of the amount of damage, respectively. In some cases, $\tilde{p}$ is distributed on other kinds of distribution forms such as binomial distribution and Poisson distribution. However, if we consider the case in which there are many consumers in the market, these distributions can approximate the form of the normal distribution. It is assumed that the consumer can know both parameters in the normal distribution when he or she determines the amounts of the insurable asset and the insurance coverage. In addition, assume that $\mu_p \equiv \partial\mu/\partial p > 0$ and that $\sigma_p^2 \equiv \partial\sigma^2/\partial p > 0$. This assumption indicates that the amount of the insurable asset affects both the mean and the variance in the normal distribution. The consumer can purchase insurance coverage in order to receive compensation in the case of the accident. $\alpha \in [0,1]$ and $\beta > 0$ represent the insurance coverage rate and insurance premium rate, respectively.

The consumer is assumed to be weakly risk averse and the form of his or her utility function is specified as follows:

$$u = -\exp(-rW), \tag{1}$$

where $r \geq 0$ represents the consumer's degree of absolute risk aversion and W indicates the consumer's aggregated income, which is shown as:

$$W = w + v - p - (1-\alpha)\tilde{p} - \alpha\beta p. \tag{2}$$

There are two reasons to specify the consumer's utility function in the equation (2). First reason is that the consumer's utility function in the equation (2) is that degree of absolute risk aversion is constant. In other words, we do not need to consider the relation between aggregated income and degree of absolute risk aversion. Second reason is to simplify the computation. It is well known that the combination between normal distribution and the utility function in the equation (2) can simplify to compute consumer's certainty equivalent[1]. From equations (1) and (2), the consumer's certainty equivalent, which is denoted by CE, can be derived as follows:

$$CE = w + v - p - (1-\alpha)\mu - \alpha\beta p - \frac{1}{2}r(1-\alpha)^2\sigma^2. \tag{3}$$

Then, the consumer simultaneously chooses p and α to maximize its own certainty equivalent. In order to derive the optimal amounts of the insurable asset and the insurance coverage, the first-order conditions with respect to p and α can be written as[2]:

$$\frac{\partial CE}{\partial p} = v_p - 1 - (1-\alpha)\mu_p - \alpha\beta - \frac{1}{2}r(1-\alpha)^2\sigma_p^2 = 0, \tag{4}$$

$$\frac{\partial CE}{\partial \alpha} = \mu - \beta p + r(1-\alpha)\sigma^2 = 0. \tag{5}$$

From equation (5), we have:

$$\alpha = 1 - \frac{\beta p - \mu}{r\sigma^2}. \tag{6}$$

In addition, from equations (4) and (6), we have:

$$v_p = 1 + \beta + \frac{(\beta p - \mu)\{-2\sigma^2(\beta - \mu_p) + \sigma_p^2(\beta p - \mu)\}}{2r(\sigma^2)^2}. \tag{7}$$

2. Compare the amount of optimal insurable assets

In this section, we compare the optimal amounts of insurable assets when the insurance premium rates are actuarially fair and unfair. If an insurer is assumed to be risk neutral and it sets an actuarially fair insurance premium rate, the insurance premium rate can be written as:

$$\alpha\beta p - \alpha\mu = 0 \Rightarrow \beta = \frac{\mu}{p}. \tag{8}$$

Substituting equation (8) into equation (6), we have:

$$\alpha = 1. \tag{9}$$

Thus, the consumer purchases full insurance coverage even when he or she simultaneously chooses the amounts of the insurable asset and the insurance coverage. Substituting equation (8) into equation (7), we have:

$$v_p = 1 + \beta. \tag{10}$$

In order to shed light on the implications of equation (10), we derive the optimal amount of the insurable asset that is not subject to any risk of damage as follows:

$$\frac{\partial(v-p)}{\partial p} = v_p - 1 = 0 \Rightarrow v_p = 1. \tag{11}$$

From a comparison of equations (10) and (11), the optimal amount of the insurable asset in a risky situation is smaller than it is in a non-risky situation because $v_{pp} \leq 0$. The larger is β, the smaller is

[1] In contrast, if we use other distribution form or utility function form, exact certainty equivalent cannot be derived.

[2] We assume that the second-order conditions are always satisfied. Second-order conditions are always satisfied when r is sufficiently high and when the conditions $\mu_{pp} \equiv \partial^2\mu/\partial p^2 \geq 0$ and $\sigma_{pp}^2 \equiv \partial^2\sigma^2/\partial p^2 \geq 0$ are met.

the optimal amount of the insurable asset in a risky situation because β reflects the amount of risk. In addition, from equation (10), we find that the accident probability distribution indicated by μ and σ^2 does not affect the optimal amount of the insurable asset because the consumer purchases full insurance coverage.

This result has some actual implications. It indicates that if the seller can alleviate some or all risks about the selling products, the consumers purchase more amount of products. Actually, consumers can cover the damages without their payments thanks to the manufacturers' guarantee to repair the damages of their products in a certain period. Such guarantee system can alleviate risks from the consumer and promotes to increase selling of these products.

Next, consider the situation in which the insurance premium rate is actuarially unfair. The insurance premium rate is shown as:

$$\alpha\beta p - \alpha\mu > 0 \Rightarrow \beta > \frac{\mu}{p}. \tag{12}$$

Substituting equation (12) into equation (6), we have that:

$$\alpha < 1. \tag{13}$$

The consumer purchases partial insurance coverage and then the accident probability distribution indicated by μ and σ^2 affects the optimal amount of insurance coverage.

From equations (7) and (10), whether the optimal amount of the insurable asset in the actuarially unfair case is larger than that in the actuarially fair case depends on the sign of the third term on the right-hand side of equation (7). Then, we can show:

$$Sign\left[\frac{(\beta p - \mu)\left\{-2\sigma^2(\beta - \mu_p) + \sigma_p^2(\beta p - \mu)\right\}}{2r(\sigma^2)^2}\right] = \\ = Sign\left[-2\sigma^2(\beta - \mu_p) + \sigma_p^2(\beta p - \mu)\right]. \tag{14}$$

The following equation can be derived from equation (12) using the implicit function theorem:

$$\alpha\beta - \alpha\mu_p > 0 \Rightarrow \beta > \mu_p. \tag{15}$$

From equations (12) and (15), the sign of equation (14) cannot be uniquely decided. In other words, whether the optimal amount of the insurable asset in the actuarially unfair case is larger than that in the actuarially fair case is ambiguous. The main reason that an unambiguous result cannot be derived is that there are two opposing effects in relation to the optimal amount of the insurable asset.

In order to shed light on the first effect, we transform equation (4) as follows:

$$v_p = 1 + \mu_p + \alpha(\beta - \mu_p) + \frac{1}{2}r(1-\alpha)^2\sigma_p^2. \tag{16}$$

In the actuarially unfair case, the fourth term on the right-hand side of equation (16), which represents the amount of the consumer's risk premium, does not disappear because $\alpha < 1$. In other words, the consumer bears the risk premium even after purchasing insurance coverage. In this situation, an increase in the amount of the insurable asset leads to an increase in the amount of the risk premium because $\sigma_p^2 > 0$. From this effect, the consumer has an incentive to lower the amount of the insurable asset.

In order to shed light on the second effect, the following equation is derived from equation (4).

$$\frac{\partial^2 CE}{\partial p \partial \alpha} = \mu_p - \beta + r(1-\alpha)\sigma_p^2. \tag{17}$$

Equation (17) represents the relationship between the amounts of the insurable asset and the insurance coverage. If the sign of equation (17) is positive (negative), this relationship is complements (substitutes). Substituting equation (6) into equation (17), we find:

$$-\sigma^2(\beta - \mu_p) + \sigma_p^2(\beta p - \mu). \tag{18}$$

Because of equations (12) and (15), the sign of equation (18) cannot be uniquely decided. However, it is easy to understand that a change from the actuarially fair case to the unfair case lowers the advantages of purchasing insurance coverage and, hence, lowers the optimal insurance coverage. At the same time, the consumer wants to shift his or her purchasing power away from insurance coverage towards the insurable asset if equation (17) becomes negative, which means the amounts of the insurable asset and the insurance coverage are substitutes. From this effect, the consumer has an incentive to raise the amount of the insurable asset.

Also, this result implicitly shows the effect of risk aversion to choose the amount of insurable asset. It is easy to imagine that actual insurance premium rate seems to be unfair because there are some kinds of costs such as management and marketing costs. From the result we mentioned before, whether the optimal amount of the insurable asset is larger than that in actuarially fair case is unclear. However, when the degree of absolute risk aversion is higher, risk premium, which represents $(1/2)r(1-\alpha)^2\sigma_p^2$, becomes larger and the relationship between insurable asset and insurance coverage has a tendency to be complement because the equation (17) has a tendency to be positive. Thus, in this case, unfair insurance premium rate lower the optimal amount of insurable asset. In contrast, when the degree of ab-

solute risk aversion is very small, risk premium is very small and the equation (17) has a tendency to be negative because $\beta > \mu_p$. Thus, in this case, unfair insurance premium rate raise the optimal amount of insurable asset. In a nutshell, the effect of unfair insurance premium rate on the optimal amount of insurable asset is closely related to each consumer's risk aversion.

Conclusion

This study investigated the situation in which the consumer simultaneously purchases both an insurable asset and insurance coverage. The main results of this study are as follows. First, the optimal amount of the insurable asset in a risky situation is smaller than it is in a non-risky situation. Second, when the insurance premium rate is actuarially fair, the accident probability distribution does not affect the optimal amount of the insurable asset because the consumer purchases full insurance coverage. Third, when the insurance premium rate is actuarially unfair, it is ambiguous whether the optimal amount of the insurable asset is larger than it is in the actuarially fair case.

There are some possible extensions to this study. In particular, the insurance premium rate is assumed to be an exogenous variable in this study. However, in the real world, insurers can choose their insurance premium rates unless an insurance market is heavily regulated or extremely competitive. Future research could analyze the case in which the insurance premium rates are endogenous variables.

References

1. Barigozzi, Francesca and Henriet, Dominique (2011). "Genetic Information: Comparing Alternative Regulatory Approaches When Prevention Matters", *Journal of Public Economic Theory*, 13, pp. 23-46.
2. Cooper, Russell and Hayes, Beth (1987). "Multi-Period Insurance Contracts", *International Journal of Industrial Organization*, 5, pp. 211-231.
3. de Garidel-Thoron, Thomas (2005). "Welfare-Improving Asymmetric Information in Dynamic Insurance Markets", *Journal of Political Economy*, 113, pp. 121-150.
4. Dionne, Georges and Lasserre, Pierre (1985). "Adverse Selection, Repeated Insurance Contracts and Announcement Strategy", *Review of Economic Studies*, 52, pp. 719-723.
5. Doherty, Neil A. and Thistle, Paul D. (1996). "Adverse Selection with Endogenous Information in Insurance Markets", *Journal of Public Economics*, 63, pp. 83-102.
6. Doherty, Neil A. and Posey, Lisa L. (1998). "On the Value of a Checkup: Adverse Selection, Moral Hazard and the Value of Information", *Journal of Risk and Insurance*, 65, pp. 189-211.
7. Ehrlich, Isaac and Becker, Gary S. (1972). "Market Insurance, Self-Insurance, and Self-Protection", *Journal of Political Economy*, 80, pp. 623-648.
8. Hoel, Michael, Iversen, Tor, Nilssen, Tore, and Vislie, Jon (2006). "Genetic Testing in Competitive Insurance Markets with Repulsion from Chance: A Welfare Analysis", *Journal of Health Economics*, 25, pp. 847-860.
9. Hoy, Michael and Polborn, Mattias (2000). "The Value of Genetic Information in the Life Insurance Market", *Journal of Public Economics*, 78, pp. 235-252.
10. Janssen, Maarten C.W. and Karamychev, Vladimir A. (2005). "Dynamic Insurance Contracts and Adverse Selection", *Journal of Risk and Insurance*, 72, pp. 45-59.
11. Pauly, Mark V. (1968). "The Economics of Moral Hazard: Comment", *American Economic Review*, 58, pp. 531-537.
12. Rothschild, Michael and Stiglitz, Joseph, E. (1976). "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information", *Quarterly Journal of Economics*, 90, pp. 629-649.
13. Vázquez, Francisco J. and Watt, Richard (1999). "A Theorem on Multi-Period Insurance Contracts without Commitment", *Insurance: Mathematics and Economics*, 24, pp. 273-280.