“The out-of-sample forecasts of nonlinear current depth of recession model of the output in the United Kingdom”

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The out-of-sample forecasts of nonlinear current depth of recession model of the output in the United Kingdom

Abstract

Although the Current Depth of Recession (CDR) could be used as an indicator of the business cycle, it cannot be a threshold variable because of some existing statistical problems. This study aims at modifying the original CDR to prove that the modified CDR (MCDR) is an appropriate threshold variable compared to the original CDR. The quarterly data from the United Kingdom (UK) from 1959 to 2006 are used to construct two types of TAR models which adopt the CDR and the modified CDR (MCDR) as the threshold variables. By using Root Mean Square Error (RMSE), Theil's Inequality Coefficient, and DM (Diebold and Mariano, 1995), the researchers examine the efficiency of out-of-sample forecast, and the results show that MCDR is more appropriate for being the threshold variable than CDR is.

Keywords: current depth of recession, business cycle, out-of-sample forecasting, threshold model.
JEL Classification: E32, C22, C53.

Introduction

The strength or weakness of business cycle indicates the growing circumstance of the economy. Delong and Summers (1986), Hamilton (1989), Hussey (1992), Beaudry and Koop (1993), and Henry et al. (2004) stressed, the asymmetry of output and economic growth exists under different business cycle regimes. Öcal (2006) also found the evidence of business cycle regime asymmetries. Therefore, it is appropriate to adopt the linear model without considering the nonlinear characteristic of economic growth, which might result in a biased conclusion.

To avoid the bias indicated above, some researchers switched the models from linear design to nonlinear design, and the studies are grouped into two categories – parametric and non-parametric estimations. In addition, the nonlinear models created with parametric estimation are separated into two types. One is Markov switching model, and the other is the threshold model that carries out the regime switching in accordance with certain threshold variable.

Hamilton (1989) first introduced the Markov switching model to examine the business cycle regime asymmetries, some other researchers modified or applied the Markov switching model to investigate various subjects[1]. Furthermore, Tong (1978) and Tong and Lim (1980) used some valuable and stationary variables as the threshold variable to develop the Threshold Autoregressive Model (TAR). The different regimes of model are therefore decided based on whether the value of threshold variable is greater, smaller or equal to a certain threshold value.

The threshold model not only has the capability to investigate the outcomes estimated under different regimes, but also to discover the economic senses contained in relation between threshold variable and threshold value. The threshold model provides a wide range for application; therefore, many researchers apply this model to analyze several subjects, such as Tsay (1989, 1998), Hansen (1996, 1999), Weise (1999), Chen et al. (2003), Huang and Yang (2004), and Huang et al. (2005). This study focuses on the threshold model because we can apply the framework of univariate and multivariate model to analyze different subjects such as finance, output, and impulse respond of prices. Also, the process of estimation to probe the best threshold variable is conducted by means of endogenesis rather than exogenesis, which is able to improve the model efficiency. These reasons help explain why the threshold model is selected as a framework for this research.

Whether the threshold model could actually emerge the core of subject does not depend on the preciseness of regimes created in the model. In addition, we are concerned with the abundance of economic senses contained in the threshold variable itself; so that, we are able to differentiate varied regimes using this variable. It is necessary to accurately select appropriate threshold variable for the model to improve the efficiency of the threshold model. Hence, choosing the business cycle as the variable in a threshold model is able to improve the analysis process. However, business cycle actually is an incorporeal economic concept; therefore, it requires another variable to represent the economic concept. Several studies apply the economic growth rate or stock return as the proxy variable for business cycle (Huang et al., 2005). Other researchers (Henry & Olekalns, 2002, and Henry et al., 2004) select the characteristics of "Current Depth of Recession" (CDR) to their empirical studies, which specially use CDR as a switching factor of the regime model to create the empirical models under different regimes of business cycle. Henry and Olekalns (2002) and Henry et al. (2004) confirmed that CDR itself has a good condition to be

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[1] For example, Filardo (1994), Hamilton and Lin (1996), Filardo and Gordon (1998) and Layton and Katsuura (2001) applied Markov switching model to investigate various subjects related to business cycle.

the proxy variable of "business cycle"; in addition, CDR has the characteristic to differentiate varied regimes of business cycle.

CDR being the indicator of the business cycle which is created by Beaudry and Koop (1993) was applied to determine the asymmetry of business cycle. It is sorted into two states CDR=0 and CDR>0. CDR=0 represents the expansive period of economy, while CDR>0 represents the recessional period of economy. The equation is indicated as follows:

$$CDR_{i,t} = \max\{Y_{i,t-s}\}_{s\geq 0}^{t} - Y_{i,t}, \quad (1)$$

where $Y_{i,t}$ indicates the output of period t; CDR is the gap between the historical maximum level of output (within period t to previous period s) and the level at period t.

How does CDR measure the expansive and recessional period of the economy? It is based on the trend of real output during expansions and recessions. The economic system is implied to step into a new recessive period as soon as there is a decrease in real output. Therefore, the operation of CDR based on above characteristics is able to identify the business cycle. Regarding the operation of CDR, the first step is to select the historical maximum level of real output, which is regarded as the determinant of business cycle regimes, then, is the difference between this historical maximum real output and the current real output. Therefore, CDR>0 indicates that the economic system breaks away the original trend of economic growth, implying the recession of business cycle. Consistently, CDR=0 represents the expansion of business cycle[1].

The main purpose of this study is to involve CDR into the threshold model so that CDR becomes the best proxy variable to predict business cycle; thus, it is necessary to modify CDR which we call modified CDR (MCDR). MCDR not only possesses the characteristic of the original CDR but also enlarges the value range of CDR in order to use in expansive period. In other words, MCDR possesses value of both recession and expansion at the same time; therefore, the value of the MCDR can be positive or negative, which enables the researchers to conduct linear test before constructing the threshold model. Through these steps, MCDR can be used as the variable in the threshold model, and the estimation becomes more conscientious.

Through the threshold model and MCDR, the threshold model provides the best efficiency of the estimation under different circumstances of the business cycle. As described above, we are able to recognize the strength of the MCDR. However, it is improper to address the adequacy of using MCDR before comparing the efficiency of the MCDR and CDR. Therefore, this study uses the real GDP from UK as the research sample to construct the TAR model with CDR threshold variable and MCDR threshold variable, so that we can recognize the difference between endogenous threshold variable and exogenous threshold variable. By using root mean square error (RMSE) and Theil's inequality coefficient (Thei-U), this study tries to compare the out-of-sample predictive efficiency between two models. Furthermore, the research applies DM test presented by Diebold and Mariano (1995) to compare the out-of-sample forecasting efficiencies between two models. The results indicate that using MCDR as the threshold variables provides better forecasting efficiency than using CDR, which implies that MCDR threshold variable is an appropriate proxy variable better than CDR.

This study consists of four sections, which are introduction, sampling and research method, data analysis, and finally conclusion.

1. Sampling and research methods

1.1. Sampling. The sample of the study was gathered from the first season in 1959 to the last season in 2006 from the UK, and research model consists of Gross Domestic Product (GDP, Code 99B), GDP deflator (Code 99Bir), and population (Code 99Z) three variables. The database is created by International Financial Statistics (IFS) of the International Monetary Foundation (IMF). The use of the data is to construct the real GDP per capita.

1.2. Research method. It is sensitive to use CDR to determine the recession, and the CDR possesses the characteristic to be the proxy variable for the business cycle. However, it might be too strict. Also, it is too subjective to differentiate the good or bad of the economy based on the "0" point, and which is given through exogenous model. Because the exogenous model is a fixed one, which cannot be adjusted to the change of environment, there might have model estimation bias if we apply original CDR as the threshold variable to the threshold model. Therefore, it is necessary to adjust the period for good or bad of economy.

The way to modify the CDR in this study is to make sure that modified CDR is able to show the value in both recessive period and expansive period. Since the range containing both positive and negative values is broader in MCDR, it is not only suitable for linear test, but also to verify the adaptability of using nonlinear model. Besides, through the process of estimating the threshold model, the optimal thresh-

[1] Based on the literature review, such as Pesaran and Potter (1997), Altissimo and Violant (2001) who also attempted to modify CDR and used the modified CDR as the threshold variable for the model. However, the method used in this study is different from those models.

old value is searched out from given endogenous model with MCDR. The limitation of threshold values from exogenous model is therefore improved. Consequently, MCDR not only maintains the original characteristic but also eliminates the imperfections of CDR.

In addition, in order to distinguish from the New CDR developed by Bradley and Jansen (1997)[1], in which CDR1 represents decline within recessive period, and CDR2 represents recovery within recessive period, we name our unstandarized modified CDR as CDR3, and the equation is stated as follows:

$$CDR3_{i,t} = \max\{Y_{i,t-s}\}_{s>0}^{t} - Y_{i,t}, \quad (2)$$

where $Y_{i,t}$ indicates the output of period t; *CDR3* represents the difference between the historical maximum level of output (within period t to previous period s) and the level at period t[2]; *CDR3* not only retains the original *CDR* but also quantifies the expansive state that has formerly been zero.

CDR3 and CDR exhibit the same framework; however, the only difference between CDR3 and CDR is whether the s-value is equal or larger than zero. In other words, CDR3 is no other than the CDR enlarged bilaterally. CDR3 not only maintains the original characteristic of CDR, but also allows to display the value for expansion which is not allowed in CDR. In order to make sure both positive value and negative value are consistent with the viewpoint of the business cycle so that we can compare the difference between CDR and CDR3 conveniently, this study tries to standardize the CDR3, that is, CDR3 values are normalized by its standard deviation. CDR3, after being normalized, is named Modified CDR (MCDR), which is exhibited as follows.

$$MCDR_{i,t} = \frac{CDR3_{i,t}}{STD_{i,CDR3}} = \frac{CDR3_{i,t}}{\sqrt{\sum (CDR3_{i,t} - \mu_{i,CDR3})^2 / N_i - 1}}, \quad (3)$$

where STD_{CDR3} indicates the standard deviation of *CDR3*; μ_{CDR3} is the mean of *CDR3*; N_i is the sample size of country i.

Based on Figure 1, the values of MCDR are distributed positively and negatively within a certain interval and these values exhibit the different stages of the business cycle. For instance, MCDR=+1~+3 (times of the standard deviation) implies that this economic system is under recession, and the bigger the value is, the stronger the recession appears to be. Contrarily, MCDR=-1~-3 (times of the standard deviation) infers an expansion, and the bigger negative values imply the stronger expansion. Thus, the regimes of business cycle are easily defined by MCDR values.

For further testing the difference among original CDR, CDR3 and MCDR, the results are shown with three indicators in Figure 1. The results indicate that there are recessive values and expansive values within CDR3 and MCDR. The process of approaching MCDR is similar to that of CDR3, and the only difference is the measurement. Through the standardized MCDR, it can be applied for different countries in the future to determine the different segment values of the business cycle for those countries.

1.3. TAR model. Another focus of this study is integrating CDR into the threshold model. If we are able to get the optimal threshold value through the endogenous process for the threshold model, and to recognize the different segment of the business regime by threshold value, the MCDR therefore can achieve the anticipant improvement.

Tong (1978) and Tong and Lim (1980) developed the TAR model, which uses the "variable" as the breakpoint of the model. The different regimes of the models are determined according to whether the threshold variable is greater, smaller, or equal to the specific threshold value. For instance, two regimes of TAR model with uni-variable under lag p period can be shown as follows:

$$W_t = \varphi_{0,1} + \varphi_{1,1} W_{t-1} + ... + \varphi_{p,1} W_{t-p} + \varepsilon_t \qquad Z_{t-d} > \gamma \quad (4)$$

$$= \varphi_{0,2} + \varphi_{1,2} W_{t-1} + ... + \varphi_{p,2} W_{t-p} + \varepsilon_t \qquad Z_{t-d} \le \gamma, \quad (5)$$

where p represents lag period number; Z_{t-d} represents threshold variable; d represents delay period number; γ represents threshold value; $\varepsilon_t \sim iid$ is the error; and $E(\varepsilon_t | \Omega_{t-1}) = 0$, $E(\varepsilon_t^2 | \Omega_{t-1}) = \sigma^2$, while Ω_{t-1} is referred to as the assemblage of data for the last period.

The model shown above implies that equation (4) is established when the threshold variable is larger

[1] Bradley and Jansen (1997) address that the recessive period of CDR represents a contaminated recessive period but not pure recessive period. For resolving this limitation, they tried to modify CDR and develop another new CDR (NCDR) to improve the limitation of CDR for analyzing subjects related to pure and contaminated recessive periods. NCDR applied the part of CDR>0 to differentiate the business cycle to expansive and recessive periods based on the positive or negative of economic growth.

[2] For differentiating past GDP and current GDP, this study assumed s=1, which not only keeps the value in recessive period unchanged, but also reconstructs the value in expansive period and accomplishes the objective for modifying CDR.

than threshold value, while equation (5) is established when the threshold variable is smaller than or equal to threshold value. Under the assumption regarding the normal distribution of ε_t, two regimes of TAR model can be reset as follows:

$$W_t = (\varphi_{0,1} + \varphi_{1,1}W_{t-1} + ... + \varphi_{p,1}W_{t-p})I(Z_{t-d} > \gamma) + (\varphi_{0,2} + \varphi_{1,2}W_{t-1} + ... + \varphi_{p,2}W_{t-p})I(Z_{t-d} \le \gamma) + \varepsilon_t, \quad (6)$$

where *I( )* is the index function for regime; *I*( ) = 1 implies building up a regime; whereas *I*( ) = 0 implies that the regime was not formed.

Furthermore, equation (6) can be expressed as follows:

$$W_t = \varphi_1' x_t I(Z_{t-d} > \gamma) + \varphi_2' x_t I(Z_{t-d} \le \gamma) + \varepsilon_t, \quad (7)$$

where $\phi_j = (\phi_{0,j}, \phi_{1,j}, ..., \phi_{p,j})'$, j = 1,2 and $x_t = (1, W_{t-1}, ..., W_{t-p})'$.

When the threshold value γ is fixed, we can estimate $\phi = (\phi_1', \phi_2')'$ by the Least Squared Estimation.

TAR can estimate the threshold value through the grid search, the concept regarding grid search is to probe potential changing point of the structure through searching the minimal value of the SSE (Sum Square of Error), while

$$\hat{\phi}(\gamma) = (\sum_{t=1}^{n} x_t(\gamma) x_t(\gamma)')^{-1} (\sum_{t=1}^{n} x_t(\gamma) W_t), \quad (8)$$

where $x_t(\gamma) = (x_t' I(W_{t-1} > \gamma), x_t' I(W_{t-1} \le \gamma))'$; $\hat{\varepsilon}_t(\gamma) = W_t - \hat{\phi}(\gamma)' x_t(\gamma)$ is the error and $\hat{\sigma}^2 = \sum_{t=1}^{n} \hat{\varepsilon}_t(\gamma)^2 / n$ is the variance. Therefore, we can get the threshold value through searching the minimal variance, which is $\hat{\gamma} = \arg\min \hat{\sigma}^2(\gamma)$.

1.4. The estimation of model efficiency. According to the assumption as above, we recognize that the MCDR threshold model with endogenous threshold values should be better than model with exogenous threshold values concerning the estimation efficiency and the appropriateness of using non-linear model. However, for recognizing the significant difference between CDR and MCDR, this study will verify the significance through the TAR model and the process of out-of-sample forecasting.

$$\Delta Y_t = \varphi_1' x_t I(CDR_{t-d} > 0) + \varphi_2' x_t I(CDR_{t-d} \le 0) + \varepsilon_{CDR,t}, \quad (9)$$

$$\Delta Y_t = \varphi_1' x_t I(MCDR_{t-d} > \gamma) + \varphi_2' x_t I(MCDR_{t-d} \le \gamma) + \varepsilon_{mCDR,t}, \quad (10)$$

where ΔY represents the growth rate for real GDP per capita; $\phi_j = (\phi_{0,j}, \phi_{1,j}, ..., \phi_{p,j})'$ is the estimated parameter; j = 1,2; $x_t = (1, \Delta Y_{t-1}, ..., \Delta Y_{t-p})'$; $I(\cdot)$ is the indicator function of the regime; $I(\cdot)$ = 1 implies that the regime is formed; MCDR, CDR are the threshold variables; γ is the threshold value from endogenous approach; d is delay period number of threshold variable.

Equation (9) is the TAR model by using CDR as the threshold variable, while equation (10) is the TAR model by using MCDR as the threshold variable. In this study, we use RMSE and Theil-U two indicators to conduct the out-sample-forecasting, and the indicator was set as follows:

$$RMSE = \sqrt{\frac{1}{K}\sum_{i=1}^{K}(Y_{t+i}^f - Y_{t+i}^a)^2}. \quad (11)$$

In the equation (11) Y^f_{t+i} and Y^α_{t+i} indicate the forecasting value and real value of the period $t + i$. K is the sample size of the forecasting period. Through the mean square error of the model, we can determine the forecasting effect for the model. The smaller the *RMSE*, the better the forecasting effect of the model will be.

We can define the statistics Theil-U as follows:

$$U = \sqrt{\frac{\sum (Y_{t+i}^f - Y_{t+i}^a)^2 / T}{\sum (Y_{t+i}^a)^2 / T}}, \quad (12)$$

where Y^f_{t+i} and Y^α_{t+i} indicate the forecasting value and real value of the period $t + i$. T is the sample size of the forecasting period. The value exists between 0 and ∞ ($0 \le U \le \infty$). When U is closed to 0, which implies that the forecasting value is closed to the real value, so, we can assure that the forecasting value is equal to real value when $U = 0$.

Two measured indicators shown above apply the absolute value to examine the model efficiency without any statistical theories or structures; however, it is impossible to recognize the significant differences among values. Furthermore, this study applies the third measured indicator – DM test developed by Diebold and Mariano (1995). DM statistics is used to test significance of the forecasting efficiency for the models.

$$DM = \frac{\bar{l}}{\sqrt{2\pi \hat{f}(0)/T}}, \quad (13)$$

where l indicates the mean of a general loss function; T is the number of the observed value; $f(0)$

implies the consistent estimate of the spectral density when the frequency of a general loss function equals zero. According to Diebold and Mariano (1995), *DM* is a normal distribution statistics.

The process to verify the efficiency of out-sample-forecasting in this study is similar to that in the study by Claveria et al. (2007)[1]. However, the TAR model in this study belongs to non-linear structure, which is different from the study by Diebold and Mariano (1995). We cannot make sure that DM statistics possesses the characteristics of the normal distribution; therefore, by following Chung (2006), this study applies bootstrap method to get the threshold limit value of the statistics, and to investigate the capability of the out-of-sample forecast for the model. This process is different from that by Claverial et al. (2007). Also, during the process of DM test, we apply the "loss function" to conduct further DM test, and the "loss function" implies the difference of the forecasting deviation square between CDR model and MCDR threshold model. That is, $f_i = fe^2_{CDR,\,t+i} - fe^2_{MCDR,\,t+i}$, where *i* implies period number of the out-of-sample forecast. The null hypothesis for the DM model is that there is no significantly efficient difference between two models, whereas the alternative hypothesis becomes that the forecasting efficiency for one model is better than that for the other models.

2. Empirical results and analysis

This study first follows equations (1) and (3) to get the CDR series and MCDR series from the UK. In order to avoid the estimation false for the regression[2], we should make sure the variables of the model possess the characteristic of stationarity; thus, we conduct unit root test for real GDP per capita. The ADF (augmented Dickey-Fuller) unit root test developed by Said and Dickey (1984) assumed that errors acquire homogeneity and white noise, while PP (Phillips and Perron) unit root test allows that errors possess heteroskedasticity and weakly dependence. This study applies PP unit root test for each variable because the results of the PP test are more consistent and stationary.

The results shown in Table 1 indicate the real GDP per capita unit root test in the UK, we use constant model and constant plus time trend variable model, and the results indicate that the one-stage error difference of the real GDP per capita in the UK conforms to the requisition of stationarity. Besides, the results of series unit root-test for both CDR and MCDR possess the characteristic of stationarity as well, and meet the criterion for the threshold variable. For comparing and forecasting the models, the samples are chosen for the period from the first season in 1959 to the fourth season in 2001 as well as from the first season in 2002 to the fourth season in 2006; totally it results in 20 periods to conduct out-of-sample forecast for model.

During the empirical process, we use CDR and MCDR as the threshold variables to construct the TAR model with real GDP per capita growth rate furthermore to compare the out-of-sample forecasting power difference between two models. The threshold value for the TAR model with CDR threshold variable investigate the good or bad economy based on whether the value is "0"; therefore, it is not necessary to conduct linear test before estimating the threshold model. However, the optimal threshold value for a TAR model with MCDR threshold variables is solved through endogenous method, we should firstly conduct a linear test to confirm the non-linear characteristics of the model. When deciding the lag-p period of the model, we apply AIC (Akaike's Information Criterion) rule to select the optimal number of lag-p, and the results show that lag-p 4 is the best one for the threshold model.

The linear test results shown as Table 2 indicate that we should reject the null hypothesis regarding MCDR is linear model in the UK. Besides, when deciding the delay period for threshold variable of the TAR model, we use linear-test based on the maximum value in F-statistics.

The TAR model estimation results shown as Table 3 indicate that the TAR mode with MCDR threshold variable is more appropriate than that with CDR, no matter measured with adjusted-R-square value, sum of error square, or AIC. It is no surprise to get these results because MCDR possesses whole range of values compared to CDR, which only includes positive value, plus the threshold value endogenously developed by model, the model efficiency in TAR with MCDR threshold variable should be better than that with CDR. In addition, Figure 2 depicts that the regime of the business cycle is differentiated by endogenous threshold value. The above part of the dotted line represents the regime with economic recession, and the under part of the dotted line represents the regime with economic expansion, where the periods of regime with economic recession are less than those with economic expansion, and which is significantly different from applying exogenous (CDR=0) model.

[1] When Claveria et al. (2007) first applied RMSE to compare the out-sample-forecasting effects among several models, next, they applied the method developed by Diebold and Mariano (1995) to investigate the significance regarding forecasting effect between two models. However, Claveria et al. (2007) did not apply bootstrap method to get the critical value of DM statistics.

[2] It might create spurious regress issue if we use non-stationary data; for detail please see Grange and Newbold (1974).

However, we cannot prove which model is the best only based on the results of the model estimation. In order to compare two threshold models, we take the observed values from the last 5 years (2002 to 2006) and conduct the out-of-sample forecast for the models. The out-of-sample forecasting test consists of five forecasting periods in order to verify the forecasting power for the whole model, and which are out-forecast periods 1, 2, 3, 4, and 5; and we use rolling forecasting to get the out-of-sample forecasting value. The sample periods are from the first season in 2002 to the fourth season in 2006. The window of TAR keeps unchanged during the process of out-of-sample forecast.

The out-of-sample forecast comparing results shown as Table 4 indicate that there is a significant difference between CDR and MCDR regarding forecasting efficiency no matter through RMSE test or Theil-U test. However, TAR model with MCDR threshold variable has significantly greater forecasting efficiency than TAR model with CDR threshold variable in the result of out-of-sample forecast periods 1-5. The efficiencies are displayed on the conscientious use of statistical process and econometric model, for instance, the process of constructing non-linear TAR model. The efficiencies are also exhibited on the capability of out-of-sample forecast. The results in this study show, TAR model using MCDR as the threshold variable can significantly improve the forecasting efficiency.

However, can we approve above concerns just based on RMSE and Theil-U shown in Table 4? For conscientious concerns of the statistics, we cannot do it because the standard for judging the good or bad situation is only based on the value. Therefore it is necessary to examine the difference between the values, and to determine the criterion of the value to solve the statistical significance.

Therefore, we further use DM statistics to test the significant difference between two models regarding the forecasting power. Based on Diebold and Mariano, DM test is standard normal distribution only under linear model. However, the model in this study is non-linear model; thereby, we follow Chung (2006) and use bootstrap method to get the threshold limit value, and this value becomes the criterion to make decision.

Table 5 shows the results of DM test, from which we get five threshold limit values, which are 97.5%, 95.0%, 90.0%, 10.0%, 5.0%, and 2.5%. Under α=10%, when DM value is greater (or equal to) than the threshold limit value under 90%, MCDR model has significantly better forecasting power than CDR model. Whereas, when DM value is smaller (or equal to) than the threshold limit value under 10%, this implies that CDR model has significantly better forecasting power than MCDR model. The results indicate that four DM values with out-sample-forecast, f1, f2, f3, f4, are positive ones, and all of them exist inside of the threshold limit value under 90%. These results imply no significant difference between two models regarding forecasting power. With regard to DM with out sample-forecast f5, the value not only is positive, but also exists outside of the threshold limit value under 90%. This result implies that MCDR model has significantly better forecasting power than CDR model. Also, all DM values with period 1-5 are all not significant in terms of negative efficiency. Above all, regarding the forecasting power, TAR model with MCDR threshold variable is significantly better than TAR model with CDR threshold variable.

Conclusion

The main purpose of this study is to compare the difference between MCDR and CDR regarding the out-sample-forecasting power with the use the threshold variable. We take real GDP per capita in the UK as our research sample, and use CDR and MCDR as the threshold variables to construct two TAR models, to conduct model estimation and out-of-sample forecasting efficiency comparison between two models. The results of this empirical study conclude: Modified CDR (MCDR) has better potential to be the threshold variable for the TAR model compared to CDR.

This study only applies uni-variate threshold TAR model so that it is simple to analyze and the results are easier to be understood. However, the application to integrate the MCDR as the threshold variable into the model is not limited to TAR model or bio-regimes threshold models. We can use the MCDR as the threshold variable if only the threshold model meets the requirement regarding model construction and empirical process, and the model should take business cycle as the threshold variables for the model

The contributions of this study are as follows (a) regarding empirical process, this study verifies that MCDR and CDR are all appropriate variables to represent the business cycle, and MCDR has greater forecasting efficiency than CDR; (b) concerning research method, we integrate MCDR into threshold model to enlarge the application of original CDR, to enforce the appropriate power of using non-linear threshold model to estimate. Whenever conducting the study related to business cycle, we can get MCDR through the process of real GDP, because MCDR is an appropriate variable to represent business cycle.

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Appendix

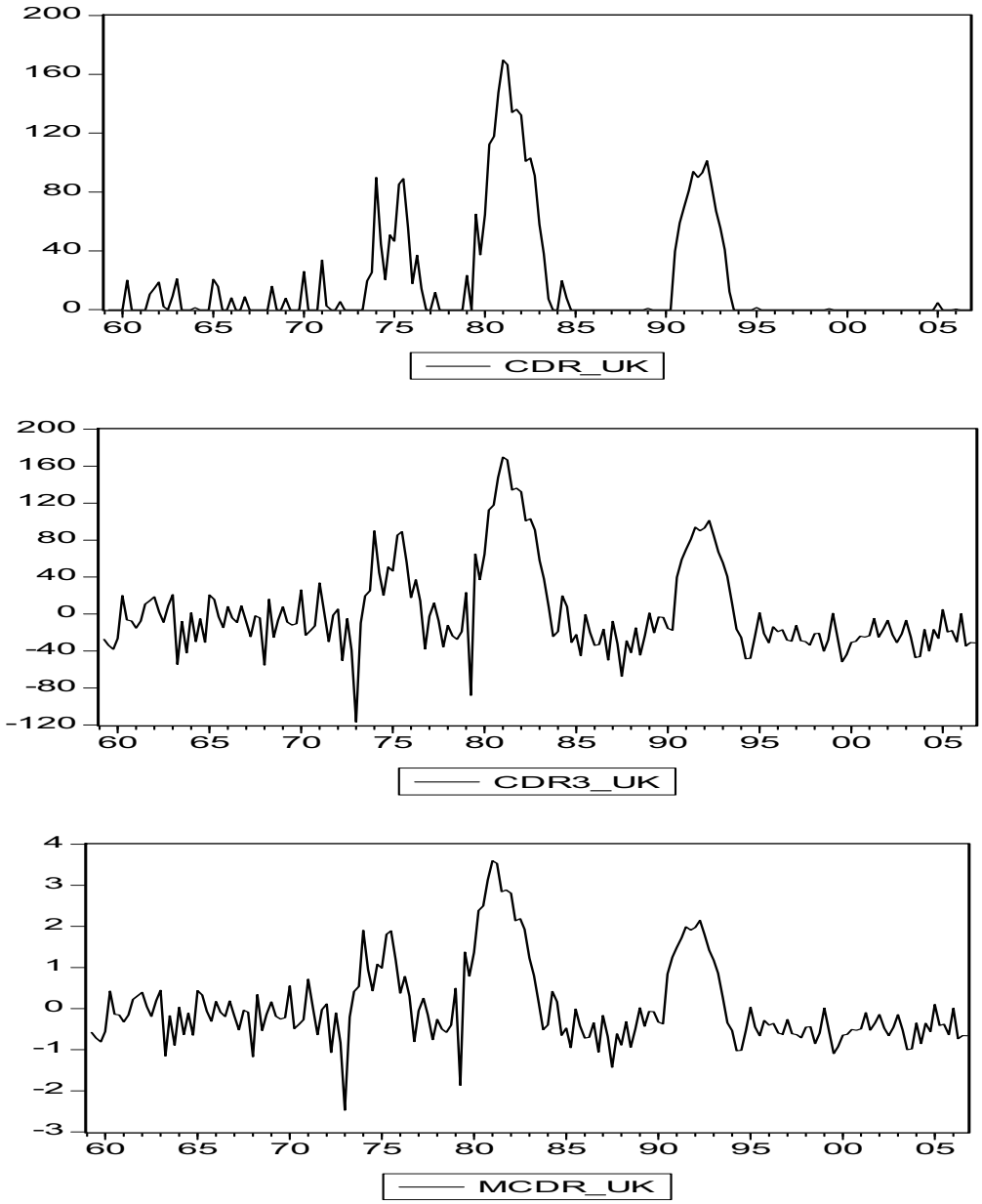

Fig. 1. Time series for CDR, CDR3, & MCDR in the UK

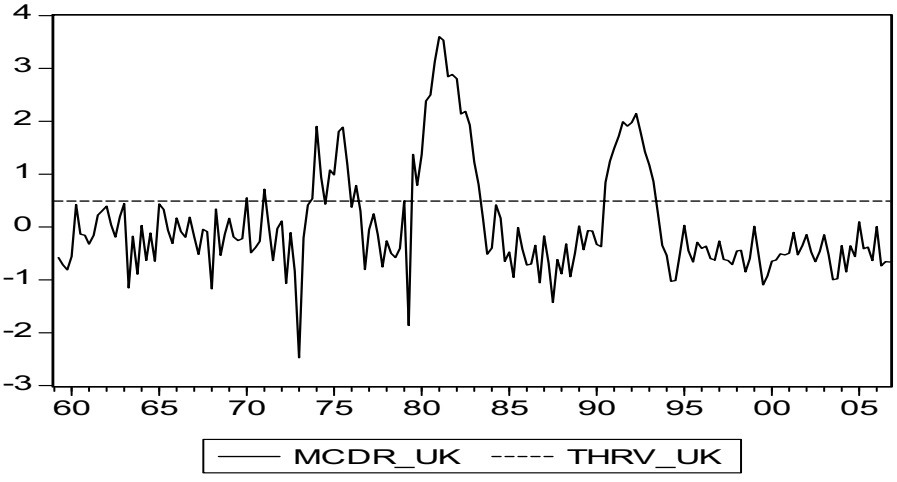

Fig. 2. MCDR and time series for threshold value in the UK

Table 1. Results for unit roots test

	Level		First difference	
Variable	Constant term	Constant & time trend	Constant term	Constant & time trend
Real GDP per capita	1.79	-0.63	-14.40**	-14.37*
CDR	-3.37**	-3.36		
MCDR	-4.71**	-4.74**		

Notes: This study applies seasonal data; therefore, we can get higher correlation. The study applies PP unit roots test because it allows that errors display heteroskedasticity and weakly dependence and the results are more stationary and consistent. The null hypothesis for PP (Phillips-Perron) test is that unit roots exist, ** and * denote the 5% and 10% significance level.

Table 2. Results for linear test

		D = 1	D = 2	D = 3	D = 4
UK AR(4)	F-test	**4.61***	2.90	3.28	3.40
	TRV	**0.4914***	0.3938	0.5463	-0.494
	P_value	**(0.01)***	(0.16)	(0.11)	(0.08)

Notes: AR(N) indicates the (N) lag period number for the model, and the longest period is 4. Basically, the study applies AIC as the criterion to decide the lag period, and D implies the delay period for the threshold variable. F-test is to test F distribution statistics. TRV indicates the optimal threshold value for TAR, "()" is the P-value for F statistics. * implies that the model setting is based on the biggest F statistics.

Table 3. Results for TAR model

Threshold variables	*CDR*	*MCDR*
Regime 1	$CDR_{t-1} > 0$	$MCDR_{t-1} > 0.4914$
Constant	**0.45 (0.00)***	**0.38 (0.02)***
ΔY_{t-1}	0.00 (0.08)	0.05 (0.78)
ΔY_{t-2}	0.16 (0.17)	**0.34 (0.02)***
ΔY_{t-3}	0.19 (0.06)	0.10 (0.48)
ΔY_{t-4}	-0.04 (0.72)	**-0.22 (0.05)***
Regime 2	$CDR_{t-1} \le 0$	$MCDR_{t-1} \le 0.4914$
Constant	**0.61 (0.02)***	**0.68 (0.00)***
ΔY_{t-1}	-0.21 (0.13)	**-0.23 (0.02)***
ΔY_{t-2}	-0.04 (0.75)	-0.12 (0.24)
ΔY_{t-3}	0.04 (0.77)	0.03 (0.74)
ΔY_{t-4}	0.15 (0.29)	**0.20 (0.05)***
Adjusted R^2	0.00	0.07
Sums of square error	165.90	153.86
Likelihood Ratio	-236.41	-230.12
AIC value	2.95	2.88
Q(4)	0.33 (0.99)	0.48 (0.98)
Q(8)	10.17 (0.25)	12.86 (0.12)
Q(12)	13.38 (0.34)	14367 (0.26)
ARCH(4)	10.51 (0.03)	4.98 (0.29)

Table 3 (cont.). Results for TAR model

Threshold variables	*CDR*	*MCDR*
ARCH(8)	10.57 (0.23)	6.38 (0.60)
ARCH(12)	13.36 (0.34)	8.68 (0.73)

Notes: $\triangle$ is the notation of the first difference, *Y* implies Real Personal GDP, Q(K) indicates the correlation *Q* test for errors developed by Ljung and Box (1979), *K* implies the lag period number, ( ) is the P-value of the coefficient, ARCH(K) is the statistics to test the heterogeneity of the error variance.

Table 4. Results for out-sample-forecast effect comparing

	RMSE statistic value		Theil-U statistic value	
Threshold variable	CDR	MCDR	CDR	MCDR
Out-of-sample forecast period numbers				
1	0.276	0.273*	0.404	0.401*
2	0.263	0.257*	0.382	0.372*
3	0.268	0.262*	0.385	0.377*
4	0.263	0.254*	0.379	0.366*
5	0.256	0.237*	0.364	0.336*

Notes: * implies that the value of the indicator is smaller.

Table 5. Results for DM test

	f_i				
Critical values	f_1	f_2	f_3	f_4	f_5
97.5%	0.015	0.013	0.008	0.010	0.013
95%	0.013	0.009	0.007	0.008	0.010
90.0%	0.010	0.007	0.006	0.006	0.007
DM statistics value	0.001	0.004	0.003	0.005	0.009*
10.0%	-0.011	-0.007	-0.006	-0.005	-0.008
5.0%	-0.014	-0.009	-0.008	-0.007	-0.010
2.5%	-0.017	-0.011	-0.009	-0.008	-0.013

Notes: The results show the critical values for DM statistics under critical values for 97.5%, 95.0%, 90.0%, 10.0%, 5.0%, and 2.5%; all critical values are selected through bootstrap method, and the loss function $f_i = fe^2_{CDR,\,t+i} - fe^2_{MCDR,\,t+i}$, f_i : i=1, 2, 3, 4, and 5 indicates the DM results for out-of-sample forecast from 1 to 5 period. The null hypothesis for the DM test is that the mean of the function f_i is 0, which implies that no significant difference exists between two models regarding their out-of-sample forecast effect. We should reject the null hypothesis when the forecasting effect is better in one model than that in another model. Also, *** indicates that DM value is larger than (or equal to) the critical value of 97.5%, ** implies that DM value is between the critical value of 97.5% and 95%, and * implies that DM value is between the critical value of 95.0% and 90.0%.