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ARTICLE INFO	Robert Verner and Gabriel Herbrik (2017). Bond yields and stock returns comparison using wavelet semblance analysis. *Investment Management and Financial Innovations*, *14*(2-1), 281-289. doi:10.21511/imfi.14(2-1).2017.12
DOI	http://dx.doi.org/10.21511/imfi.14(2-1).2017.12
RELEASED ON	Thursday, 27 July 2017
RECEIVED ON	Wednesday, 11 January 2017
ACCEPTED ON	Friday, 02 June 2017
LICENSE	This work is licensed under a Creative Commons Attribution-NonCommercial 4.0 International License
JOURNAL	"Investment Management and Financial Innovations"
ISSN PRINT	1810-4967
ISSN ONLINE	1812-9358
PUBLISHER	LLC “Consulting Publishing Company “Business Perspectives”
FOUNDER	LLC “Consulting Publishing Company “Business Perspectives”

NUMBER OF REFERENCES	NUMBER OF FIGURES	NUMBER OF TABLES
28	4	0

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BUSINESS PERSPECTIVES

LLC "CPC "Business Perspectives"
Hryhorii Skovoroda lane, 10, Sumy, 40022, Ukraine

www.businessperspectives.org

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BOND YIELDS AND STOCK RETURNS COMPARISON USING WAVELET SEMBLANCE ANALYSIS

Abstract

Various measures of resemblance are increasingly applied in confrontation of data samples obtained by different sources. Semblance analysis aims at comparison of two sets of data based on their phase and frequency. Conventional semblance analysis following the Fourier transform has several deficiencies resulting from the transform. To overcome these obstacles, another type of semblance analysis was developed applying the wavelet transform. This paper focuses on semblance analysis of stock prices and government bond yields of two major global economies using continuous wavelet transform regarding both scale and time.

Keywords bond yields, wavelets, stock indices, time-series

JEL Classification C22, G17

Received on: 11th of January, 2017
Accepted on: 2th of June, 2017

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INTRODUCTION AND LITERATURE REVIEW

Financial time series have been the subject of academic research for several decades. Because of their specific nature, most of financial data is nonstationary, which means that the statistical properties of the data change over time. Its probabilistic distribution can vary rapidly, and during a short period investors might gain large profits, as well as significant losses. Aiming at government bond yields, Moessner (2015) quantified the impact of policy rate guidance of Federal Open Market Committee at zero on US treasury yields with maturities up to 10 years. His outcomes indicated that open-ended and time-contingent forward guidance announcements caused a reduction in forward yields at a wide range of horizons. On the other hand, Poghosyan (2014) analyzed determinants of government bond yields in 22 European economies between 1980 and 2010 using panel cointegration techniques. His estimations suggested that while in the aftermath of the crisis spreads in some peripheral countries exceeded the level indicated by fundamentals, North European countries have benefited from their safe-haven character which is in line with results of Gonzalez-Rosada and Yeyati (2008). Confirming Diebold and Li (2006), Hautsch and Ou (2012) proposed a Nelson-Siegel interest rate term structure model with the crucial yield variables following autoregressive processes with stochastic volatility. They estimated the model on US treasury yields applying Markov chain Monte Carlo methods and argued that the factor volatilities followed highly persistent processes. Moreover, they showed that yield factors and factor volatilities were closely related to macroeconomic state factors.

Following Lee et al. (2013) and Yang and Shigeyuki (2015), Jammazi et al. (2015) explored the dependence relationship between long-term government bonds and stock returns for a range of countries over the last twenty years using a dynamic DCC-GARCH-copula model which allows obtaining a flexible and comprehensive description of the time variation of various patterns. Their results indicated positive stock-bond association during the 1990s, and negative from the early 2000s. Additionally, authors found no evidence of asymmetric and tail dependence for the majority of countries. Antonakakis and Vergos (2013) examined government bond yield spread spillovers among European countries during the global financial as well as the Euro zone debt crisis. They applied the VAR-based spillover index based on Diebold and Yilmaz (2012) and impulse response analyses. Their outcomes suggested that on average, bond yield spreads shocks tend to increase future bond yield spreads, and that they are connected to policy changes and news announcements. Moreover, bond yield spreads shocks from the peripheral countries have significantly more destabilizing force on other countries than shocks coming from the core countries. Asymmetry in the distribution of government bond returns in developed countries was studied by Fujiwara et al. (2013). Authors provided evidence for asymmetry in government bond returns with short maturities and found that the probability of a large and negative excess return is more likely in a less liquid market.

While Koeda (2013), Chionis et al. (2014), Afonso and Nunes (2015) and Kiley (2016) also analyzed dependence of government bond yields on fundamental factors, Kim and In (2007) focused on the relationship between changes in bond yields and stock prices in the G7 economies. They stated that given relationship might be positive, as well as negative. To properly explore the relationship, they proposed wavelet correlation analysis and their results showed that the correlation between changes in bond yields and stock prices can vary in different countries and time scales.

Following previous research, this paper focuses on semblance analysis of government bond yields and stock prices in US and Germany using continuous wavelet transform.

1. METHODOLOGY

There exist various methods of how to compare two time series, such as cross-correlation, however standard techniques can often result into outcomes that are difficult to exploit or do not give sufficient perspective about the position of the datasets in time. Reboredo et al. (2017) therefore explored causality and co-movement between oil and renewable energy stock prices using continuous and discrete wavelets, Akoum et al. (2012) examined the short and long term dependencies between stock market returns and OPEC basket oil utilizing the wavelet coherency methodology, and Samadder (2015) investigated the periodicity of the two prime Indian and American stock market indices applying Ferraz-Mello method of date-compensated discrete Fourier transform.

Semblance analysis has been commonly utilized in seismology, however, its construction seems appropriate for financial time series as well (Cooper & Cowan, 2008). It confronts two samples regarding the correlations between their phase angles, as a function of frequency. Blackedge (2006) and Ahmed and Rao (2012) describe the Fourier transform $H(f)$ of a sample $h(t)$ as:

$$H(f)=\int_{-\infty}^{\infty} h(t)e^{-2\pi ift}dt, \tag{1}$$

where t represents the time and f denotes the frequency. Generally, H is complex, therefore it has both a phase and amplitude at each frequency. After the Fourier transforms, the difference in their phase angles at each frequency can be calculated as (Christensen, 2003; Cooper & Cowan, 2008):

$$S=\cos\theta=$$
$$=\frac{R_1(f)R_2(f)+I_1(f)I_2(f)}{\sqrt{R_1^2(f)+I_1^2(f)}+\sqrt{R_2^2(f)+I_2^2(f)}}, \tag{2}$$

where $R_1(f)$ and $I_1(f)$ are the real and imagi-

nary components of the Fourier transform of the first sample, while $R_2(f)$ and $I_2(f)$ are the components of the second sample, expressed as a function of frequency f. S can have values between –1 and 1, where 1 means absolute phase correlation, –1 implies perfect negative correlation and 0 represents no correlation. Semblance procedure divides each input sample into two output samples comprising of the set of the input sample that is correlated, and the set that is not, which is done in the frequency domain. After the selection of the threshold correlation, the Fourier transform of each sample is divided into two groups. First one consists of the Fourier coefficients with a semblance above the threshold and the second group consists of the remainder. In each case are the missing coefficients substituted by zeros. Consequently, the inverse Fourier transform is performed to each group, so that there are two output samples for each input sample.

In contrast to the Fourier transform, the wavelet-based techniques provide the possibility to cover temporal variability in spectral character. Even though the wavelets are relatively young method, their applications have expanded to many different areas. In the field of finance, Liu et al. (2017) investigated the evolution of mean and volatility spillovers between oil and stock markets in the time and frequency dimensions using a wavelet-based GARCH-BEKK method and showed that spillover effects vary across wavelet scales in terms of strength and direction. Similar studies focusing on financial issues were also presented by Khalfaoui et al. (2015), Ferrer et al. (2016) and Shahzad et al. (2016). Detailed description of wavelet theory and applications can be found in Brémaud (2013), Debnath (2012) and Teolis (2012). Mallat (1999) defines the continuous wavelet transform of a sample $h(t)$ as:

$$CWT(u,s)=\int_{-\infty}^{\infty} h(t)\frac{1}{|s|^{0,5}}\psi^*\left(\frac{t-u}{s}\right)dt, \qquad (3)$$

where t is the time, u describes the displacement, s represents the scale, ψ enotes the mother wavelet, and * stands for complex conjugate. Same as in (Cooper & Cowan, 2008), the complex Morlet wavelet was applied in this work, which Teolis (2012) defines as:

$$\psi(x)=\frac{1}{\pi f_b}e^{2\pi i f_c x}e^{\frac{-x^2}{f_b}}, \qquad (4)$$

where f_b determines the bandwidth and f_c the wavelet centre frequency. Contrary to conventional semblance analysis based on Fourier transform, the wavelet transform does not assume that the frequency content of a sample is constant. Two time series might be compared applying wavelets with cross-wavelet transform described in Torrence and Compo (1998) or Grinsted et al. (2004).

2. RESULTS

As it has already been stated, continuous wavelet transform provides better temporal resolution than semblance models based on Fourier transform. To explore the abilities of continuous wavelet transform based semblance analysis in financial time series research, we examined two major global economies, i.e., US and Germany. In order to measure the semblance between German stocks and government bond yields we focused on daily returns of stock index DAX, as well as daily 10 year Bund yield returns. Both time series are benchmark values defining the Euro zone performance. Since stock and bonds are usually considered as investment substitutes, we might expect positive relationship between stock indices returns and changes in government bond yields as it is explained below.

Our sample consisted of 2238 daily observations between 28th February 2008 and 16th November 2016. On the other hand, US sample was represented by 10 year Treasury yield returns and Standard & Poor's 500 stock index comprising of 2228 observations also between 28th February 2008 and 16th November 2016. Daily return $r(t)$ in time t was calculated as

$$r(t)=\frac{p_t-p_{t-1}}{p_{t-1}},$$

where p_t defines the value of given financial instrument in time t. Different size of samples can be contributed to variant public and national holidays in both countries. It is worth to notice that $r(t)>0$ in case of government bond yields de-

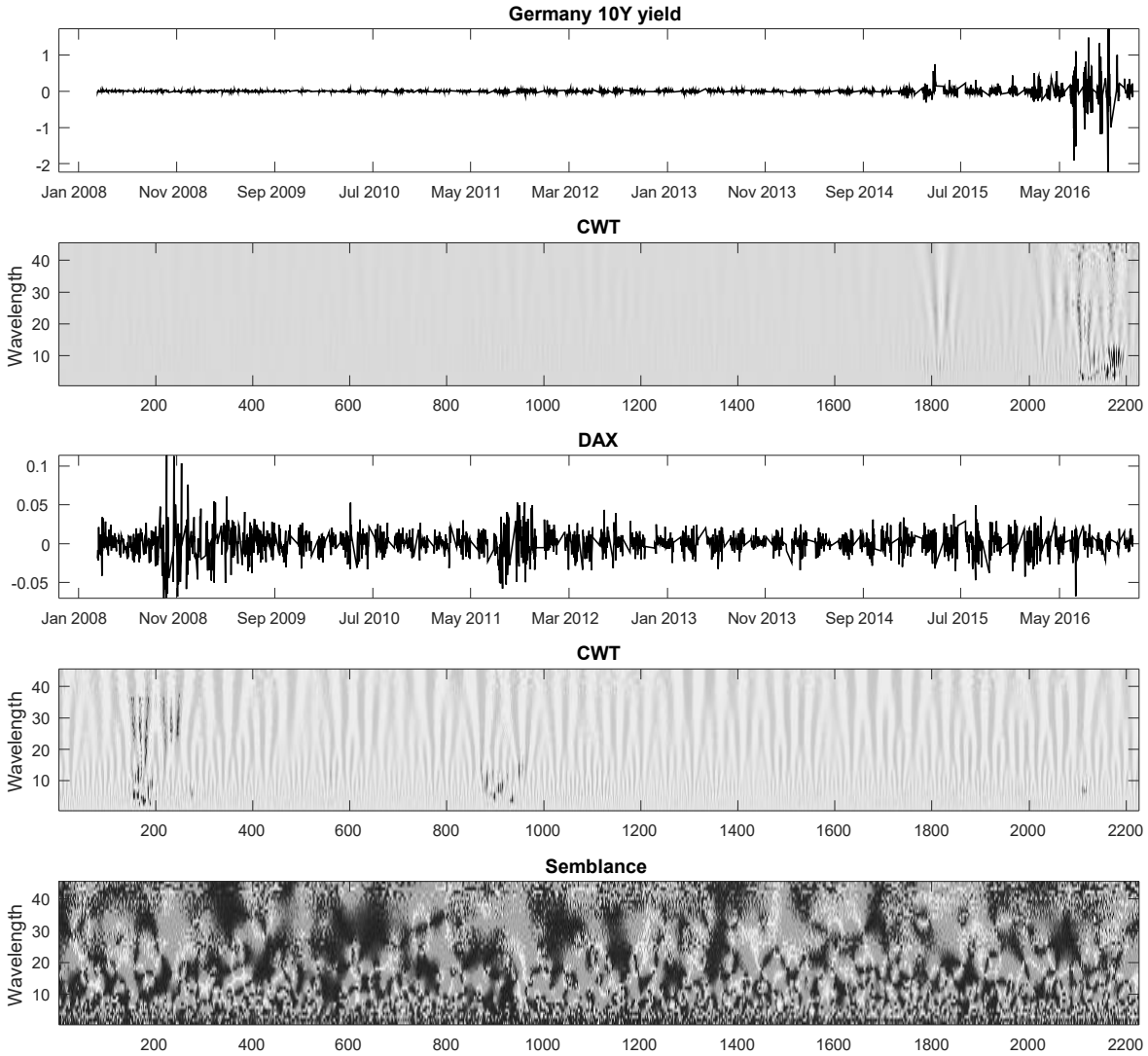

Figure 1. German 10 year bond yield and DAX analysis

scribes decline in bond prices, therefore positive semblance between stock returns and bond yields means negative relationship regarding their market price movements.

Figure 1 investigates the sample of German 10 year government bond yields and DAX. First part of Figure 1 shows daily change of 10 year German government bond yield in period between 28th February 2008 and 16th November 2016. Second chart depicts the real part of complex continuous wavelet transform given bond yield sample. Dark grey color in 2016 indicates a positive amplitude and black at the beginning of 2015 suggests a negative amplitude. Following chart presents daily returns of stock index DAX, while fourth chart shows the real part of its complex continuous wavelet transform. Again, dark grey color in 2008 indicates a positive amplitude and black indicates a negative amplitude. Finally, semblance *S* is presented in last chart, where dark grey corresponds to a semblance of +1, light grey corresponds to a semblance of 50% and black corresponds to a semblance of –1. It is interesting that daily changes of 10 year German government bond yield and daily DAX returns are usually correlated on scale of 20-40 days, while scale of 10-20 days indicates significantly lower correlation in examined data sample. Most of the time, the rise of German government 10 year bond yields was accompanied by the rise of stock index DAX. However, after the peak of the European debt crisis in the 2013, both type of financial assets exhibited

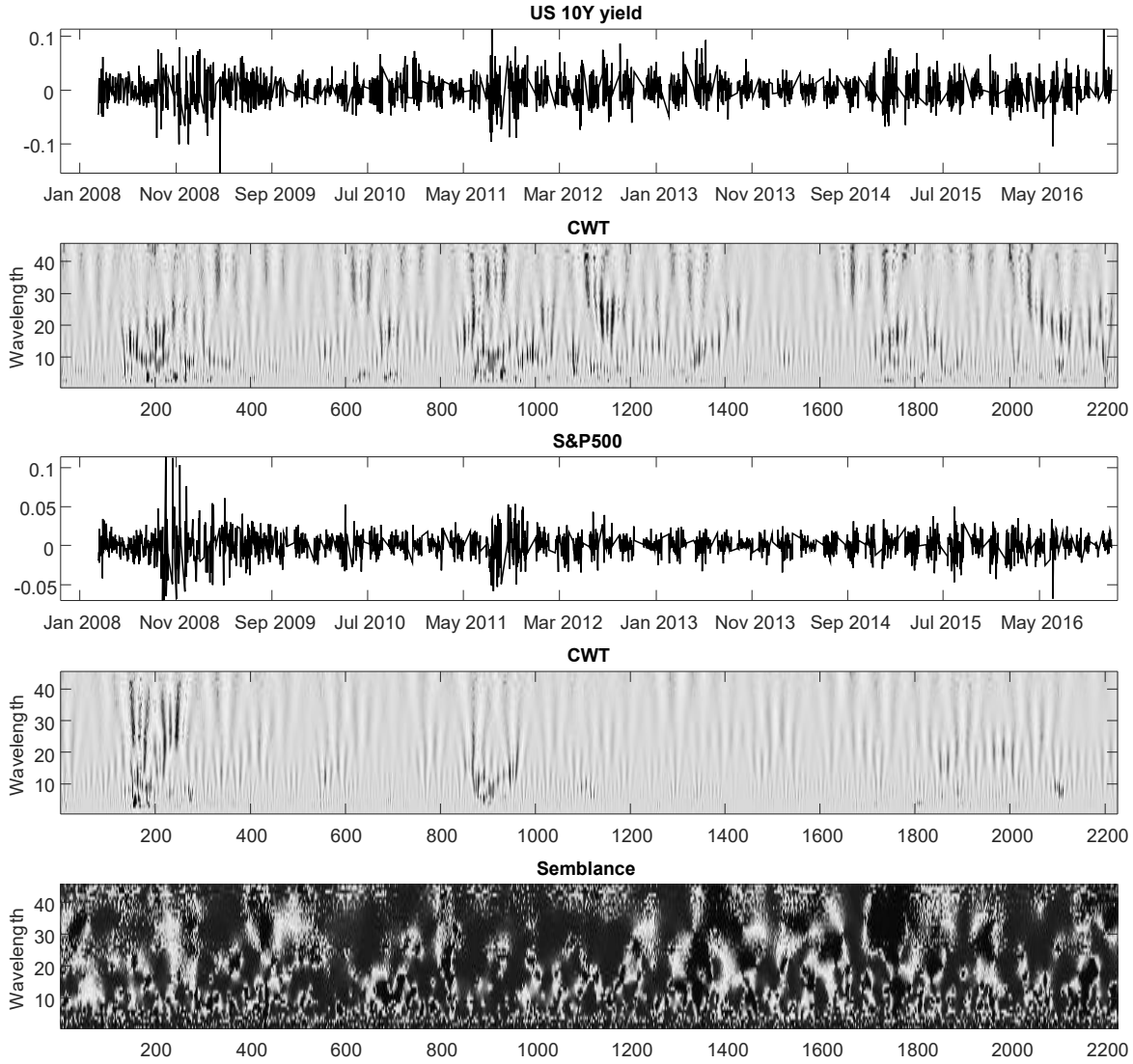

Figure 2. US 10 year treasury yield and S&P500 analysis

significantly larger correlation (increased black area in the right part of the chart). It might be explained by unprecedented Troika policy negotiations which influenced European financial markets over all segments and types of financial instruments.

Focusing on the US markets, Figure 2 explores the relationship between Standard & Poor's stock index and 10 year US treasury yields. Alike as in case of German economy, first part of Figure 2 exhibits the daily change of 10 year treasury yields in period between 28th February 2008 and 16th November 2016, while the second chart shows the real part of complex continuous wavelet transform given bond yield sample. Again, semblance is presented in last chart, where dark grey corresponds to a semblance of +1 and black corresponds to a semblance of –1. Light grey color corresponds to a semblance of 50%. We might conclude that besides the period between 2013 and 2014, daily changes of 10 year treasury yield and daily S&P500 returns were negatively correlated primarily on scale of 10-40 days, in period of 2013 to 2014 we found negative correlation between US stock prices and treasury yields, i.e., returns of both type of usual substitutes in US moved in same direction (black color on the fifth segment of Figure 2).

In order to extend our analysis, we compared also the relationship between government bond yields and stock market returns in Germany and US. Figure 3 depicts the semblance between

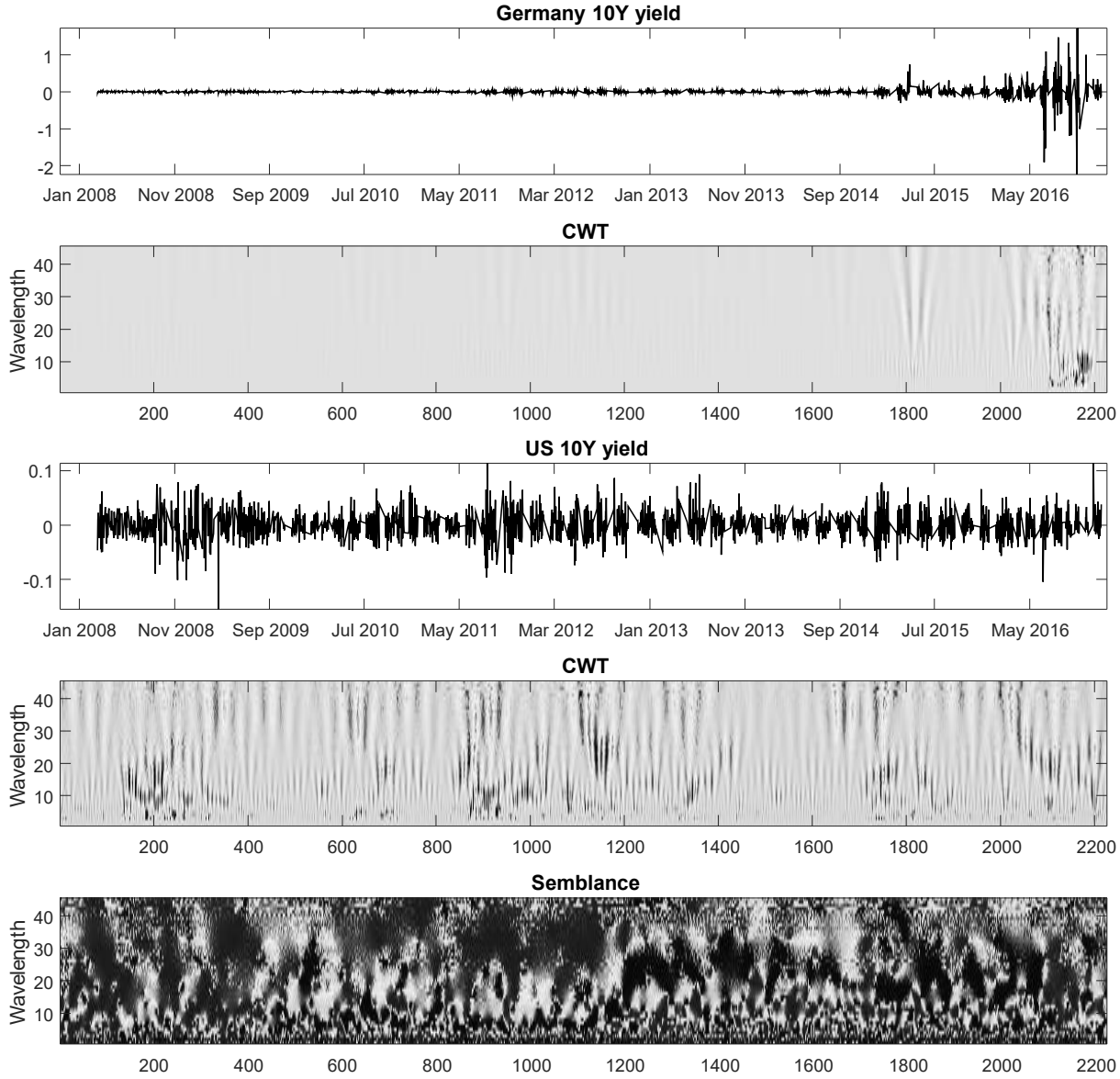

Figure 3. German 10 year bond and US 10 year treasury yield analysis

10 year German and US government bond yields in given period between 28th February 2008 and 16th November 2016. We might see that until the European debt crisis, yields were significantly correlated on almost all scales. Since second half of 2012, there have been observed negative relationship between US and German government bond yield which might be contributed to monetary policy measures implemented by FED and ECB, as well as to significantly higher economical growth and lower unemployment in US.

Figure 4 focused on the comparison of the relationship between German stock index DAX and US stock index S&P500 in period between 28th February 2008 and 16th November 2016. There is a semblance very close to 1 (dark grey color on the left) during the period of financial crisis on all scales, however, since second half of 2009 the semblance turn to negative on the scale of 5-15 days. After the expansion of European debt crisis in 2012, stock indices returns of both countries started to significantly differ and semblance was close to –1 on almost all scales.

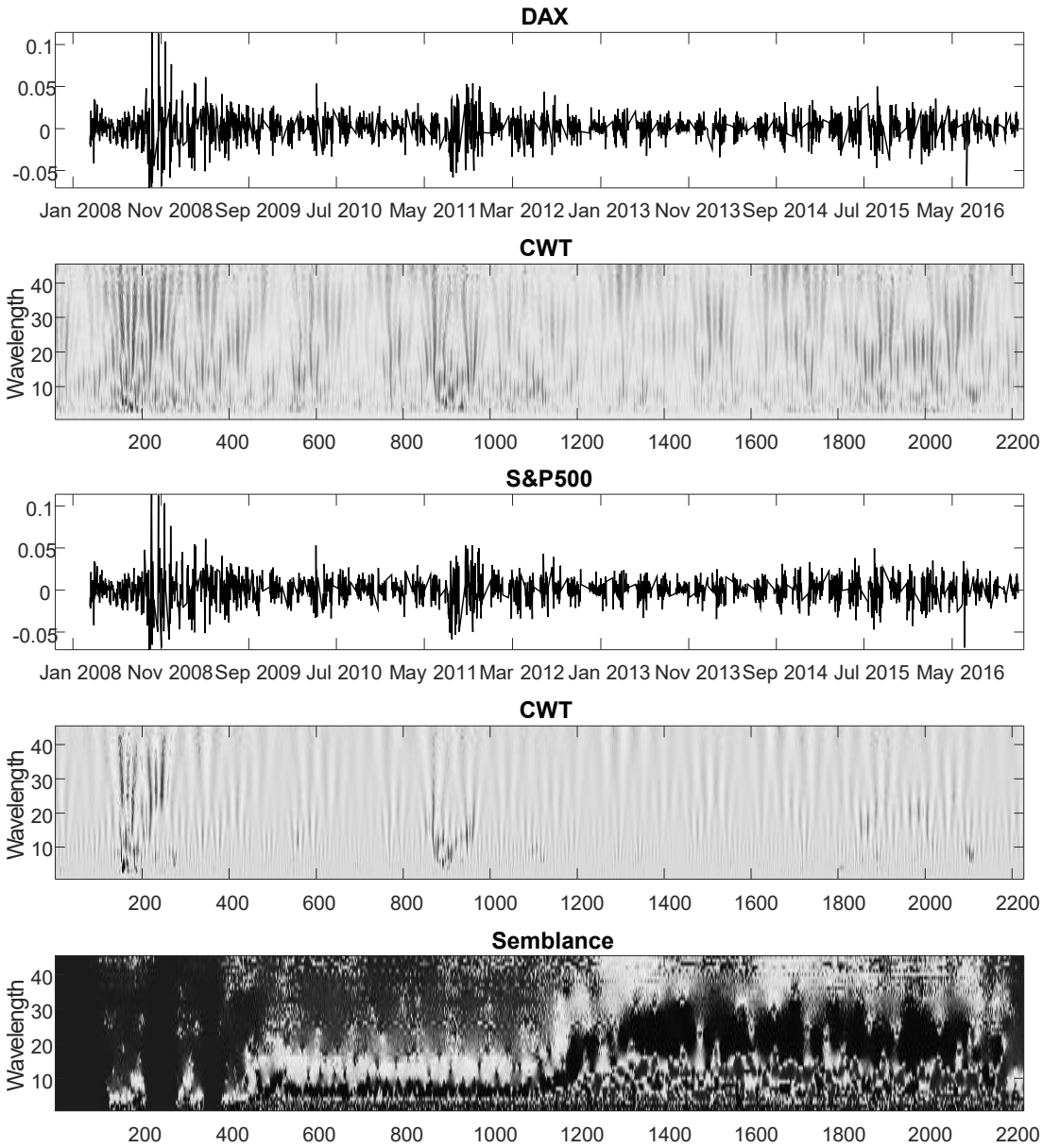

Figure 4. DAX and S&P500 analysis

CONCLUSION

The relationship between stock returns and bond yield changes has been extensively explored in academic research. Even though different methods of resemblance analysis were applied in comparison of financial data samples, only small number of studies focused on wavelets as a perspective approach to enhance conventional semblance analysis. Conventional semblance analysis is based on the Fourier transform, however, it has several shortcomings resulting from its nature. This work aims at semblance analysis of stock prices and government bond yields of US and Germany using continuous wavelet transform. To examine the semblance between German government bond yields and stocks, we focused on the sample of daily returns of stock index DAX and 10 year Bond yield returns consisting of 2238 daily observations between 28th February 2008 and 16th November 2016.

US sample was presented by 10 year Treasuries and S&P500 stock index comprising of 2228 observations in the same period.

Our results indicate that daily DAX returns and daily changes of 10 year German government bond yields are usually correlated on scale of 20-40 days, while scale of 10-20 days suggests significantly lower correlation on the explored data sample. However, the rise of German government 10 year bond yields was usually accompanied by the rise of stock index DAX. Focusing on the relationship between Standard & Poor's stock index and 10 year US treasury yields we might conclude that besides the period between 2013 and 2014, daily changes of 10 year treasury yield and daily S&P500 returns were negatively correlated primarily on scale of 10-40 days, in period of 2013 to 2014 we found negative correlation between US stock prices and treasury yields.

Moreover, we also compared the relationship between government bond yields and stock market returns in both countries. Until the European debt crisis, bond yields and stock returns were significantly correlated on almost all scales, however, after the expansion of the crisis in 2012, development of financial assets in both countries started to significantly differ with semblance close to –1 on almost all scales in case of stock indices and with semblance close to –1 on scale of 15-35 days in case of bond yields.

ACKNOWLEDGEMENT

This paper is part of young scientific workers' project number I-16-110-00 optimization of financing of European enterprises using methods of artificial intelligence.

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