“An investigation over the strategic implications of environmental monitoring”

| AUTHORS          | Alberto Casagrande
<table>
<thead>
<tr>
<th></th>
<th>Marco Spallone [ORCID: 0000-0003-4990-8872]</th>
</tr>
</thead>
<tbody>
<tr>
<td>RELEASED ON</td>
<td>Tuesday, 11 December 2012</td>
</tr>
<tr>
<td>JOURNAL</td>
<td>“Environmental Economics”</td>
</tr>
<tr>
<td>FOUNDER</td>
<td>LLC “Consulting Publishing Company “Business Perspectives”</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>NUMBER OF REFERENCES</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>NUMBER OF FIGURES</td>
<td>0</td>
</tr>
<tr>
<td>NUMBER OF TABLES</td>
<td>0</td>
</tr>
</tbody>
</table>

© The author(s) 2020. This publication is an open access article.
Alberto Casagrande (Italy), Marco Spallone (Italy)

An investigation over the strategic implications of environmental monitoring

Abstract

A firm’s decision of obeying environmental regulatory standards depends crucially on its chances of being detected and on the costs it must bear in case of detection. The paper investigates the relationship between the amount of resources devoted to environmental monitoring and the extent of non-compliance, using a game theoretical model to capture the strategic implications of the monitoring process. In the model a population of firms, each of whom decides whether or not to be compliant, and a monitoring agency, that can detect non-compliance only by monitoring signals, strategically interact (more precisely, each firm interacts both with the monitoring agency and all other firms). In particular, each firm produces a signal, the distribution of which is (not perfectly) correlated with its behavior, while the agency, that is resource constrained, chooses some (optimal) fraction of the signals to monitor; hence, the probability of being monitored for each firm depends crucially on the behavior of both the monitoring agency and all other firms. Simply put, if a large fraction of firms chooses not to obey regulatory standards, the probability of being monitored for non-compliant firms is small. The main consequence of the strategic interaction among firms is that a more aggressive monitoring policy may end up relaxing the resource constraint of the monitoring agency as long as the firms, perceiving a higher chance of being detected, become compliant. In fact, while in a framework with no strategic interaction a more aggressive monitoring policy simply induces a larger fraction of firms to be compliant (this effect, recognized by Becker and Stigler in their seminal contributions, is called the “impact effect”), in the model presented in the paper a more aggressive monitoring policy also implies a higher probability of being monitored for the remaining non-compliant firms, and, in turn, implies a further switch to compliancy. The authors show that this further switch, that is called call “magnification effect”, can be very relevant; hence, when monitoring policies are to be designed, the advice is to take strategic interaction among firms in the right consideration.

Keywords: environment, monitoring, strategic interaction.

JEL classification: Q50, Q58.

Introduction

Environmental protection is a highly considered issue in many countries. In fact, since all economic activities generate negative environmental externalities, such as pollution, governments usually impose environmental taxes and emission standards to producers. The main problem with this regulatory approach is that firms’ compliance is not guaranteed.

We assume that environmental regulation is not fully enforceable because monitoring is costly and only a fraction of the firms can be monitored by a dedicated monitoring agency. We deal only with intentional violations of emission standards and we do not consider accidental violations, due, for example, to negligence. Moreover, we do not deal with environmental taxes.

This paper addresses the following questions: what is the equilibrium fraction of non-compliant firms within a population of heterogeneous firms (with varying propensities to violate emission standards) when the monitoring agency is budget constrained? What impact can an increase in the monitoring agency budget have on the equilibrium fraction of non-compliant firms?

In our model, a population of firms with varying propensities to violate regulatory standards faces a discrete choice, either to obey the regulatory standards or to violate them. Each action generates a random signal, and we assume that the higher the signal (for example, a higher concentration of pollutants in the air, or in the water), the more likely a non-compliant behavior has generated it. The monitoring agency is able to observe all the signals, but can detect non-compliant firms only by monitoring them. Since resources are constrained and monitoring is costly, the monitoring agency must choose some (optimal) fraction of the firms (i.e., signals) to monitor.

We show that under plausible monotonicity assumptions the monitoring agency chooses to monitor the sub-set of firms generating the highest signals. The key property of this model is that each firm strategically interacts with both the monitoring agency and all other firms. In fact, given the optimal strategy of the monitoring agency, the (endogenous) probability of being monitored for each firm depends not only on the amount of resources available to the monitoring agency, but also on the behavior of all other firms; simply put, in our model if a large fraction of firms chooses not to obey regulatory standards, the probability of being monitored for non-compliant firms is small.

© Alberto Casagrande, Marco Spallone, 2012.

1 Environmental compliance topics are reviewed in Cohen (1999).

2 A classification of environmental violations can be found in Cropper and Oates (1992).

Moreover, we show that multiple equilibria may arise, each one of them implying a different equilibrium level of compliancy.
The main consequence of the strategic interaction among firms is that a more aggressive monitoring policy may end up relaxing the resource constraint of the monitoring agency as long as enough firms, perceiving a higher chance of being detected, become compliant. In fact, while in a framework with no strategic interaction a more aggressive monitoring policy simply induces a larger fraction of firms to be compliant (we call this effect, recognized by Becker and Stigler in their seminal contributions, “impact effect”), in our model a more aggressive monitoring policy also implies a higher probability of being monitored for the remaining non-compliant firms, and, in turn, implies a further switch to compliance. We show that this further switch, that we call “magnification effect”, can be very relevant; hence, when monitoring policies are to be designed, our advice is to take strategic interaction among firms in the right consideration.

The paper is organized as follows. Section 1 summarizes the related literature. Section 2 describes the theoretical model and the results. The final section concludes the paper. The Appendix contains stability analysis performed over the potential multiple equilibria of the model.

1. Related literature

The economic literature on law enforcement and environmental violations provides complete references for our paper. As pointed out in the introduction, our paper proves that: (1) an aggressive monitoring policy induces compliance; and (2) the effects of an aggressive policy are amplified by the strategic interaction among each firm, the monitoring agency, and all other firms operating in the same regulatory environment.

While the first result is different from that obtained by Harford (1978), who argues that firms’ compliance (with waste/emission standards) does not depend on the monitoring agency budget, it is consistent with the conclusions of Becker (1968), Stigler (1970), and many others (see, for example, Heyes (2000)).

However, the result that monitoring induces compliance is often derived within theoretical frameworks that consider only the interaction between each single firm and a dedicated monitoring/enforcement agency. We believe that the decisions of each single firm about environmental issues are affected by the behavior of both the monitoring/enforcement agency and all other firms operating in the same regulatory environment. In our paper this complex interaction is rigorously modeled and drives all the results: in particular, we prove that it may lead to multiple equilibria and that it seriously affects the effectiveness of monitoring policies by amplifying their benefits in terms of compliance.

An approach similar to ours can be found in Stadler and Perez-Castrillo (2006). However, while they consider the interaction among a population of heterogeneous firms and a monitoring agency (taking into account both regulatory standards and environmental taxation), they do not let the firms interact among themselves, thus underestimating the relevance of such interaction. Moreover, while they adopt a principal-agent approach assuming perfect commitment of the enforcement agency, we model the monitoring problem as a Bayesian game where the monitoring agency has no commitment capacity.

Finally, the strategic interaction among firms and the consequent magnification effect provides a further interpretation of a robust empirical evidence (Dasgupta et al., 2001; Foulon et al., 2002) that proves that not only monitoring, but also the threat of monitoring is useful in reducing the level of pollution.

Lately, Arguedas and Rousseau (2012) proposed a framework in which inspection agencies gather information about firms pollution levels and this information may allow agencies to differentiate their monitoring strategies in the future. They argue that, if a firm is less successful than its peers in reducing emissions, it may find convenient to mimic low-abatement cost firms by choosing lower emission levels in order not to face stricter inspections. However, while they find that the ongoing signaling game between firm types might lead to firms over-complying with the emission standard, in our model firms never over-comply since there is no convenience in doing so. Actually, full compliance is possible either when the resource constraint of the agency is not binding or when the individual cost of getting apprehended is very high.1

2. The model

There is a monitoring agency willing to apprehend as many non-compliant firms as possible given the amount of resources available for monitoring. For simplicity, it is assumed that only a fixed fraction of the firms $R$ can be monitored, with $0 < R < 1^2$. It is also assumed that every non-compliant firm is apprehended if monitored. Hence, the specific objective of the monitoring agency is to maximize the probability of apprehending non-compliant firms by choosing a subset of firms for monitoring. Generally speaking, the

---

1 Finally, Chu-Chuan and Yu-Bong Lai (2012) argue that if environmental regulation is subject to the influence of lobbying, a stricter enforcement policy may result in a larger amount of pollution emissions due to the higher political pressure exerted by polluting firms. Our result holds since we do not consider political effect in our model.

2 Here, we are assuming that the level of expenditures is given and that the cost of monitoring is the same across firms. These assumptions allow us to express the budget constraint of the agency in terms of the fraction of firms that can be monitored.
objective of a monitoring/enforcement agency is to maximize social welfare by choosing monitoring/enforcement expenditures, or, given the level of expenditures, by choosing the probability of detection (see for further discussion Polinsky and Shavell (2000)). Here, we are assuming that, given the level of expenditure, maximizing the efficiency of the monitoring process implies maximizing social welfare.

There is a continuum of firms. Each firm has a firm-specific cost of getting apprehended, which is private information and is independently distributed across the population of firms with cumulative distribution function $F$. Each firm can choose whether to be non-compliant ($D$) and break the law or to be compliant ($H$). The firms who decide to be compliant receive a positive payoff equal to BH with certainty. If non-compliant, they get a higher benefit, $BD > BH$, but they suffer their cost $c$ if apprehended. The firm-specific cost may be thought as the sum of a common pecuniary fine and a non-pecuniary penalty that depends on community pressure and social norms (Pargal and Wheeler, 1996; Hettige et al., 1996; Arora and Cason, 1996; Brooks and Sethi, 1997). In either case, firms send a one dimensional signal that is observable by the monitoring agency. However, since the signal is not perfectly correlated with the behavior of firms, the agency cannot establish a priori whether signals are generated by a willful violation of regulatory standards, or by a random act of nature. It is assumed that the signals can take any value $s$ in some connected subset of $R$. Let $g(s \mid D)$ and $g(s \mid H)$ be the density functions of the signals, given compliant and non-compliant behavior respectively. It is assumed that $g(s \mid D)$ and $g(s \mid H)$ are positively defined over all $s$. Denote by $G(s \mid D)$ and $G(s \mid H)$ the respective cumulative distribution functions.

We assume that ex ante each firm doesn’t know what its signal is going to be. Nonetheless, each firm knows that when it does not comply, its signal is likely to be relatively higher than when it does not, as Assumption 1 below clarifies.

Assumption 1. Monotone likelihood ratio property. The likelihood ratio is $\frac{g(s \mid H)}{g(s \mid D)}$ decreasing in $s$.

Figure 1 below provides a graphical intuition of Assumption 1.

![Fig. 1. Graphical intuition of Assumption 1](image)

The following Lemma is an immediate implication of Assumption 1 and fully defines the monitoring strategy of the agency.

Lemma 1. Let $P(D)$ be the fraction of the population of firms that does not comply. Then, the marginal density of the signals $h(s)$ may be defined as:

$$h(s) = g(s \mid D)P(D) + g(s \mid H)(\log c - P(D)).$$

If $0 < P(D) < 1$, then the agency monitors the highest signals and chooses the unique $s^*$ satisfying:

$$(1 - G(s^* \mid D)) P(D) + (1 - G(s^* \mid H))(1 - P(D)) = R$$

In view of Lemma 1, a monitoring agency policy may be defined by $s^*$.

Proof. Define $P(D \mid s)$ as the probability of non-compliant behavior associated with each signal $s$. We show first that $P(D \mid s)$ is increasing in $s$. Let $s_l$ and $s_h$ be two signals with $s_l < s_h$. Then:

$$\frac{P(D \mid s_l)}{P(D \mid s_h)} = \frac{g(s_h \mid D)}{G(s_h \mid D)} \frac{h(s_l)}{g(s_l \mid D)} > \frac{g(s_h \mid H)}{G(s_h \mid H)} \frac{h(s_l)}{g(s_l \mid H)} = \frac{P(H \mid s_h)}{P(H \mid s_l)}$$

the strict inequality being a consequence of Assumption 1. Since for all $s$, $P(H \mid s) + P(D \mid s) = 1$, the result follows. Since $P(D \mid s)$ is an increasing function of $s$, then the monitoring agency is going to monitor the highest signals. By monotonicity of $G(s^* \mid D)$ and $G(s^* \mid H)$ there is a unique $s^*$ satisfying equation 1.

It is worth noticing that equation (1) is just a reaction function. In fact, $s^*$ that satisfies equation (1) is expressed as a function of $P(D)$ (i.e., the fraction of non-compliant firms), which in turn depends on the behavior of each individual firm.

Now, even if the monitoring policy is fully defined by $s^*$, each firm does not know whether its signal is going to be higher or lower than $s^*$. In fact, the choice of $s^*$ by the monitoring agency and the choice of each firm about compliance are simulta-
neous. Then, for any choice \( s^* \) by the monitoring agency, the probability of apprehension for firms that are cheating is \( 1 - G(s^* | D) \).

Each firm must decide whether to be compliant or not. A strategy \( \sigma \) maps each firm-specific cost into the binary set \( \{D, H\} \). Let \( B_s = BD - BH > 0 \) denote the net benefit from being non-compliant. For any \( s^* \), let \( c^*(s^*) \) be such that:

\[
c^*(s^*) = \frac{B_s}{1 - G(s^* | D)}. \tag{19}
\]

Then, the best strategy for each firm with cost \( c \) is given by:

\[
\sigma(c) = \begin{cases} 
H, \forall c \geq c^*(s^*) \\
D, \forall c < c^*(s^*) 
\end{cases} \tag{20}
\]

As it is obvious, the strategy of each firm depends on the monitoring policy as firms perceive any monitoring policy into their probability of getting apprehended. Moreover, it depends on the behavior of all other firms since it affects such probability.

From now on, \( c^* \) will denote \( c^*(s^*) \). Hence, the fraction of non-compliant firms in the population, \( P(D) \), is endogenous and equal to \( F(c^*) \).

2.1. The equilibria. Roughly speaking, an equilibrium is a monitoring policy and a strategy for all firms such that each firm does not want to change its action, given the monitoring agency policy, while the monitoring agency makes an efficient use of all its resources, given the firms’ behavior. An equilibrium is defined as a pair of \( \{s^*, c^*\} \) which satisfies both equations (1) and (2). To eliminate equilibria in which the entire population of firms chooses either to be compliant or non-compliant, we introduce the following assumption:

**Assumption 2.** \( 0 < F(c) < 1 \) for all \( c > B_s \).

Simply put, Assumption 2 prevents individual costs of getting apprehended from being too high or too low with respect to the net benefit of cheating. Assumption 2 implies the following Lemma:

**Lemma 2.** In equilibrium \( 0 < P(D) < 1 \).

Now, we can solve a simultaneous system of reaction functions: equation (2) may be seen as the reaction function of the firms to any monitoring agency policy, while equation (3) below is the response of the monitoring agency to any \( c^* \).

Since \( P(D) = F(c^*) \), we may rewrite equation (1) as:

\[
(1 - G(s^* | D))F(c^*) + (1 - G(s^* | H))(1 - F(c^*)) = R. \tag{30}
\]

It is possible to solve for \( c^* \) in equation (2) and substitute into equation (3) in order to obtain the following equation in \( s^* \):

\[
J(s^*; R) = F \left( \frac{B_s}{1 - G(s^* | D)} \right) (1 - G(s^* | D)) + \\
\left[ 1 - F \left( \frac{B_s}{1 - G(s^* | D)} \right) \right] (1 - G(s^* | H)) - R = 0. \tag{31}
\]

We can interpret \( J(s^*; R) \) as the excess demand of resources by the monitoring agency. Given that firms are choosing their optimal \( c^* \), whenever \( J(s^*; R) > 0 \), the monitoring agency exceeds its budget constraint, while \( J(s^*; R) < 0 \) implies that the monitoring agency is not using all the available resources.

2.2. Multiplicity of equilibria. Depending on the properties of \( F \) and \( G \), equation (4) may be satisfied by more than one \( s^* \) for some feasible \( R \). Note that the limits of \( J(s^*; R) \) as \( s^* \) approaches its upper and lower bound are \(-R \) and \( 1 - R \), respectively. Thus, a sufficient condition for the uniqueness of solution for any possible \( R \) requires \( J(s^*; R) \) being monotone increasing in \( s^* \). Let \( f \) be the density function of the firm-specific costs. Then, by differentiating, and using equation (2) to replace \( \frac{B_s}{G(s^* | D)} \) with \( c^* \), we have

\[
J'(s^*; R) = F(c^*) (-g(s^* | D)) + (1 - F(c^*)) (-g(s^* | H)) + \\
\left[ \frac{g(s^* | D) B_s}{1 - G(s^* | D)} \right] f(c^*) (G(s^* | H) - G(s^* | D)). \tag{32}
\]

We can interpret the first two terms in equation (5) as the additional resources needed by the monitoring agency to monitor an increasing fraction of firms (both compliant and non-compliant), given the original cutoff cost \( c^* \). The third term in equation (5) is the source of multiple equilibria, since under the usual monotonicity conditions it is always positive. It measures the amount of resources available to the monitoring agency since a fraction of the population of firms becomes compliant. It has two components.

The first component \( \frac{g(s^* | D) B_s}{(1 - G(s^* | D))^2} \) represents the adjustment in the cutoff cost of the firms due to the change of the monitoring agency policy \( s^* \). The second component \( f(c^*) (G(s^* | H) - G(s^* | D)) \) measures the corresponding budget relief for the monitoring agency. So, as more signals are monitored, an increasing fraction of the firms, by perceiving tougher controls, becomes compliant. The effect is a relief of the budget constraint of the monitoring agency.

We address the stability issues about the equilibria described above in the Appendix.
2.3. Comparative statics. Within the theoretical framework described above, it is also possible to analyze the effects of a change in $R$ on the equilibrium fraction of non-compliant firms. Here, we do not discuss whether increasing the budget of the monitoring agency is an optimal decision or not. In other words, we do not compare the costs and the benefits of such increase; we just want to describe what are the consequences of a more aggressive monitoring policy, with a particular focus on the role played by strategic interaction.

In our setup it is possible to isolate an impact effect as well as a complementary magnification effect, which is the strategic effect generated by our model. To compute these, we show again the system of equations that characterizes the model. Equations (2) and (3) are rewritten by splitting the endogenous ($c^*$ and $s^*$) and the exogenous variables ($R$ and $B_0$) of the system. We get:

$$
\begin{align*}
(1 - G(s^* | D))c^* &= B_0, \\
(1 - G(s^* | D))F(c^*) + (1 - G(s^* | H))(1 - F(c^*)) &= R.
\end{align*}
$$

The total effect (TE) of the change in $R$ on the fraction of non-compliant firms can be measured, using total derivatives over the above system of equations:

$$
TE = \frac{dF(c^*)}{dc^*} \cdot \frac{dc^*(R)}{dR} = f(c^*) \cdot \frac{B_0 g(s^* | D)}{1 - G(s^* | D)} \cdot \frac{1}{J'(s^*; R)}.
$$

As mentioned before, the total effect can be split into the impact effect and the magnification effect. By the impact effect (IE) it is meant the change in the fraction of non-compliant firms following a change in $R$ if no further (strategic) reactions by firms were considered. A monitoring agency, which does not take into account general equilibrium considerations, would think of it as the total effect.

After an increase in $R$, given the cutoff level cost $c^*$, the monitoring agency is able to decrease $s^*$; as a consequence, firms decrease $c^*$; since the probability of apprehension increases, more firms decide to be compliant, and the fraction of non-compliant firms decreases.

The impact effect is given by:

$$
IE = \frac{f(c^*)}{F(c^*)[-g(s^* | D)] + (1 - F(c^*))[-g(s^* | H)]} = \frac{B_0 g(s^* | D)}{1 - G(s^* | D)}.
$$

The magnification effect (ME) is defined as the further decrease in the fraction of non-compliant firms due to strategic interaction:

Because of the impact effect, the signals sent by the firms have shifted downward, and less signals are now above the actual policy; so, the monitoring agency is able to further decrease $s^*$; hence, firms further decrease $c^*$, and the fraction of non-compliant firms further decreases.

We can measure this magnification effect as follows:

$$
ME = \frac{J'(s^*; R)}{1 - m} = \frac{1}{1 - m}.
$$

where

$$
m = \frac{B_0 g(s^* | D)}{1 - G(s^* | D)} \cdot \frac{f(c^*)(G(s^* | H) - G(s^* | D))}{1 - F(c^*)[-g(s^* | D)] + (1 - F(c^*))[-g(s^* | H)]}.
$$

Notice that the magnification effect takes values always greater than 1, as long as $m < 1$. We can interpret $m$ as a strategic multiplier¹. The numerator and the denominator of $M$ are the positive and the negative component of $J'(s^*; R)$, respectively. So, the higher the budget relief by the monitoring agency following any decrease in $s^*$, the lower the additional resources needed to monitor more signals, the higher $M$. Furthermore, as $M$ approaches 1, the magnification effect increases and becomes very relevant.

So, our policy indication is that one should seriously consider strategic implications when evaluating the costs and the benefits of a more aggressive monitoring policy because benefits may be underestimated if the reaction of the universe of firms operating in a certain regulatory environment is overlooked.

Conclusion
The main theoretical result of this paper is that when heterogeneous firms are allowed to strategically interact over their decisions about obeying regulatory standards multiple equilibria with different levels of non-compliancy may arise. Moreover, we showed that the strategic interaction among firms amplifies the effects of a more aggressive environmental monitoring policy. An obvious policy implication of our model is that one should seriously consider strategic implications when evaluating the costs and the benefits of a more aggressive monitoring policy because benefits may be underestimated if the reaction of the universe of firms operating in a certain regulatory environment is overlooked.

Our results rely on two main assumptions: first, on the hypothesis that the monitoring agency is willing to spend its money efficiently; second, on the assumption that the agency itself is not corrupt, or that beyond corrupt officers there exists a clean public

¹ Alternatively, one may regard the magnification effect as the difference between a partial equilibrium analysis and a general equilibrium one.
authority responsible for monitoring them. If more public resources do not imply either better or higher numbers of controls at the same (high) quality, and/or if the agency is itself corrupt, there is obviously no advantage in increasing public budgets.

Furthermore, we assume that the agency has an exogenous amount of resources that can be spent to reduce non-compliance. These resources may be thought of as the result of some political process. This work can be extended in several other ways. First, one could allow social networks as a channel for propagating either honest or dishonest behavior, using frameworks such as local interaction analysis. Second, one can look at different methodologies or technologies of monitoring. Third, one can adopt this framework to analyze other phenomena such as direct or indirect tax evasion, or corruption.

References


Appendix. Stability analysis

We suppose that both the agency and the firms act in a myopic incremental perspective. More specifically we suppose that, for given \( c^* \), the agency decreases its policy \( s^* \) over time in order to correct any budget surplus and increases it to adjust any budget deficit. Likewise we assume that, for given \( s^* \), firms revise \( c^* \) to keep track of any agency policy change. They increase \( c^* \) if net benefits exceed the expected costs at the actual threshold.

The above assumptions ensure that all stable equilibria are plausible. The agency does not have a first mover advantage and cannot commit to the highest stable equilibrium policy. The motivations behind these assumptions are two. On the one hand, usually firms are conservative in their behavior and monitoring agencies do not dramatically alter their policies. On the other hand, it would be difficult, even for the monitoring agency, to infer the distribution of the non-compliance costs.
Let \( s^* \) and \( c^* \) be the derivative of \( s^* \) and \( c^* \) with respect to time, respectively. The following system of equations describes the above behavior:

\[
\begin{align*}
\dot{s} &= \left[ 1 - G(s^*(t) | D) \right] F(c^*(t)) + \left[ 1 - G(s^*(t) | H) \right] \left[ 1 - F(c^*(t)) \right] - R, \\
\dot{c} &= \frac{B_{sg}}{1 - G(s^*(t) | D)} \cdot \dot{c}^*(t).
\end{align*}
\]

The agency increases \( s^* \) as non-compliancy increases, and firms decrease \( c^* \) as monitoring policy gets more aggressive.

At any locus \((s^*, c^*)\) below \( s^* = 0 \) the agency decreases \( s^* \) to correct its budget surplus, whereas below \( c^* = 0 \) firms increase \( c^* \) since expected costs at the current \( s^* \) are less than net benefits.

We define an equilibrium \((s^*, c^*)\) to be stable if there exists a sufficiently small neighborhood of it such that any initial pair \((s^*, c^*)\) in the neighborhood asymptotically converges to \((s^*, c^*)\). Generically speaking, extreme equilibria are stable, and between any two stable equilibria there is an unstable one, as it is clear from the following figure.

![Fig. 1. Equilibria](image)

In fact, through the linearization of our system of equations around any steady state \((s^*, c^*)\), we get:

\[
\begin{align*}
\dot{s} &= A \cdot \dot{s}^* + B \cdot \dot{c}^* + 0, \\
\dot{c} &= C \cdot \dot{s}^* + 1 \cdot \dot{c}^* + 0,
\end{align*}
\]

where

\[
\begin{align*}
A &= -g(s^* | D) F(c^*) - g(s^* | H)(1 - F(c^*)), \\
B &= f(c^*) [G(s^* | D) - G(s^* | H)], \\
C &= \frac{B_{sg}(s^* | D)}{1 - G(s^* | D)}.
\end{align*}
\]

The trace and the determinant of the Jacobian matrix is all we need for inferring over the stability of any equilibrium. It easily turns out that the trace is always negative, and the sign of the determinant is positive if and only if \( J'(s^*; R) > 0 \).

Then, we may conclude that whenever \( J'(s^*; R) > 0 \), the equilibrium is stable, and whenever \( J'(s^*; R) < 0 \), the equilibrium is unstable. Since, as we said above, the limits of \( J'(s^*; R) \) as \( s^* \) approaches its upper and lower bound are \( R \) and \( 1 - R \), respectively, extreme equilibria are stable, and between any two stable equilibria there is an unstable one.