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Estimating sovereign risk by a structural approach: the role of Forex reserves for emerging market countries

Abstract

We apply the structural approach as first introduced by Merton (1974) to quantify the probability that a sovereign defaults on repayment obligations in foreign currency. Thereby, we focus on the process of the sovereign’s ability to pay, as approximated by Forex reserves. The process and its parameters are inferred from fundamental economic as well as bond market data of the considered countries. The estimated default probabilities for a sample of 17 emerging market and transition countries for which the necessary input data are given are evaluated in different ways.

Keywords: sovereign risk, probability of default, Forex reserves.
JEL Classification: F34, G15, G12.

Introduction

Sovereign risk means the risk that the government of a state is going to declare its inability to pay and suspend or even stop its debt servicing. Sovereign risk plays a crucial role for the decision on lending to emerging market and transition countries since it has a great influence on the expected return on investments.

There is a huge scientific literature on this topic. The estimation of default risk mostly is based on Logit models, Probit models or discriminant analysis\(^1\). Such approaches are not without problems. On one hand, these models are backward looking by nature since they assume that the interrelation between default probability and explaining factors, as specified for the past, is valid for the future, too. On the other hand, not all potentially influencing factors are quantifiable and (publicly) observable.

However, among practitioners sovereign ratings relying – partly – on subjective and qualitative evaluation, play a prominent role. Examples for underlying methodologies are country reports, scoring models and the assessment by experts. The high degree of subjectivity of these methods can be seen as a main drawback. Another problem is the assessment of risk on an ordinal scale.

In this article, we use a method to measure sovereign risk that avoids these drawbacks. In particular, we quantify the sovereign risk by calculating the probability of default for sovereigns and hence, measure the risk in a cardinal manner. Our approach is based on an adaptation of the Merton model (1974) for evaluating corporate liabilities to the issue of sovereign risk and to the evaluation of liabilities of states. The basic idea of the structural approach is that there exists an underlying stochastic state variable (which describes the firm value in the Merton case of corporate liabilities) that triggers a default if it falls below a certain threshold value (the amount of debt repayments in the Merton case).

To quantify the stochastic state variable and the parameters of its stochastic process, different approaches are possible. Whereas in some papers only macroeconomic fundamental data are used (see, for example, Clark, 1991 or Claessens and van Wijnbergen, 1993), another strand of the literature uses market data from government bond markets only (see, for example, Claessens and Pennacchi, 1996; Lehrbass, 2000; Keswani, 2000; Maltritz, 2006; or Huschens et al., 2007). In this paper we combine both types of data to quantify the state variable and its parameters.

In contrast to earlier contributions (see, Karmann, 2000, and Karmann and Maltritz, 2004) focusing on selected countries, we consider a broad sample of countries. In fact, all emerging market and transition countries for which the necessary input data had been available are taken into account. This allows to evaluate the resulting default probabilities, hereby calculating the quadratic probability score.

We proceed as follows. The next section deals with the description of our model. Section 2 explains the application of the model. Section 3 is concerned with the evaluation of the results and the last section concludes.

1. The model

As explained in the introduction, structural models rely on the idea that there exists a structural state variable which triggers a default event by hitting a default threshold. In the Merton model, the state variable describes the value of the firm.

One of the central questions in evaluating sovereign risk by the use of structural models is: How is the state variable to determine? Or to put it another way:
What does the repayment of liabilities depend on? In the case of sovereign risk, besides the sovereign’s ability to pay his willingness to pay plays an important role (see Eaton, Gersovitz and Stiglitz, 1986, for an overview), since there is still no juridical way for the creditors to enforce their payment claims towards foreign countries. Whether payments are made or not, depends on the decision of the sovereign and on his willingness to pay. In making this decision, the sovereign will weigh out the costs and benefits of default or payment, respectively. A government does indeed benefit from a default since the saved capital will be available for other purposes. However, the costs of default are the disruption of commercial activities and high costs due to raising considerable interest rates for further borrowing or even the inability to receive new capital.

Assumption 1:

It is assumed that the costs of a default are so high that they go beyond the benefits and therefore the sovereign will repay as long as he is able to do so.

This means that the sovereign’s ability to pay is the variable that decides whether or not repayments will be made, whereby the ability to pay is defined as follows:

Definition 1:

The ability to pay, \( A \), equals the sum of all foreign exchange that a sovereign can raise at time \( t \) in order to meet his debt servicing obligations.

The country’s ability to pay is an unobservable variable, just as the value of a firm in the Merton model. Its determination is explained in the next section. Now we describe the assumptions regarding the stochastic characteristics of the ability to pay.

Assumption 2:

The process of the sovereign’s ability to pay, \( A \), is given by:

\[
\mathrm{d}A_t = \mu A_t \, \mathrm{d}t + \sigma A_t \, \mathrm{d}W_t, \tag{1}
\]

where \( \mu \) and \( \sigma \) are constant and \( W \) is a Standard Wiener Process.

In the next step, the default threshold is to identify. As in the Merton case, we consider a single debt servicing payment which is securitized by a (default-risky) zero bond. Regarding the condition for a default we make the following assumption:\(^1\):

Assumption 3:

We assume that a default occurs when total amount of repayment requirements, \( B_T \), at a time \( T \) is higher than the ability to pay, \( A_T \), at time \( T \):

\[
A_T < B_T. \tag{2}
\]

With these assumptions the probability of a default event in \( T \) can be estimated in \( t ( < T) \) by:

\[
\text{PoD}_{t,T} = P(A_T < B_T | A_t) = N\left(\frac{\ln(B_T / A_t) - (\mu - \sigma^2 / 2)(T - t)}{\sigma \sqrt{T - t}}\right). \tag{3}
\]

Thereby, \( N(x) \) describes the value of the cumulative standard normal distribution for the argument \( x \).

For the estimation of the input data necessary to calculate the default probability, we adopt the Black-Scholes formula for the case considered here. This formula is based on the typical assumptions of structural models concerning the tradability of securities:

Assumption 4:

Securities are traded without arbitrage opportunities on perfect markets.

That means, for instance, that there are no transaction costs, and that selling (including short selling) and buying of any fraction of any security are possible in continuous time.

Furthermore, it is assumed that a risk-free zero bond with repayment amount \( B_T \) and maturity \( T \) exists, whereby for the risk-less interest rate, \( r_o \), holds:

Assumption 5:

There is one risk-less interest rate for borrowing and lending which is constant over time to maturity.

Hence, we have to consider the following securities:

- a risk-less bond;
- the underlying state variable, i.e. the ability to pay; and
- a derivative security, as for example a zero bond, which securitizes the debt of the country, respectively a (hypothetical) put option, which can be used to secure this zero bond.

Of course, the ability to pay is not tradable (and not observable). But assumption 4 is needed for the derivation of the pricing equation since it requires that it is possible to build and maintain a riskless portfolio by combining the debt contract with other securities (see Merton, 1974, p. 451). The possibility of building and maintaining such a portfolio (without addi-

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\(^1\) The single-payment model considered here can be extended to the case of multiple payments by use of compound option theory. This is done in Geske (1977) and Delianedis and Geske (1998) for corporate borrowers and in Maltritz (2006) for sovereign borrowers. The main problem is that no detailed information is available on the term structure of debt repayments.
tional costs) can be ensured by assumption 4. But, instead of the default trigger, other securities (which depend on the default trigger) could be used to build and maintain a risk-less portfolio. For example, credit default swaps or other credit derivatives can be combined with debt contracts to build such a risk-less portfolio. In the last decade, liquid markets for credit default swaps as well as for government bonds have been developed. So, in principle there exist tradable assets to build a risk-less portfolio. This justifies to employ assumption 4. These assumptions allow to derive the following pricing formula for a hypothetical put option, which secures the default-risky debt:

$$P_t = B_t e^{-r(T-t)} N(-d_2) - A_t N(-d_1),$$  

with:

$$d_1 = \frac{\ln(A_t / B_t) + (r + \sigma^2 / 2)(T-t)}{\sigma \sqrt{T-t}},$$

$$d_2 = d_1 - \sigma \cdot \sqrt{T-t}.$$  

2. Application

This section describes how the necessary input data are derived to estimate the default probability by (3). With respect to debt and term structure of necessary payments, publicly available information is incomplete\(^1\). Only for short-term debt (with duration up to one year) the repayment amount is known. For long-term debt, there are no detailed and current data available. Hence, like in a number of other papers (see, for example, Clark, 1991; Claessens and van Wijnbergen, 1993; or Karmann, 2000), the single payment model explained above is used. Thereby, only the short-term debt and the short-term default probability are considered. The debt data describe the repayment amount of the whole country, whereas time series separately accounting for private and public short-term debt are not accessible. The data are available on an annual base. Hence, our calculations are based on annual time series. For the calculation of the default probability, we assume that payments on later dates do not influence the repayment of short-term debt. Hence, the default threshold equals the repayment amount of short-term debt which is assumed to be due in one year.

As explained in the last section, the stochastic state variable is assumed to be equal to the ability to pay. In our application, the ability to pay is approximated by the foreign exchange reserves which the government, resp. central bank, already has. Thereby, it is neglected that the ability to pay can be higher when the government is capable to acquire foreign exchange through capital imports\(^2\). But, in the short run and for the short-run default probability, this procedure seems to be appropriate.

Besides the state variable, we need the parameters describing the stochastic process. The drift, \(\mu\), is estimated from historical time series of foreign exchange reserves. Assumption 2 implies that the log changes of the ability to pay for equidistant time intervals, \(\Delta t\), are independently identically normally distributed:

$$a_i = \ln A_i - \ln A_{i-\Delta t} \sim i.n.n(\mu - \frac{\sigma^2}{2})\Delta t, \sigma \sqrt{\Delta t}. \tag{5}$$

Hence, if we estimate the mean of a time series of log changes by using a simple mean value estimator, we can calculate the drift as\(^3\):

$$\mu_i = \frac{1}{N} \sum_{i=0}^{N} a_{i-\Delta t} + \frac{\sigma^2}{2}. \tag{6}$$

In principle, we could estimate the volatility, \(\sigma\), in the same way by considering eq. (5) and using an estimator for the standard deviation for the \(a_i\)'s. Instead, we use the Black Scholes formula (4) to calculate \(\sigma\) as implicit volatility as resulting from the spread of government bonds. This approach is based on the idea that the spread mainly consists of a risk premium reflecting the risk assessment of the market participants. I.e., we use this market risk assessment to infer the risk parameter of the stochastic process.

The basic idea is described in Karmann and Maltritz (2004) showing that the price spread between default risky bonds of the observed countries and risk-free US-government bonds with similar attributes is equal to the value of a hypothetical put option. Thereby, the face value equals the strike price while the ability to pay equals the underlying of the option. Hence, the option can be priced by the Black-Scholes formula (4). Since the value is given by the observed price spread and the other variables are derived as explained above, the formula can be solved for the implicit volatility (iteratively).

3. Results and evaluation

Our approach is applied to a sample of 17 emerging markets for the years from 1994 to 2002. The country sample and time period are determined by the availability of market data. We start the calculations in 1994, since then for a number of developing and transition countries markets for government bonds had

\(^1\) Our data source is Datastream® provided by Thomson Financial.

\(^2\) By contrast, the approach in Karmann and Maltritz (2004) also takes into account potential capital imports resulting from future trade activities.

\(^3\) In our application, we use a time window of one year before \(t\).
been established. However, even today for most developing countries there exist no traded government bonds in foreign currency. But, the countries for which such bonds exist are typically the major debtors among all developing countries.

For our evaluation, the estimated country-specific default probabilities on January 1st of every year are compared with a dummy describing whether a default occurs in this country during the forecast period, i.e. the respective year. In this case, the dummy takes the value one. Otherwise it is zero. The crises dummies, KD, and the default probabilities calculated for the beginnings of each year are shown in Table 1 and Table 2 respectively.

<table>
<thead>
<tr>
<th>Table 1. Crises dummies</th>
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<tbody>
<tr>
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<tr>
<td>Argentina</td>
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<tr>
<td>Brazil</td>
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<tr>
<td>Chile</td>
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<tr>
<td>China</td>
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<td>Colombia</td>
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<tr>
<td>Ecuador</td>
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<td>Indonesia</td>
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<td>Malaysia</td>
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<td>Mexico</td>
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<td>Peru</td>
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<td>Philippines</td>
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<td>Poland</td>
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<tr>
<td>Romania</td>
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<tr>
<td>Russia</td>
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<tr>
<td>South Africa</td>
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<tr>
<td>South Korea</td>
</tr>
<tr>
<td>Turkey</td>
</tr>
<tr>
<td>Venezuela</td>
</tr>
</tbody>
</table>

The determination of the dummy variable is inspired by Manasse et al. (2003) where a country is defined to be in default if, on one hand, it is classified as defaulted by Standard & Poor’s (i.e. the country Rating is “D” [default] or “SD” [selected default]) or, on the other hand, a large IMF-emergency-credit is announced. The latter is applied to cover situations where a country is de facto in default, while a default in the legal sense is avoided with the help of the international community. In our sample, examples for such situations are Mexico (1994), Korea (1997), Brazil (1998 and 2001) and Turkey (2000) while Ecuador (1999), Russia (1998) and Argentina (2001) are classified as defaulted according to S&P.

Years where a country already is in default at the first of January are not included in our sample since we aim to estimate the probability of default rather than the probability that an already existing debt crisis continuous. Thereby, we assume that a debt crisis persists as long as the country has the rating D or SD or as long as the IMF arrangement goes on. If, subsequently, a new arrangement is met (which satisfies the 100%-of-quota criterium), we assume the crisis to persist as long as the new arrangement persists. Hence, after the end of a debt crisis, we include a waiting period of one year before we continue to evaluate. Periods which are excluded from evaluation are symbolized by missing values in Tables 1 and 2. Furthermore, missing values may also result from lack of input data for the calculation of default probabilities. This holds especially for the interest rates and spreads of government bonds for several countries at the beginning of our evaluation period.

To evaluate the quality of our estimated default probabilities, we employ the quadratic probability score (QPS), as frequently used (see, e.g., Berg and Pattillo, 1999, or Ho, 2004):

\[
QPS = \frac{1}{N} \sum_{n=1}^{N} 2(PoD_n - KD_n)^2.
\]

\(^2\) The threshold is that the credit has to be higher than 100% of quota.
Table 2. Estimated annual default probabilities

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>0.055</td>
<td>0.139</td>
<td>-</td>
<td>-</td>
<td>0.289</td>
<td>0.315</td>
<td>0.425</td>
<td>0.524</td>
<td>-</td>
</tr>
<tr>
<td>Brazil</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.384</td>
<td>0.521</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Chile</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.055</td>
<td>0.066</td>
<td>0.053</td>
<td>-</td>
</tr>
<tr>
<td>China</td>
<td>0.026</td>
<td>0.001</td>
<td>0.007</td>
<td>0.003</td>
<td>0.006</td>
<td>0.004</td>
<td>0.015</td>
<td>0.014</td>
<td>0.011</td>
</tr>
<tr>
<td>Colombia</td>
<td>-</td>
<td>-</td>
<td>0.29</td>
<td>0.346</td>
<td>0.47</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Ecuador</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.149</td>
<td>0.771</td>
<td>-</td>
<td>-</td>
<td>0.515</td>
</tr>
<tr>
<td>Indonesia</td>
<td>-0.008</td>
<td>-</td>
<td>-</td>
<td>0.03</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Malaysia</td>
<td>0.347</td>
<td>0.047</td>
<td>0.07</td>
<td>0.02</td>
<td>0.107</td>
<td>0.055</td>
<td>0.016</td>
<td>-0.038</td>
<td>0.027</td>
</tr>
<tr>
<td>Mexico</td>
<td>-</td>
<td>0.997</td>
<td>-</td>
<td>0.658</td>
<td>0.414</td>
<td>0.501</td>
<td>0.662</td>
<td>0.549</td>
<td>0.429</td>
</tr>
<tr>
<td>Peru</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.11</td>
<td>0.211</td>
<td>0.066</td>
<td>0.033</td>
<td>0.142</td>
</tr>
<tr>
<td>Philippines</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.014</td>
<td>0.01</td>
<td>0.044</td>
<td>0.063</td>
<td>0.035</td>
</tr>
<tr>
<td>Romania</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.023</td>
<td>0.437</td>
<td>0.124</td>
<td>0.06</td>
<td>0.023</td>
<td>-</td>
</tr>
<tr>
<td>Russia</td>
<td>-</td>
<td>0.962</td>
<td>0.023</td>
<td>0.16</td>
<td>0.132</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.068</td>
</tr>
<tr>
<td>South Africa</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.01</td>
<td>0.161</td>
<td>0.025</td>
<td>0.072</td>
<td>0.073</td>
</tr>
<tr>
<td>South Korea</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.014</td>
<td></td>
</tr>
<tr>
<td>Turkey</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.121</td>
<td>0.307</td>
<td>0.12</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Venezuela</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.029</td>
<td>0.126</td>
<td>0.083</td>
<td>0.06</td>
<td>0.074</td>
</tr>
</tbody>
</table>

The QPS is similar to the mean squared error where the true value of a quantity is subtracted from its estimated value. Instead of the true value of the default probability, which is an (even ex post) unobservable variable, our dummy variable, KD, is used to indicate whether a default occurs in the forecast period. Since the default probabilities are estimated for an annual forecast period, we evaluate the probabilities with annual data. Thereby, we consider single calendar years and compare the dummy indicating whether a default has occurred in the year considered (and the respective country) with the default probability being estimated for the beginning of this year.

Our QPS takes a value of 0.20. This value is relatively small in comparison to other studies (see Table 3). For example, Berg and Pattillo (1999) obtain a QPS between 0.226 and 0.237 by the use of a Probit model and a QPS between 0.267 and 0.270 by the use of a signal approach. The QPS values in Ho (2004) are between 0.198 and 0.314 for Markov switching models and between 0.192 and 0.314 for Logit models. These results concern in-sample estimations. By contrast, our estimation methodology needs no future data which are unknown at the date of estimation. Hence, comparing with out-of-sample studies is more appropriate. As can be seen in Table 3, the results for out-of-sample estimations reported by Berg and Pattillo are even worse than the results just reported. So, we conclude this study to present quite reliable estimations of default probabilities.

Table 3. QPS values for several studies

<table>
<thead>
<tr>
<th>Study</th>
<th>Methodology</th>
<th>QPS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present study</td>
<td>Structural approach</td>
<td>0.201</td>
</tr>
<tr>
<td>Berg and Pattillo (1999)</td>
<td>Signal approach</td>
<td>0.267 - 0.270</td>
</tr>
<tr>
<td></td>
<td>Probit model</td>
<td>0.226 - 0.237</td>
</tr>
<tr>
<td>Berg and Pattillo (1999)</td>
<td>Signal approach</td>
<td>0.398 - 0.402</td>
</tr>
<tr>
<td></td>
<td>Probit model</td>
<td>0.281 - 0.325</td>
</tr>
<tr>
<td>Ho (2004)</td>
<td>Markov-Switching-model</td>
<td>0.198 - 0.440</td>
</tr>
<tr>
<td></td>
<td>Logit-model</td>
<td>0.192 - 0.314</td>
</tr>
</tbody>
</table>

This comparison has to be taken with care since papers cited in Table 3 focus on financial crises, which are defined in the sense of a currency crisis rather than a debt crisis (and, beside this, they consider different data samples). Nevertheless, the results are quite promising and give incentives for further work on the estimation of sovereign default risk with structural models.

Especially, the determination of the state variable requires further research. The simple approach based on foreign exchange reserves, as proposed here, is not without problems. On one hand, the ability to pay can

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1 It is worth mentioning that the occurrence of debt and currency crisis is highly correlated (see Reinhart, 2002). In fact, the most debt crises in our sample are accompanied by a currency crisis.
be underestimated since the government has the possibility to acquire more foreign capital by raising new credits if possible (see Karmann and Maltritz, 2004). So, higher developed countries, as for example Hungary, have relatively low foreign exchange reserves (in relation to GDP) when compared to less developed countries. This may be explained by the fact that a country like Hungary is not forced to hold high amounts of foreign exchange reserves (and to bear the resulting costs) since it is able to raise new foreign capital if necessary. By contrast, other countries may face considerable problems to acquire international liquidity. Hence, they may hoard foreign exchange reserves. Indeed, in the last years we observe a change in policy since quite a number of developing countries have accumulated high amounts of foreign exchange reserves (see, e.g., India).

On the other hand, the state variable, i.e. the effective payments, can be lower than the existing amount of foreign exchange reserves in case of problems of unwillingness to pay which plays a crucial role for the occurrence of sovereign defaults. So, the high amounts of foreign exchange reserves, accumulated by emerging countries during the last years, may be misleading when calculating the true volume of assets available for international debt servicing.

The problem for quantification is that not all influencing factors are observable and quantifiable. Hence, it is difficult to estimate the state variable based on its causes. One possibility to overcome this problem is to simultaneously estimate the state variable and its parameters based on market data, as done in Maltritz (2006).

Furthermore, the evaluation of the default probabilities, based on statistical tests, requires future research. Thereby, and in contrast to corporate risk, typically just a few heterogeneous observations are available for evaluation. This problem is addressed in Huschens, Karmann, Maltritz and Vogl (2006) where statistical tests for heterogeneous default probabilities are developed.

**Conclusion**

We estimate the default risk of sovereign states by using a structural model. Structural models assume that a stochastic state variable determines a default to occur if the state variable falls below a certain threshold value.

The application is based on macroeconomic fundamentals as well as market data from government bond markets. The stochastic state variable is identified as the ability to pay and approximated by the foreign exchange reserves. The volatility is estimated based on market data, using the Black-Scholes formula for put options.

The model is applied to a sample of 17 emerging market countries for the years from 1994 to 2002. The evaluation of the estimated default probabilities for this data sample yields reliable and promising results. So, the QPS value is on the lower bound of QPS values reported in similar papers.

This supports future work on the topic of quantifying sovereign risk by structural models.

**References**