The International Fisher Effect: theory and application

Abstract
This paper uses an asset pricing based approach to derive an international version of the Fisher effect, denoted the “International Fisher Effect”, and tests it for the US and the UK interest rates and inflation rates differentials. We apply the casewise bootstrap technique that is robust to heteroscedasticity and non-normality, which usually characterize financial data. We also allow for a structural break in October 1987 in our estimations. The results show that the international Fisher effect is slightly less than unity. This means that nominal interest rates differential responds less than point-for-point to the changes in the inflation rates differential. The implication of this empirical finding is explained in the main text.

Keywords: interest rates, inflation rates, international Fisher effect, casewise bootstrap, structural break.

JEL Classification: G15, E40, E43, C22.

Introduction
The relationship between the nominal interest rates and the expected inflation is of fundamental importance in financial markets. In his seminal book Fisher (1930) establishes the foundation of the underlying relationship between the nominal interest rate and the purchasing power of money measured by the inflation rate. The response of the nominal interest rate to the inflation rate is known as the Fisher effect in the literature and it is of paramount importance pertinent to the efficiency of the financial markets and the performance of the monetary policy. The Fisher effect predicts that the real interest rate is not affected by the changes in the expected inflation rate because it results in equal changes in the nominal interest rate. This implies the nominal interest rate responds one-for-one to the expected inflation rate, which in turn implies the long-run real interest rate is established in the real sector of the market by means of “technology and preferences”.

Despite its sound theoretical foundation, the full Fisher effect has not been strongly supported empirically (Hatemi-J and Irandoust, 2008). The estimated slope coefficients in regressions of nominal interest rates on different measures of expected inflation rates are significantly different than the theoretical value of unity. Fama and Gibbon (1982), Huizinga and Mishkin (1986) and Kandel et al., (1996) found that real interest rates were negatively related to the expected inflation rates. Crowder and Wohar (1999) showed the Fisher effect is similar for taxable and non-taxable interest rates in the US and the Fisher effect was found to usually be less than unity. According to the literature, there are several reasons for not finding a full Fisher effect. In a seminal paper Tobin (1969) argues that investors shift their portfolios towards real assets if the expected inflation rate becomes remarkably high. The money illusion phenomenon is expressed as another explanation for not finding a full Fisher effect (Modigliani and Cohn, 1979; Tanzi, 1980; Summers, 1983). The existence of peso problems in the market for nominal debt is additional argument put forward by Evans and Lewis (1995)1. The fourth reason, suggested by Fried and Howitt (1983), is the existence of a liquidity premium included in financial assets that increases when expected inflation rate increases. It can also be argued that one of the reasons for not finding a full Fisher effect might be due to parameter instability. Previous studies take usually for granted that the estimated parameter values remain constant across the time. However, there are many reasons to expect that structural breaks can take place. Changes in peoples’ preferences and their behavior, major technological advancements, financial crises, policy alteration, institutional and organizational development can result in structural breaks. In addition, both the interest rates and the inflation rates are usually non-normal and heteroscedastic, which calls into question the application of standard methods.

The aim of this paper is to derive an international version of the Fisher effect using an asset pricing based approach. We call this phenomenon the international Fisher effect. To our best knowledge, this is a notation that has been introduced in this paper. It should be pointed out that all previously mentioned arguments for not finding a full Fisher effect might be valid for an international Fisher effect also. The exchange rate risk and the existence of transaction costs, especially for trading across international markets, might be additional factors for not finding a full international Fisher effect. We also test whether this effect is empirically supported between

the US and the UK economies. The international Fisher effect has implications for market integration and market efficiency. A full international Fisher effect would imply that arbitrage possibilities across economies do not exist. A casewise bootstrap approach that is insensitive to the presence of heteroscedasticity and non-normality is utilized to obtain more precise estimates. The impact of a potential structural break due to October 1987 is also taken into account in the estimations.

The rest of this paper consists of four sections. Section 1 derives an international version of the Fisher equation. Section 2 describes the data and the econometric methodology. Section 3 presents the empirical findings, and the last section offers conclusions.

1. The International Fisher Effect

According to the literature, a standard asset pricing model in which both nominal and real bonds are traded in the domestic market or the foreign market provides the following conditions:

\[ i_t^m = r_t^m + E \left( \pi_{t+m} | \Omega_t \right) + \frac{1}{2} Var_t (\pi_{t+m}) - Cov_t (\pi_{t+m}, d_{t+m}), \]  

\[ i_t^m = r_t^m + E \left( \pi_{t+m}^* | \Omega_t^* \right) + \frac{1}{2} Var_t (\pi_{t+m}^*) - Cov_t (\pi_{t+m}^*, d_{t+m}^*), \]

where \( i_t^m \) is the nominal return on the \( m \)-period bond, \( r_t^m \) represents the return in real terms, \( \pi_t^m \) signifies the inflation rate between the periods \( t \) and \( t + m \). Both nominal and real returns are assumed to be continuously compounded. The notation \( d_{t+m} \) stands for the real discounting factor that is a function of the consumption growth in the consumption based capital asset pricing model. The expectation operator based on information set available in period \( t \), i.e. \( \Omega_t \), is denoted by \( E \). The denotations \( Var_t \) and \( Cov_t \) represent the variance and covariance measures, which are also based on the information available in period \( t \). A star above a variable indicates that variable being a foreign variable. Equation (1) states that the nominal interest rate is a linear function of the real interest rate, the inflation rate and a risk premium that is measured by the second movements of \( \pi_{t+m}^m \) and \( d_{t+m}^m \). A similar result holds for the foreign market according to equation (2). By applying the rational expectations hypothesis we have

\[ E \left( \pi_{t+m} | \Omega_t \right) = \pi_{t+m} + \epsilon_{t+m}, \]  

\[ E \left( \pi_{t+m}^* | \Omega_t^* \right) = \pi_{t+m}^* + \epsilon_{t+m}^*. \]

where \( \epsilon_{t+m} \) and \( \epsilon_{t+m}^* \) are white noise error terms. Assuming an insignificant risk premium in each market and applying equations (3) and (4) we can express equations (1) and (2) as:

\[ i_t^m = r_t^m + \pi_{t+m} + \epsilon_{t+m}, \]

\[ i_t^m = r_t^m + \pi_{t+m}^* + \epsilon_{t+m}^*. \]

If the steady state value of the real interest rate is constant, then the nominal interest rate responds one-for-one to the expected inflation rate according to the Fisher effect formulated in equations (5) and (6). In an international version of the Fisher effect the interest rate differential between the two countries should be equal to their expected inflation differential. This means that we can combine the equations (5) and (6) to obtain the following international version of the Fisher equation:

\[ \left( i_t^m - i_t^m \right) = \left( r_t^m - r_t^m \right) + \left( \pi_{t+m} - \pi_{t+m}^* \right) + \left( \epsilon_{t+m} - \epsilon_{t+m}^* \right). \]

It should be pointed out that the international Fisher effect can be higher than one if the interest income is imposed to taxation. This point is shown in the Appendix.

2. Data and methodology

The source of the data used in this study is the International Financial Statistics (CD-ROM). The data frequency is monthly and it covers observations on short-term nominal interest rates and CPI inflation for the US and the UK economies. The sample period is 1964:M1-2007:M1.

By putting \( m = 1 \) we can represent equation (7) in the form of the following regression relationship:

\[ \Delta i_t = a + b \Delta \pi_{t+1} + \epsilon_t, \]  

where \( \Delta \) represents the difference between the domestic and the foreign variable, and \( a \) and \( b \) are parametric coefficients to be estimated. The error term is denoted by \( \epsilon_t \) and it is assumed to be a white noise process. To explore the international Fisher effect when there is a potential structural break, we extend equation (8) as:

\[ \Delta i_t = a_1 + a_2 I_t + b_1 \Delta \pi_{t+1} + b_2 I_t \Delta \pi_{t+1} + v_t, \]

where \( a_1, a_2, b_1 \) and \( b_2 \) are parametric constants to be estimated. \( I_t \) is a dummy indicator that is equal to
zero for the period before October 1987 and it is equal to one for each observation during the period after the break. The denotation \( v \) is a stochastic error term, which does not have necessarily to be homoscedastic or non-normally distributed. The break period is selected to be at 1987:M9 due to the Black Monday stock market crash, which took place on October 19, 1987. By the end of this month, stock markets in the UK and the US had fallen 26.4% and 22.68%, respectively.

It is widely accepted that the probability of extreme events in the financial markets is much higher than what a normal distribution would suggest. To take this issue into account in our estimations we apply a casewise bootstrap approach, which has been developed recently by Hatemi-J and Hacker (2005). This method is robust to heteroscedastic and non-normal error term in the regression and it is also used to test the statistical significance of these estimated coefficients. In order to make the presentation more compact we represent equation (9) in matrix format as the following:

\[
Y = ZB + \nu , 
\]

\[
Y = \begin{bmatrix} \Delta i_1 \\ \Delta i_2 \\ \vdots \\ \Delta i_T \end{bmatrix} \text{ a } (T \times 1) \text{ vector,}
\]

\[
Z = \begin{bmatrix} 1 & \Delta \pi_2 & I_1 & I_1 \Delta \pi_2 \\ 1 & \Delta \pi_3 & I_2 & I_2 \Delta \pi_3 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \Delta \pi_{T+1} & I_T & I_T \Delta \pi_{T+1} \end{bmatrix} \text{ a } (T \times 4) \text{ matrix,}
\]

\[
B = \begin{bmatrix} a_1 & a_2 & b_1 & b_2 \end{bmatrix} \text{ a } (4 \times 1) \text{ vector,}
\]

\[
v = \begin{bmatrix} \nu_1 \\ \nu_2 \\ \vdots \\ \nu_T \end{bmatrix} \text{ a } (T \times 1) \text{ vector.}
\]

The ordinary least squares estimator for the parameter vector is obtained by calculating the following:

\[
\hat{B} = (Z'Z)^{-1} Z'Y . 
\]

By using these denotations, we describe the casewise bootstrap technique to be performed via the following steps:

1. Create \( Y \) and \( Z \) by resampling with replacement and denote them \( Y^* \) and \( Z^* \), i.e. generate:

\[
Y^* = \{Y_1^*, Y_2^*, \ldots, Y_n^*\},
\]

\[
Y_i^* \in Y \forall i , \text{ where } i = 1, \ldots, n.
\]

\[
Z^* = \{Z_1^*, Z_2^*, \ldots, Z_n^*\},
\]

\[
Z_i^* \in Z \forall i , \text{ where } i = 1, \ldots, n. \text{ The denotation } n \text{ represents the bootstrap sample size.}
\]

2. Calculate the parameter vector by using \( Y^* \) and \( Z^* \) and denote it \( \hat{B} \), that is, estimate

\[
\hat{B} = \left(Z^* Z^* \right)^{-1} Z^* Y^* .
\]

3. Repeat steps one and two \( N \) times, where \( N \) is the number of bootstrap iterations, which is 10000 in this study.

4. Calculate the casewise bootstrap coefficient vector \( \hat{B}^* \) via:

\[
\hat{B}^* = \frac{\sum_{j=1}^{N} \hat{B}_j}{N} .
\]

The casewise bootstrap method is also used to obtain the p-values for all elements in the parameter vector \( \hat{B}^* \). For example, let us concentrate on the situation in which the null hypothesis that is tested is \( a_i = 0 \). We obtain the p-value for this hypothesis by ranking the calculated values for \( \hat{B}^* \) as the first step. If the estimated value of the median is a positive value for \( a_i^* \), then the p-value is the percentage of elements in the bootstrap distribution for \( a_i^* \) that are negative added to those that are greater than twice the median. If the estimated median for \( a_i^* \) is negative, the p-value is the percentage of elements in the bootstrap distribution for \( a_i^* \) that are positive plus the percentage of elements in \( a_i^* \) that are less than twice the median.

The cut-off point of twice the median of \( a_i^* \) is equivalent to p-values that are similar to those symmetric two-sided tests in a traditional hypothesis testing approach as shown by Hatemi-J and Hacker (2005). The p-values for the other parameters are estimated in a similar way. All the bootstrap simulations in this paper are conducted by using a GAUSS program code, which is accessible upon request.
3. The results

Table 1 presents the estimation results applying the casewise bootstrap method. Based on these results we can observe that the intercept as a measure of risk premium is positively significant. There is also a significant break in this intercept, which suggests a statistically significant increase in the intercept after October 1987 period. The slope parameter that represents the international Fisher effect is statistically significant. There is no statistically significant break in this slope. Thus, the October 1987 event has resulted in a break in the risk premium but not in the international Fisher relation between the US and the UK. However, the estimated value of the slope is slightly less than one. Finding an international Fisher effect less than unity might be explained by the arguments expressed in the introduction of this paper.

Table 1. The estimation results using the casewise bootstrap method

<table>
<thead>
<tr>
<th>Estimated value</th>
<th>Intercept (a1)</th>
<th>Change in intercept (a2)</th>
<th>Slope (b1)</th>
<th>Change in slope (b2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>p-value</td>
<td></td>
<td>0.449</td>
<td>0.772</td>
<td>-0.291</td>
</tr>
</tbody>
</table>

Conclusions

This paper derives the international Fisher effect analytically using an asset pricing approach and tests it empirically using the casewise bootstrap method, which performs accurately when the error term in the regression is heteroscedastic and non-normally distributed. We also allow for a break due to the October 1987 stock market crash. The sample covers the period of 1964:M1-2007:M1. Monthly data for the US and the UK markets are used. The estimated results reveal that the October 1987 stock market crash has resulted in a significant break in the risk premium but has not resulted in any break in the international Fisher relation between the two economies. The interest rates differential between these countries does positively and significantly respond to their inflation rates differential by the same amount for the pre-break and post-break periods. Nevertheless, this response is slightly less than unity. However, the parameter value that we obtained is close to unity. Hence, taking into account the transaction costs and the existence of an exchange rate risk premium, earning arbitrage profits may still not be possible. Thus, the markets may still be considered as efficient.

References


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1. It should be pointed out that we tested each differential variable for unit roots. The results, not reported, were supporting the stationary property of each variable.


**Appendix. The tax-adjusted International Fisher Effect**

According to Darby (1975), if the interest income is imposed to taxation then the Fisher effect might be higher than one. To see this point analytically, we assume that the interest income is taxed by \( \tau \) decimal points in the domestic market and by \( \tau^* \) decimal points in the foreign market. In this case the nominal returns are equal to \((1-\tau)m^d\) and \((1-\tau^*)m^f\), respectively. By substituting these values into equations (5) and (6) we can obtain the following tax-adjusted Fisher equations:

\[
i_{t}^{m} = \left[ \frac{1}{1-\tau} \right] r_{t}^{m} + \left[ \frac{1}{1-\tau} \right] \pi_{t+m} + \left[ \frac{1}{1-\tau} \right] \varepsilon_{t+m}, \tag{A1}\]

\[
i_{t}^{f} = \left[ \frac{1}{1-\tau^*} \right] r_{t}^{f} + \left[ \frac{1}{1-\tau^*} \right] \pi_{t+m} + \left[ \frac{1}{1-\tau^*} \right] \varepsilon_{t+m}. \tag{A2}\]

Equations (A1) and (A2) show that the Fisher effect is higher than one in the domestic market if \( \tau > 0 \) and it is higher than one in the foreign market if \( \tau^* > 0 \). By taking the interest rate differential using equations (A1) and (A2) we obtain the following tax-adjusted international Fisher equation:

\[
\left( i_{t}^{m} - i_{t}^{f} \right) = \left( \left[ \frac{1}{1-\tau} \right] r_{t}^{m} - \left[ \frac{1}{1-\tau^*} \right] r_{t}^{f} \right) + \left( \left[ \frac{1}{1-\tau} \right] \pi_{t+m} - \left[ \frac{1}{1-\tau^*} \right] \pi_{t+m} \right) + \left( \left[ \frac{1}{1-\tau} \right] \varepsilon_{t+m} - \left[ \frac{1}{1-\tau^*} \right] \varepsilon_{t+m} \right). \tag{A3}\]

Assuming that the tax rates are equal to each other in the two markets we can express equation (A3) as

\[
\left( i_{t}^{m} - i_{t}^{f} \right) = \left[ \frac{1}{1-\tau} \right] \left( r_{t}^{m} - r_{t}^{f} \right) + \left[ \frac{1}{1-\tau} \right] \left( \pi_{t+m} - \pi_{t+m} \right) + \left[ \frac{1}{1-\tau} \right] \left( \varepsilon_{t+m} - \varepsilon_{t+m} \right). \tag{A4}\]

Thus, the tax-adjusted international Fisher effect can be higher than one.