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Income distribution, quality differentiation and product line design

Abstract

The purpose of this paper is to characterize the relationship between quality differentiation, income distribution and product line design. According to the findings, the author can explain the trend of quality differentiation and the phenomenon of extreme product diversity in different conditions of income distribution. When the middle class of consumers reach a certain critical few number, the quality for them will descend. Only the high-end and low-end quality left for this extreme condition; that is, the product quality spectrum will shrink. The product quality for middle and low class will gradually get worse and worse, even lower than original quality. The product line design will be reconsidered to the opposite extremes.

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Introduction

In economics, income distribution is how a nation's total GDP is distributed amongst its population, and it has always been a central concern of economic theory and policy. Modern economists have also addressed this issue, but have not been more concerned with the distribution of income across individuals and households. Major important theories and policies concern the relationship between income inequality and economic growth.

However, our focus is to discuss the relationship between income distribution and quality differentiation. We are interested in situation where what is the effect on quality differentiation and product line design of firm when the income distribution in society becomes more inequal. This paper suggests a model accounting for such situation and characterizes the subgame equilibriums for a monopoly firm. We prove that when the income distribution is growing into more inequality, the quality differentiation will become more extreme. Besides, when the middle class of consumers becomes few enough, then, the product quality for them will descend, that is the quality spectrum will shrink and the product line design will be reconsidered to the opposite extremes.

In the seminal paper by Mussa and Rosen (1978), they consider a monopolist face a pool of privately informed consumers, which differ in terms of their willingness-to-pay for quality of product. Their conclusion is that in equilibrium: (1) Each consumer is allocated a distinct quality. (2) Quality provision is distorted: all consumers except for the highest type consume inefficiently low qualities and the monopolist enlarges the quality range, relative to the efficient outcome, with the broadening occurring at the low end of quality at the same time.

There is a large theoretical literature on quality differentiation which extends the work of Mussa & Rosen (1978) and Maskin, E. & J. Riley (1984) in various directions. Examples include Moorthy (1984) who considers non-linear preferences and shows that this could induce the monopolist to aggregate distinct consumer segments into one segment; Srinagesh and Bradburd (1989) examined a case where there is a negative correlation between total and marginal utility of quality across consumer types. They show that quality may be enhanced, not degraded, at the higher ends of quality, while there is no distortion at the lowest quality.

Rochet and Stole (1999) proposed a more general framework where consumers have random outside options. They find that the profit-maximizing quality provision produces either no distortion on the boundaries in a fully separating equilibrium or efficiency-on-the-top with bunching over a range of low types. Johnson and Myatt (2003) emphasized its usefulness in analyzing product line and pricing choices of multi-product firms in monopoly and Cournot duopoly contexts.

Product line design is a critical decision that determines some successes of firms (Hauser, Tellis, and Griffin, 2006). These studies investigate product line design from a marketing perspective (e.g., Balakrishnan, Gupta, and Jacob, 2004, 2006; Belloni et al., 2008; Lan Luo, 2010). Orhun (2009) studies optimal product line design when consumers exhibit choice-set-dependent preferences. Liu and Cui (2010) allow the monopolist to extend its

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product line depending on whether it sells through a centralized channel or a decentralized channel; and Guo and Zhang (2012) study optimal product line design when consumers must incur deliberation costs to uncover their valuations for quality.

Our analysis builds on these studies and considers a discrete number of product quality like in the Mussa-Rosen framework. We analyze the effects on the product line of introducing the income distribution and complement them by showing that, if the middle class of consumers are small enough, there will be a bunching equilibrium. The product line of firm will incline to the high-end or the low-end and the middle class will decline gradually.

The remainder of this paper is organized as follows. Section 1 describes the basic model. Section 2 develops some structural properties, and uses them to characterize some interesting points and presents the main results, including the seller's optimal policy. Finally, last section offers concluding remarks and discusses various ways our basic model can be extended. All proofs are presented in the Appendix.

1. The model

The used model is an extension of the well-known model of Mussa and Rosen (1978) and Guo and Zhang (2012). We consider a single product monopolist operating in a market for a vertically differentiated good where consumers are characterized by a marginal willingness to pay for quality $\Theta = \{\underline{\theta}, \hat{\theta}, \overline{\theta}\}$ with $\underline{\theta} < \hat{\theta} < \overline{\theta}$ and $\underline{\theta} > 0$; $\underline{\theta}$ is the lowest degree of willingness to pay, $\hat{\theta}$ is the middle degree of willingness to pay and $\overline{\theta}$ is the highest degree of willingness to pay. The willingness to pay of consumers is private information. For convenience, we suppose $\overline{\theta} - \hat{\theta} = \hat{\theta} - \underline{\theta} = \Delta\theta$; that is, the differences among consumer's types are simply the same. The population of consumer is uniformly distributed over the interval with density 1, so that the total number of consumers in this market is also 1. The corresponding probability is, respectively, $\underline{\nu}, \hat{\nu}, \overline{\nu}$ and $\underline{\nu} + \hat{\nu} + \overline{\nu} = 1$.

Individuals derive a net surplus from consumption defined as follows:

$$U = \theta q - p,$$

where q is the quality level and p is the price at which such a quality is being supplied. So, for different types of consumers, the net surplus will be $\underline{U} = \underline{\theta}\underline{q} - \underline{p}$, $\hat{U} = \hat{\theta}\hat{q} - \hat{p}$, $\overline{U} = \overline{\theta}\overline{q} - \overline{p}$. The production cost of firm is $C = C(q)$, we assume that $C'(q) > 0$ and $C''(q) > 0$; that is, the higher the quality of the product, the higher the marginal cost. We can imagine the above situation as follows: the firm faces a group of consumers who have different willingness to pay and it offers the following choices $(\underline{p}, \underline{q})$, $(\hat{p}, \hat{q})$ and $(\overline{p}, \overline{q})$ to the consumers. The goal of the firm is to set the price in order to maximize the profit. Since the firm doesn't know the real type of consumers, the choices it offers definitely satisfy the incentive constraints and participation constraints of consumers. For the consumer with highest degree willingness to pay, the incentive constraints must satisfy

$$\overline{U} \geq \hat{U} + \Delta\theta\hat{q}, \tag{1}$$

$$\overline{U} \geq \underline{U} + 2\Delta\theta\underline{q}. \tag{2}$$

For the consumer with middle degree willingness to pay high, the incentive constraints must satisfy

$$\hat{U} \geq \underline{U} + \Delta\theta\underline{q}, \tag{3}$$

$$\hat{U} \geq \overline{U} - 2\Delta\theta\underline{q}. \tag{4}$$

For the consumer with lowest degree willingness to pay high, the incentive constraints must satisfy

$$\underline{U} \geq \hat{U} - \Delta\theta\hat{q}, \tag{5}$$

$$\underline{U} \geq \overline{U} - 2\Delta\theta\overline{q}. \tag{6}$$

Then, we will consider which incentive constraints are binding. First, we add (1) to (4), and we get $\overline{q} \geq \hat{q}$; then, (3) and (5) are being put together, and we get $\hat{q} \geq \underline{q}$. In sum, we can get

$$\overline{q} \geq \hat{q} \geq \underline{q}. \tag{7}$$

This is the monotonicity constraint and it helps us to simplify the incentive constraints. We can get

$$\overline{U} \geq \underline{U} + \Delta\theta(\hat{q} + \underline{q}). \tag{8}$$

The participation constraints of consumers for different types are

$$\overline{U} \geq 0, \tag{9}$$

$$\hat{U} \geq 0, \tag{10}$$

$$\underline{U} \geq 0. \tag{11}$$

In the above participation constraints, the only binding constraint is (11). If there is no binding at all, firm will reduce the reservation utility of all consumers and it will increase the profit without violating participation constraints. We insert the binding conditions (1), (3) and (11) into the optimal problem. Now we can show the optimal behavior of the firm in the following formula:

$$\underset{\{\underline{q},\hat{q},\overline{q}\}}{Max}\ \overline{\nu}\cdot(\overline{\theta}\overline{q}-C(\overline{q}))+\hat{\nu}\cdot(\hat{\theta}\hat{q}-C(\hat{q}))+\underline{\nu}\cdot(\underline{\theta}\underline{q}-C(\underline{q}))-\overline{\nu}\cdot\Delta\theta(\hat{q}+\underline{q})-\hat{\nu}\cdot\Delta\theta\underline{q},\quad s.t.\ \overline{q}\geq\hat{q}\geq\underline{q}.$$

1.1. Under complete information. We characterize the first-best outcome as a benchmark. Suppose now that the firm has complete information about the true θ value of the consumers and, therefore, it can price-discriminate perfectly. It only needs to satisfy the participation constraints of consumers. After calculation, we can get the optimal condition of firm:

$$\overline{\theta}=C'(\overline{q}^{*}),\tag{12}$$

$$\hat{\theta}=C'(\hat{q}^{*}),\tag{13}$$

$$\underline{\theta}=C'(\underline{q}^{*}).\tag{14}$$

The above condition is the complete information efficient production level. That is, the consumer's willingness to pay equals to the marginal cost of the firm, no matter what kind of the type it is. This is the first best equilibrium.

1.2. Under incomplete information. When the types of the consumers are private information, the optimal condition entails the following:

(1) when $\hat{\nu}>\overline{\nu}\underline{\nu}$, the monotonicity conditions are strictly satisfied:

$$\overline{\theta}=C'(\overline{q}^{SB}),\tag{15}$$

$$\hat{\theta}=C'(\hat{q}^{SB})+\frac{\overline{\nu}}{\hat{\nu}}\Delta\theta,\tag{16}$$

$$\underline{\theta}=C'(\underline{q}^{SB})+\frac{\overline{\nu}+\hat{\nu}}{\underline{\nu}}\Delta\theta.\tag{17}$$

In the above condition of incomplete information, we get $\overline{q}^{SB}=\overline{q}^{*}$, but $\hat{q}^{SB}<\hat{q}^{*}$ and $\underline{q}^{SB}<\underline{q}^{*}$, that is the firm will offer the same quality to the highest level consumers, whereas provide middle and lowest level consumers with lower quality products. The highest type consumer level is not distorted and his information rent is $\overline{U}=\Delta\theta(\hat{q}+\underline{q})$, depends now on the consumption levels of all types who are less efficient than the efficient type, those consumption levels must be distorted downward to reach the optimal rent extraction-efficiency trade-off. The reason for this expression of the agent rent is that all the local upward incentive constraints, and only those constraints, are binding. While the information rent of the middle class consumers is $\hat{U}=\Delta\theta\underline{q}$, because he can pretend to be a lowest class consumer. The lowest class consumers get a zero rent $\underline{U}=0$. Such information rent all meets the downward incentive constraints of (4), (5) and (6);

(2) when $\hat{\nu}\leq\overline{\nu}\underline{\nu}$, some bunching equilibrium emerges. We still have $\overline{q}^{SB}=\overline{q}^{*}$, but now $\hat{q}^{SB}=\underline{q}^{SB}=q^{P}<\overline{q}^{*}$, with $\hat{\theta}=C'(\hat{q}^{P})+\frac{1+\overline{\nu}}{1-\overline{\nu}}\Delta\theta$. It means that the highest class consumers will have the information rent $2\Delta\theta q^{P}$, the middle class consumers will have the information rent $\Delta\theta q^{P}$, such different information rent also meets the downward incentive constraints of (4), (5) and (6).

2. Discussion and analysis

Last section we discuss when information is incomplete, the firm will provide relatively poor quality products to the low class consumers. This argument also be found in Mussa & Rosen (1978), Srinagesh & Bradburd (1989) and Srinagesh et al. (1992). It's because the firm does not understand the true type of consumers, so the spectrum of qualities will be larger under asymmetric information than under complete information. The highest and middle class consumers will have some information rent, respectively, $\Delta\theta(\hat{q}+\underline{q})$ and $\Delta\theta\underline{q}$.

Proposition 1: With the reduction of middle class consumers, the middle class and low class consumer product quality will be getting closer and closer.

Proof: see Appendix 2.

It is an interesting point from the equilibrium. Under asymmetric information, the firm will provide $\overline{\nu}\cdot\Delta\theta(\hat{q}+\underline{q})+\hat{\nu}\cdot\Delta\theta\underline{q}$ information rent to the highest class and middle class consumer. When the middle class consumers become less and less, it means that the consumers that the firm faces are either those highest class or those lowest class. The firm will reduce the product quality for middle class consumers to decrease the information rent for the highest class consumer, at the same time, the firm will let the middle class consumers to have more information rent $\Delta\theta\underline{q}$ for the differentiation whether it is the middle class consumers or not.

This tendency seems to correspond to the current phenomenon in the society. We often see that with uneven income distribution, brand-name products or

high quality goods with high price still sell well and cheap commodity remain all right, whereas middle class products usually lose its market.

If the middle class consumers keep decreasing and, as a result, the society grows to be M-shape, then, the firm doesn't need to differentiate the middle class consumers. We have the following proposition, therefore:

Proposition 2: When the society has been the M-shape society, product quality offered by the firm will develop to two extremes and the product quality for the middle and the lowest class consumers will get worse.

Proof: see Appendix 3.

This proposition explains that as the middle class consumers get fewer and fewer, the society grows extremely as M-shape, the firm will provide the products of same quality to the middle and low class consumers in order to reduce the information rent. In this case, the information rent for high class consumers is $2\Delta\theta\hat{q}^P$, while the information rent for middle class consumers is $\Delta\theta\hat{q}^P$.

For the middle and lowest class consumers, their product quality is $\hat{q}^P$, which is worse than what they got in non-M-shape society $\underline{q}^{SB}$, because the firm doesn't need to differentiate the middle class consumers anymore. To save information rent, the firm cut down the product quality for the middle and low class consumers.

The firm will offer different product qualities for the consumers of different classes, but the quality is insufficient relative the first-best. With the reduction of the middle class, the product quality for the lowest class gets better and better, so the product qualities offered for the middle and lowest class consumers are gradually similar. However, once reaching the critical point, the product quality for middle and lowest class consumers will worsen at the same time, even worse than second-best. The reason is that the firm doesn't need to differentiate the middle and lowest class consumers in this situation. To obtain bigger profit and save information rent, they provide middle and lowest class consumers with the lowest product quality. This is the conclusion and contribution we make here, which can't be found in other studies.

Conclusion

Following the hypothesis about quality differentiation and consideration of income distribution, we think that the different income distributions will have some effects on product qualities and product line design. We can explain (1) when the numbers of the middle class consumers decrease, there is no influence on highest quality products, while the product quality for middle class consumers will get worse, but the lowest class will get better, so the product quality for middle and lowest class consumers tends to be close. (2) When the population of middle class consumers are lower than some certain critical point, the product for middle class consumers will disappear, with only the highest and lowest quality left for extremes and the product quality for middle and lowest class consumers will gradually get worse and worse, even lower than the original quality.

The future studying direction of our research is to discuss the relationship between income distribution, product quality and product line design in the framework of oligopoly market. By studying the interaction of firm behavior, our research will reach for the more reality. Another direction of our future research is the effect of different degrees of income distribution on product quality when the utility of consumers in different classes interact. Also, we could develop a data simulation approach that handles both discrete and continuous design variables, and might extend this paper by guaranteeing global optimality in a considerably large-scale product line design problem. We think these extensions will be very interesting in the future.

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Appendix A. Proof of the equilibrium (1)

We use the following Lagrange function to solve the equilibrium of firm.

$$\underset{\bar{q},\hat{q},\underline{q}}{\text{Max}}\ \bar{\nu}\cdot(\bar{\theta}\bar{q}-C(\bar{q}))+\hat{\nu}\cdot(\hat{\theta}\hat{q}-C(\hat{q}))+\underline{\nu}\cdot(\underline{\theta}\underline{q}-C(\underline{q}))-\bar{\nu}\cdot\Delta\theta(\hat{q}+\underline{q})-\hat{\nu}\cdot\Delta\theta\underline{q}$$

$$s.t.\ \ \bar{q}\geq\hat{q}\ \ \&\ \ \hat{q}\geq\underline{q}$$

We denote by y_i the multiplier of the constraints. The Lagrange function is, then,

$$Z=\bar{\nu}\cdot[\bar{\theta}\bar{q}-C(\bar{q})]+\hat{\nu}\cdot[\hat{\theta}\hat{q}-C(\hat{q})]+\underline{\nu}\cdot[\underline{\theta}\underline{q}-C(\underline{q})]-\bar{\nu}\cdot\Delta\theta\cdot(\hat{q}+\underline{q})-\hat{\nu}\cdot\Delta\theta\cdot\underline{q}\ldots+y_1\cdot[\bar{q}-\hat{q}]+y_2\cdot[\hat{q}-\underline{q}]\quad(A.1)$$

Using the Kuhn–Tucker condition, that is:

$$\Rightarrow\frac{\alpha Z}{\alpha q_i}\leq 0,\ q_i\geq 0,\ \frac{\alpha Z}{\alpha q_i}\cdot q_i=0\ \ \&\ \ \frac{\alpha Z}{\alpha y_i}\leq 0,\ y_i\geq 0,\ \frac{\alpha Z}{\alpha y_i}\cdot y_i=0$$

So we have

$$\Rightarrow\frac{\alpha Z}{\alpha\bar{q}}=\bar{\nu}\cdot[\bar{\theta}-C'(\bar{q})]+y_1\leq 0,\ \bar{q}\geq 0\ \&\ \{\bar{\nu}\cdot[\bar{\theta}-C'(\bar{q})]+y_1\}\cdot\bar{q}=0\qquad(A.2)$$

$$\frac{\alpha Z}{\alpha\hat{q}}=\hat{\nu}\cdot[\hat{\theta}-C'(\hat{q})]-\bar{\nu}\cdot\Delta\theta-y_1+y_2\leq 0,\ \hat{q}\geq 0\ \&\ \{\hat{\nu}\cdot[\hat{\theta}-C'(\hat{q})]-\bar{\nu}\cdot\Delta\theta-y_1+y_2\}\cdot\hat{q}=0\qquad(A.3)$$

$$\frac{\alpha Z}{\alpha\underline{q}}=\underline{\nu}\cdot[\underline{\theta}-C'(\underline{q})]-\bar{\nu}\cdot\Delta\theta-\hat{\nu}\cdot\Delta\theta-y_2\leq 0\ ,\ \underline{q}\geq 0\ \ \&\ \ \{\underline{\nu}\cdot[\underline{\theta}-C'(\underline{q})]-\bar{\nu}\cdot\Delta\theta-\hat{\nu}\cdot\Delta\theta-y_2\}\cdot\underline{q}=0$$

(A.4)

$$\frac{\alpha Z}{\alpha y_1}=\bar{q}-\hat{q}\geq 0,\ y_1\geq 0\ \&\ y_1\cdot(\bar{q}-\hat{q})=0\qquad(A.5)$$

$$\frac{\alpha Z}{\alpha y_2}=\hat{q}-\underline{q}\geq 0,\ y_2\geq 0\ \&\ y_2\cdot(\hat{q}-\underline{q})=0\qquad(A.6)$$

We have a little different conditions:

1. If $\frac{\alpha Z}{\alpha y_1}=\bar{q}-\hat{q}>0$ ， $y_1=0$ & $\frac{\alpha Z}{\alpha y_2}=\hat{q}-\underline{q}>0$ ， $y_2=0$ ：

Then, we can get $\bar{\theta}=C'(\bar{q})$ from (A.2); $\hat{\theta}=C'(\hat{q})+\frac{\bar{\nu}}{\hat{\nu}}\Delta\theta$ from (A.3); $\underline{\theta}=C'(\underline{q})+\frac{\bar{\nu}+\hat{\nu}}{\underline{\nu}}\Delta\theta$ from (A.4). We compare them with the first-best condition, that is $\bar{\theta}=C'(\bar{q}^*)$, $\hat{\theta}=C'(\hat{q}^*)$ & $\underline{\theta}=C'(\underline{q}^*)$,

So we will have $\bar{q}^{sb}=\bar{q}^*$, but $\hat{q}^{sb}<\hat{q}^*$ and $\underline{q}^{sb}<\underline{q}^*$.

2. If $\frac{\alpha Z}{\alpha y_1}=\bar{q}-\hat{q}>0,\ y_1=0$ & $\frac{\alpha Z}{\alpha y_2}=\hat{q}-\underline{q}=0,\ y_2\geq 0$

Then, we can get

$\bar{\theta}=C'(\bar{q})$ from (A.2), $\hat{\theta}=C'(\hat{q})+\frac{\bar{\nu}\Delta\theta-y_2}{\hat{\nu}}$ from (A.3), $\underline{\theta}=C'(\underline{q})+\frac{\bar{\nu}+\hat{\nu}}{\underline{\nu}}\Delta\theta+\frac{y_2}{\underline{\nu}}$ from (A.4).

Because $\hat{q}=\underline{q}$ ，so we have $\hat{\theta}-\frac{\bar{\nu}\Delta\theta-y_2}{\hat{\nu}}=\underline{\theta}-\frac{(\bar{\nu}+\hat{\nu})\Delta\theta+y_2}{\underline{\nu}}$

$$\Rightarrow \frac{\bar{\nu}\Delta\theta - y_2}{\hat{\nu}} = \frac{\Delta\theta + y_2}{\underline{\nu}} \Rightarrow \bar{\nu}\underline{\nu}\Delta\theta - \underline{\nu}y_2 = \hat{\nu}\Delta\theta + \hat{\nu}y_2 \Rightarrow (\bar{\nu}\underline{\nu} - \hat{\nu})\Delta\theta = (\underline{\nu} + \hat{\nu})y_2 \geq 0$$

That is if $\hat{\nu} \leq \bar{\nu}\underline{\nu}$, there will be the bunching equilibrium. We use $y_2 = \frac{(\bar{\nu}\underline{\nu} - \hat{\nu})}{\underline{\nu} + \hat{\nu}}\Delta\theta$

$$\hat{\theta} = C'(\hat{q}) + \frac{\bar{\nu}\Delta\theta}{\hat{\nu}} - \frac{1}{\hat{\nu}} \cdot \frac{(\bar{\nu}\underline{\nu} - \hat{\nu})\Delta\theta}{\underline{\nu} + \hat{\nu}} = C'(\hat{q}) + \frac{\bar{\nu}(\underline{\nu} + \hat{\nu})\theta - (\bar{\nu}\underline{\nu} - \hat{\nu})\Delta\theta}{\hat{\nu}(\underline{\nu} + \hat{\nu})}$$

$$= C'(\hat{q}) + \frac{\bar{\nu}\hat{\nu}\Delta\theta + \hat{\nu}\Delta\theta}{\hat{\nu}(\underline{\nu} + \hat{\nu})} = C'(\hat{q}) + \frac{\Delta\theta(\bar{\nu} + 1)}{\underline{\nu} + \hat{\nu}} = C'(\hat{q}) + \frac{1 + \bar{\nu}}{1 - \bar{\nu}}\Delta\theta$$

So when $\hat{\nu} \leq \bar{\nu}\underline{\nu}$, we have a bunching equilibrium as given below

$\bar{q}^{SB} = \bar{q}^*$, but $\hat{q}^{SB} = \underline{q}^{SB} = q^P < q^*$, & $\hat{\theta} = C'(\hat{q}^P) + \frac{1 + \bar{\nu}}{1 - \bar{\nu}}\Delta\theta$.

Appendix B. Proof of the proposition 1

We have $C'(\hat{q}^{SB}) = \hat{\theta} - \frac{\bar{\nu}}{\hat{\nu}}\Delta\theta$, using total differential, then

$C''(\hat{q}^{SB})d\hat{q}^{SB} = \frac{\bar{\nu}}{\hat{\nu}^2}\Delta\theta d\hat{\nu}$, and we get $\frac{d\hat{q}^{SB}}{d\hat{\nu}} = \frac{\bar{\nu}}{\hat{\nu}^2 C''(\hat{q}^{SB})}\Delta\theta$.

Because the cost function $C(q)$ is convex function, so $\frac{d\hat{q}^{SB}}{d\hat{\nu}} > 0$; that is, with the reduction of middle class consumers, the product quality that the firm sold to middle class consumers will be getting worse and worse.

Besides $C'(\underline{q}^{SB}) = \underline{\theta} - \frac{\bar{\nu} + \hat{\nu}}{\underline{\nu}}\Delta\theta$, still using total differential, then $C''(\underline{q}^{SB})d\underline{q}^{SB} = -\frac{\Delta\theta}{\underline{\nu}}d\hat{\nu}$, and we have $\frac{d\underline{q}^{SB}}{d\hat{\nu}} = \frac{\Delta\theta}{\underline{\nu}C''(\hat{q}^{SB})}$.

Because the cost function $C(q)$ is convex function, so $\frac{d\underline{q}^{SB}}{d\hat{\nu}} < 0$, that is, with the decrease of middle class consumers, the product quality that the firm sold to lowest class consumers will be getting better and better.

Appendix C. Proof of the proposition 2

As $\hat{q} = \underline{q}$ ，so $\hat{\theta} - \frac{\bar{\nu}\Delta\theta - y_2}{\hat{\nu}} = \underline{\theta} - \frac{(\bar{\nu} + \hat{\nu})\Delta\theta + y_2}{\underline{\nu}} \Rightarrow \frac{\bar{\nu}\Delta\theta - y_2}{\hat{\nu}} = \frac{\Delta\theta + y_2}{\underline{\nu}}$

$\Rightarrow \bar{\nu}\underline{\nu}\Delta\theta - \underline{\nu}y_2 = \hat{\nu}\Delta\theta + \hat{\nu}y_2 \Rightarrow (\bar{\nu}\underline{\nu} - \hat{\nu})\Delta\theta = (\underline{\nu} + \hat{\nu})y_2 \geq 0$

That is, when $\hat{\nu} \leq \bar{\nu}\underline{\nu}$, there will be a bunching equilibrium.

Using $y_2 = \frac{(\bar{\nu}\underline{\nu} - \hat{\nu})}{\underline{\nu} + \hat{\nu}}\Delta\theta$, so we have

$$\hat{\theta} = C'(\hat{q}) + \frac{\bar{\nu}\Delta\theta}{\hat{\nu}} - \frac{1}{\hat{\nu}} \cdot \frac{(\bar{\nu}\underline{\nu} - \hat{\nu})\Delta\theta}{\underline{\nu} + \hat{\nu}}$$

$$= C'(\hat{q}) + \frac{\bar{\nu}(\underline{\nu} + \hat{\nu})\theta - (\bar{\nu}\underline{\nu} - \hat{\nu})\Delta\theta}{\hat{\nu}(\underline{\nu} + \hat{\nu})} = C'(\hat{q}) + \frac{\bar{\nu}\hat{\nu}\Delta\theta + \hat{\nu}\Delta\theta}{\hat{\nu}(\underline{\nu} + \hat{\nu})}$$

$$= C'(\hat{q}) + \frac{\Delta\theta(\bar{\nu} + 1)}{\underline{\nu} + \hat{\nu}} = C'(\hat{q}) + \frac{1 + \bar{\nu}}{1 - \bar{\nu}}\Delta\theta$$

When $\hat{\nu} \leq \bar{\nu}\underline{\nu}$, we have $\bar{q}^{SB} = \bar{q}^*$, $\hat{q}^{SB} = \underline{q}^{SB} = q^P < q^*$ and $\hat{\theta} = C'(\hat{q}^P) + \frac{1 + \bar{\nu}}{1 - \bar{\nu}}\Delta\theta$

When the middle class consumers are small enough, the product quality that the firm sold to all-class consumers will be to the development of both ends.

Besides $\underline{\theta} = C'(\underline{q}^{SB}) + \frac{\bar{\nu} + \hat{\nu}}{\underline{\nu}}\Delta\theta$ ，so $C'(\underline{q}^{SB}) = \underline{\theta} - \frac{\bar{\nu} + \hat{\nu}}{\underline{\nu}}\Delta\theta$, and $\hat{\theta} = C'(\hat{q}^P) + \frac{1 + \bar{\nu}}{1 - \bar{\nu}}\Delta\theta$, so $C'(\hat{q}^P) = \hat{\theta} - \frac{1 + \bar{\nu}}{1 - \bar{\nu}}\Delta\theta$ 。

We can compare $\underline{\theta} - \frac{\bar{\nu} + \hat{\nu}}{\underline{\nu}}\Delta\theta$ and $\hat{\theta} - \frac{1 + \bar{\nu}}{1 - \bar{\nu}}\Delta\theta$, so

$\hat{\theta} - \frac{1 + \bar{v}}{1 - \bar{v}}\Delta\theta - \underline{\theta} + \frac{\bar{\nu} + \hat{\nu}}{\underline{\nu}}\Delta\theta = \frac{\hat{v} - \bar{v}\underline{v}}{\underline{v}(1 - \bar{v})}\Delta\theta$. Because $\hat{\nu} \leq \bar{\nu}\underline{\nu}$, so $C'(\hat{q}^P) \leq C'(\underline{q}^{SB})$ and the cost function $C(q)$ is convex function, so $\hat{q}^P \leq \underline{q}^{SB}$.