“The topology of Danish interbank money flows”

AUTHORS	Kirsten Bonde Rørdam Morten L. Bech
ARTICLE INFO	Kirsten Bonde Rørdam and Morten L. Bech (2009). The topology of Danish interbank money flows. *Banks and Bank Systems*, *4*(4)
RELEASED ON	Wednesday, 16 December 2009
JOURNAL	"Banks and Bank Systems"
FOUNDER	LLC “Consulting Publishing Company “Business Perspectives”

NUMBER OF REFERENCES	NUMBER OF FIGURES	NUMBER OF TABLES
0	0	0

Kirsten Bonde Rørdam (Denmark), Morten L. Bech (USA)

The topology of Danish interbank money flows

Abstract

This paper presents the first topological analysis of Danish money market flows. We analyze the structure of two networks with different types of transactions. The first network is the money market network, which is driven by banks' behavior on the interbank market, the second is the network of customer driven transactions, which is driven by banks' customers' transactions demand. We show that the structure of these networks differs.

This paper adds to the new and growing literature on network topological analysis of payment systems.

Keywords: network, topology, payment system, money market.
JEL Classification: E42.

Introduction

The recent financial turmoil has highlighted the central role played by the interbank money markets for the smooth functioning of the financial system and implementation of monetary policy. Liquidity evaporated from many parts of the interbank money market and central banks have intervened in force and have de facto replaced private intermediation with public intermediation.

Thus, understanding the inner workings of the money market is of paramount importance in terms of analyzing and responding to financial turmoil.

Theoretical contributions have discussed whether a complete financial structure, where all banks have cross-holdings on each other, or an incomplete structure, where banks only keep the cross-holdings needed, is optimal for hindering contagion from arising, cf. Allen and Gale (2000), Freixas and Parigi (1998) and Freixas et al. (2000). Basically, this is a choice between liquidity saving (banks can keep smaller liquidity reserves if they can raise liquidity via the interbank market) and contagion risk (banks become fragile towards disturbances – in other banks or the network as a whole – if they use the interbank market). In theoretical models, central banks are assumed to make optimal interventions in the interbank market whereby they can hinder contagion from arising, cf. Freixas (2000). But the risk of contagion effects and central banks' possible actions depend crucially on the actual structures on the interbank market.

The large-value payments system is in general the settlement platform for the interbank money market. The lion share of the money market transactions are settled on this platform. Therefore, disruptions in the large-value payment systems can in and by themselves create dislocations in the money market. Moreover, disruptions for a single bank can affect all other banks in the network. Thus, resiliency is crucial. Besides the size of interbank exposures on the money market, the risk of contagion effects also depends on the size of banks and these banks' locations in a network, cf. Lublóy (2006) and Upper and Worms (2004).

Network topology provides a framework for analyzing the inner working of interbank money flows. During the last couple of years, the physical theory of networks has developed rapidly as it has been shown that many physical networks have several characteristics in common. That is, payment systems have many things in common with other physical networks like the internet or networks for electricity or water supply. In recent years, a new and growing literature on the functioning of payments systems has emerged using the network topological approach. This has led to important new insights into the functioning of financial networks in the US, Japan, Austria and Hungary among others, cf. Soramäki et al. (2007), Inaoka et al. (2004), Boss et al. (2004), and Lublóy (2006).

Data from the transaction journal of the Danish large-value payment system are used to analyze two economically different networks of interbank money flows. The first network consists of money market transactions, the second of all other transactions. That is, the primary transactions in the payments network are banks' proprietary transactions and customer driven transactions. In contrast to this, the money market network consists of overnight money market loans.

We find that the structure of these networks differs considerably. In the payments network, two commercial banks are responsible for a rather large share of the total activity, whereas there are several major banks in the money market. Both networks are rather concentrated as 10 banks are responsible for most of the transactions in both networks. Seasonal effects are important for the size of the networks. The payments network extends by the turn of the month and quarter and on the first business day following a holiday. In contrast to this, weekday effects drive the calendar effects observed in the money market. Event studies of an operational disruption do not indicate any troubles with regard to the workings of the large-value payment system, whereas payments disruptions by a major participant change the structure of the networks and the level of their activities.

This paper is organized as follows. In section 1 we describe the data and the algorithm used for dividing the data into money market transactions and other transactions. We analyze the network topologies of these economically different networks, which are labelled money market network respectively payments network. Illustrations of these networks are presented in section 2, and section 3 is devoted to a components analysis of the active banks in each network on daily basis. In section 4, the summary statistics of topological measures are presented and we analyze the permanency of links and nodes, which are of importance for the stability of the networks. Moreover, correlations between basic topological measures and seasonal effects are discussed. The final part of the analysis in section 5 is devoted to event studies of two recent incidents in the Danish large-value payment system. Finally, the last section concludes.

1. The data set

We have access to all transactions, originated over the Danish large-value payment system (Kronos) in 2006[1]. The system was open daily from 7.00 a.m. to 3.30 p.m. and 130 banks, including the central bank and branches of foreign banks, were members of the system in 2006.

Banks use large-value payment systems to settle obligations on behalf of their customers as well as their own obligations arising from proprietary operations. An important component of the latter is overnight money market activities. We use an algorithm similarly to Furfine (1999) to separate out from the transaction data set the deliveries and returns of overnight money market loans. We refer to all other transactions as payments.

The algorithm defines a transaction as an overnight money market loan if there is a transaction from bank A to bank B on day t and a reverse transaction from B to A on the same amount plus interest on the following day. The details of the algorithm are explained in the appendix.

A couple of caveats are appropriated as the algorithm's selection criteria do not select overnight money market transactions perfectly. First, the algorithm can only capture overnight loans, transferred via the payment system. Second, we can only observe the settlement time of the transactions but not the actual point in time where a bank enters into an agreement on an uncollateralized overnight loan with another bank. An uncollateralized money market loan can be agreed upon earlier in the day of settlement or on previous days[2]. Third, the algorithm does not identify term loans. However, this market is small in Denmark as more than 90% of the banks lending in the money market for deposits have maturity less than 7 days[3]. Fourth, the borrower and lender, registered by the payment system, may not be the final ones due to correspondent banking. Despite these drawbacks, the algorithm has been used on similar Danish data by Amundsen and Arnt (2005). Thus, we will adopt this algorithm and analyze the network topology for the money market on the available data.

We identify two economically different networks by the algorithm's division of our data:

1) money market network, which consists of overnight money market loans identified by the algorithm;
2) payments network consisting of all other transactions, primarily the settlement of customer driven transactions and banks' proprietary transactions[4].

The basic characteristics for the money market network and the payments network are shown in Table 1 along with the results for the full data set.

[1] We exclude transfers to and from auxiliary systems such as the Continuous Linked Settlement for FX trades, CLS, the Danish automated clearing house (Sumclearing) and the Danish central securities depository (VP). The purpose, value and timing of these settlements differ fundamentally from bank to bank transfers.

[2] Tomorrow-next and spot-next trades, which also imply pairs of transactions between two banks on two consecutive days, are agreed upon 1 respectively 2 days before the settlements of the trades.

[3] This is calculated from data on turnover and interest rates in the Danish market for uncollateralized overnight money market lending. In 2006, 12 banks reported these data to the Danish central bank. The central bank estimates an average tomorrow-next interest rate, which is published daily to the market. See Damm and Pedersen (1997) for a detailed description.

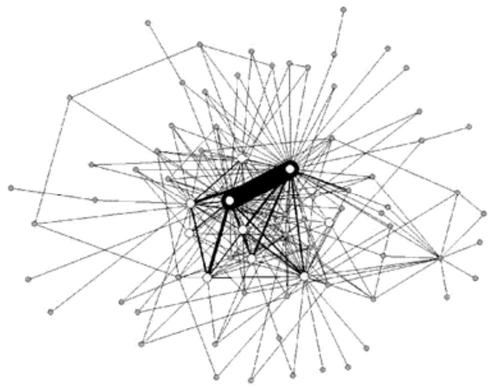
[4] All transactions to/from the central bank are in this network since the central bank does not engage in unsecured overnight lending.

Table 1. Characteristics of the networks, totals for 2006

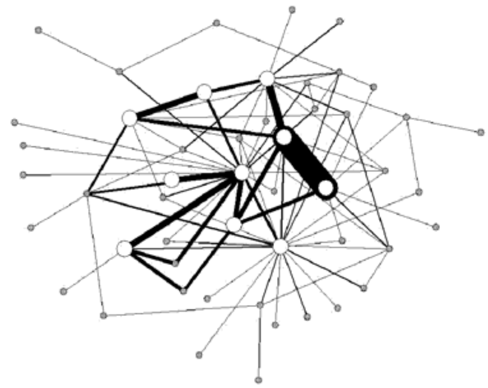
	Trans-actions	Payments	Money market
Active banks	130	130	70
Volume of transactions (thousands)	602.7	574.6	28.2
Value of transactions (trillion DKK)	33.3	26.8	6.5
Mean value of transactions (million DKK)	55.3	46.7	230.7
Volume of transactions (per cent)	100.0	95.3	4.7
Value of transactions (per cent)	100.0	80.5	19.5
10 largest banks' share of			
- Volume of transactions	87.3	88.9	53.7
- Value of transactions	91.1	93.1	83.0

Note: "Transactions" denotes the results for the full data set. Outgoing volume and value from the banks are used to estimate the shares reported.

For each of the business days in 2006 we construct a money market network and a payments network and we use these to obtain aggregated annual results. Each network consists of a number of nodes and links. The banks are nodes and the transactions form links between banks. Two banks are said to be linked if there is at least one transaction between them. Links can be directed, where the direction follows the flow of money, i.e. from lender to borrower and from payer to payee, or undirected. If there are more transactions via the same link, the transactions in a network are weighted. The weights are the sum of either value or the number of transactions between two banks.



a: Payments



b: Money market

Note: The top-10 banks (large and white colored) are identified from total value of outgoing payments in 2006. Links are undirected and weighted by value.

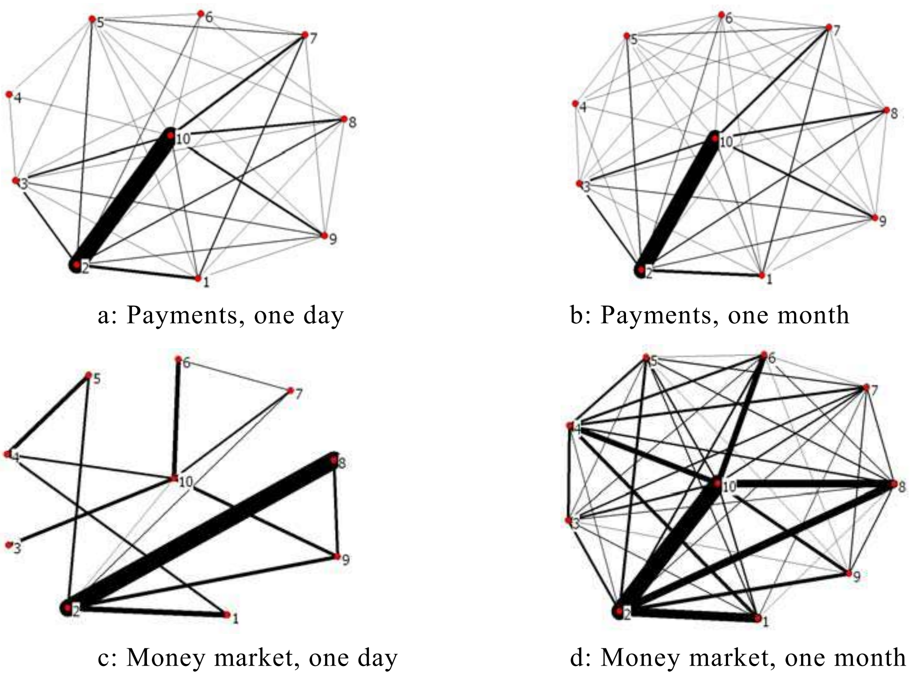
Fig. 1. Payments and money market networks

2. Illustration of the networks

The payments and money market networks for a single day in 2006 are illustrated in Figure 1. The thickness of the links is scaled by the value transferred across and the ten banks, which transferred the most money in either network, are highlighted by larger white nodes. Three structural features are immediately obvious. First, more banks are active in the payments network than in the money market network. Second, two large commercial banks play a major role in both networks, but somewhat surprising the important bank-pair in the payments network is *different from* the major bank-pair in the money market. Third, the top-10 banks account for a significant share of the turnover in terms of values in both networks (93.1 and 83.0 per cent, respectively), which is quite natural, as large banks tend to be more connected than other banks. However, the top-10 banks' market share in terms of volume is 53.7 per cent in terms of the number of loans in the money market network, cf. Table 1. This reflects that the average loan size of the top-10 banks is substantially larger than for other banks in the money market (the average loan size for top-10 banks is 356.6 million DKK and 84.5 million DKK for other banks).

In order to better understand the structure of flows among large banks, we plot the network of only the ten largest banks in Figure 2. We do so in two ways. In the first column of Figure 2 we show networks based on transactions for one day, whereas the second column shows the networks based on transactions for an entire month. The structural differences between the payments and the money market networks are striking. The one day center of the payments network is almost complete[1], whereas the degree of completeness is 20.0 per cent on average in the center of the one day money market.

[1] The degree of completeness is at its maximum of 100 per cent in a complete network and at its minimum in a tree network, where the degree of completeness is equal to 1 divided by the number of nodes. Complete and tree networks are stylized networks, which are not observed empirically. See the appendix for an illustration of stylized networks.



Note: Data for total payments between the ten largest banks in March 2006 are used in the one-month-figures. Since the weighting of links in each network depends on the total value of transactions in each network, the thickness of the links is not comparable between networks. The center of each network consists of the 10 largest banks measured by the total outgoing value of transactions. The top-10 banks are all commercial banks and bank 2-10 are the same in both networks, whereas bank 1 differs between the payments network and the money market network.

Fig. 2. Graphical illustration of the center of the networks (measured in value)

3. Components

Nodes in a network can be divided into groups depending on how they connect to other nodes. A network is comprised by a set of disconnected components within which nodes are linked by an undirected path and do not have links to nodes outside the component. Many empirical investigations find that one of the disconnected components is several orders of magnitude larger than the other disconnected components, cf. Dorogovtsev and Mendes (2002), Albert and Barabási (2002), Soramäki et al. (2007). In contrast, we find that the payment and interbank money market networks consist only of a single component on every day.

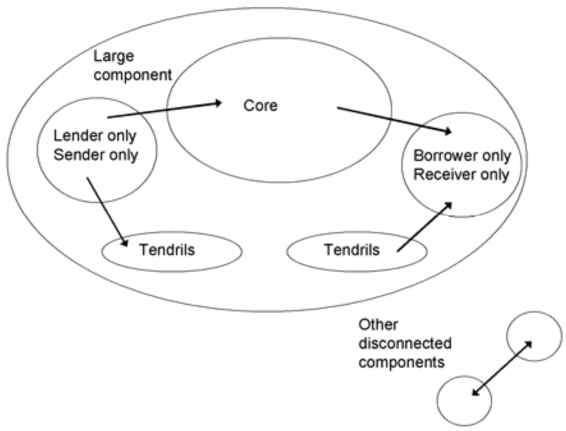


Fig. 3. A network and its components

Table 2. Components in the networks

	Payments	Money market
Nodes connected by a directed path	The core	The core
Nodes on a directed path to core/tendril	The sender only	The lender only
Nodes on a directed path from core/tendril	The receiver only	The borrower only
Other nodes	Tendril	Tendril

Note: The lender/sender (borrower/receiver) only component can submit (receive) transactions to (from) either the core of the network or to (from) a tendril.

Table 3. Components of the networks, 2006

Component	Comp.'s shares	Mean	Median	Max	Min	Std	Value		Capital, average
							Out	In	
Payments	Per cent	Number of nodes					Per cent		Billion DKK
Network	100.0	89.0	89.0	76.0	113.0	5.3	100.0	100.0	23.1
Core	67.7	60.3	60.0	48.0	86.0	6.2	99.6	99.6	33.2
Sender only	16.2	14.4	14.5	3.0	24.0	3.8	0.3	0.0	2.6
Receiver only	15.1	13.5	13.0	4.0	29.0	4.0	0.0	0.3	2.7
Tendrils	0.9	0.9	0.0	0.0	5.0	1.1	0.0	0.0	2.0
Money market	Per cent	Number of nodes					Per cent		Billion DKK
Network	100.0	43.6	44.0	32.0	53.0	4.1	100.0	100.0	42.3
Core	62.9	27.4	28.0	3.0	43.0	6.8	93.5	93.3	61.6
Lender only	16.9	7.4	6.0	0.0	24.0	5.1	5.5	0.7	11.0
Borrower only	16.9	7.4	6.0	0.0	27.0	4.9	0.7	5.7	10.9
Tendrils	3.3	1.4	1.0	0.0	24.0	2.3	0.3	0.2	8.3

Note: The components' shares (Comp.'s shares) of the network are calculated from the mean of the number of nodes. The shares of the value are calculated for in- respectively outgoing payments and the last column contains the average level of capital for the banks in each component. The large maximum value of tendrils in the money market occurs on the first business day in 2006.

We divide the networks into four subcomponents[1], cf. Table 2. First, we have the core which consists of banks that are connected to each other via a directed path. Attached to the core are two peripheral sets of banks that are on a directed path to or from the core. As such, the core facilitates the circulation (or intermediation) of funds within the network, whereas banks in the peripheral groups are either senders or receivers of funds only. Finally, a limited number of banks belong to so-called tendrils, which consist of nodes that are on a directed path to or from the peripheral components.

Our results show that 89.0±5.3 (the mean plus/minus the standard deviation across days) banks are active in the payments network on average in 2006. 60.3±6.2 banks belong to the core, cf. Table 2. The money market network is smaller with only 43.6±4.1 banks being active on an average day in 2006. The size of the core in the money market was 27.4±6.8.

In both networks, most of the transactions are transferred within the core, cf. Table 3. As measured by capital[2], banks in the core are larger than banks in other components in both networks. As a number of smaller banks are active in the payments network only, the average capital level of banks is larger in the money market than in the payments network.

The lion share of value in both networks is transferred within their respective cores. For the payments network the share is 99.6 per cent of the total value, whereas in the money market network it is 93.5 per cent. Banks in the peripheral groups comprise almost all of the remaining values in both networks.

4. Summary statistics for the network topologies

A detailed analysis of the structural differences between the networks across time is difficult by visualization. Therefore, we consider a set of statistical measures common in the network topological approach in this section[3]. We will focus on statistics of network activity in the core of the networks as the core plays a key role in determining the activity and the well-functioning of a payment systems network due to its intermediary role in distributing liquidity among banks in demand and supply of it, cf. Table 3. Furthermore, this is in line with the approach in Soramäki et al. (2007), Bech and Atalay (2008) and Pröpper et al. (2008).

4.1. Characteristics of the networks. The turnover in the payments network[4] is larger both in value and volume than in the money market network and number of active banks are largest in the payments network, cf. Table 4. The average value transferred via a link in the money market network is slightly lower than in the payments network (the link weight in value is 322.7 million DKK respectively 374.7 million DKK on average), whereas the volume transferred via a link in the payments network is significantly larger than in the money market network. This explains the difference in the average size of a transaction in the two networks in Table 1.

[1] In the network topology methodology the large component is known as the Giant Weakly Connected Component. The core of the network is denoted the Giant Strongly Connected Component and the lender/sender (borrower/receiver) only components as the Giant In-Component (Giant Out-Component). Finally, the other disconnected components are denoted Disconnected Components, cf. Dorogovtsev and Mendes (2002).

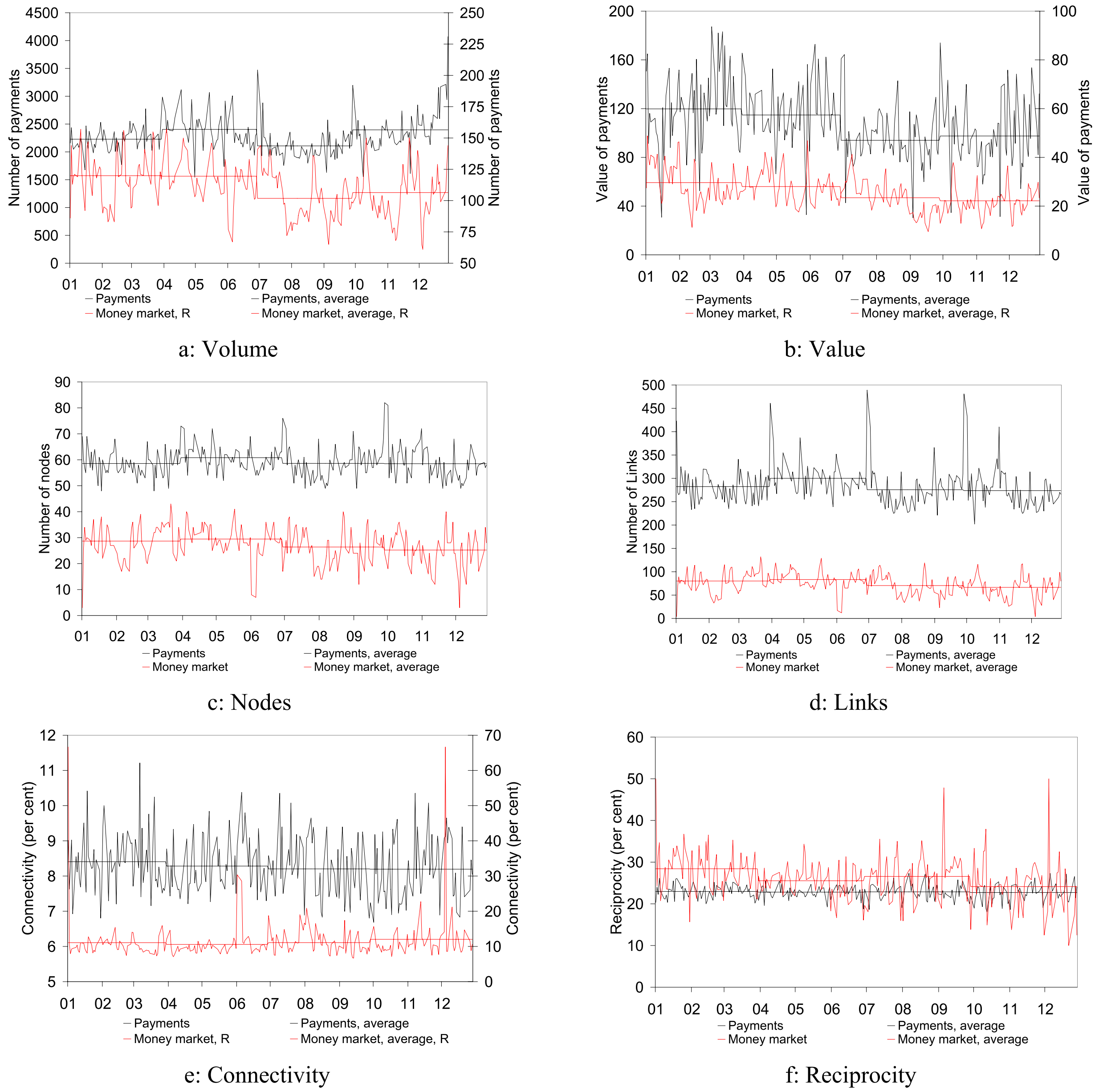
[2] Banks' capital is their productively employed capital, which comprises deposits, issued bonds, subordinated capital contributions and equity capital. Banks' productively employed capital is used to determine the membership fee in the Danish large-value payment system.

[3] The topological measures we use are explained in the appendix.

[4] The summary statistics for the payments network are in line with the results for the transactions network (the whole data set) since most of the observations in the transactions network are the same as in the payments network, see Table A4 in the appendix.

40 banks were active on each business day in the payments network and they handled 99.2 per cent of the total value (30.8 per cent of the total volume). Moreover, 26 links were permanent as they existed on each business day and these accounted for 74.6 per cent of the value transferred (77.6 per cent of the volume). Thus, most links only exist for few business days. This is in contrast with the Hungarian large-value payment system, where a larger fraction of the value is transferred via permanent links, cf. Lublóy (2006). One reason might be that the Hungarian system has larger banks as its members, whereas banks are of different size in the Danish RTGS-system. In the money market, 7 banks were active on all business days and they handled 10 per cent of the volume and 66.7 per cent of the value. The most permanent link existed for 189 days out of 252 business days and this link handled 19.2 per cent of the total turnover in the money market. Thus, in both networks a number of banks handle a large share of the total value of transactions and most links exist for a few days only as illustrated in Figure 5.

Although the top-10 banks in both networks tend to form links with almost all other top-10 banks in Figure 2, the actual number of links formed is substantially smaller than the potential number of links when we consider the networks in general.



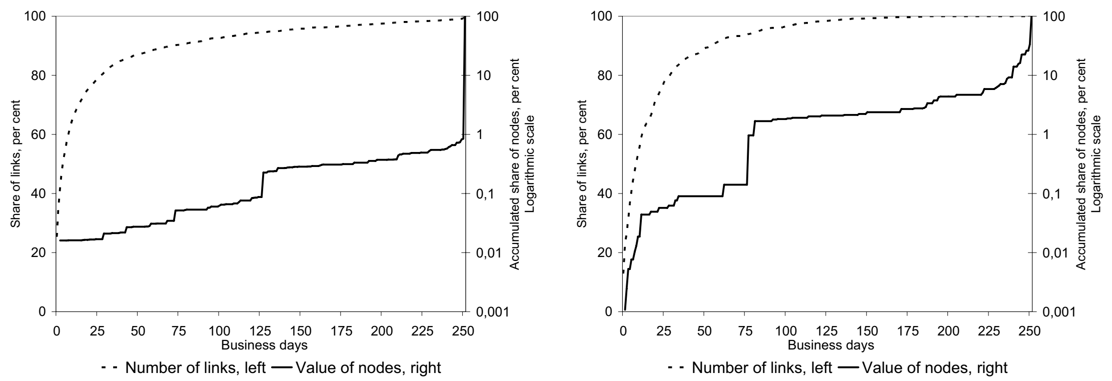
Note: Data for the core in 2006. Quarterly averages are included in the figures. Months are labelled with numbers from 1 to 12. The value is in billion DKK. Although the value (volume) is downward (upward) sloping during the year in the payments network, the average value of a payment has been almost unchanged in the period 2003-2007.

Fig. 4. Activity of the payments and money market networks, 2006

Table 4. Summary statistics, payments and money market networks, 2006

	Mean	Median	Min	Max	Std	Mean
Payments						Fedwire
Volume	2,162.4	2,127.0	1,493.0	3,434.0	283.8	436
Value	105.5	101.3	29.5	186.9	27.3	1.3
Nodes	60.3	60.0	48.0	86.0	6.2	5,086
Links	282.6	277.0	202.0	489.0	40.9	76,614
Connectivity, per cent	8.3	8.2	6.7	11.2	0.8	0.3
Reciprocity, per cent	22.8	22.8	18.0	27.3	1.8	21.5
Clustering	0.5	0.5	0.4	0.7	0.1	0.5
Average path length	2.5	2.5	2.3	2.7	0.1	2.6
Average node degree, k	4.8	4.7	4.0	6.4	0.4	15.2
Link weight, value	0.4	0.4	0.1	0.7	0.1	15.2
Link weight, volume	7.7	7.7	5.0	10.1	0.8	5.2
Node strength, value	1.8	1.7	0.5	3.4	0.5	n.a.
Node strength, volume	36.7	36.3	24.1	54.6	4.3	n.a.
Money market						Fed Funds Market
Volume	86.4	88.0	4.0	144.0	26.0	2.6
Value	22.9	22.1	0.3	45.2	8.1	0.3
Nodes	27.4	28.0	3.0	43.0	6.8	470.2
Links	75.0	76.0	4.0	132.0	23.3	1,543
Connectivity, per cent	11.2	10.2	6.7	66.7	5.8	0.7
Reciprocity, per cent	26.2	26.4	10.0	50.0	5.5	6.5
Clustering	0.2	0.2	0.0	0.5	0.1	0.1
Average path length	2.9	2.9	1.3	4.6	0.4	2.7
Average node degree, k	2.7	2.7	1.3	3.5	0.3	3.3
Link weight, value	0.3	0.3	0.1	1.5	0.1	219
Link weight, volume	1.2	1.1	1.0	1.8	0.1	1.7
Node strength, value	0.9	0.8	0.1	2.0	0.3	719
Node strength, volume	3.1	3.1	1.3	4.3	0.4	5.5

Note: Average statistics of daily observations for the core. The value, link weight and node strength in value are in billion DKK. Clustering and average path length are based on payments submitted from a node. Data for Fedwire and the Fed Funds Market are from Soramäki et al. (2007) and Bech and Atalay (2008) and the volume is in thousands, the value in trillion USD and the link weight and node strengh in value are in million USD. Node strength is not available for Fedwire in Soramäki et al. (2007).



a: Payments b: Money market

Note: The value of nodes (number of links) is measured in per cent and accumulated and plotted against the number of business days. The value of nodes is measured on a logarithmic scale. The data are for the whole network of interconnected banks.

Fig. 5. Frequency of links and nodes

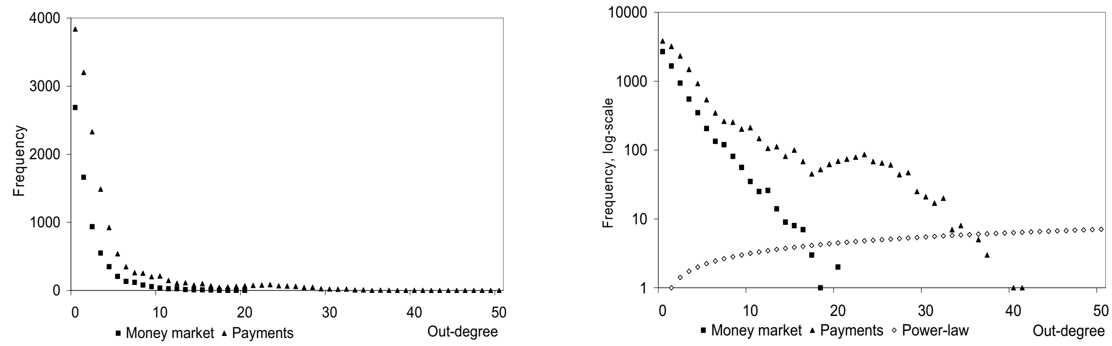
For both networks only 1 out of 10 possible links is formed on a given day with a slightly lower connectivity in the payments network (8.3±0.8 per cent) than in the money market network (11.2±5.8 per cent). Thus, banks in the periphery of a network tend to form fewer links than the banks in the core.

The reciprocity, which measures the share of links between banks for which there is a link in the opposite direction, is virtually the same in the two networks, as 1 out of 4 links has transactions in both directions. The reciprocity in the money market network is substantially larger than in the Fed Funds

Market, whereas the reciprocity in the payments network is a bit larger than in Fedwire[1]. In the payments network, there is a 50 per cent chance that two neighbors of a node are also linked to each other, whereas there is only a 1 out of 5 chance in the money market network. In both networks, the clustering coefficient is much higher than the connectivity, so neither of the networks is random[2].

An important characteristic of a node in a network is the number of links, which originate from a node and the number of links terminating in a node. The average number of links per node in the payments network is 4.8±0.4, which is almost double the average node degree of 2.7±0.3 in the money market network. In the payments network, the maximum number of links originating from (terminating in) an active bank is 29.0±3.9 (34.6±4.4), cf. Table A1 in the appendix. In the money market network, the number of links, originating from (terminating in) an active bank, is 10.3±3.4 (10.3±3.6). That is, banks within the money market tend to have fewer links to other banks than active banks in the payments network.

The distribution of links, originating from (terminating in) nodes (out-degrees respectively in-degrees), are fat-tailed, cf. Figure 6a. A number of studies have shown that in- and out-degrees in large-value payment systems in the US, Japan and Austria follow power-laws[3], cf. Inaoka et al. (2004), Soramäki et al. (2007) and Boss et al. (2004). In a random network, the distributions of in- and out-degrees follow a Poisson distribution, cf. Dorogovtsev and Mendes (2002) and Newman (2005). Neither a power-law distribution, nor a Poisson distribution capture the distribution of the in- and out-degrees correctly in the Danish case, cf. Figure 6b and 7. In the payments network, the exponential distribution or the negative binomial distribution capture the actual distributions of in- and out-degrees quite well, whereas the exponential distribution is closest to the actual values of in- and out-degrees for the money market network, cf. Figure 7.

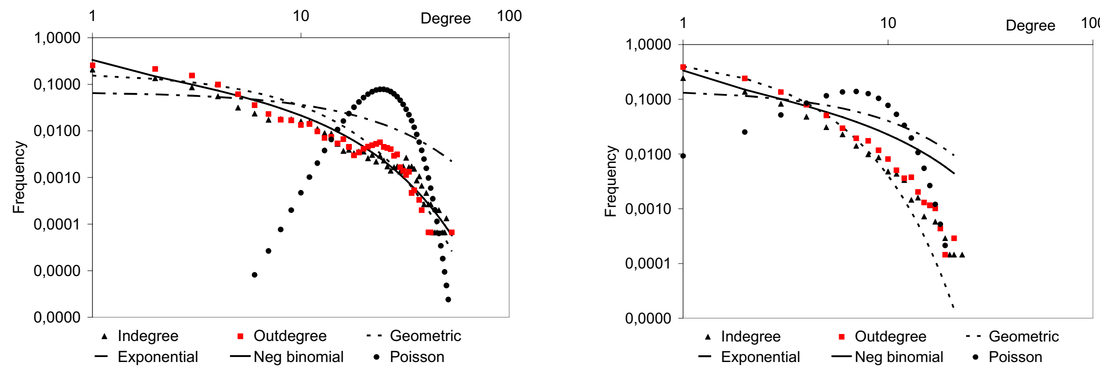


a: Distribution of out-degrees

b: Log-scale including power-law

Note: The y-axis is in log-scale in panel (b). The data are for the whole network of interconnected banks. Only out-degrees are shown here, but figures for in-degrees are similar.

Fig. 6. Distributions of out-degrees



a: Payments

b: Money market

Note: Both x- and y-axes are logarithmic. The data are for the whole network of interconnected banks. The leftwing tail of the Poisson distribution in panel (a) has been cut off to keep a clear picture. This choice is reasonable since the in- and out-degrees for the payments network are clearly not Poisson distributed.

Fig. 7. Distribution of in- and out-degrees

[1] Overnight loans between banks are borrowed or lent in the Market for Federal Funds (Fed Funds Market). The Fedwire Funds Service (Fedwire) is a real-time gross settlement system operated by the Federal Reserve System in the US.

[2] The clustering coefficient is equal to the connectivity in a random network. A random network is constructed by adding links at random to a given set of nodes. This is a stylized type of network, which is unobserved in reality.

[3] A power-law is a distribution for which there is a scale effect, i.e. $P(X = x) \approx x^{-y}$.

An omnipresent question in network theory is the relative importance of different nodes and links usually referred to as centrality. We have already discussed the notion of degree above. The most connected bank on any given day in our sample had 53 outgoing (55 incoming) links for the payments network and 21 outgoing (24 incoming) links for the money market network. Another way to measure importance is node strength which measures the amount (or number) of payments or loans processed by a participant. According to this measure, the largest node across all days processed outgoing payments worth 74.2 billion DKK in the payments network and lent out loans worth 21.1 billion DKK in the money market network on any given day. The largest (directed) link between any two banks in the two networks transferred 58.8 billion DKK worth of payments and 12.2 billion DKK worth of loans. In a relative sense the largest node and link in the payments network accounted for 52.9 and 43.7 per cent, respectively, of the total value transferred on any day. In the money market the equivalent "market share" numbers were 71.0 and 64.5 percent, respectively.

Another measure of centrality is betweenness, which is a measure of the number of paths between other nodes that run through node i. The more paths node i handles, the more central is this node in the network. The measure can also be applied for links to identify the most important links between banks. Results in Table A1 in the appendix show that the average betweenness for links is almost identical in both networks (29.2±8.4 in the money market and 30.3±3.7 for the payments network), whereas the betweenness for nodes in the money market network is 40 per cent lower than in the payments network, i.e. each node in the money market handles fewer paths than banks in the payment network.

The average path length is the average number of links, which connects two banks via the shortest possible path, i.e. the average path length measures across how many links 1 DKK must pass to reach another bank. Our results show an average path length of 2.5±0.1 in the payments network and 2.9±0.4 in the money market, cf. Table 4. The corresponding values for Fedwire and the Fed Funds Market are 2.6 and 2.7, respectively, cf. Soramäki et al. (2007) and Bech and Atalay (2008). The maximum distance between two banks (measured by the number of links) is the diameter, which is 5.5±0.7 for the payments network and 6.7±1.3 for the money market network, cf. Table A1 in the appendix. This is substantially smaller than the diameter in Fedwire of 6.6 on average and the diameter in the Fed Funds Market of 7.3, cf. Soramäki et al. (2007) and Bech and Atalay (2008).

More than half of the other banks in the payments network can be reached within 2 nodes, cf. Table A1. Increasing the distance to 3 implies that 91.2±2.7 per cent of the nodes can be reached and by the distance 5 almost all banks are reachable. In a study for the Fedwire, Soramäki et al. (2007, Table 3) find that the mass distribution function reaches almost 100 per cent within the distance 4. The larger distance between banks in the money market implies that only 42.1±9.5 (71.6±10.0) per cent of the banks can be reached within a distance of 2 (3).

4.2. Correlations of network statistics and seasonal effects. The correlation coefficients between the basic network statistics confirm the patterns in Figure 4, where the activity in volume and value tend to covariate with the size of the networks (nodes and links) in both networks, see Table 5. In the payments network, connectivity is negatively correlated with the number of active banks and links. This result is contrary to Soramäki et al. (2007), who find that the correlation between nodes (links) and connectivity is quite strong and positive in Fedwire. Moreover, the reciprocity is uncorrelated with the activity (value and volume), so the payments network does not become denser as the activity increases. The connectivity in the money market is negatively related to any measure of activity (value and volume) and size (nodes and links). In general, the denseness of the money market network (reciprocity) is uncorrelated with any other measure with the possible exception of the slightly positive correlation between reciprocity and connectivity. This reflects that a bank, which becomes active in the money market, tends to have only a few links to other banks.

Table 5. Correlations of basic network properties, 2006

Payments						
	Value	Volume	Nodes	Links	Connectivity	Reciprocity
Value		0.58	0.25	0.44	0.14	0.26
Volume			0.50	0.68	-0.02	0.09
Nodes				0.86	-0.72	-0.36
Links					-0.28	-0.17
Connectivity						0.48
Reciprocity						
Money market						
	Value	Volume	Nodes	Links	Connectivity	Reciprocity
Value		0.62	0.49	0.55	-0.34	0.04
Volume			0.92	0.98	-0.57	0.09
Nodes				0.95	-0.69	0.05
Links					-0.57	0.07
Connectivity						0.25
Reciprocity						

There seems to be a seasonal pattern in Figure 4, especially around quarter ends. To test for this we regress 8 different topological measures on a set of dummies for holidays, weekdays and liquidity provisions by the Danish central bank in addition to the regular liquidity adjustments on Fridays. Results are shown in Tables A2 and A3 in the appendix.

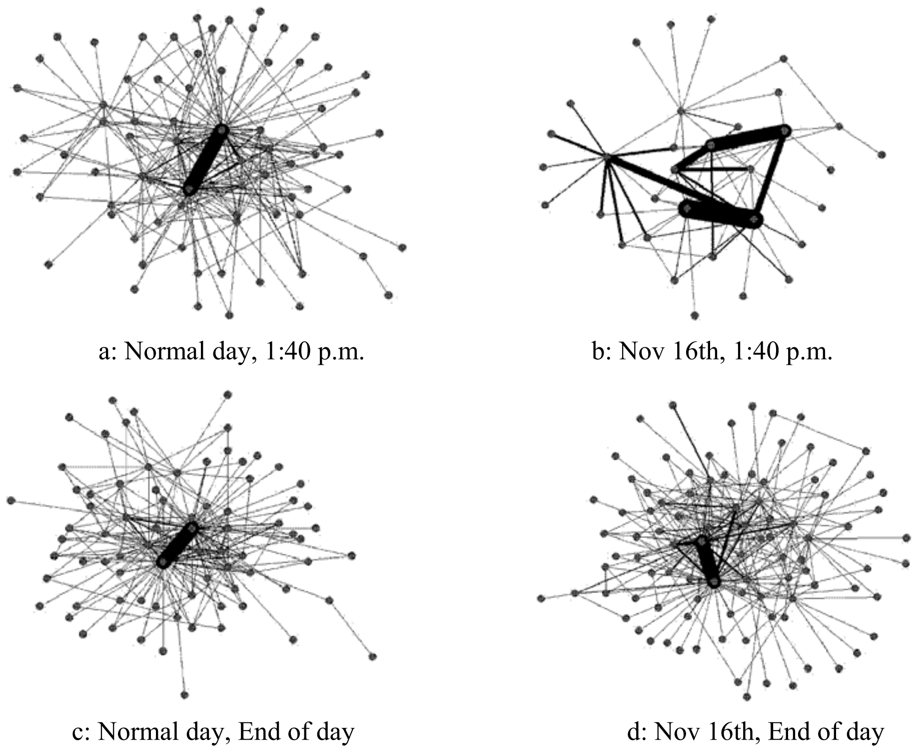
For the payments network, the effects on the first business day following Danish or US holidays are significant for links, value, volume and average node degree. This network is extended at every turn of month and quarter both considering the number of active banks, links, value, volume and average node degree. Moreover, the connectivity decreases significantly by the turn of quarter. These effects are due to large quarterly interest and repayment on mortgage loans, which is the prime source of funds in the Danish housing market, and monthly payments of salaries, social benefits and taxes etc., which initiate more transactions than usual. The network is largest on Fridays (in nodes, links and value) and smaller at the beginning of the week (in nodes, links, volume and average path length). Both planned and unexpected liquidity adjustments increase the number of links and the average node degree significantly.

For the money market there are significant weekday effects for nodes, links and volume, especially on Fridays, which is the first day in the weekly liquidity schedule. This affects connectivity positively. The same pattern is observed by the turn of the month, but only the average node degree and the connectivity are positively affected at the turn of quarter. Unexpected liquidity adjustments increase the average node degree and decrease the average path length. In contrast to this, there are no effects from expected liquidity adjustments or from holidays.

5. Event studies

In order to investigate how the networks respond to disturbances we consider two case studies of operational events[1]. The first event is an intraday operational disruption of the Danish large-value payment system; the second is payment disruptions by a major participant on multiple days.

5.1. Operational disruption of the system. On Thursday, November 16th, 2006, the Danish large-value payment system experienced an intraday operational failure, cf. Danmarks Nationalbank (2007). The system opened as usual, but due to an unsuccessful software update the settlement process stopped after the first few minutes and the system remained down for more than 6 hours. When the system came up again later that day, a large bulk of transactions was settled immediately. As a consequence of this event, the Danish central bank extended the closing of the system with 15 minutes but only two transactions took place after the official closing time at 3:30 p.m. Furthermore, the central bank provided extra liquidity to the market by repurchases of certificates of deposit.



a: Normal day, 1:40 p.m. b: Nov 16th, 1:40 p.m.

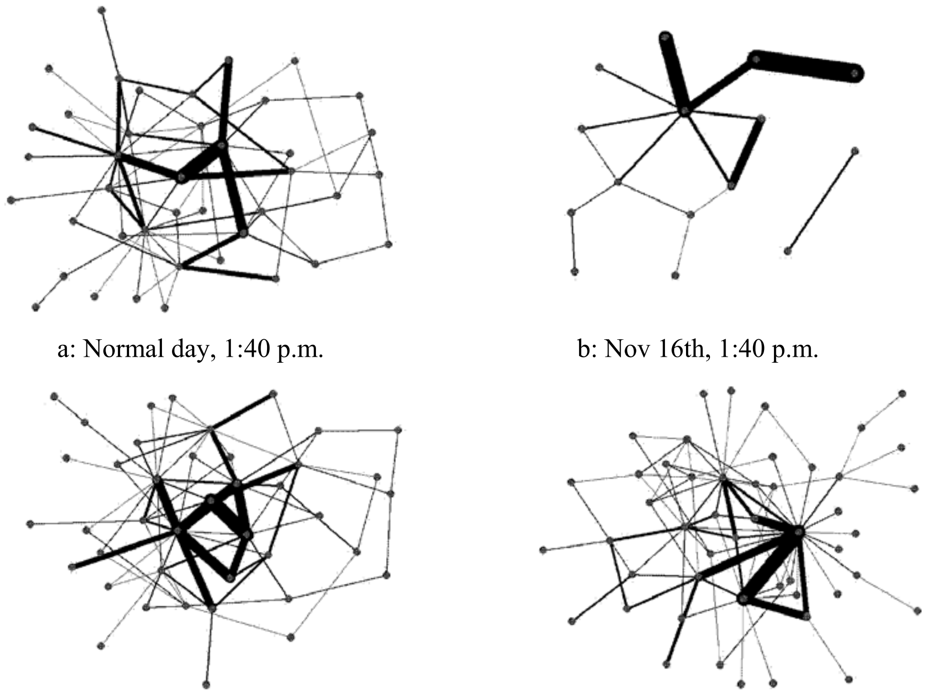
c: Normal day, End of day d: Nov 16th, End of day

Note: The figures are weighted by value.

Fig. 8. Operational disruption in the payments network

[1] Event studies are useful to analyze whether banks change behavior and if this benefits the functioning of a network. Both the subprime crisis in 2007 and effects from Sept. 11th, 2001 had substantial influence on the network topology of the US financial market, cf. Soramäki et al. (2007) and Kroszner (2007). The Danish payments and money market networks were unaffected by the subprime crisis in a data set for the period of July-September 2007. Pröpper et al. (2008) reach the same conclusion in a similar study for credit markets in the Netherlands.

The operational disturbance implied a different structure of the networks during the day, cf. Figure 8 and 9 and Table 6. By the end of the day, almost all of the topological measures were significantly different from the 2006 average, cf. Table 6. The activity and size of the payments network decreased significantly. The average path length had decreased significantly by the end of the day, whereas the connectivity and clustering of the payments network increased significantly. That is, the payments network became narrower.



a: Normal day, 1:40 p.m.

b: Nov 16th, 1:40 p.m.

c: Normal day, End of day

d: Nov 16th, End of day

Note: The figures are weighted by value.

Fig. 9. Operational disruption in the money market network

In opposition to this, the activity in terms of volume and the size of the money market increased, although the average value of each money market loan had decreased significantly by the end of the day. The connectivity of the money market network decreased significantly, whereas the average path length and average node degree increased. Thus, although the actual number of links out of the potential number of links decreased, the average number of links per active bank increased in the money market. All in all, the money market became wider during this event.

The drop in payments network activity and boom in overnight money market loans are in opposition to the seasonal effects by the turn of the month, cf. Table A2 and A3.

Although the operational disruption of the system had a large impact on the topologies of the payments and the money market networks, these effects were temporary. If the operational event had lasted longer, these effects might have been even more pronounced.

Table 6. Effects of an operational breakdown in the networks

	2006 Average	Confidence limits		Operational breakdown	
		Lower	Upper	End of day	1:40 p.m.
Payments					
Volume	2,162.4	2,127.1	2,197.6	1,883.0*	220.0*
Value, billion DKK	105.5	102.2	108.9	80.6*	6.7*
Nodes	59.1	58.4	59.8	55.0*	33.0*
Links	282.6	277.5	287.7	260.0*	70.0*
Connectivity, per cent	8.3	8.2	8.4	8.8*	6.6*
Clustering	0.53	0.53	0.54	0.55*	0.27*
Average path length	2.48	2.47	2.49	2.46*	1.75*
Average node degree	4.77	4.73	4.81	4.73	2.12*

Table 6 (cont.). Effects of an operational breakdown in the networks

	2006 Average	Confidence limits		Operational breakdown	
		Lower	Upper	End of day	1:40 p.m.
Money market					
Volume	86.4	83.2	89.7	103.0*	19.0*
Value, billion DKK	22.9	21.9	23.9	20.9*	3.5*
Nodes	27.4	26.6	28.3	29.0*	15.0*
Links	75.0	72.1	77.9	82.0*	16.0*
Connectivity, per cent	11.2	10.5	11.9	10.1*	7.6*
Clustering	0.17	0.16	0.18	0.18	0.08*
Average path length	2.94	2.90	2.99	3.01*	0.54*
Average node degree	2.69	2.64	2.73	2.83*	1.07*

Note: Mean values of selected summary statistics for the core. Confidence limits for the 95 % confidence interval are used to determine the significant variables, which are marked with *. Clustering, average path length and average node degree are reported with 2 decimals.

5.2. Payment disruption by a major participant. One of the largest commercial banks in Denmark, Danske Bank, was not able to send payments in the large-value payment system on two successive days in March 2003. This was caused by a major it-problem[1]. The Danish central bank supplied the banks with extra liquidity to overcome a potential lack of liquidity in the markets as the major participant was able to receive, but could not send transactions to other banks.

The effects on the networks' structures were most pronounced on the first day of the event, Wednesday, March 12th. The activity and size of the payments network decreased, whereas the activity in terms of volume and the size of the money market network increased by around 50 per cent on this day, although the average size of an overnight money market loan decreased, cf. Table 7. Connectivity and clustering increased significantly in the payments network, whereas the average path length and the average node degree decreased. In the money market, the effects on these four variables were opposite.

On the second day of this event, the major participant informed the public about the it-problem and its implications for the bank's business. Together with the significant boom in activity and size of the money market of the first day of payment disruptions by a major participant, this led to a decrease in activity and size of the money market network on the second day of this event. This decreased the average path length and average node degree.

Table 7. Effects of payment disruptions by a major participant

	Average	Confidence limits			
	March 2003	Lower	Upper	March 12	March 13
Payments					
Volume	2,306.5	2,267.4	2,345.6	1,505.0*	1,625.0*
Value, billion DKK	145.8	142.1	149.5	119.7*	128.6*
Nodes	56.5	55.9	57.2	49.0*	58.0*
Links	281.2	275.8	286.5	227.0*	254.0*
Connectivity, per cent	9.0	8.9	9.1	9.7*	7.7*
Clustering	0.50	0.50	0.51	0.58*	0.43*
Average path length	2.46	2.45	2.47	2.44*	2.57*
Average node degree	4.95	4.91	4.99	4.63*	4.38*
Money market					
Volume	64.4	61.3	67.5	92.0*	46.0*
Value, billion DKK	18.7	17.8	19.6	8.8*	7.8*
Nodes	24.1	23.0	25.1	37.0*	19.0*
Links	56.9	54.2	59.7	87.0*	44.0*
Connectivity, per cent	12.8	11.7	13.9	6.5*	12.9
Clustering	0.17	0.16	0.18	0.10*	0.17
Average path length	3.03	2.97	3.09	3.70*	2.96*
Average node degree	2.24	2.19	2.28	2.35*	2.32*

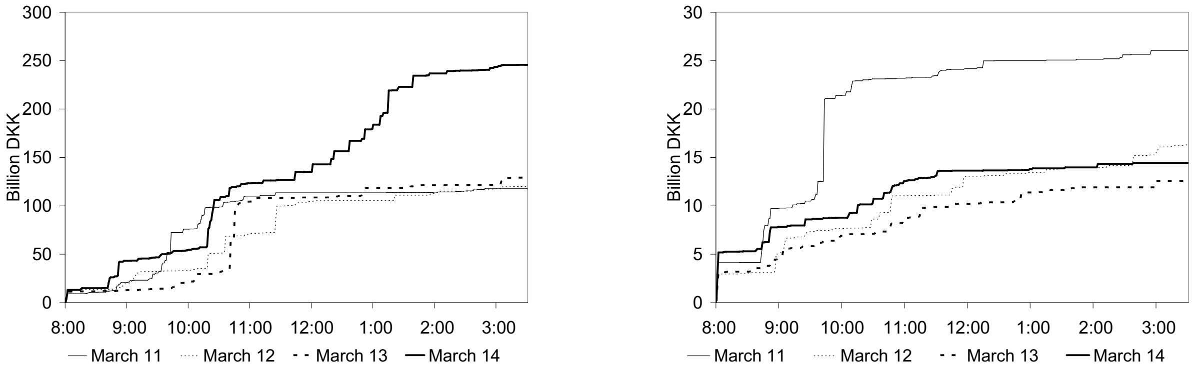
Note: The note to Table 6 also applies here. The average of March 2003 excludes data from March 12th and March 13th.

[1] For a description of this event see Berlingske (2003a, 2003b).

The activity and the size in terms of links of the payments network remained significantly lower than the average for March 2003, but more banks became active in the payments network on the second day of payment disruptions. This is reflected in the significant drop in connectivity, clustering and average node degree. The average path length increased, i.e. transactions had to pass more links to reach the final recipient of a transaction.

The disruptions by a major participant also caused an accumulated settlement demand in the payments network and this led to a sharp increase in the value settled within this network on the first normal business day after the event, cf. Figure 10.

Compared with the operational breakdown, the effects of payment disruptions by a major participant are larger in both networks. The structural changes in the networks' topologies were temporary. And it seems as if the other banks took precautionary actions towards the disturbance and continued settlements as far as possible in both networks.



a: Payments b: Money market

Note: Amounts settled in the networks during the day on selected dates in March 2003. March 11th (March 14th) was the last (first) business day before (after) the payment disruption by a major participant, while this event had effect on March 12th and 13th. The values of overnight loans in the money market increase by coincidence on March 11 as there are no holiday effects or effects of additional liquidity adjustments by the central bank this day. The opening time of the large value payment system was 8.00 a.m.-3.30 p.m. until June 1st 2003.

Fig. 10. Large bank payment disruption

Conclusion

The topological analysis shows that the structure of the Danish money market is different from the structure of the payments network. This is a consequence of the difference in the nature of transactions in the networks. Transactions in the money market network are driven by banks' behavior, whereas transactions in the payments network arise from banks' proprietary transactions as well as customer driven transactions. In the payments network, two commercial banks are responsible for a rather large share of the total activity, whereas the banks in the core of the money market are of more equal size. Both networks are rather concentrated.

Our results show that the distributions of in- and out-degrees follow the exponential or the negative binomial distributions in the payments network, while the exponential distribution captures the distribution of in- and out-degrees quite well in the money market. In other countries, in- and out-degrees follow power-law distributions, but power-law distributions are clearly rejected in our data set.

We find clear evidence of seasonal effects for both networks. The results show that the payments network becomes wider by the turn of the month and quarter and on the first business day following a holiday. In contrast to this, weekday effects drive the calendar effects observed in the money market.

Event studies of an operational disruption imply a different structure of the networks during the day. Although the structure of the networks is almost normal by the end of the day, the daily activity of the payments network decreased considerably. In contrast to this, the daily activity of the money market increased. The topological effects of this event are in line with the seasonal effects by the turn of the month but with the opposite signs. The effects of the operational event were temporary, but might have been more pronounced in case the operational event had lasted longer than it did. Payment disruptions by a major participant decreases (increases) the level of activity in the payments (money market) network; especially on the first day of the event. An accumulated settlement demand was built up in the payments network, which was released on the first normal business day after the payment disruption by a major participant leading to a sharp increase in the value settled in the payments network.

It could be interesting to see if the payments network builds up in a different way than the money market network during the day. At the moment a rather large fraction of the settlements takes place before noon both in the money market and in the payments network, but the effect of a different timing of settlements on the structure of the networks is a question for further research.

Acknowledgement

We thank Danmarks Nationalbank for providing us with the data set for the analysis. This paper has benefited from helpful comments from Enghin Atalay, Asger Lau Andersen, Bjarne Astrup Jensen and seminar participants in the Annual Workshop, Danish Graduate Programme in Economics, 2007; the Annual Workshop, The Danish Doctoral Educational Network in Finance, 2008; The Danish Economic Society's Annual Meeting, 2008; Danmarks Nationalbank, 2008 and Norges Bank, 2008.

References

1. Albert, Réka and Albert-László Barabási (2002). Statistical mechanics of complex networks, *Reviews of Modern Physics*, Vol. 74, January 2002, pp. 48-97.
2. Allen, Franklin and Douglas Gale (2000). Financial Contagion. *Journal of Political Economy*, 2000, Vol. 108, No.1.
3. Amundsen, Elin and Henrik Arnt (2005). Contagion Risk in the Danish Interbank Market. *Working Paper* 29/2005, Danmarks Nationalbank.
4. Bech, Morten L. and Enghin Atalay (2008). The Topology of the Fed Funds Market, *Staff Report* No. 354, November 2008, Federal Reserve Bank of New York.
5. Berlingske (2003a). Straarups nedbrud, Berlingske Tidendes Nyhedsmagasin, No. 11, March 24th, 2003.
6. Berlingske (2003b). Efter Danske Bank-nedbruddet: Undskyld og skal vi så komme videre, Berlingske Tidendes Nyhedsmagasin, No. 14, April 14th, 2003.
7. Boss, Michael, Helmut Elsinger, Martin Summer and Stefan Thurner (2004). Network topology of the interbank market, *Quantitative Finance*, 4:6, pp. 677-684.
8. Damm, Birgitte and Anne Reinhold Pedersen (1997). New Money-Market Statistics, Monetary Review, 3. Quarter 1997, Danmarks Nationalbank.
9. Danmarks Nationalbank (2007). Report and Accounts 2006.
10. Dorogovtsev, S.N. and J.F.F. Mendes (2002). Evolution of networks, Advances in Physics, 51:4, pp. 1079-1187.
11. Freixas, Xavier and Bruno Parigi (1998). Contagion and Efficiency in Gross and Net Interbank Payment Systems. *Journal of Financial Intermediation*, Vol. 7, pp. 3-31, 1998.
12. Freixas, Xavier, Bruno M. Parigi and Jean-Charles Rochet (2000). Systemic Risk, Interbank Relations, and Liquidity Provision by the Central Bank. *Journal of Money, Credit and Banking*, Vol. 32, No. 3, August 2000 (Part 2).
13. Furfine, Craig H. (1999). *The Microstructure of the Federal Funds Market, Financial Markets, Institutions & Instruments*, Vol. 8, No. 5, December 1999.
14. Hekmat, Ramin (2006). Ad-hoc Networks: Fundamental Properties and Network Topologies, Springer, 2006.
15. Inaoka, Hajime, Takuto Ninomiya, Ken Taniguchi, Tokiko Shimizu and Hideki Takayasu (2004). Fractal network derived from banking transaction. An analysis of network structures formed by financial institutions. *Bank of Japan Working Paper Series*, No. 04-E-04, April 2004.
16. Kroszner, Randall S. (2007). Recent Events in Financial Markets. Speech at the Institute of International Bankers Annual Breakfast Dialogue, Washington D.C., October 22, 2007. http://www.federalreserve.gov/newsevents/speech/kroszner20071022a.htm.
17. Newman, MEJ (2005). Power laws, Pareto distributions and Zipf's law. Contemporary Physics, Vol. 46, No. 5, pp. 323-351.
18. Lublóy, Ágnes (2006). Topology of the Hungarian large-value transfer system. Magyar Nemzeti Bank (Central Bank of Hungary), MNB, Occasional Papers No. 57 /2006.
19. Pröpper, Marc, Iman van Lelyveld and Ronald Heijmans (2008). Towards a Network Description of Interbank Payment Flows, DNB Working Paper No. 177/May 2008, De Nederlandsche Bank
20. Soramäki, Kimmo, Morten L. Bech, Jeffrey Arnold, Robert J. Glass and Walter E. Beyeler (2007). The topology of interbank payment flows. Physica A 379 (2007), pp. 317-333.
21. Upper, Christian and Andreas Worms (2004). Estimating bilateral exposures in the German interbank market: Is there a danger of contagion?, *European Economic Review*, Vol. 48, No. 4, August 2004, pp. 827-849.

Appendix

1. More summary statistics

Table A1. More summary statistics, payments and money market networks, 2006

	Mean	Median	Min	Max	Std	Mean
Payments						Fedwire
Distance measures						
Diameter	5.5	5.0	4.0	8.0	0.7	6.6
MDF, M(2)	54.6	54.8	44.7	65.4	4.2	41.6
MDF, M(3)	91.2	91.4	83.2	97.8	2.7	95.9
MDF, M(4)	99.1	99.3	94.9	100.0	0.8	99.9
MDF, M(5)	99.9	100.0	96.9	100.0	0.3	n.a.

Table A1 (cont.). More summary statistics, payments and money market networks, 2006

	Mean	Median	Min	Max	Std	Mean
Payments						Fedwire
Degree distribution						
max k^{in}	34.6	34.0	24.0	51.0	4.4	2,097
max k^{out}	29.0	29.0	22.0	53.0	3.9	1,922
Centrality measures						
Betweenness, links	30.3	30.3	20.7	38.4	3.7	n.a.
Betweenness, nodes	86.0	85.8	61.9	125.4	10.5	n.a.
Money market						Fed Funds Market
Distance measures						
Diameter	6.7	7.0	2.0	10.0	1.3	7.3
MDF, M(2)	42.1	40.6	26.6	100.0	9.5	n.a.
MDF, M(3)	71.6	71.4	47.3	100.0	10.0	n.a.
MDF, M(4)	89.7	91.0	61.6	100.0	7.3	n.a.
MDF, M(5)	96.6	98.0	71.7	100.0	4.2	n.a.
MDF, M(6)	98.9	99.9	82.0	100.0	2.3	n.a.
MDF, M(7)	99.7	100.0	90.1	100.0	1.1	n.a.
Degree distribution						
max k^{in}	10.3	10.0	2.0	24.0	3.6	127.6
max k^{out}	10.3	10.0	2.0	21.0	3.4	48.8
Centrality measures						
Betweenness, links	29.2	29.4	2.0	51.1	8.4	n.a.
Betweenness, nodes	52.1	52.5	0.7	92.6	16.5	n.a.

Note: Data for the core of the networks. Mass Distribution Functions, MDF, are based on payments submitted from a node. max k^{in} (max k^{out}) is the maximum number of links ending in (starting from) a node. Data for Fedwire and the Fed Funds Market are from Soramäki et al. (2007) and Bech and Atalay (2008). n.a. means not available.

2. Seasonal effects. This section contains the regression results regarding seasonal effects in the core of the networks. As mentioned above, we test for seasonal effects due to holidays, turn of month and quarter and liquidity adjustments by the central bank. Holiday effects are measured on the first business day following a closing day. American holidays are holidays in addition to Danish holidays and European holidays are captured by the dummies for Danish holidays and turn of month. The turn of month (quarter) includes the first and the last business day in each month (quarter). Liquidity adjustments by the central bank are adjustments in addition to the regular adjustments on Fridays, e.g. the Danish central bank adjusts liquidity extraordinarily on days with receipts of tax payments, reimbursements of social benefits etc. Additional adjustments will normally be announced in advance (expected adjustments), but a few adjustments are not announced (unexpected adjustments).

Table A2. Seasonal effects, payments network, 2006

	Intercept	Holidays in		End of		Liquidity adjustment		Weekdays				R^2
		Denmark	US	Quarter	Month	Expected	Unexpected	Monday	Tuesday	Thursday	Friday	
Nodes	59.9**	5.7*	1.1	17.1**	6.1**	2.5*	1.7	-3.6**	-1.4	-1.9*	2.8*	31.9
	0.8	2.4	1.4	2.2	1.2	1.3	1.5	1.1	1.1	1.1	1.3	
Links	279.2**	73.9**	18.5*	146.3**	48.5**	18.1**	24.6*	-13.2*	-13.4*	-5.2	38.7**	40.7
	4.8	15.4	6.9	10.0	7.0	7.1	10.6	6.9	5.6	6.4	11.4	
Value, billion DKK	95.1**	23.8**	15.9*	43.2**	22.0**	5.4	0.5	6.1	-2.5	0.7	22.9**	20.8
	3.2	8.9	6.9	8.8	6.8	6.1	7.9	6.4	4.8	5.0	4.8	
Volume, thousands	2.2**	0.6*	0.4**	0.7**	0.2*	0.2**	0.1	-0.2**	-0.2**	-0.1*	0.0	26.7
	0.0	0.3	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	
Average node degree	4.7**	0.7**	0.2**	0.9**	0.3**	0.1*	0.3**	0.1	-0.1*	0.1	0.4**	34.6
	0.0	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.0	0.1	0.1	
Connectivity, per cent	8.0**	0.4	0.2	-0.8*	-0.3	-0.2	0.2	0.6**	0.0	0.4*	0.2	9.5
	0.1	0.5	0.3	0.4	0.2	0.2	0.2	0.2	0.2	0.2	0.2	
Average path length	2.50**	-0.08	-0.05*	-0.01	-0.01	0.00	-0.04**	-0.03*	0.03*	-0.02	0.00	7.8
	0.01	0.06	0.02	0.03	0.02	0.02	0.01	0.02	0.02	0.02	0.01	
Clustering, per cent	53.7**	2.6	2.7	-1.1	0.3	1.7*	1.7	0.3	-0.2	0.8	-4.5**	17.2
	0.6	2.3	1.9	2.3	1.2	0.9	2.3	1.1	0.9	0.9	0.9	

Note: For each variable, the first line of results is parameter estimates and the second robust standard errors. Significant parameters on a 1 (5) per cent level are marked with ** (*) in a one-tailed t-test (df=200). Average path length is reported with two decimals.

Table A3. Seasonal effects, money market network, 2006

	Intercept	Holidays in		End of		Liquidity adjustment		Weekdays				R²
		Denmark	US	Quarter	Month	Expected	Unexpected	Monday	Tuesday	Thursday	Friday	
Nodes	30.6**	-4.7	1.5	-2.1	-5.0**	-0.9	3.1	-5.8**	-3.6**	0.5	-5.6**	22.6
	0.8	4.2	1.5	1.8	1.3	1.3	3.5	1.2	1.2	1.1	1.2	
Links	84.1**	-12.4	5.3	2.5	-14.9**	0.1	14.8	-18.9**	-9.6*	1.8	-17.4**	18.6
	2.9	13.3	6.1	6.9	4.0	4.7	12.0	4.2	4.2	4.1	4.0	
Value, billion DKK	24.4**	-3.4	2.8	-3.6	2.2	-1.0	-1.7	-4.2**	-1.7	1.1	-2.7	7.8
	1.1	4.3	3.7	2.2	2.4	1.4	2.1	1.5	1.6	1.5	1.5	
Volume, thousands	0.1**	-0.0	0.0	0.0	-0.0**	0.0	0.0	-0.0**	-0.0*	0.0	-0.0**	17.6
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
Average node degree	2.7**	-0.3	0.1	0.4**	-0.1	0.1	0.2*	-0.1*	-0.0	0.0	-0.1	8.6
	0.0	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	
Connectivity, per cent	9.2**	10.6	-1.2	2.3*	4.1*	0.4	0.7	2.8**	2.3*	-0.0	2.3**	17.3
	0.4	7.6	1.3	1.1	2.5	0.7	2.0	1.0	1.4	0.5	0.7	
Average path length	3.05**	-0.35	-0.11	-0.16	-0.12	-0.09	-0.16**	-0.10	-0.12	-0.03	-0.12*	6.9
	0.05	0.26	0.10	0.11	0.13	0.07	0.05	0.07	0.07	0.06	0.06	
Clustering, per cent	15.4**	-2.6	3.8	1.4	0.3	2.4	2.1	1.1	1.7	2.2	2.4	2.8
	1.1	3.3	3.7	2.1	2.3	2.0	2.4	1.6	1.6	1.7	1.6	

3. Summary statistics of the transactions network. This section contains the network topological measures of the transactions network, i.e. the network based on the full data set.

Table A4. Summary statistics, transactions network, 2006

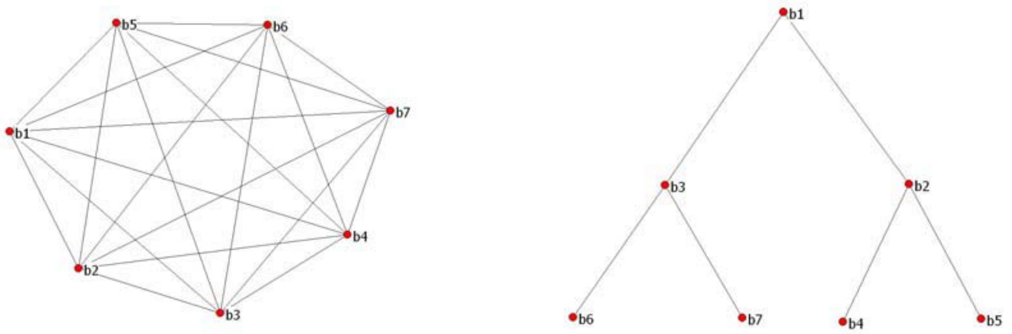
	Mean	Median	Min	Max	Std
Basic network properties					
Volume	2,355.9	2,337.5	1,607.0	4,171.0	323.6
Value	131.7	128.9	46.9	224.8	30.8
Nodes	67.8	67.0	57.0	88.0	4.8
Links	373.7	368.0	283.0	713.0	48.4
Connectivity, per cent	8.3	8.2	6.9	10.1	0.7
Reciprocity, per cent	24.0	24.0	19.1	28.4	1.9
Clustering	0.5	0.5	0.4	0.6	0.0
Average path length	2.4	2.4	2.2	2.6	0.1
Average node degree, k	5.5	5.5	4.6	8.1	0.4
Link weight, value	0.4	0.4	0.1	3.4	0.5
Link weight, volume	6.3	6.3	4.3	8.8	0.6
Node strength, value	1.9	1.9	0.8	3.4	0.5
Node strength, volume	34.7	34.4	23.0	48.8	3.9
Distance measures					
Diameter	5.1	5.0	4.0	8.0	0.6
MDF, M(2)	58.1	58.1	48.7	69.8	4.2
MDF, M(3)	94.3	94.4	87.1	98.8	2.1
MDF, M(4)	99.7	99.8	96.4	100.0	0.5
MDF, M(5)	100.0	100.0	98.1	100.0	0.1
Degree distribution					
max k^{in}	40.9	40.0	31.0	57.0	4.7
max k^{out}	34.5	34.0	24.0	54.0	4.2
Centrality measures					
Betweenness, links	29.4	29.0	22.5	38.0	3.2
Betweenness, nodes	93.9	93.5	74.3	121.0	9.1

Note: The notes to Table 4 and Table A1 also apply to this table.

4. The Furfine algorithm. The Furfine algorithm is used to identify overnight money market loans in order to split our data set into transactions stemming from two economically different networks.

The algorithm defines a transaction as an overnight money market loan if 1) the borrowed amount is at least 1 million DKK in integer numbers, 2) the borrowed amount is repaid with interest the next business day, and 3) the interest amount is within an acceptable range, i=[i_{Low},i_{High}]. The lower (upper) bound of this interval is the minimum interest rates on unsecured overnight lending reported by a panel of Danish banks minus (plus) 25 basis points. The acceptance range is extended with ±25 basis points since "interest rates charged are likely to vary across transactions", cf. Furfine (1999, p. 26). The acceptance range on Danish data is smaller than the ±50 basis points Furfine (1999) uses on Fedwire transactions. But broadening (decreasing) the acceptance range to ±50 basis points (±0 basis points) on Danish data gives almost the same classification of unsecured overnight lending by the algorithm.

5. Stylized networks. Two different extremes of stylized networks are illustrated in Figure A1. In a complete network, a bank has links to all other banks in the network such that each bank submits and receives transactions to/from all other banks within the network. In a tree network bank 1 submits transactions to banks 2 and 3, which submit transactions to banks 4 and 5 and banks 6-7 respectively. Another type is random networks, which is constructed by adding links at random to a given set of nodes. Stylized networks are not observed empirically but they are useful as benchmarks for analytical purposes.



a: Complete network b: Tree network

Fig. A1. Stylized networks

6. Topological measures used. The list below gives a short description of the topological measures used above in the same order as they appear in the text. Hekmat (2006) gives a thorough description of the physical concepts in network topology.

Connectivity. Connectivity, p is given by $p = \frac{m}{n(n-1)}$, where n is the number of nodes and m is the number of links in a network, e.g., the ratio of actual links formed to the number of potential links. p is a measure of the degree of completeness of a network and it varies between $\frac{1}{n}$ (tree network) and 1 (complete network).

Reciprocity. Reciprocity measures the share of links for which there is a link in the opposite direction (per cent). It varies between 0 (tree network) and 1 (complete network).

Clustering. Clustering is the probability that two banks, where each of them has a link to bank x, also have a link to each other. The clustering coefficient of node i is $C_i = \frac{m_{nn,i}}{k_i(k_i - 1)}$, where $m_{nn,i}$ is the number of links between the neighbors of node i and k_i is the number of payments terminating in (or originating from) node i. In other words, the clustering coefficient measures the number of links existing between the neighbor-nodes of node i divided by the potential number of links between the neighbors, e.g., clustering varies between 0 (tree network) and 1 (complete network). The clustering coefficient for the whole network is $C = \frac{1}{n}\sum_{i=1}^{n} C_i$. In a random network, i.e., where the links between banks are distributed randomly, the clustering coefficient is equal to the connectivity of the network, $C = p$. Clustering can be estimated both using the payments received in a node and payments submitted from a node and the latter measure is presented in this paper.

Average path length. The average path length for a network measures the number of links a transaction must pass to reach another bank in the network. Formally, $l = \frac{1}{n}\sum_{i=1}^{n} l_i$, where l_i is the average path length of node i given by $l_i = \frac{1}{n-1}\sum_{j \neq i}^{n-1} d_{ij}$ where the distance d_{ij} between nodes i and j is 1 if node i has a link to node j. The average path length can be estimated using payments received in or submitted from a node. The average path length measures in this paper are based on payments submitted from a node.

Average node degree, which is a measure of the average number of links per node, $k = \frac{1}{n}\sum_{i=1}^{n} k_i^{\ in} = \frac{1}{n}\sum_{i=1}^{n} k_i^{\ out} = \frac{m}{n}$,

where $\sum_{i=1}^{n} k_i^{\ in}$ ($\sum_{i=1}^{n} k_i^{\ out}$) is the sum of links that terminate in (originates from) a node.

Link weight. Links can be weighted by either the volume or value of payments through a link, e.g., a link, which handles 10 transactions, is more important than a link, which handles 1 transaction and vice versa for links weighted by values transferred. Formally, the w_{ij} is the weight assigned to the link between nodes i and j .

Node strength. Node strength for node i is defined as $s_i^{out} = \sum_{j=1}^{n} w_i^{out}$ for payments submitted from a node (we use this measure in this paper) or $s_i^{in} = \sum_{j=1}^{n} w_i^{in}$ for payments received in a node. That is, the larger the strength of a node is, the more important is the node in the network.

Diameter. The maximum distance between two nodes in a network. Defined as $\mathrm{D} = \max_{\mathrm{i}}(\max_{\mathrm{j}} \mathrm{d}_{\mathrm{ij}})$.

MDF(x), Mass Distribution Function, where *x* is the distance from a node. That is, MDF(2) says how large a share of all the nodes in the network, which can be reached within the distance 2 from a node. Formally, $M(x) = \frac{1}{n(n\text{-}1)}\sum_{i=1}^{n}\sum_{j \neq i} 1(d_{ij} \leq x)$, where $1(\cdot)$ has the value 1 if $d_{ij} \leq x$ and 0 otherwise. The mass distribution function can be estimated using payments submitted to or received from the nodes in a network. The MDF(x)-measures in this paper are based on payments submitted from a node.

Maximum in-degree of a node (max k^{in}). This is a measure of the maximum number of links that terminate in a node.

Maximum out-degree of a node (max k^{out}). This is a measure of the maximum number of links that originates from a node.

Betweenness (for nodes or links). Betweenness is a centrality measure, where the idea is that node i (link *ij*) is more central, the more paths are between nodes that run via node i (run via link *ij*) in the network.