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R ratio optimization with heterogeneous assets using genetic algorithm

Abstract

This paper presents a framework to portfolio optimization that is superior to the mean-variance approaches utilized for asset allocation. We show how a portfolio with heavily differing asset types in various market phases can be managed efficiently by using a ratio-based portfolio optimization approach and provide a general solution to related optimization problems and the technical challenges arising from them. Portfolio optimization is done by using a modified version of the R ratio in a benchmark-free setting for real estate funds of funds (FoFs). We use a genetic algorithm to solve the non-quasi-convex optimization problem and propose the use of genetic algorithms for related ratio-based optimization problems. Our results show the appropriateness of both the modified R ratio and the genetic algorithm used to optimize the fund portfolios in the benchmark-free environment. The algorithm efficiently solves the non-quasi-convex type of problem and related approaches of portfolio optimization are outperformed by the R ratio focused approach.

Keywords: portfolio optimization, genetic algorithm, R ratio, funds of funds, real estate funds, expected tail loss, non-quasi-convex.
JEL Classification: G11, C61.

Introduction

In this paper, we propose a framework to portfolio optimization that is superior to the mean-variance approaches utilized for asset allocation. We show how a portfolio with heavily differing asset types in various market phases can be managed efficiently by using a ratio-based portfolio optimization approach and provide a general solution to related optimization problems and the technical challenges arising from them.

Since the formulation of the portfolio selection theory, as formulated by Markowitz (1952), portfolio selection has been among the most discussed finance topics in both the theory and practice of finance. As a result, a large body of research work has emerged. Although the mean-variance approach allows a portfolio manager to identify the efficient frontier, risk-reward measures must be utilized to select the optimal portfolio given the investor's risk aversion. The most commonly used measure is the Sharpe ratio proposed by Sharpe (1964) and its extension (Sharpe, 1994). The Sharpe ratio focuses on portfolio compositions of assets that maximize the ratio of expected portfolio returns to the variability of the returns.

While the combination of the basic objectives of investing – maximizing reward and minimizing return variability or risk – is still the baseline for portfolio optimization approaches and frameworks, the measures and tools employed have changed. The mean-variance framework and the Sharpe ratio generally refer to the trade-off between reward and uncertainty (or variability); however, measures that try to capture risk instead of uncertainty have become increasingly popular. While there is still considerable debate on the most desirable and important properties of risk measures in portfolio theory[1], recent approaches mainly share the same crucial characteristic, namely a focus on the tails of the return distributions. Among those measures, ratios that relate portfolio reward to portfolio (tail) risk have gained greater attention[2].

In this paper, we contribute to the existing literature by providing a portfolio optimization method that is both independent of any distributional assumptions and may be used with any combination of assets, not being limited to benchmark-related problems. These properties are especially important when considering flexible and complex financial market products and the active management of portfolios containing them. One example is the funds of funds (FoFs) product because it requires careful allocation of capital by FoF managers in order to achieve value added for investors[3]. This stems from the fact that for FoFs normally a very large universe of target funds may be available, depending on the products' specification. If the universe of possible fund investments is very heterogeneous, the task of portfolio management is even more complicated. We use such a heterogeneous set of target funds with a sample of two very different types of real estate invest-

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[1] See Rachev et al. (2007) for an extensive study of risk and reward measures in portfolio management.
[2] See Farinelli et al. (2009) for applications and comparisons of tail ratio measures.
[3] See Stein et al. (2008) for a general introduction to funds of funds.

ment funds that are highly suitable for our study. The framework presented in this paper may be applied to any combination of assets though, for example, for bond and equity portfolios or even for direct investments rather than fund investments.

We use a modified version of the Rachev ratio (R ratio hereafter), which is a reward-to-risk ratio that is free from distributional assumptions. However, optimizing this ratio makes the solution of a non-quasi-convex optimization problem necessary. As this technical issue is very general and applies to all ratio problems which may have a negative denominator, we propose genetic algorithms as a general solution method for all ratio problems being non-quasi-convex. We show that although the span of possible solutions is very large due to the heterogeneous fund types that are candidates for inclusion in the portfolio, genetic algorithm solves the optimization problem efficiently and for all periods without the problem of numerical instability for the solution.

The paper is organized as follows. In section 1 we explain the methodology used in our study, namely the statistical measures, the optimization approach, and the genetic algorithm for solving the problem at hand. We introduce the data and the implications of the differing fund type properties in section 2. The portfolio optimization results are presented in section 3 and our findings are summarized in the last section.

1. Rachev ratio, portfolio optimization and the genetic algorithm

We begin with the R ratio[1]. This return-risk measure uses the expected tail loss (equivalent to the conditional value at risk, CVaR for continuous distributions), generally being defined as:

$$ETL_{1-\alpha}(r_p) = E\left(\max\left(-r_p, 0\right) \middle| -r_p > VaR_{1-\alpha}\left(r_p\right)\right),$$

where $ETL_{1-\alpha}\left(r_p\right)$ is the expected tail loss with tail probability α for portfolio returns r_p. Common choices for α are 1% or 5% in accordance with common choices of the confidence levels 99% and 95% used for value-at-risk (VaR) and other risk measures. ETL goes beyond traditional VaR by providing information on the expected loss in the case of a tail event instead of furnishing information only on the loss not be exceeded with the respective confidence level[2].

For the R ratio, the measure of expected tail loss is used in the following way: The nominator is defined as the ETL with probability α of the negative of the excess return of a portfolio over the benchmark. Conversely, the denominator is the ETL with probability β of the excess return of a portfolio over the benchmark. Defining the ratio this way, one obtains a measure of the estimated outperformance controlled for the severity of underperformances of a portfolio against the benchmark:

$$R(r_p) = \frac{ETL_{1-\alpha}\left(r_b - r_p\right)}{ETL_{1-\beta}\left(r_p - r_b\right)}.$$

In this study, we do not use a benchmark because we combine very different fund types, so we set r_B to zero and therefore have the modified R ratio being defined as:

$$R(r_p) = \frac{ETL_{1-\alpha}\left(-r_p\right)}{ETL_{1-\beta}\left(r_p\right)}.$$

By using this ratio, one obtains a measure for absolute expected gains at a given probability level divided by the absolute expected losses at another probability level. Sensible percentages for probability level α are, for example, 30-40% to get a reward term that focuses on the upper 30-40% of the return distribution, while probability level β could be chosen to be 1% or 5% to take the highest expected losses into account and to be in accordance with common risk metrics.

Having defined the ratio to optimize the FoFs, we need to impose sensible restrictions and bounds prior to solving the problem. As normally FoF is of the long only type, we impose the typical no short-selling constraint. Furthermore, we restrict the maximum weight of any fund to 20% to obtain sensible results that are in accordance with practical portfolio management and often seen regulatory or compliance restrictions. In addition, we impose the classical full investment constraint and restrict the outcomes to portfolios with positive expected returns[3].

The problem therefore takes the following form:

$$\max_{w} R(r_p) = \frac{ETL_{1-\alpha}\left(-w^T r\right)}{ETL_{1-\beta}\left(w^T r\right)},$$

$$\sum w_i = 1 \text{ (full investment constraint)},$$

[1] For extensive discussions and applications concerning the R ratio and related risk and performance measures see Biglova et al. (2004), Rachev et al. (2005), Okuyama and Francis (2007), Rachev et al. (2008) and Farinelli et al. (2009).

[2] See Sortino and Sachell (2001) and Rockafellar (2002) among others concerning VaR and CVaR / ETL.

[3] The decision whether to impose the restriction for positive expected returns of a portfolio needs to be based on the available asset types, since depending on the market situation no solution may be obtained if all or most assets had a negative return in the estimation period. In our case, it is always possible to obtain positive expected portfolio returns.

$0 \le w_i \le 0.2$ (long-only constraint and upper limit of 20%),

$w^T r > 0$ (positive expected return)

with $r_p = w^T r$ being the portfolio returns for the vector of fund weights w and the vector of fund returns r. We have chosen to maximize the R ratio with probability levels of 33% in the nominator, i.e. the upper third of the return distribution and 1% for the denominator, i.e. the lower 1% of the return distribution. Defining the ratio that way, we obtain a moderate and not very aggressive measure for the reward, controlled for the most severe expected losses during one period:

$$\max_w R(r_p) = \frac{ETL_{67\%}\left(-w^T r\right)}{ETL_{99\%}\left(w^T r\right)}.$$

We will contrast the results with other optimizations, for which the same restrictions and bounds were applied. The following optimizations were performed, thereby setting benchmark values as well as riskless rates of return to zero for achieving comparable results:

Sharpe-ratio (SR) optimization:

$$\max_w S(r_p) = \frac{w^T r}{\delta\left(w^T r\right)};$$

Expected Tail Loss (ETL) minimization:

$$\min_w ETL_{99\%}(w^T r);$$

Expected Tail Gain (ETG) maximization:

$$\max_w ETL_{67\%}(-w^T r).$$

Optimizations, as presented above, were performed due to the following considerations: The Sharpe Ratio is used to check whether the distribution and tail focused measures are truly superior to their mean-variance counterparts. The minimization of the expected tail loss has become a popular approach in portfolio optimization in the recent past and the expected tail loss is the denominator of our non-benchmark related R ratio, i.e. the risk part of the ratio. As the risk part of the ratio is used for a stand-alone optimization, it is natural to use the reward term as a single objective too, in order to analyze whether it is one term or the interplay of the two terms that delivers the best result.

While the SR, ETL, and ETG optimizations can be done using derivative based solving routines or linear programming routines (the solutions may lead to local minima however), the R ratio introduces more challenging computational issues. Generally, performance ratio optimizations may cause several issues related to solving the problem at hand. The ratio may turn out to be unbounded, which is a very general argument that is valid for all performance ratios with a possibly negative denominator.

For the R ratio in particular, there are additional complications because the problem is not quasi-convex. This means it cannot be reduced to a convex problem with the usual techniques, implying there may be many local extrema. However, even if problems are not quasi-convex, they can still be solved with traditional convex techniques (we have to keep in mind that the solution is only local nevertheless) but on the condition that the ratio is continuously differentiable twice. As the ETL function used in the R ratio does not have a first derivative for all portfolios as well as for small sample sizes and/or low tail probabilities, the issue of numerical instability may arise nevertheless. Thus, the optimization may not converge generally because of two reasons – either we have a case in which the ratio is unbounded, or the derivatives which the traditional convex optimization methods require are numerically unstable.

We resort to the class of genetic algorithms to solve the optimization problem outlined above. Classified as heuristic methods for global search problems, genetic algorithms are procedures that behave like natural, evolutionary processes. The origin of genetic algorithms dates back to the 1950s with Barricelli (1954 and 1957), Fraser (1957) and Fraser and Burnell (1970) heavily influencing the use of genetic algorithms in computer applications. Over the course of time, genetic algorithms have found their way to applications and research in finance and economics. For recent examples, see Dempster and Jones (2001), Hryshko and Downs (2004), Lai and Li (2008), and Lin and Liu (2008), among others.

Generally, optimization using genetic algorithms is done by successively generating "populations" of solutions. Starting the search, random combinations of individuals are formed, for which all individuals are evaluated concerning their fitness, i.e. their contribution with respect to the objective function. In any following iteration, the current population is used to build the next generation. This is done by selection based on the fitness of individuals, randomly recombining populations and mutating individuals. In our case the fitness function is the R ratio as a function of the return vectors and of the weights of the funds in the FoF, the population is the portfolio composition. This means that the genetic algorithm is successively building fund compositions and the evaluation of any fund's contribution to the fitness (i.e. to the maximization of the R ratio) is indicative on the following compositions.

While the use of genetic algorithms is often induced by computational necessities as in our case, they have a very beneficial side effect: The genetic algorithms search for global minima and therefore one obtains a very robust solution to the problem at hand and is not left with a local minimum or corner solutions.

2. Real estate funds: data and implications for portfolio optimization

In this section we describe our data sample and the implications of the data properties. The two types of funds used in this study are real estate mutual funds and German open-ended real estate funds. The former funds invest in companies in the real estate sector and in real estate-related companies. These companies need not be Real Estate Investment Trusts (REITs). Candidate companies are those doing business mainly through the development, management or trading of real estate properties. In addition, real estate companies that are qualified as REITs are tax-exempt under the requirement of an almost complete distribution of their capital gains. As with any type of stock, the stocks of real estate companies that the mutual fund managers invest in are traded on exchanges and are therefore priced through demand and supply interactions in the equity market. The share value can trade at a premium to or discount to net asset value of the properties held by the company. According to the share price of the target stocks, the daily net asset value of the real estate mutual funds is derived, at which fund shares may be redeemed on a daily basis.

The second asset type used in this study is the German regulated open-ended real estate funds. According to German investment law, the special type of open-end fund must invest directly in property, and most funds focus on commercial real estate. As with U.S. open-end funds (mutual funds), the fund issues shares at net asset value; that is, there is no premium or discount as in the case of a closed-end fund and redemptions are also possible at net asset value on every trading day[1]. Daily net asset values of the funds are determined via rents received, re-valuations of property held (normally once per year for each building), sales and acquisitions of properties as well as on costs and fees (from property management, consulting services, construction, refurbishments). In addition, the funds need to hold large amounts of liquidity (mainly cash, overnight money and very conservative bond investments) because of their investments in very illiquid assets and the daily fund inflows and outflows. Due to the German practice of valuation, the changes in property values are small and provide a stable and smooth pattern over time. This is caused by basing the valuations on the long-term expected rents to be received (a long-term sustainable rental income method) by holding the property and is in contrast with mark-to-market oriented valuation methods seen in many other jurisdictions. In addition, especially for large portfolios, the smoothing effect is even greater because the assets re-valuation is distributed over the year, rather than taking place at one time for all properties held. For these reasons, open-ended real estate funds typically exhibit a very stable and non-volatile pattern over time.

Using these two kinds of real estate investments results in a very heterogeneous sample what represents a common problem for FoF managers. The problem of not having a benchmark for portfolio selection is apparent in this case, too. While FoFs investing in these two types of real estate funds (and in related fund types of the real estate sector) are spreading in Europe at the time of writing of this study, the combination of safe-haven investments and more risky and volatile assets is also common for other asset classes. Balanced funds or mandates comprising both bonds and stocks or bond and equity funds are examples of related problems. The nature of those changes primarily with respect to the combination of the differing asset types and the respective weightings.

As indicated above, the two types of real estate funds differ significantly with respect to their return characteristics and statistical properties. Apart from some exceptions the typical open-ended real estate fund is returning between 3% to 6% per year with small daily movements in the net asset value and an annualized standard deviation of less than 1%. In contrast, the real estate mutual funds are exhibiting high volatility and leptokurtotic, skewed return distributions, and are prone to tail events that are typical for equity investments.

For each class we have included 10 funds with Europe as their main investment region. Using weekly total return data from Thomson Financial DataStream until end of October 2008, we have chosen end of October 2003 as our beginning date to have five years of data. As we use a rolling window of 52 weeks, we have 209 periods and therefore four years with largely differing market periods for the fund portfolio optimization.

Tables 1a and 1b show the used funds and the descriptive statistics. From the statistics it is evident that the two fund types are very different from each other and that any assumption of normality of the return distributions fails.

[1] If any, there was only very little trading volume of these funds in secondary markets during normal market phases. However, the temporary suspension of redemptions by some funds (caused by large outflows of money and deteriorating liquidity) in October 2008 has led to trading activity on stock exchanges.

Table 1a. Statistics of data for German open ended real estate funds

German open ended real estate funds	Mean	Standard deviation	WeeklyMin	WeeklyMax	ETL 99%	Max. drawdown	Jarque-Bera
AXA Immoselect	4,78%	0,60%	-0,21%	0,68%	-0,16%	-0,21%	2352,18***
Commerz Real Hausinvest Europa	4,27%	0,83%	-0,19%	0,72%	-0,18%	-0,36%	691,43***
Credit Suisse Euroreal	4,21%	0,31%	0,00%	0,23%	0,00%	-0,00%	59,51***
Deutsche Bank Grundbesitz Europa	6,59%	4,77%	-6,33%	4,32%	-3,19%	-6,33%	26969,81***
DEGI Europa	3,18%	0,90%	-0,08%	1,71%	-0,05%	-0,08%	193716,98***
DEKA Immobilien Europa	4,27%	0,77%	-0,19%	0,72%	-0,18%	-0,19%	1400,83***
iii Euro Immoprofil	-0,57%	1,66%	-2,81%	0,69%	-1,61%	-3,61%	92949,90***
UBS Euroinvest Immobilien	5,89%	0,98%	-0,14%	1,24%	-0,11%	-0,14%	5421,51***
Union Investment Uniimmo Deutschland	3,83%	1,66%	-1,45%	2,63%	-0,78%	-1,45%	60209,68***
WestInvest 1	2,87%	1,06%	-1,32%	0,66%	-0,80%	-1,32%	12933,74***

Notes: Annualized (linear) returns and standard deviation. ***, **, and * denote significance at the 1%, 5%, and 10% levels (rejection of the normal distribution).
Source: Thomson Financial Datastream.

Table 1b. Statistics of data for real estate equity funds

Real estate equity funds	Mean	Standard deviation	WeeklyMin	WeeklyMax	ETL 99%	Max drawdown	Jarque-Bera
Amadeus European Real Estate Securities Fund	-9,63%	22,36%	-21,86%	6,60%	-15,74%	-72,36%	1316,72***
Credit Suisse European Property	-4,74%	21,31%	-17,60%	7,09%	-14,66%	-64,72%	492,47***
Dexia European Property Securities	-4,09%	20,62%	-19,00%	7,18%	-15,13%	-62,84%	1036,17***
Henderson Horizon Pan European Equities Fund	-5,23%	20,33%	-18,26%	5,73%	-14,70%	-68,86%	874,33***
Morgan Stanley European Property Fund	-6,38%	21,62%	-19,48%	5,81%	-15,22%	-66,67%	794,58***
AXA Aedificandi	0,87%	21,30%	-20,92%	6,73%	-15,57%	-58,79%	1462,06***
ESPA Stock Europe Property	-0,55%	18,14%	-13,69%	5,49%	-10,92%	-58,94%	243,19***
Pioneer Eastern Europe Stock	-2,16%	30,49%	-24,56%	18,87%	-19,58%	-68,11%	633,07***
ING Invest European Real Estate	-2,28%	20,93%	-16,65%	6,96%	-13,24%	-60,57%	347,92***
Constantia European Property	-5,14%	20,58%	-12,84%	8,24%	-10,65%	-64,94%	84,28***

Notes: Annualized (linear) returns and standard deviation. ***, **, and * denote significance at the 1%, 5%, and 10% levels (rejection of the normal distribution).
Source: Thomson Financial Datastream.

Furthermore, Figure 1 is displaying the very time-dependent performance of the real estate equity funds and the fairly stable return patterns of the German open ended real estate funds.

3. Optimization results

We show the results of the dynamically optimized fund portfolios in this section. As the algorithm is seeking to minimize the fitness function, we took the negative of the R ratio to maximize it. It is clear that the possible results can be very dispersed when considering the minimum (0.0587) and maximum (infinite for the fund with zero ETL, 21.391 for remaining funds) values of the R ratio of the 20 funds during the testing period. Even though the imposed boundaries greatly reduce the span of possible results, the dispersion is, of course, still huge.

First, we checked whether a common derivative-based optimization routine would find solutions to the problem. In almost all periods this approach failed, although the maximum allowed iterations and function evaluations have been set to almost impractically high values. This comes as no surprise when keeping in mind the numerical problems discussed in section 1. We therefore went on with the analysis using the genetic algorithm to optimize fund portfolios with respect to the R ratio.

Figure 2 shows an arbitrarily picked example (from the week ending September 15, 2006) of the 209 optimizations. From the subplot bottom left showing the cause of termination we see that the algorithm found a solution to the problem after only 19 generations, which was within the span of maximum iterations allowed (set to 100). Furthermore, one can see that with the ongoing process of building fund compositions the algorithm approached both the minimum of the fitness function (the maximum attainable R ratio in our case, subplot top left) as well as the fulfilling of the constraints by minimizing the constraint violations (subplot bottom right). The population providing the best solution to the R ratio maximization problem is depicted in the subplot at top right, showing the composition of the expected R ratio-optimal FoF for the next period. For every period, the genetic algorithm converged to an optimum without exceeding the limits or constraints, showing the usefulness of its application to the problem.

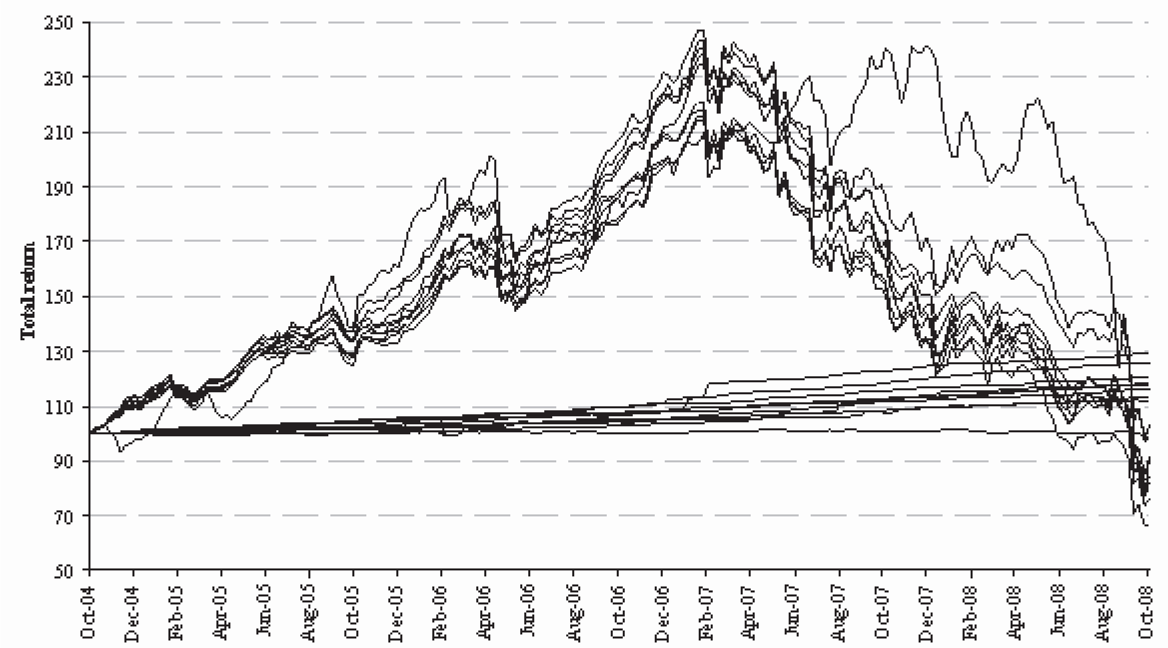

Fig. 1. Total returns of the 20 used funds

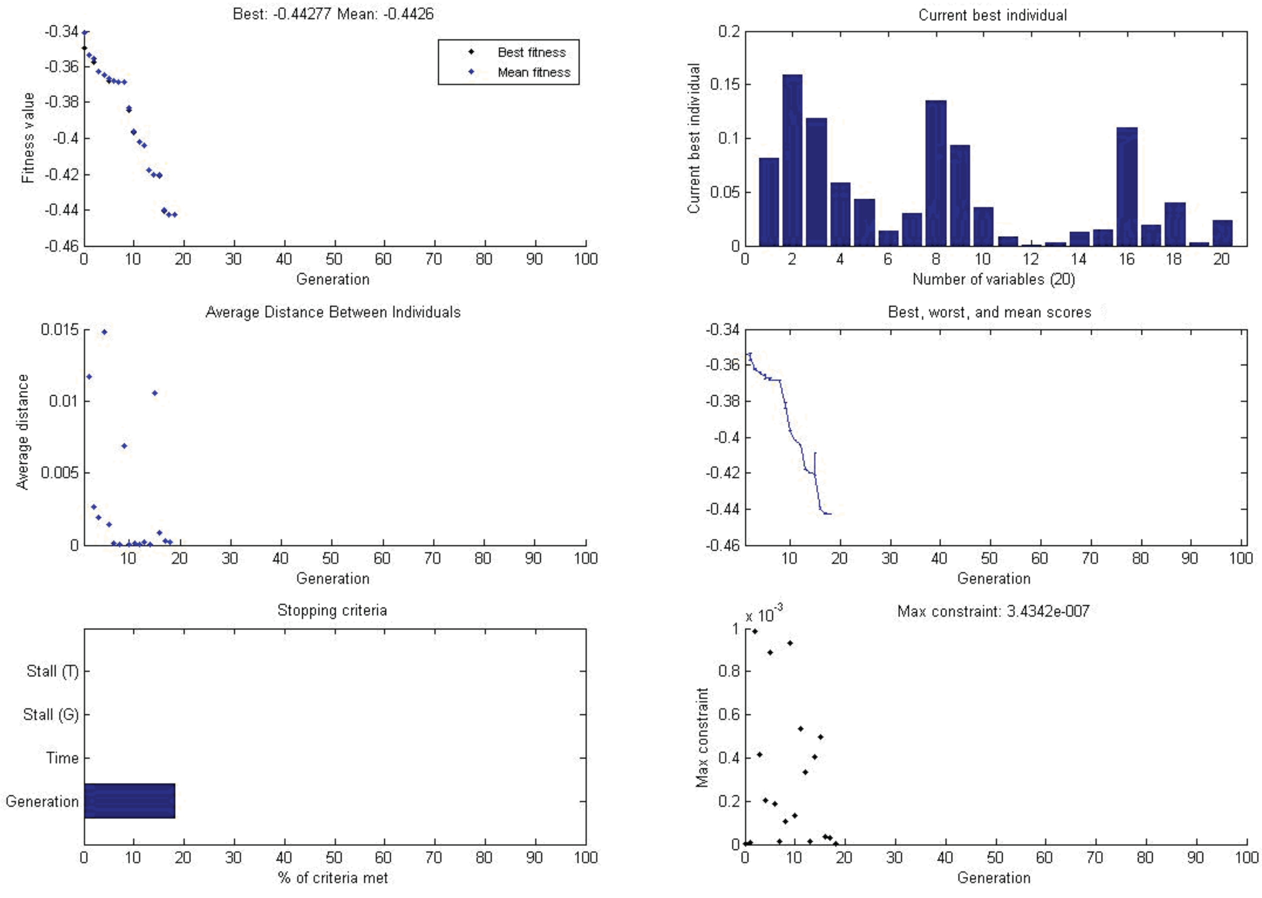

Fig. 2. Example of genetic algorithm for solving the R ratio optimization for the estimation period September Week 2, 2005 until September Week 2, 2006

The SR optimization was done by a standard derivative-based optimization. For only a handful of periods, optimal portfolios were violating a constraint; we then used the previous allocation for that period, not significantly influencing the results. For the ETL and ETG optimizations, standard derivative-based solving methods were also sufficient and delivered results for all 209 periods for both approaches, we did not experience numerical instabilities in any of the rebalancing periods.

By calculating the portfolio returns when investing the portfolio as indicated by the weekly ratio maximization, the performances shown in Figure 3 and summarized in Table 2 are obtained. The R ratio optimized portfolio clearly outperforms both its Sharpe ratio counterpart that focuses on returns to variability as well as the two approaches using either the reward or the risk term. As expected, the R ratio FoF has a higher standard deviation than the Sharpe ratio portfolio, but only a slightly higher one than the risk reduction focused minimum ETL portfolio (the ETG oriented FoF has the highest dispersion, of course, as it does not control for either variability or risk). It is particularly interesting that the R ratio optimal portfolio has a somewhat smaller ETL than the portfolio focusing exclusively on that measure. This means that the orientation of the R ratio to realize gains and thereby to control for the highest risks works very well for our set of heterogeneous assets. A reward to risk ratio as used here is therefore highly effective on realizing risk-adjusted returns. This became even more clear when calculating the R ratio for all four approaches after the optimizations were done. As the ratio should naturally be the highest for the approach focusing on it, we can see that indeed this outcome is obtained, with a 42% (0.27 to 0.19) higher ratio when being compared with the Sharpe and ETL portfolio and a 29% (0.27 to 0.21) higher ratio when being compared to the ETG portfolio.

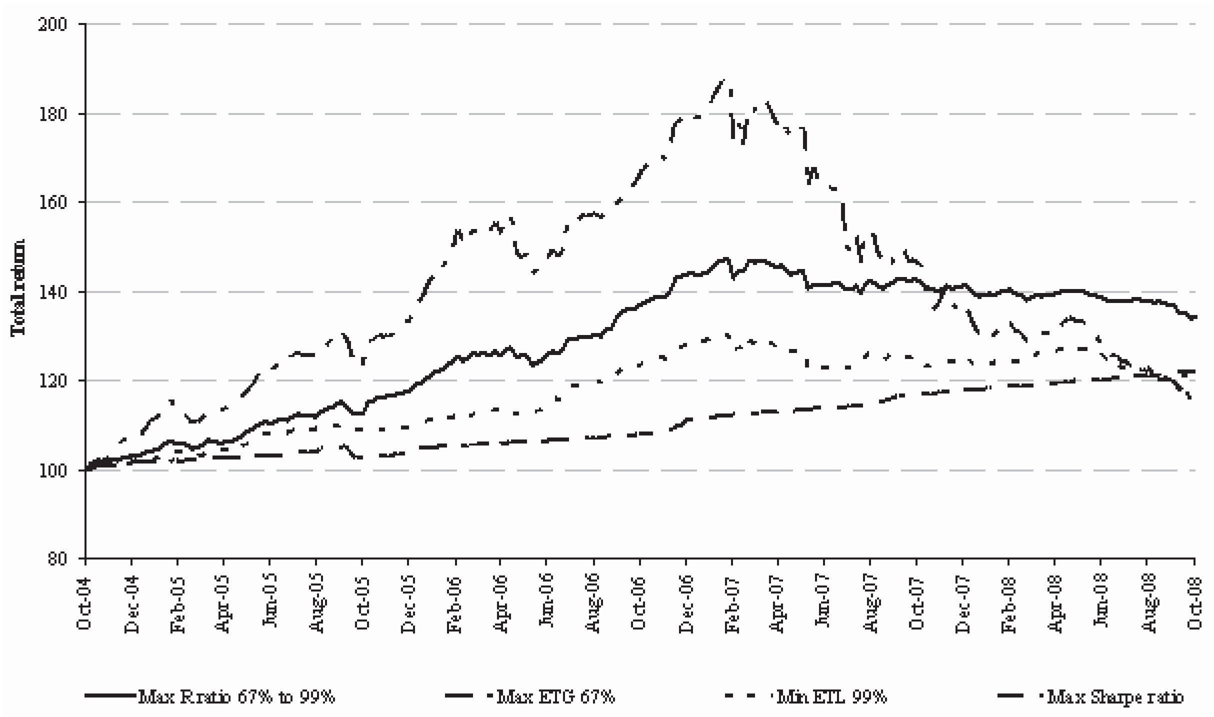

Fig. 3. Total returns of the four portfolio optimization approaches

Table 2. Statistics of optimized portfolios

Optimized funds of funds over time	Mean	Standard deviation	WeeklyMin	WeeklyMax	ETL 99%	Max drawdown	R ratio (67% to 99%)
R ratio optimized portfolio (67% to 99%)	7,61%	4,68%	-2,93%	2,31%	-2,76%	-9,08%	0,27
Sharpe ratio optimized portfolio	5,06%	1,69%	-1,43%	1,23%	-1,31%	-2,38%	0,19
Expected Tail Gain optimized portfolio (67%)	3,20%	12,59%	-7,81%	4,89%	-7,71%	-39,53%	0,21
Expected Tail Loss optimized portfolio (99%)	4,89%	4,36%	-3,51%	2,11%	-3,11%	-7,91%	0,19

Notes: Annualized (linear) returns and standard deviation.

As the statistics of the FoFs discussed, so far, focused on the weekly measures and the distributions, the inter-temporal measures also deserve attention. As we can see from Figure 3, the four approaches led to very different return patterns over time. While the ETG portfolio generates large returns during the bull phase of the real estate equity markets, the same portfolio took a large hit during the correction in the market and the ongoing financial market crisis, since no control for risk is implemented. On the other side, the large standard deviations of the real estate mutual funds lead to very defensive FoF allocations when using the Sharpe ratio. The return pattern merely resembles the ones of the German open-ended real estate funds, i.e. the Sharpe ratio is missing the upside possibilities due to investing heavily in the safe-haven funds. While all three approaches result in a lower terminal wealth than the R ratio FoF, the comparison between the portfolios based on the R ratio and the ETL turns out to be most interesting again. After the R ratio portfolio has realized far more upside returns in the bull phase of the real estate equity markets, the drawdown in the following post-peak phase (which was in February 2007) was only slightly worse than that of the ETL FoF (-9.08% versus -7.91%). This shows again that R ratio optimized portfolios may be able to realize upside potentials and, on the other hand, limit the severity of losses during downward phases as well.

However, none of the approaches delivered a return pattern that realized the good performance of the equity markets and switched completely into safe-haven investments during the drawdown period, but this is merely a fact that is due to the chosen exemplary estimation window of 52 weeks. Although it is questionable that perfectly fitting portfolios are realistic, shorter durations, higher frequencies, and other estimation methods for the tails or combinations of estimation periods could further enhance the return patterns of all four approaches.

Conclusion

In this paper, we propose a framework to portfolio optimization that is superior to the mean-variance approaches utilized for asset allocation. Using a very heterogeneous set of funds for which we used real estate funds as an example, we show how a portfolio can be managed efficiently by using a ratio-based portfolio optimization approach. We also provide a general solution to related optimization problems and the technical challenges arising from them.

The modified R ratio approach used for our benchmark-free optimization delivers an FoF performance that is superior to the one obtained when performing a Sharpe ratio-based optimization approach as well as when employing other tail-dependent optimization frameworks. Our results show the appropriateness of the approach that is due to the capability of taking into account tail risks and simultaneously realizing gains on the upside.

Arising computational challenges caused by the non-quasi-convex type of the optimization problem are addressed by using a genetic algorithm. The genetic algorithm solved the optimization problem efficiently and resulted in robust optima, while classical derivative-based algorithms, which in addition may result in local minima, failed to solve the problem at hand. As the problem of non-quasi-convexity of the optimization is apparent for all ratio-based optimizations to may have a negative denominator, we propose to use genetic algorithms for solving such problems in general.

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