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## The water and diamond paradox and green NNP as a welfare indicator

### Abstract

A classical structure that is used to analyze the water and diamond paradox provides an intuitive underpinning to the modern theory of welfare measurement in a growth context. John Law's and Adam Smith's concepts of value-in-use and value-in-exchange have modern aggregated counterparts. Complemented with Dupuit's extension in terms of a utility function with a declining marginal utility, they are close to enough to provide the intuition behind important aspects of modern dynamic welfare measurement. We answer four modern questions: (1) Will an increase in the level of *NNP* indicate a welfare improvement? (2) Will *NNP* growth indicate a local welfare improvement? (3) If the answers to (1), (2) are no, what are the underlying reasons? (4) How do the correct welfare indicators look like? At least Dupuit, as an inventor of the consumer surplus, may perhaps have agreed with some of the answers to the modern dynamic approach.

**Keywords:** the water and diamond paradox, green NNP, welfare measurement.

**JEL Classification:** B12, C61, O44.

### Introduction

It has been long known that traditional GDP (Gross Domestic Product), or for that matter traditional NNP (Net National Product), are not exact welfare indicators. The text-book arguments behind this view contain a couple of obvious reasons. One is related to the definition of net investments: the only information about net investments in the conventional NNP refers to physical, "man-made capital". This means that changes in other important stocks, such as natural resource stocks, environmental stocks and the stock of human capital are not included. Another – although related – flaw in NNP is that external effects are not handled in an appropriate manner. When present, the market data on which NNP is based are flawed because prices do not reflect the true underlying scarcities. A third example is that traditional NNP, because it is an aggregate number, does not reveal how consumption opportunities are distributed between individuals or generations.

However, all three of the above reasons can be assumed away by moving to an ideal situation, where it is assumed that all types of capital stocks are correctly priced and included in NNP. We can also assume that all consumption services produced by capital goods are included in the consumption vector, and that the corresponding correct rental prices are available. Moreover, we can exclude externalities, and duck distributional issues by assuming that an intertemporal welfare function supports the efficient market solution. Now, in what sense will an augmented NNP concept, comprehensive (or Green) NNP ( $NNP^c$ ), which does not include the above listed flaws, be a welfare indicator? More specifically, will a higher  $NNP^c$  indicate a welfare improve-

ment? And will  $NNP^c$  growth indicate a (local) welfare improvement? And, if not, what are the underlying reasons? Or even more important, what would the correct welfare indicator look like?

### 1. Some old key results – the value paradox

To answer the array of questions posed at the very beginning of this paper, we will start by moving to a classical and very simple framework. The idea is to convey the intuition behind the answers to the reader in a simple manner. The discussion shows that the classical economists had important insights into matters that even today still causes some confusion<sup>1</sup>.

At the end of the fourth chapter of book one in Adam Smith's celebrated volume *The Wealth of Nations* (1776), he brings up a valuation problem that is usually referred to as *The Value Paradox*<sup>2</sup>. He writes: writes:

"The world VALUE, it is to be observed, has two different meanings, and sometimes expresses the utility of some particular object, and sometimes the power of purchasing other goods which the possession of that object conveys. The one may be called "value in use"; the other, "value in exchange". The things which have the greatest value in use have frequently little or no value in exchange; and, on the contrary, those which have the greatest value in exchange have frequently little or no value in use. Nothing is more useful than water: but it will purchase scarce anything; scarce anything can be had in exchange for it. A diamond on the contrary, has scarce any value in use; but a very great quantity of other goods may frequently be in exchange for it<sup>3</sup>."

<sup>1</sup> For an elegant summary of the issues involved see Asheim (2005).

<sup>2</sup> First mentioned by John Law (1705).

<sup>3</sup> Smith (1776), reprinted as *Penguin Classics*, 1986, pp. 131-132).

He is unable to credibly resolve the paradox – although he uses three chapters to convince the reader that it can be resolved by the components of the natural price, i.e., essentially the notion that the long-run price is determined by the production costs. Some of the reasons behind the “failure” are not farfetched. Adam Smith was aware of supply and demand without being able to produce anything fresh about the fundamental ideas upon which these concepts rest. He was not aware of the idea to model the total utility value of consumption in terms of a utility function, and the related idea of assuming that the utility function exhibits a declining marginal utility. To the late Scholastic tradition represented by e.g. Ferdinando Galiani (1750/1977) the water and diamond paradox would not have represented any serious difficulty. The Scholastics were used to discuss the relationship between utility, scarcity and relative value. Galiani may have claimed that the high total value of water is counteracted by its lack of scarcity, while the reverse is true for diamonds. In other words, the high relative price of diamonds in relation to the price of water can be explained by an extreme high scarcity of diamonds and an extreme low scarcity of water. Although his formal analysis does not make sense in all respects, it is clear that he was close to discovering a decreasing marginal utility. He writes<sup>1</sup>: “there is nothing more useless than bread for one who is sated.”

Adam Smith wrote his treatise 60-80 years ahead of Cournot (1838), Dupuit (1844) and Gossen (1854), who founded the modern utilitarian framework in Economics. His distinction between value-in-use and value-in-exchange nevertheless contains a non-trivial insight, which is fundamental for the answer to the questions under consideration. In fact, he touched upon the answer to the first question, since he pointed out that the value-in-use of a good – its contribution to total welfare – is not the same thing as its value-in-exchange.

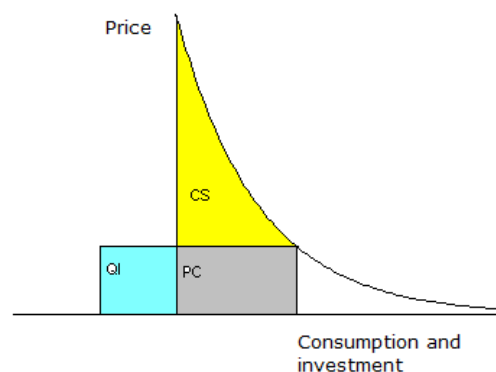
Today a first/second semester student of Economics would say: the demand and supply curves are such that they intersect at a very low price for water, and a very high price for diamonds. Although essentially correct, Adam Smith would perhaps have asked: why do supply and demand intersect at such a low level for water? Now, a first semester student could be in trouble. A clever one would say that price is determined by the value of the last unit sold, which in a perfect market equals marginal cost. Since consumers are free to buy or not buy, the price must coincide with the value of the last unit. If water is priced above its marginal utility, the last unit cannot

be sold. Therefore the price must fall until it coincides with the value of the last unit sold. Moreover, all units of water are homogeneous and as there is only one price in a competitive market, all units will sell for the price (marginal utility) of the last unit. As one of Paul Samuelson’s students pointed out: “The theory of economic value is easy to understand if you just remember that the tail wags the dog: concentrate on marginal and not total utility”<sup>2</sup>. As we will show below, this is not the whole truth. Adam Smith was right that the adverb *total* matters for value.

**1.1. A simple scheme of analysis – the Smith-Dupuit answer.** Let us now use the insights provided by Smith and later by Dupuit to give intuitive answers to the following four questions:

1. Will an increase in the level of  $NNP^c$  indicate a welfare improvement?
2. Will  $NNP^c$  growth indicate a local welfare improvement?
3. If the answers to (1) and (2) are no, what are the underlying reasons?
4. What do the correct welfare indicators look like?

Let us start from an extended “dynamic” version of the standard supply and demand diagram. In Figure 1 the first quadrant contains the standard demand curve. For simplicity, and to highlight the particular structure of the problem, we have assumed that the supply curve is vertical.



**Fig. 1. A dynamic version of value-in exchange and value-in-use**

Since there are only one good and one period, we can interpret the demand curve as either measured in utility (marginal utility) or in a money metrics, which means that it is a Marshallian demand curve. Although Adam Smith did not use any diagrams, it is reasonable to connect the rectangular area  $PC$  with the term value-in-exchange, and the rectangular area plus the triangular area  $CS$  with the term value-in-use. This is the interpretation made by Dupuit (1844), who writes:

<sup>1</sup> Galiani (1750/1977, p. 26).

<sup>2</sup> Samuelson (1964, pp. 433-434).

“Doctor Smith, who recognizes two values in an object – *its value-in-use*, which is its utility as we understand, the value to him who has a need to consume the product; and its *value-in-exchange*, which is the value of the same product to him who has a need to sell it<sup>1</sup>”.

The value-in-exchange is also what the consumer has to pay in terms of utility in order to consume the good. To interpret the diagram as  $NNP^c$  we have added net investment in the second quadrant. As the reader can see from the diagram the price of the investment good coincides with the price of the consumption good. This will be the case in a one good dynamic economy where the consumption good can be used also for investment. In a dynamic context the current value of investment represents what net investment will yield in utility in terms of future consumption goods. In particular, along an efficient path consumption and future consumption (investment) should be allocated so that the marginal utilities of future and present consumption coincide. In Figure 1 this insight has been used to illustrate a dynamic version of the concept value-in-use which consequently contains the value of net investment.

Hence, if we use the intuition conveyed by the Smith-Dupuit framework to guess how welfare should be measured in a first best market economy, an increase in  $NNP^c$  will, in general, not indicate a welfare improvement. The value-in-exchange does not measure total utility, it measures the value of total consumption and total net investment evaluated at the marginal value of consumption.

In other words, although Smith did not recognize marginal utility or the consumer surplus, his idea – inspired by the value paradox – about the distinction between the value-in-exchange and the value-in-use, can, together with Dupuit’s insights (even diagrams) on marginal utility and the consumer surplus, help us to give a non-formal answer to all four questions listed earlier. Neither  $NNP^c$ , nor its growth seem to be good welfare indicators<sup>2</sup>. The reason is that  $NNP^c$  is only one component of value-in use; consumer surplus is the other.

From Figure 1, which formally only covers a one good economy, it is clear that comprehensive NNP is linear in consumption and, in general, unable to handle consumer surpluses – the triangle  $CS$ . An informed guess, based on the classical ideas, is therefore that  $NNP^c$  plus terms reflecting the consumer surpluses in all markets will be the basis for

both a global – and a local welfare indicator in a dynamic economy.

## 2. Some modern key results – Weitzman’s theorem

There is a classical result in Weitzman (1976) on the welfare significance of NNP. It tells us that, in a Ramsey growth model with a utility function that coincides with an aggregate consumption good and a comprehensive set of capital goods,  $NNP^c$  is proportional to future utility along the first best growth path. The factor of proportionality is the utility interest rate (the rate of time preference). This may sound as good news for  $NNP^c$  as a welfare indicator, but Weitzman uses a suggestive “trick” by choosing a linear homogeneous utility function that creates no consumer surplus.

With a more general utility function, the result can be expressed in a utility metrics by saying that the value of the Hamiltonian of the optimal control problem, i.e. the current value of the utility function plus the future utility value of the net investment vector, measured along an optimal path, is directly proportional to the sum of future utility. The intuition is that at each instant in time, consumption is allocated such that the marginal utility of consumption equals what a unit of investment would yield in terms of utility from future consumption. Moreover, due to a non-arbitrage condition (the Euler condition) on the value of future investment, it is not profitable to move investment from one point in time to another. Given a constant utility discount factor, integrating the differential equation for the development of the Hamiltonian along an optimal path yields the result.

This general version of Weitzman’s theorem is not practical, since we cannot observe utility. Later researchers in the comprehensive NNP-Green Accounting tradition, such as Hartwick (1990) and Mäler (1991) have partly circumvented this problem by linearizing the Hamiltonian (the utility function) and dividing by the marginal utility of the consumption good<sup>3</sup>, thereby generating NNP like linear money metric indexes which have been referred to as Green-NNP. This is, an approximation of the true money value of the total utility, and, under a strictly concave utility function, it may be a bad approximation.

The first conclusion from the modern approach is that  $NNP^c$  is not a welfare indicator in the sense that we can conclude that an increase (decrease) in comprehensive NNP, will mean an increase (decrease) in welfare. It is not quite clear what can be said

<sup>1</sup> Dupuit (1952), English translation.

<sup>2</sup> Note that Figure 1 describes a most simple setting and the value-in-exchange still does not work.

<sup>3</sup> The utility functions depend on aggregate consumption.

about  $NNP^c$ -growth as a local welfare indicator, but the first conclusion sounds like bad news.

### 3. A correct money metrics welfare indicator

Translated into a Ramsey growth model with  $n$  consumption goods and  $m$  capital goods<sup>1</sup>, Weitzman's fundamental theorem tells us that

$$\begin{aligned}
 H^*(t) &= u(c^*(t)) + \lambda^c(t)i^*(t) = \\
 &= \theta \int_t^\infty u(c^*(s))e^{-\theta(s-t)} ds = \theta W^*(t).
 \end{aligned}
 \tag{1}$$

In other words, the current value Hamiltonian at time  $t$  is proportional to the sum of future utility measured along the first best path of the economy (here denoted by the top-index\*) or the optimal value function  $W^*$ . The consumption vector  $c(t) = [c_1(t), \dots, c_n(t)]$ , and the net investment vector  $i(t) = [i_1(t), \dots, i_m(t)]$  are comprehensive in the sense that they contain all consumer and investment goods that are relevant for the consumptive and productive capacity of the economy. The factor of proportionality,  $\theta$ , is the utility discount rate,  $u(c^*(t))$  is the instantaneous utility function, and  $\lambda^c(t)i^*(t)$  denotes the vector product of the future current utility value of investments<sup>2</sup>,  $\lambda^c(t)$ , and the vector of net investments,  $i^*(t)$ . This means that all the entities in equation (1) are measured in utility. In other words, the Hamiltonian or "Utility  $NNP^c$ ", is a perfect welfare indicator in a utility metrics. Moreover, growth in Utility  $NNP^c$  is the corresponding perfect local welfare indicator.

Since most of the literature after Weitzman (1976) has stayed in the unpractical utility metrics or linearized the Hamiltonian (the utility function) to approximate utility  $NNP^c$ , one may wonder what the problems are to move into a money metrics. To see this we rewrite the instantaneous utility function in the following manner

$$\begin{aligned}
 u(c^*(t)) &= \int_0^{c^*(t)} u_c(c^*)dc = \\
 &= \lambda^m(t) \left[ p^*(t)c^*(p^*(t)) + \int_{p^*(t)}^{\bar{p}(t)} c^*(p)dp \right].
 \end{aligned}
 \tag{2}$$

The first equality in equation (2) follows immediate-

ly from Figure 1 by interpreting the demand curve in a utility metrics as the marginal utility of consumption,  $u_c(c)$ . The integral denotes the sum of the vector of marginal utilities from zero to the optimal consumption vector, i.e., it corresponds to the value in use. In the last component of the equation,  $\lambda^m(t)$  is the marginal utility of money at time  $t$ ,  $p^*(t)$  denotes the price vector that supports the optimal path, and  $c^*(p^*(t))$  is the demand vector at time  $t$ . The latter does not contain any income arguments that typically generates "integrability problems", i.e. the value of the (total) consumer surplus will depend on the integration (price) path. The reason is that the utility function in the Ramsey growth model is additively separable over time<sup>3</sup>. Hence, there are no income effects, and the consumer surplus is well defined.

The second equality is obtained after some technicalities, changing integration variables by putting

$$dc = \left( \frac{dc}{dp} \right) dp$$

and partially integrating the resulting expression. This means, among other things, that  $\bar{p}$  is the choke of price vector, i.e.  $c(\bar{p})=0$ . The first term on the right hand side of (2) corresponds to the utility cost of consumption and the second one to the utility value of the consumer surpluses in the economy. However, since the marginal utility of money is not necessarily constant over time, this expression cannot be used to transform the right hand side of equation (1) into a money metrics. The marginal utility of money cannot be moved outside the integral in equation (1), i.e. it would remain value-in-use times the exponential discount factor.

The key to a money metrics transformation of equation (1) was provided by Weitzman (2001, 2003), who introduced the following price index formula

$$\pi(t) = \frac{p(t;c)c}{p(t_0;c)c} \tag{3}$$

The notation on the left hand side,  $\pi$  depends only on time, indicates that the index is independent of the marker basket<sup>4</sup> – "benchmark independent". Formally this means that

$$\lambda^m(t_0) = \pi(t)\lambda^m(t) \tag{4}$$

The interpretation of the price vectors is that  $p(t, c)$

<sup>1</sup> In a Ramsey model, one maximizes the present value of utility  $\int_0^\infty u(c(s))e^{-\theta(s-t)} ds$ , subject to consumption and net investment, under a convex technology.

<sup>2</sup> Mathematically  $\lambda^c(t)$  is a vector of adjoint variables or co-state variables.

<sup>3</sup> For a formal derivation, see e.g. Weitzman (2001).

<sup>4</sup> To show that the index is independent of the market basket  $c$ , i.e. benchmark independent, and why this is important, we note that since the utility function is stationary over time it follows from the first order conditions for an optimal path that  $u_c(c) = \lambda^m(t_0)p(t_0;c) = \lambda^m(t)p(t,c)$ .

Multiplying through by the market basket  $c$ , and solving for the marginal utility of money at time  $t_0$  yields  $\lambda^m(t_0) = \pi(t)\lambda^m(t)$  which is a constant.

and  $p(t_0, c)$  denote the “imputed” market clearing prices that would be observed at the two points in time if the market basket of goods being consumed in the economy were  $c$ ; the vector  $p(t_0, c)$  is also the actual price vector at  $t_0$ . The name “ideal measure” is chosen by Weitzman (2001) to denote the direction toward which the formulators of a CPI- or PPP-type index strive when they try to select a representative market basket straddling two economies at a given point in time, or the same economy at two different points in time. The imputation problem is difficult in both cases. The scalar  $\pi(t)$  measures the price level at time  $t$  relative to that at time  $t_0$ . Put differently, the index seems to demand a great deal of information, but it is hard to see that any other idea will work. A simple way out would be to assume that the marginal utility of money is constant over time. This will, in general, only hold in a steady state, where the utility and money discount rates coincide<sup>1</sup>. However, using the Euler equations for the first best dynamic optimization problem, one can show that the marginal utility of income follows the following differential equation

$$\lambda^m(t) = \lambda^0(t_0) e^{\int_{t_0}^t R(\tau) d\tau} \tag{5}$$

Here  $R(t)$  is the nominal interest rate. Equations (4) and (5) means that the index can be written

$$\pi(t) = \lambda^0(t_0) / \lambda^m(t) = e^{-\int_{t_0}^t R(\tau) d\tau} \tag{6}$$

In other words, since a series of nominal interests typically exist the index is identified if one can come up with a measure of the utility discount rate  $\theta$ .

We can now re-scale the left hand side of equation (1) to read

$$H_r^*(t) = \lambda^m(t) \pi(t) \left[ \frac{y^*(t) + cs^*(t)}{\pi(t)} \right] = \lambda^m(t_0) [y_r^*(t) + cs_r^*(t)] \tag{7}$$

where  $y_r^*(t) = y^*(t) / \pi(t) = p_r^*(t) c^*(t) + q_r^*(t) i^*(t)$  is the real comprehensive NNP and  $cs_r^*(t) = cs_r^*(t) / \pi(t)$  the consumer surplus both expressed in real terms (in the prices of period  $t_0$ ). The real prices for consumer and investment goods are  $p_r^*(t) = p^*(t) / \pi(t)$  and  $q_r^*(t) = q^*(t) / \pi(t)$ , respectively. The expression in (6) has been called Generalized Comprehensive NNP (GCNNP)<sup>2</sup>. A similar operation can now be executed

on the right hand side of equation (1) and one ends up with the following money metrics results by putting the constant marginal utility of money at the base year equal to one:

*Theorem (Weitzman-Li & Löfgren):* The Generalized Comprehensive (Green) Net National Product (GCNNP) in (6) is a stationary equivalent of the future value of consumption plus the consumer surplus in real terms such that

$$\int_t^\infty H_r^*(t) \exp(-\theta(s-t)) ds = \int_t^\infty \left\{ p_r^*(s) c^*(s) + \int_{p_r^*(s)}^{\bar{p}_r} d_0(p_r) dp_r \right\} \exp(-\theta(s-t)) ds$$

or equivalently

$$H_r^*(t) = \theta M_r^*(t) \tag{8}$$

where

$$M_r^*(t) = \int_t^\infty \left[ p_r^*(s) c^*(s) + \int_{p_r^*(s)}^{\bar{p}_r} d_0(p_r) dp_r \right] \exp(-\theta(s-t)) ds$$

can be interpreted as the generalized welfare (wealth) in real terms.

Again returning to Smith and Dupuit,  $M_r^*(t)$  can also be interpreted as the total current value-in-use in real money terms. In the same spirit, we may call GCNNP the instantaneous value-in-use in real money terms. The Theorem tells us that there is a direct proportionality between the two money metrics value-in-use concepts, and that both are correct welfare indicators under first best conditions. It also tells us that  $NNP^c$  can only be a welfare indicator under special circumstances. One special case emerges if the utility function is linear homogeneous, i.e., when doubling consumption doubles utility. A special “special” case is when the utility function consists of an aggregate consumption good like in Weitzman’s original paper.

**3.1. Growth and welfare.** Given the Theorem, it should be obvious that growth in  $NNP^c$ , without further ado, cannot indicate a local welfare improvement. To see exactly why it fails to do so, we differentiate equation (7) ( $\lambda^m(t_0) = 1$ ) with respect to time to get

$$\dot{H}_r^*(t) = \dot{y}_r^*(t) + \dot{cs}_r^*(t) = \dot{y}_r^*(t) - \dot{p}_r^* c^*(t) \tag{8}$$

where the first term on the right hand side represents growth in real Green NNP. The second term represents the growth in consumer surplus at time  $t$ . Obviously, as long as the latter term is different from zero, we cannot conclude that  $NNP^c$ -growth  $\dot{y}_r^*(t) > 0$ , indicates a local welfare improvement. The reason for its appearance is that changes in relative prices will take place along the endogenously

<sup>1</sup> Nevertheless, applied consumer price index theory is based on the assumption that the marginal utility of income remains constant over time. It also neglects the change in the asset position of consumers (saving).

<sup>2</sup> See Li and Löfgren (2002) or Aronsson et al. (2004). Li and Löfgren (2002) contains a formal proof of the Theorem, and Weitzman (2001) contains a proof of an analogous result that compares two economies at the same point in time.

determined growth path of the economy, and change the value of the consumer surplus.

What we can do to make comprehensive NNP-growth a welfare indicator is to condition growth on some other aspect of the economic conditions at the time of measurement. In a seminal paper, Asheim and Weitzman (2001) show that if  $NNP^c$  is deflated by a Divisa consumer price index, which means that the last term in equation (8) becomes zero through the properties of this consumer price index, then growth in Green NNP can serve as a welfare indicator provided that the real interest rate is positive<sup>1</sup>. Li and Löfgren (2006) show that growth in Green NNP at constant prices (measured at time  $t$ ) indicates a welfare improvement provided that the “overall marginal rate of return of investment” is positive. The latter concept corresponds to the weighted average of the own- rates of return to capital goods using the corresponding net investment values as weights. The result holds independently of the price index, and the sign of the overall rate of return is observable<sup>2</sup>.

The key to this result is that there exists a simple local welfare criterion that always works, namely genuine saving. It is straightforward to show, by differentiation of the optimal value function, that

$$\dot{W}^*(t) = \dot{H}^*(t)\theta^{-1} = \lambda^c(t)\dot{i}^*(t) = \lambda^m(t)[q^*(t)\dot{i}^*(t)], \quad (9)$$

i.e., genuine saving coincides with the sum of comprehensive net investment multiplied by the marginal utility of money. The overall rate of return on investment is defined as

$$\rho(t) = R(t) - \dot{q}^*(t)i(t)/q^*(t)i^*(t) = \dot{y}^*(t)[q^*(t)\dot{i}^*(t)], \quad (10)$$

where  $R(t)$  is the nominal interest rate<sup>3</sup>. Clearly,  $\rho(t) > 0$ , and  $\dot{y}^*(t) > 0$  implies that genuine saving is positive, indicating a local welfare improvement.

There is, however, a situation where NNP growth breaks down as a welfare indicator, independently of any (nontrivial) conditioning. Say that we are dealing with a spaceship economy<sup>4</sup> in which the

resource stock dynamics is given by the differential equation  $\dot{x}(t) = -c(t)$ , with the limited food supply,  $x(0) = x_0$ , at time zero. The maximized current value Hamiltonian can be written

$$H^{c^*}(t) = u(c^*(t)) - \lambda^{c^*}(t)c^*(t). \quad (11)$$

Linearizing by taking the first differential of the utility function and using the first order conditions for an optimal path,  $u_c(c^*(t)) = \lambda^c(t)$ , means that we can approximate the Hamiltonian and, hence, future welfare by the expression

$$H^{c^*}(t) \approx u_c(c^*(t))c^*(t) - \lambda^{c^*}(t)c^*(t) = \lambda^{m^*}(t)p^*(t)[c^*(t) - c^*(t)] = 0. \quad (12)$$

In other words, NNP defined as the value of consumption plus net investment, is identically zero for the spaceship economy, independently of the size of the packed lunch,  $x_0$ . This means that the rate of return concept defined in equation (7) breaks down as a welfare indicator, since NNP growth is identically zero, and hence  $\rho = 0$ .

Genuine saving tells us that local welfare is decreasing over time. The results in equation (5) and the Theorem are of-course still valid, and it is straight forward to show that the size of packed lunch matters for welfare, that the value of the consumer surplus is positive, and that it decreases over time<sup>5</sup>.

### Conclusion

The above analysis shows how the classical structure that is used to analyze the water and diamond Paradox provides an intuitive underpinning to the modern theory of welfare measurement in a growth context. John Law’s and Adam Smith’s concepts of value-in-use and value-in-exchange have modern aggregated counterparts in a Generalized Comprehensive (Green) NNP concept and a comprehensive (Green) NNP, respectively. Complemented with Dupuit’s extension in terms of a utility function with a declining marginal utility, they are enough to provide the intuition behind important aspects of modern dynamic welfare measurement.

In fact, although much more detailed and technical, the modern theory seems to add very little in terms

<sup>1</sup> Note that the Divisa consumer price index cannot be used to transfer Weitzman’s (1976) result into an exact money metrics welfare index. Hence, even if the last term of (8) disappears the claim requires conditioning on a positive real interest rate.

<sup>2</sup> The problem of indicating a welfare improvement by means of growth in NNP is discussed by Dasgupta and Mäler (2000) and Dasgupta (2001). A comprehensive survey of the current state of the art is provided by Asheim (2005).

<sup>3</sup> Equation (9) can be decomposed in the following manner:

$$\rho(t) = \sum_{j=1}^m \alpha_j [r(t) - \dot{q}_j(t)/q_j(t)], \quad (8a)$$

where  $\alpha_j = q_j(t)i_j(t)/\sum_{j=1}^m q_j(t)i_j(t)$  is the weight share in total net investment of capital component  $j$ , and  $r(t) - \dot{q}_j(t)/q_j(t)$  is its own rate of interest. Hence,  $\rho(t)$  can be interpreted as the net-investment weighted own rate of interest.

<sup>4</sup> The term originates from Boulding (1966). He conducts an insightful verbal discussion of the planet earth’s sustainability problem. An econ-

omy with a similar problem to define comprehensive NNP would be Kuwait, which is extremely dependent upon its oil resources.

<sup>5</sup> A similar problem occurs with the Asheim-Weitzman welfare indicator, since growth in NNP at changing prices will be zero. To see this, we note that their welfare indicator has the following shape:

$R(t)q_d^*(t)\dot{i}^*(t) = p_d^*(t)\dot{c}^*(t) + q_d^*(t)\dot{i}^*(t) + \dot{q}_d^*(t)i^*(t)$ . In the cake eating model  $p^d(t) = q^d(t)$  and  $\dot{y}^*(t) = 0$ , i.e., NNP at fixed prices is identically equal zero. This means that  $\dot{c}^*(t) = -\dot{i}^*(t)$  for all  $t$ , implying, since initial conditions are the same, that  $c^*(t) = -i^*(t)$  all  $t$ . From this and the properties of the Divisa index, we can conclude that  $\dot{q}_d^*(t)i^*(t) = -\dot{p}_d^*(t)c^*(t) = 0$ , implying that NNP at varying prices also equals zero.

of fundamental new insights. One reason behind this may be that we over-interpret what Smith and Dupuit have accomplished. As a matter of fact, it would have been hard for them to come up with the idea to condition on the real interest rate, or the overall rate of return on investment, in order to make growth in Green NNP a local welfare indicator. The

main reason is, of course, that the concepts were invented years after Dupuit wrote his insightful paper. However, as the inventor of consumer surplus, he would perhaps have agreed, if presented with the modern dynamic approach, that the only possibility to make growth in Green NNP a reliable welfare indicator is to condition on something.

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