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Research on the risky convertible bond with reset clause: an application of finite difference method

Abstract

Convertible bond is difficult to value since it is a hybrid financing instrument. The concept commonly accepted is that the convertible bonds can be divided into pure bonds and call options. However, it makes the pricing of convertible bonds become complicated for more and more additional provisions embedded in the bonds. In the background, numerous methods are developed for finding the most accurate pricing model of convertible bonds. With the default risk concern, Ayache, Forsyth and Vetzal (AFV, 2003) propose a model where the stock price of the bond issue firm drops at a specified percentage and obtain a total risk hedge result. Nowadays, most convertible bonds have reset clauses. For reacting to the reality, the aim of this paper is to extend AFV model (2003) to incorporate the reset clause to price the conversion option of the risky convertible bond.

Keywords: convertible bonds, reset clause, finite difference method, default risk.
JEL Classification: G12, G13.

Introduction

Among all financing instruments, convertible bonds (CBs) have become the most commonly issued security. From 2001, 1240 bonds included European convertible bonds (ECBs), straight bonds, equity warrant bonds, exchangeable bonds, CBs and so on, are issued, and 911 of them are issued in terms of CBs. Obviously, the historical data shows that firms prefer issuing CBs to raise funds.

CBs are bonds that give the bondholders right to convert debt securities into stocks at a specified price in the future. In other words, CBs can be viewed as a mix security which comprises a straight bond and a call option. The concept has been accepted widely.

Why do most firms choose CBs to finance at the first place? First, the interest rate of CBs is usually much lower than straight bonds, some of the CBs even pay zero coupon rate. Second, Nyborg (1996) states that the management of the bond issue firm view convertible debt as delayed equity. Through issuing CBs, firms can mitigate dilution effect compared to issuing stocks directly since conversion always happen at high stock price. From the investors' point of view, since the nature of CBs is debt, they can receive coupons periodically until the bond is converted. Besides, the bondholders also benefit greatly when the stock price of the issue firm goes up by exercising the conversion right.

For avoiding the investors' loss, a reset clause is embedded into the bond. Typically, a reset clause allows the conversion price to refix when the issue firm's stock price drops over a specified percentage. Usually, a conversion price reset is restricted to be upward adjusted. Since reset clause raise the probability of conversion when the issue firm's stock price goes down, a CB with reset clause is more valuable than the one with no reset. Chang (2003) show us that reset clause has critical influence on the bond value.

Nowadays, the design of CBs has become complicated. Except reset clause, putable and callable provision also can be seen in most CBs. The putable provision protects bond holders when bond value drops at a very low value, while callable provision reduce the firm's potential loss when the bond value rises at a high level. So, when comes to pricing CBs, it becomes difficult under considering all provisions.

Originally, when there is only one option embedded in the CBs and conversion is restricted to exercise at maturity (called European type), Black-Scholes model (1973) might be a usual choice to price CBs since Black-Scholes (1973) have derived the closed-form solution of the value of the option. However, the common situation is that conversion time is uncertain. That is, the call option embedded in the CBs is American type rather than European type. After Black-Scholes (1973), there has developed several methods. And since CBs are issued by corporate issuers, default risk should be considered.

For pricing convertible bonds with default risk, two approaches with different underlying state variable of the model have been expanded. Structural-form which is originated from Merton (1974) employs firm value as the underlying state variable. However, firm value is difficult to estimate, so stock price become an alternative to represent firm value. In practice, reduced-form model is a better method than structural-form one since stock price is quiet easy to observe in the market.

In the reduced-form model, TF model is one of the commonly used methods. They divided CBs into two parts, bond and option, then price separately. AFV model (2003) correct the assumption that the

stock price jump to zero immediately when the issuer face bankruptcy. They address that stock price drops at a percentage on default should be a normal case. We can observe in the real world that the issuer' stock price drops dramatically though, not at zero.

In this paper, AFV model (2003) is extended by adding reset clause into consideration which makes the conversion ratio k become a variable rather than a constant. Finally, the Partial Differential Equation (PDE) is solved by finite difference method.

This paper is organized as follows. Section 1 presents a review of the extant literature. Section 2 reviews the previous methodologies and introduces the model of AFV (2003). The reset clause and application of the finite difference method (FDM) are incorporated into the proposed model in section 3. The last section draws the conclusions.

1. Literature review

Ingersoll (1977) and Brennan and Schwartz (BS, 1977) are the first to value CBs. Under Black-Scholes (1973), Ingersoll (1977) derived closed-form solution of the CBs with callable provision. Brennan and Schwartz extend the model of Black-Scholes (1973) and Merton (1974) by relaxing the restriction of the coupon payments and stock dividend payments to price CBs. Besides, they also permit the bondholders to convert the bond into common shares at any point of time. In their approach, firm value is employed as the underlying variable. Under the optimal strategy for call and conversion, they then derive PDE that can be solved to value CBs. For more accurate valuation, Brennan and Schwartz (1980) incorporate stochastic interest rate into the model developed in 1977. Then, they conclude that the difference between stochastic interest rate model and fixed interest rate model are very small, so for a reasonable range of interest rate, they suggest that a simpler model with fixed interest rate is preferable for practical purpose.

The model, such as BS model (1977, 1980), which the CBs value are determined by the firm value is called structural-form model. The problem is that the firm value is not traded asset, so the firm value as a variable in the model cannot be directly observable. To circumvent the problem of the structural-form model, some authors introduce stock price that can be directly observable in the market to replace firm value. Such approach is called reduced-form model which began with McConnell and Schwartz (1986). They use this method to price Liquid Yield Option Note (LYON) issued by Merrill Lynch in 1985 with the characteristic of zero coupon rate, convertible, callable and putable. In their paper, they state that "the value of the LYON depends upon the value of the issuer's common stock rather than the total market value of the firm." However, they also argue that since the value of the issuer's common stock precludes the possibility of bankruptcy, the value of LYON will be overestimated.

For reducing the difference of the model price and the market price, the default probability is proposed to be added into the model. Duffie and Singleton (1999) employed the default hazard rate to value the risky bond. Thus, the discount rate which is defined as $R_d(t) = r(t) + L(t)\lambda(t)$; $r(t)$ is the default-free rate. $L(t)$ is the fractional loss rate of market value at default, $L(t)$ is the default hazard rate.

Furthermore, they state that since the conversion of the CBs is exercised by the investors when the stock price is high, the idea can be captured that the bond issuer has lower credit risk as the stock price is higher. Under the Duffie and Singleton model (1999), Takahashi and Kobayashi (2001) model the hazard rate as a decreasing function of the stock price since the stock price is easily observed in the market. They defined the default hazard rate as $\lambda(S_t, t) = \theta + \frac{c}{S_t^b}$, where $\theta \geq 0$, b and c are some constants. Finally, they use four issues of CBs to do the empirical study and find there exists 2.25% absolute error ratio.

Tsiveriotis and Fernandes (TF, 1998) argue that if the underlying equity is that of the issuer, the equity upside has zero default risk since the issuer can always deliver its own stock. On the other hand, coupon and principle payment which are related to cash depend on the issuer's timely access to the required cash amounts, and thus introduce credit risk. Following this thinking, TF model (1998) divided the CBs into two parts: cash-only part and the equity part. Since the cash-only part is defaultable, the authors discount it at a risky rate. The equity part is discounted at a risky-free rate for it is default-free. This leads to two PDE which can be solved.

Madan and Unal (2000) and Davis and Lischka (1999) both assume that the stock price jumps to zero immediately in the event of default. However, AFV model (2003) state that the assumption that the stock price instantly jumps to zero when default event occurred is highly questionable. Stock price does decline sharply around the announcement of the default, but the range is less than 100%. So, they propose a model that the firm's stock price drops at a specified percentage (between 0% and 100%) on default. In addition, AFV (2003) model compare the difference between TF model (1998) and the hedge model they develop, and find that TF (1998) model is internally inconsistent.

The Chicago Board Options Exchange (CBOE) and the New York Stock Exchange (NYSE) start to trade S&P 500 index bear market warrants with a three-month reset in the late 1996. Gray and Whaley (1997) priced these put warrants with periodical reset. Latter, Gray and Whaley (1999) also provide a close-form solution to the put warrants with a single reset day. Besides, Cheng and Zang (2000) mentioned that an option holders always like the option be in the money. An option with periodical reset has higher possibility to keep the option value in the money.

Reset options are discussed often by latter researchers, while few studies investigate how a reset clause affects the bond value. Kimura and Shinohara (2006) use Monte Carlo method to price convertible bond with reset clause. The authors address that if the underlying stock has no credit risk of the issuer, no conversion occurs prior to maturity, i.e., conversion may only occur at maturity. However, if the bond is risky, the bond holder might convert earlier, so the conversion right becomes American type. They use classical Black-Scholes model (1973) to calculate the former case, and estimate the latter case by Monte Carlo simulation. Besides, they propose the original conversion price X is adjusted downward to mS_t when $S_t < \frac{X}{m}$ for a pre-specified constant $m \geq 1$.

2. Methodology and the review of the AFV model

2.1. Assumption. Consider a CB with no credit risk at the beginning. Assume risk free interest rates are the known function of time, and the process of stock price is as follow:

$$dS = \mu S dt + \sigma S dz, \tag{1}$$

where S is the stock price, μ is the drift rate, σ is volatility, and dz is the increment of a Wiener process. In order to improve the problem of the behavior of stock price that supposed in reduced-form, AFV model (2003) assume the drop rate of stock price be η in default, then S^+ represents the stock price immediately after default and S^- refers to the stock price before default. They assume:

$$S^+ = S^-(1-\eta), \tag{2}$$

where $0 \leq \eta \leq 1$. When $\eta = 1$, it is defined as total default; while equaling to 0, partial default is identified.

2.2. Hedging portfolio construction. Let V_t denotes the value of the CBs at time t. When there is no default risk, a hedge portfolio can be constructed as follow:

$$\Pi = V_t - \beta S_t. \tag{3}$$

Then, obtain the change of the value of this portfolio during $t \to dt$ by choosing $\beta = \frac{\partial V}{\partial S}$:

$$d\Pi = \left[\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 V}{\partial S^2}\right]dt + o(dt), \tag{4}$$

where $o(dt)$ denotes terms that go to zero faster than dt.

Now, incorporate the hazard rate denoted by $p = p(t)$ into consideration, conditional on no default event occurred prior time t. With the following assumptions:

- In default, the stock price behaves as equation (2).
- In default, the CBs holders have the option to receive: (1) the amount of RB, where B is defined as the face value of CBs here; R is the recovery rate, $0 \leq R \leq 1$; (2) shares worth $\kappa S_t(1 - \eta)$, where κ is the conversion ratio. However, κ here is not constant as the one in AFV model (2003).

In this paper, κ is changeable. Once the bond issue firm's stock price drop through certain level that defined in advance, the conversion ratio will become a new one.

Equation (4) becomes:

$$d\Pi = \left[\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 V}{\partial S^2}\right]dt - pdt\left(V - \frac{\partial V}{\partial S} S_t \eta\right) + \\ + pdt \max\left[\kappa S_t(1-\eta), RB\right] + o(dt). \tag{5}$$

In the above equation, $\beta S \eta\, pdt$ represents the gain with hedging the default risk. By assuming default risk is diversifiable, obtain:

$$E(d\Pi) = r\Pi dt, \tag{6}$$

where E is the expectation operator. Equation (6) means a risk-free portfolio can earn only risk-free interest rate. Combining (3), (4) and (5), the following PDE equation can be obtained:

$$r\left[V_t - S\frac{\partial V}{\partial S}\right] = \left[\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 V}{\partial S^2}\right] - \\ - p\left[V_t - \frac{\partial V}{\partial S} S_t \eta\right] + p\left[\mathrm{Max}\left(\kappa S_t(1-\eta), RB\right)\right], \tag{7}$$

which implies:

$$\frac{\partial V}{\partial t} + (r + p\eta)S_t \frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 V}{\partial S^2} - (r+p)V_t + \\ + p\mathrm{Max}\left[\kappa S_t(1+\eta), RB\right] = 0. \tag{8}$$

Finally, their paper uses FDM to solve the above PDE, so the boundary conditions are necessary:

$$V_t \geq \text{Max}(B_p, \kappa S_t), \tag{9}$$

$$V_t \geq \text{Max}(B_c, \kappa S_t). \tag{10}$$

Equations (9) and (10) imply that neither the call constraint nor the put constraint are binding. While this is perhaps an obvious point, it is worth remembering that in some popular existing models for convertible bonds no explicit assumptions are made regarding what happens to the stock price with feature of the reset clause.

3. Proposed model

We now consider adding the reset clause to the convertible bond model described in section 2, using the FDM discussed in subsection 3.2 for incorporating the reset clause.

3.1. Proposed model with the reset clause. We follow the same idea as described in AFV (2003) model to establish the proposed model.

Define

$$MV \equiv \{\frac{\partial V}{\partial t} + (r + p\eta - q)S_t \frac{\partial V}{\partial S} + \\ + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 V}{\partial S^2} - (r + p)V_t\}. \tag{11}$$

Add q into the equation (11), which means the stock pays a continuous dividend yield. Then, PDE equation becomes:

$$MV - p\max(\kappa S(1-\eta), RB) = 0. \tag{12}$$

According to Brennan and Schwartz (1977), they proposed that "a convertible bond can be valued only if the call and conversion strategies to be followed by the corporation and the investors respectively can be determined." For the bond holders, it is never optimal to convert the bond into stock if the conversion value is less than the call price. On the other hand, the bond will be called once the conversion value exceeds the call price. That is, both the bond issuers and the investors will make a choice under the optimal strategy.

Through the above analysis, there are two conditions to discuss separately: $B_c > \kappa S$ and $B_c \leq \kappa S$, where B_c means the call price and B_p represents the put price. When $B_c > \kappa S$, there are three situations:

$$\begin{pmatrix} MV - p\max(\kappa S(1-\eta), RB) = 0 \\ (V - \max(B_p, \kappa S)) \geq 0 \\ (V - B_c) \leq 0 \end{pmatrix}, \tag{13}$$

$$\begin{pmatrix} MV - p\max(\kappa S(1-\eta), RB) \geq 0 \\ (V - \max(B_p, \kappa S)) = 0 \\ (V - B_c) \leq 0 \end{pmatrix}, \tag{14}$$

$$\begin{pmatrix} MV - p\max(\kappa S(1-\eta), RB) \leq 0 \\ (V - \max(B_p, \kappa S)) \geq 0 \\ (V - B_c) = 0 \end{pmatrix}. \tag{15}$$

For the condition (13), the firm will not call the bonds, neither the bond holder put the bond; both sides will not take actions. Since the firm's hedge portfolio return is greater than the risk-free rate in the condition (14), the firm will not call, but the bonder may choose to put or to convert. In the condition (15), the firm's hedge portfolio return is less than the risk-free rate, so the firm will choose to call the bond and the bond value will equal to the call price. When $B_c \leq \kappa S$, the bond holder will exercise the conversion certainly, so

$$V = \kappa S. \tag{16}$$

From equations (13), (14), (15) and (16), the constraints are the same as equations (9) and (10), but we notice some slight difference for κ. Since the κ could be down adjusted as this paper discuss before, the following constraint should be considered on the reset day:

$$\text{If } S_t < X, \text{ then } \kappa = k_1, \text{ else } \kappa = k, \tag{17}$$

where S_t is the stock price on the reset day; X refers to the pre-specified conversion price; k represents the original conversion ratio and k_1 is the new conversion ratio when reset occurs.

Once the stock price on the reset day is smaller than the original conversion price, the conversion price could be reset according to equation (17). To the best of our knowledge, we are the first to incorporate the concept of the reset clause into the CB models that have been discussed before. In order to solve the PDE in equation (12) and boundary constraints in equations (9), (10) and (17), we use the FDM which has been introduced in section 3.2.

3.2. Reviews on FDM. FDM is one of the numerical analysis. The idea of this method is to solve PDE through approximating the differential equation. At first, both the forecasting period from time t to the maturity T and the underlying stock are divided into N and M parts equally. Assume a maximum stock price as Figure 1 shows, each part equal to ΔS. That is:

$$S = 0, \Delta S, 2\Delta S, \ldots, M\Delta S (= S\max).$$

The same as time $T - t$ is divided into N parts, so each part equal to Δt. Then:

$$t = 0, \Delta t, 2\Delta t, \ldots, N\Delta t (= T).$$

Therefore, we can know that the value of the CB with different stock price and time can be denoted as $V_{i,j} = V(i\Delta t, j\Delta S)$ which is shown by each grid point in Figure 1.

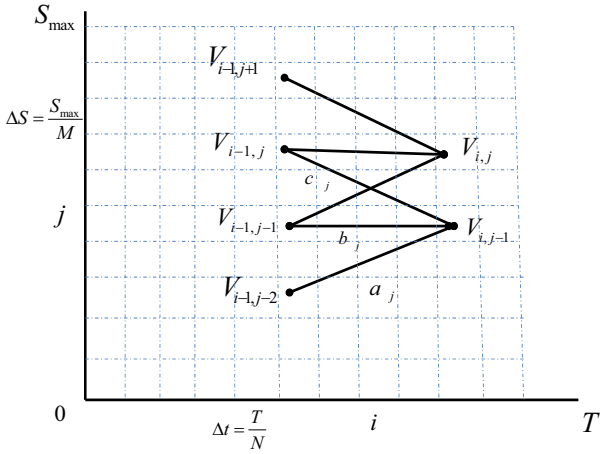

Fig. 1. $V_{i,j} = V(i\Delta t, j\Delta S)$ which is shown by each grid point

Three difference methods solve the differential equation, included Implicit Finite Difference (IFD), Explicit Finite Difference (EFD) and Crank-Nicolson Method (CNM). IFD method uses backward difference for the derivative with respect to t, and central difference for the derivate with respect to S. EFD method uses central difference for the derivative with respect to S, and forward difference for the derivate with respect to time t. However, CNM takes the average of the difference of Implicit and Explicit Finite Difference. The following is the ways to approximate the differential equation.

1. Forward difference:

$$\frac{\partial V}{\partial S} = \frac{V_{i,j+1} - V_{i,j}}{\Delta S}, \tag{18}$$

$$\frac{\partial V}{\partial t} = \frac{V_{i+1,j} - V_{i,j}}{\Delta t}. \tag{19}$$

2. Backward difference:

$$\frac{\partial V}{\partial S} = \frac{V_{i,j} - V_{i,j-1}}{\Delta S}, \tag{20}$$

$$\frac{\partial V}{\partial t} = \frac{V_{i,j} - V_{i-1,j}}{\Delta t}. \tag{21}$$

3. Central difference

$$\frac{\partial V}{\partial S} = \frac{V_{i,j+1} - V_{i,j-1}}{2\Delta S}, \tag{22}$$

$$\frac{\partial V}{\partial t} = \frac{V_{i+1,j} - V_{i-1,j}}{2\Delta t}. \tag{23}$$

For solving the proposed model, this paper adopts IFD method. Then, get the following equation:

$$\frac{\partial V}{\partial t} = \frac{V_{i,j} - V_{i-1,j}}{\Delta t}, \tag{24}$$

$$\frac{\partial V}{\partial S} = \frac{V_{i-1,j+1} - V_{i-1,j-1}}{2\Delta S}, \tag{25}$$

$$\frac{\partial^2 V}{\partial S^2} = \frac{V_{i-1,j+1} + V_{i-1,j-1} - 2V_{i-1,j}}{\Delta S^2}. \tag{26}$$

Put equations (24), (25) and (26) into equation (11). Therefore, equation (11) can be transformed into:

$$\begin{aligned} &\frac{V_{i,j} - V_{i-1,j}}{\Delta t} + (r + p\eta - q) j\Delta S \frac{V_{i-1,j+1} - V_{i-1,j-1}}{2\Delta S} + \\ &+ \frac{\sigma^2 j^2 \Delta S^2}{2} \frac{V_{i-1,j+1} + V_{i-1,j-1} - 2V_{i-1,j}}{\Delta S^2} - (r + p) V_{i,j} + \\ &+ p \max[kj\Delta S(1-\eta), RB] = 0 \end{aligned} \tag{27}$$

Then equation (27) can be rewrite as:

$$\begin{aligned} V_{i,j} &= \frac{1}{1 - \Delta t(r + p)} \times \\ &\times \left\{ \begin{array}{l} a_j V_{i-1,j-1} + b_j V_{i-1,j} + c_j V_{i-1,j+1} - \\ - p\Delta t \max[kS(1-\eta), RB] \end{array} \right\}, \end{aligned} \tag{28}$$

where

$$a_j = \frac{1}{2}\Delta t\left[j(r + p\eta - q) - \sigma^2 j^2 \right], \tag{29}$$

$$b_j = \left[1 + \Delta t \sigma^2 j^2\right], \tag{30}$$

$$c_j = -\frac{1}{2}\Delta t\left[j(r + p\eta - q) + \sigma^2 j^2 \right]. \tag{31}$$

At each time i, there are $M - 1$ equations and $M - 1$ unknown elements as follow:

$$\begin{bmatrix} b_1 & c_1 & 0 & \cdots & 0 \\ a_1 & b_2 & c_2 & 0 & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & 0 & a_{M-2} & b_{M-2} & c_{M-2} \\ 0 & \cdots & 0 & a_{M-1} & b_{M-1} \end{bmatrix} \begin{bmatrix} V_{i,1} \\ V_{i,2} \\ V_{i,3} \\ \vdots \\ V_{i,m-2} \\ V_{i,M-1} \end{bmatrix} = \begin{bmatrix} V_{i+1,1} \\ V_{i+1,2} \\ V_{i+1,3} \\ \vdots \\ V_{i+1,M-2} \\ V_{i+1,M-1} \end{bmatrix} - \begin{bmatrix} a_1 V_{i,0} \\ 0 \\ 0 \\ \vdots \\ 0 \\ c_{M-1} V_{i,M} \end{bmatrix}. \tag{32}$$

However, the boundary value is known at each time i and time T through the boundary condition prescribed. So, the unknown elements can be solved by solving the $M-1$ equations. Actually, each value of the grid point is figured out by the next period value. The bond value $V_{i,j}$ as showed in the Figure 1 can be used to calculate $V_{i-1,j+1}$, $V_{i-1,j}$ and $V_{i-1,j-1}$ through a_j, b_j and c_j. That is, the bond value at time t will be obtained first, then compute the bond value in previous period $t-1$, and so on.

As to the model in this thesis, because reset clause would be considered and the reset clause is related to the change of stock price, a stock price at each time t will be necessary. Therefore, a trinomial tree method is employed to solve the CBs price. Besides, since the EFD method is approximate trinomial tree method, an EFD method can be used to calculate the probability of stock price up, down and unchanged.

As described by Brennan and Schwartz (1977), it is more efficient to transform the stock price S as follows:

$$y(t) = \log S(t). \tag{33}$$

Then, equation (1) becomes:

$$dy(t) = (\mu - \tfrac{1}{2}\sigma^2)dt + \sigma dz. \tag{34}$$

So, PDE, equation (11), turns into:

$$\frac{\partial V}{\partial t} + \left(r + p\eta - q - \frac{1}{2}\sigma^2\right)\frac{\partial V}{\partial y} + \frac{1}{2}\sigma^2\frac{\partial^2 V}{\partial y^2} - \\ -(r+p)V + p\max[k(1-\eta), RB] = 0. \tag{35}$$

When using EFD method, the difference for derivative with respect to t and y:

$$\frac{\partial V}{\partial t} = \frac{V_{i+1,j} - V_{i,j}}{\Delta t}, \tag{36}$$

$$\frac{\partial V}{\partial y} = \frac{V_{i+1,j+1} - V_{i+1,j-1}}{2\Delta y}, \tag{37}$$

$$\frac{\partial^2 V}{\partial y^2} = \frac{V_{i+1,j+1} + V_{i+1,j-1} - 2V_{i+1,j}}{\Delta y^2}. \tag{38}$$

Puting equations (36), (37) and (38) into equation (35), we obtain:

$$\frac{V_{i+1,j} - V_{i,j}}{\Delta t} + \left(r + p\eta - q - \frac{1}{2}\sigma^2\right)\frac{V_{i+1,j+1} - V_{i+1,j-1}}{2\Delta y} + \\ + \frac{\sigma^2}{2}\frac{V_{i+1,j+1} + V_{i+1,j-1} - 2V_{i+1,j}}{\Delta y^2} - (r+p)V_{i,j} + \\ p\max[k(1-\eta), RB] = 0. \tag{39}$$

Equation (39) can be rewrited as:

$$V_{i,j} = \frac{1}{1+\Delta t(r+p)} \times \\ \times \left\{ \begin{array}{l} \left[\frac{-\Delta t}{2\Delta y}\left(r + \tfrac{1}{2}p - q - \tfrac{1}{2}\sigma^2\right) + \frac{\Delta t\sigma^2}{2\Delta y^2}\right]V_{i+1,j-1} \\ + \left[1 - \frac{\Delta t}{\Delta y^2}\sigma^2\right]V_{i+1,j} \\ + \left[\frac{\Delta t}{2\Delta y}(r + \tfrac{1}{2}p - q - \tfrac{1}{2}\sigma^2) + \frac{\Delta t\sigma^2}{2\Delta y^2}\right]V_{i+1,j+1} \\ + p\Delta t\max\left[\kappa(1-\eta), RB\right] \end{array} \right\}. \tag{40}$$

Then, equation (40) can be rewrited as:

$$V_{i,j} = \frac{1}{1+\Delta t(r+p)} \times \\ \times \left\{ \begin{array}{l} p_d V_{i+1,j-1} + p_m V_{i+1,j} + p_u V_{i+1,j+1} - \\ p\Delta t\max[k(1-\eta), RB] \end{array} \right\}. \tag{41}$$

Here, p_d refers to the probability of the drop of stock price; p_m refers to the probability of the unchanged stock price; and p_u represents the probability of the rise of the stock price. So, p_u, p_m and p_u are as follow:

$$p_u = \tfrac{1}{2}\frac{\Delta t}{\Delta y}(r + \tfrac{1}{2}p - q - \tfrac{1}{2}\sigma^2) + \tfrac{1}{2}\sigma^2\frac{\Delta t}{\Delta y^2},$$

$$p_m = \left[1 - \frac{\Delta t\sigma^2}{\Delta y^2}\right],$$

$$p_d = -\tfrac{1}{2}\frac{\Delta t}{\Delta y}(r + \tfrac{1}{2}p - q - \tfrac{1}{2}\sigma^2) + \tfrac{1}{2}\sigma^2\frac{\Delta t}{\Delta y^2}.$$

As demonstrated by Boyle and Tian (1998) that the numerical analysis literature shows:

$$\Delta y = \lambda\sigma\sqrt{\Delta t}.$$

Thus, p_u, p_m and p_d become:

$$p_u = \left[\frac{-\sqrt{\Delta t}}{2\lambda\sigma}\left(r + \tfrac{1}{2}p - q - \tfrac{1}{2}\sigma^2\right) + \frac{1}{2\lambda^2}\right], \tag{42}$$

$$p_m = 1 - \frac{1}{\lambda^2}, \tag{43}$$

$$p_d = \left[\frac{\sqrt{\Delta t}}{2\lambda\sigma}\left(r + \tfrac{1}{2}p - q - \tfrac{1}{2}\sigma^2\right) + \frac{1}{2\lambda^2}\right]. \tag{44}$$

Regarding to λ, Kamrad and Ritchken (1991) showed that the proper value for standard option would be $\sqrt{1.5}$. In fact, the EFD is equivalent to the trinomial tree approach. The expression for p_u, p_m and p_d are shown in equations (42), (43) and (44). We can interpret terms (p_u, p_m and p_d) as probability

of stock price increasing from $j\Delta S$ to $(j + 1)\ \Delta S$, probability of stock price remaining unchanged at $j\Delta S$ and probability of stock price decreasing from $j\Delta S$ to $(j - 1)\ \Delta S$ in time Δt, respectively. The concept of the trinomial tree calculation just as Figure 2 shows. We have discussed the FDM to solve the PDE.

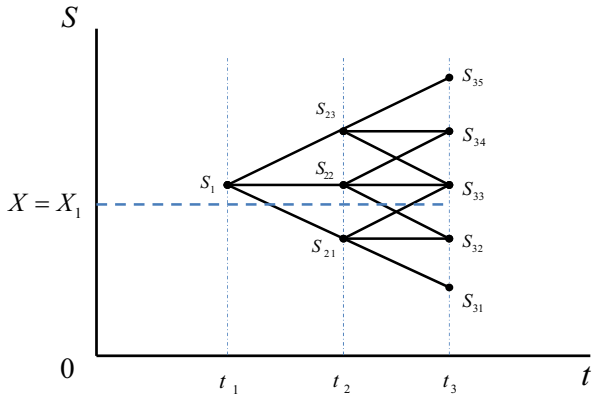

Fig. 2. Expression of the reset clause changes conversion price

3.3. FDM with reset clause. How the reset clause changes conversion price can be explained by Figure 2. Assume the conversion price X equals to X_1, if the stock price goes to S_{21} which is below X_1 at time t_2, the conversion price from X_1 turns into S_{21}. However, if the stock price goes to S_{22} or S_{23} which is above conversion price X_1, the conversion price is unchanged. Similarly, if the stock price becomes S_{31} at time t_3, then the conversion price X_1 resets to S_{31}. In the other words, once the stock price drops below the conversion price, the conversion price resets according to the provisions; while if the stock price at reset day is above the conversion price, the conversion price is unchanged.

Summary and conclusions

We provide a new pricing formula for the risky convertible bond by incorporating the reset clause. We note that the reset clause is one of the critical elements in pricing the risky convertible bond. The objective of this paper is to improve the AFV model (2003) by considering the reset clause into the model and to use IFD method for solving the AFV model (2003). When considering reset model, the EFD method is employed to solve the partial differential equation since the EFD can be approximated the trinomial tree model. Thus, the EFD is used to calculate the probability of the stock price up, down and unchanged. Then, the trinomial tree is used to obtain the model price. Finally, for comparing the proposed model with AFV model (2003), in the future we could pay more attention to use empirical data.

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