

# “Natural capital and allocation of investment sustainable perspective on endogenous growth model”

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## Natural capital and allocation of investment sustainable perspective on endogenous growth model

### Abstract

To study the sustainable development condition for long-term economic growth, the authors present an environmental-economic system using natural capital and environmental protection investment based on the endogenous economic growth model. Analyzing the evolutionary dynamics behavior, this paper discusses how the economic output is impacted by the proportion of investment allocation in this system. The results show that the complexity of environmental-economic system makes a dramatically different between the short-term and long-term. The whole environmental-economic system can grow steadily in long-term stability and develop sustainable on condition that the special proportion of environmental protection investment is satisfied.

**Keywords:** natural capital, environmental protection investment, economic growth, sustainable development.

**JEL Classification:** Q1, Q28, Q57.

### Introduction

At the beginning of the 21st century, humankind is facing many critical problems such as economic stagnation, persistent poverty, worsening environmental degradation and accelerating globalization demand attention. One key approach that has received growing attention is based on the concept of sustainable development (World Commission on Environment and Development, 1987).

Following the 1992 Earth Summit in Rio de Janeiro and the adoption of the United Nations' Agenda 21, the concept of sustainable development has become well accepted worldwide (United Nations, 1993).

In the late 1980s, the endogenous growth theory, which is represented by Paul Romer and Robert Lucas, provides a theoretical framework for analyzing the problem of sustainable development. Then many scholars discussed the problems of environmental pollution and sustainable development within the framework of endogenous growth model by incorporating pollution factors into the production function and the quality of environment into the utility function. There are many representative models, such as Bovenberg and Smulders (1995) introduced environmental factors into the production function based on Romer model; Chichilnisky (1995) presented a growth model with environmental assets as a source of utility and developed the Green Golden Rule. Stokey (1998) gave a basic framework to analyze the sustainable development and studied the outside problems of environmental pollution and the sustained development of economy by introducing pollution density index "z" as a representative consumer's control quantity into the Barro's (1990) AK model; Barbier (1999) introduced the scarcity of resources and the growth of population into Romer-Stiglitz model, exploring the optimal balanced growth path (Barbier, 1999; Stiglitz,

1974); Giuseppe (2007) distributed some productive labor into environmental resources protection department, for the environmental protection investment instead of productive activities, studying the optimal growth path of economic sustainable development.

As the complexity of environmental-economic system, the analysis of economic growth paths should include the analysis of a number of important variables' evolutionary relationships in environmental economic, and need to forecast and measure the corresponding policies on the effects of environmental-economic system (Daly, 1996). In these studies, system modeling methods (Constanza, 2002) is widely used by ecological economists. The reason is that the system modeling methods concerns the causal connection among variables and variables' interaction, and it can contain some important variables which are difficult to be measured directly and the nonlinear relation among the variables. What's more, through the computer simulation, ecological economists can understand complex behaviors of the system and identify the key factors. This paper uses the system modeling methods and the corresponding simulation software to discuss the dynamical behaviors of the whole environmental-economic system and selectively analyze the proportion of environmental protection investment impact on the evolution behavior of the system.

The aim of this paper is to contribute a more specific condition of sustainable development within the framework of neoclassical growth theory. We do so by exploring the proportion of investment allocation when natural capital plays the role of a fund (Kraev, 2002). To do so, we base ourselves on growth model of Rodrigue (2005), where physical and natural capitals are used as production factors. In the present paper, we use an allocation proportion to differentiate the investment on man-made capital or natural capital. Therefore, dynamic and structural environmental-economic interactions are endogenized into one model.

The structure of the paper is organized as follows. Section 1 establishes an endogenous economic growth model which contains natural capital and allocation of investment. Section 2 discusses dynamic evolution of the model and the conditions of sustainable development. The final Section presents conclusions and policy recommendations.

**1. An endogenous economic growth model containing natural capital**

Consider a closed economic system, the production function is the Cobb-Douglas function:

$$Y = AK^\alpha R^\beta L^{1-\alpha-\beta}, \tag{1}$$

where  $A$  is technology,  $K$  is man-made capital,  $L$  is labor,  $R$  is natural capital,  $\alpha \geq 0$ ,  $\beta \geq 0$ ,  $\alpha + \beta < 1$ .

In the following discussion, we assume that labor is a constant.

Economic output is divided into two parts, consumer goods and investment goods. The numbers of investment goods is determined by the exogenous savings rate, which is short for “s”:

$$I = sY, \tag{2}$$

“s” is a exogenous constant. Further, we assume that investment is divided into  $I_K$  (investment of man-made capital  $K$ ) and  $I_N$  (investment of natural capital  $N$ ), that is,

$$I = I_K + I_N. \tag{3}$$

In general, we can let  $I_N = pI$ , then  $I_K = (1 - p) I$ , where  $p$  is the proportion of investment allocation.

For the evolution of man-made capital, we have

$$\dot{K} = I_K - \delta K, \tag{4}$$

where  $\delta$  is the man-made capital depreciation rate,  $\delta > 0$ . And a variation of natural capital in time is determined by  $\dot{N} = N_{re} - R$  (a variation of recovery) and  $R$  (natural resources consumed in production).

A variation of natural capital recovery consists of its regeneration amount and  $I_N$  (investment of natural capital). And we take the simple linear relationship for the regeneration amount. However,  $I_N$  gives another way for the variation of natural capital recovery:

$$\dot{N}_{re} = \mu N + I_N, \mu > 0. \tag{6}$$

For the relationship between the natural resources consumed in production and natural capital, we also take the simply positive correlation:

$$R = \lambda N. \tag{7}$$

The equations formula form (1) to (7) build a closed environmental-economic growth model including natural capital and investment allocation, shown in Figure 1.

**2. Imitating dynamical behaviors**

The above equations build a nonlinear dynamic system, collating (1)-(7) can get the man-made capital and natural capital’s evolution equation:

$$\dot{K} = (1 - p)sAK^\alpha N^\beta - \delta K. \tag{8}$$

$$\dot{N} = (\mu - \lambda)N + psAK^\alpha N^\beta. \tag{9}$$

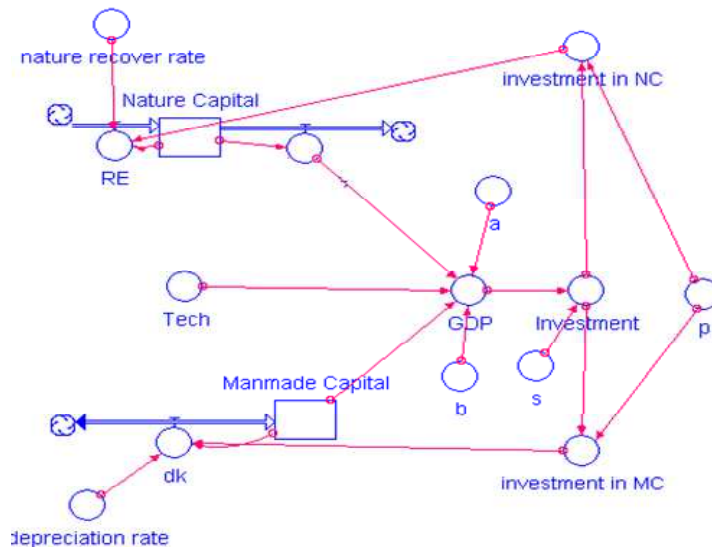


Fig. 1. The relationship between the variables and parameters of the model

By analyzing and solving the model, under the above parameters’ limiting, what we can get from the evolution results of economic output in the system will depend on the size of  $p$ , the varying propor-

tion of investment allocation of man-made capital and ecological capital (Cai, 2007). Let  $p = 0$  and  $p = 0.1$ , the evolved behavior of the short-term output can be shown in Figure 2.

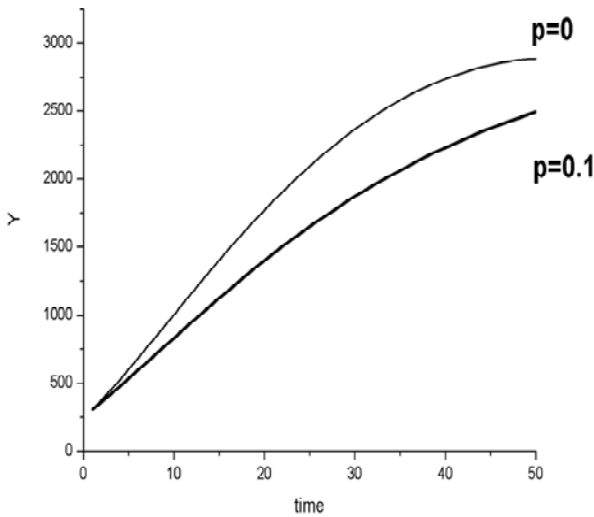


Fig. 2. Evolved behavior of short-term output in the case of  $p = 0$  and  $p = 0.1$

Since we use the specific production function, that is Cobb-Douglas function, and choose the parameters  $\alpha \geq 0$ ,  $\beta \geq 0$ ,  $\alpha + \beta < 1$ , the output of this economy system will be convergence. The evolution results show that if we allocate investment, we just only pursue the investment of man-made capital and ignore the investment of natural capital, the economic growth rate in short-term system is faster. That is, under the same parameters' limiting and in the same period, economic output without investing natural capital is higher than the output with some natural capital investment (it is shown in Figure 2 compared with  $p = 0$  and  $p = 0.1$ ).

However, the system's evolved results in long-term (see Figure 3) indicate clearly, ignoring natural capital investment, the economic growth rate in prior period is faster. But in later period, because of lacking natural capital, the economic growth rate is in recession. The reason why the system has such a huge difference in short term and long term is that the system has two feedback loops (it is shown in Figure 1). When natural capital's operating speed in production is faster than natural capital's reset speed (that is  $\lambda > \mu$ ), the stock of natural capital will decline over time if we do not invest natural capital and recover it. In the short term, natural capital is at the higher levels in the beginning, and it can satisfy the system's need. On the other hand, the rapid accumulation of man-made capital causes substitution effect and makes up that the decline of natural capital impacts on output. So in the system, the economic output is increasing (Figure 1). But in the long term, due to the declining of natural capital and the stability of man-made capital, man-made capital cannot make up the decline of natural capital, and the output is declining. What's more, the faster increase is in prior period, the faster decline is in later period. So we can see the situation in Figure 3.

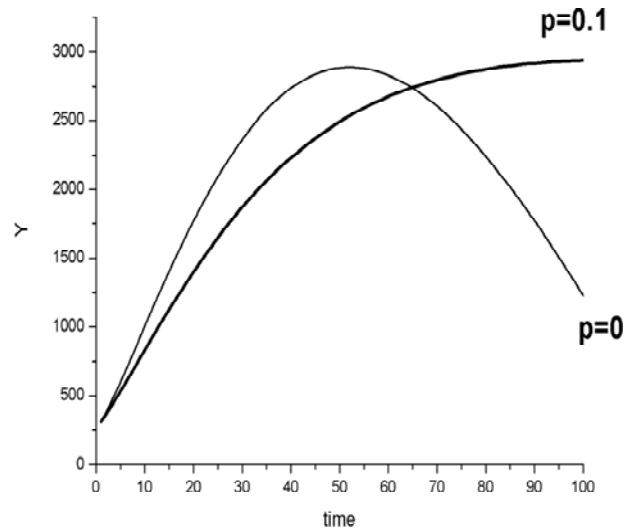


Fig. 3. Evolved behavior of long-term output in the case of  $p = 0$  and  $p = 0.1$

Based on those fundamental dynamic analyses, we can discuss sustainable development of the whole environment-economic system and the optimal proportion of environment investment by using the optimal social utility:

$$\text{Utility function: } U(C, N) = \frac{C^{1-\theta} - 1}{1-\theta} + \frac{N^{1-w} - 1}{1-w}.$$

According to Rodrigues (2005), this utility is a function of consumption,  $C(t)$  and of environmental services provided by natural capital,  $N(t)$ . So, the long-run utilitarian optimization problem is:

$$\max_{C, N, p} \int_0^{\infty} e^{-\rho t} U(C, N) dt,$$

$$\text{s.t. } \dot{K} = (1-p)sAK^\alpha N^\beta - \delta K,$$

$$\dot{N} = (\mu - \lambda)N + psAK^\alpha N^\beta.$$

The growth rates of all variables are constants in the balanced growth path. And in steady-state conditions of sustainable economic growth, from the first-order conditions of the optimal control and Euler equations we can get:

$$g_c = \frac{p(1-a-\beta)}{a-1 + \frac{1-a}{\theta} \left(1 - \frac{1-\theta}{1+w}\right) + (1-a-\beta)(1-\theta)}.$$

The equation considers the conditions of achieving sustainable economic growth under the conditions of environmental protection investment. These conditions show that only the environment-economic system with certain parameters, such as levels of technology and innovation, investment ratio in the right environment, can has sustainable development for the long term.

## Conclusions and policy recommendations

We presented an extension of the neoclassical growth model with natural capital and exogenous technological change with two main novelties: natural capital and allocation of investment. The productive processes do not provide the same positive externalities as free natural capital. Therefore, there is a trade-off between the extension of human domination of the biosphere, increasing production, and the maintenance of ecosystem services.

We found that, for some set of technological parameters and initial conditions, it is possible to experience unbounded economic growth and to keep the natural system in steady state. The constraints on the parameters state that the proportion of allocation of investment and exhaustivity speed of natural capital must be balanced, subject to technological constraints and elasticity of productive factor.

Furthermore, dynamic complexity is common in environmental-economic system, including many non-linear interactions, positive and inverse feedback and complex loops. This complexity brings a bit different evolutionary path between and short term and long term. Having fixed other parameters, we found that different proportion of allocation of investment make the three different growth paths: long-term sustainable growth, short-term temporary growth and rapid economic recession. Under the condition of sustainable development of natural capital, we get the optimal proportion of allocation of investment, which makes the present model have the optimal growth path. For a simple case, this optimal proportion equals the elasticity of productive capital when economic system has constant return scale.

These theoretical results indicate that, in the processes of economic development, China's investment strategies must be in line with the sustainable development strategy. The sustainable development strategy emphasizes compatibility of population, resources, environment, society and economy; pay attention to comprehensive coordination of generational and interregional interests. As a pointer, investment activities should change the target location only focusing on economic growth; shake off the traditional extensive increase manner and investment of high input, heavy pollution and low efficiency. Investment should be subject to the sustainable development strategy; focused on environmental and economic goals; moved to the key construction areas of sustainable development; effective supported the building for sustainable development and the 21 century's priority agenda items. At the same time, investment strategies should be followed to the principle that combines short-term interests and enhancing the capacity of sustainable development. From a strategic point of view, investment activities should take sustainable development as a goal; pay attention to the building of sustainable development; raise capital generation capacity and increase the total wealth of society. On short-term interests' view, increasing in GDP remains an important index that measures the progress of a society. Shaking off poverty and improving the level of China's economic development are not only the premise of improving the living environment and quality of life, but also the material basis of promoting capacity building. Therefore, investment activities should take into account the short-term interests and the follow-up developmental capacity, neither can be neglected.

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