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# Pension funds risk analysis: stochastic solvency in a management perspective


## Abstract

The aim of the paper is to deal with the solvency requirements for Defined Contributions Pension funds. The probability of underfunding is investigated in a stochastic framework by means of the funding ratio, which is the ratio of the market value of the assets to the market value of the liabilities. Demographic and investment risks are modelled by means of diffusion processes. Their impact on the total riskiness of the fund is analyzed via a quantile approach.


**Keywords:** pension fund, funding ratio, CIR model, MRBG model, quantile analysis.
**JEL Classification:** G22, G23, C63, J11.

## Introduction

Pension funds worldwide are in trouble. At the end of the millennium, because of the market downturn many pension plans became insolvent; some companies have been left with pension deficits larger than their stock market capitalization. Unexpected increases in life expectancy, changing accounting rules, low interest rates and very poor equity market returns have led to a steep fall in funding levels. In this context, transparency in accounting and a more efficient management of the risks involved in running a business are required. Therefore, regulators have focused their attention on the need for more transparent accounting rules and solvency requirements (Steffen, 2008).

At present, it is relevant to note that EU member States are subject to common minimum standards beyond which the majority of jurisdictions are applying their own additional standards, and these practices are based on a deterministic point of view. Although this system is very prudent, it excludes any quantification of the unexpected deviation of the risk. Therefore, the most important supervisory principles stated in the Solvency II Proposal are to be identified in the space allowed for internal models in assessing the solvency of pension systems.

Thus, the use of internal models capturing the risk profile of the business by means of stochastic processes represents the most important step in the process of shifting from simple regulatory requirements based on compact short-cut formulae to more complex calculation structures.

At this proposal, pension funds regulation typically requires that the probability of underfunding in a given time horizon is sufficiently low. A pension fund would be underfunded if its funding ratio, that is the ratio of the market value of the assets to the market value of the liabilities, is below one. In this field, the Solvency II guidelines suggest the use of the 99.5% one-year Value at Risk. We contribute to this discussion by testing another risk measure, the expected shortfall, as an instrument for controlling the solvency of the fund: in our opinion, this risk measure is particularly attractive if the attention is focused on the expected size of losses exceeding a given threshold.

In the recent actuarial literature, the problem of valuation of insurance products has been variously treated, both under its methodological aspect and applicative perspective. Ballotta and Haberman analyze the fair valuation problem of guaranteed annuity options in a stochastic mortality environment (Ballotta, Haberman, 2006), Coppola, Di Lorenzo and Sibillo deal with the problem of measuring the risk sources in a life annuity portfolio (Coppola, Di Lorenzo, Sibillo, 2000), Grosen and Jorgensen propose an analysis of the market value of insurance liabilities in a barrier option framework (Grosen, Jorgensen, 2002), Jorgensen uses a fair value approach to model insurance and pension liabilities (Jorgensen, 2004), Hari et al. study the effect of Longevity Risk on pension annuities (Hari, De Vaegenaere, Melenberg, Nijman, 2008), Milevsky and Promislow propose a stochastic approach to model the future mortality hazard rate in insurance contracts with option to annuitize (Milevsky, Promislow, 2001), Olivieri and Pitacco study the effect of solvency requirements on the probability distribution of pension annuities (Olivieri, Pitacco, 2003).

In the following we investigate the solvency requirements for a Defined Contribution Pension Fund by using the Expected Shortfall in order to analyze the impact of the financial and demographic risks on the funding ratio distribution. The choice of the Expected Shortfall as a quantile based risk measure can have multiple applications. From a managerial point of view, this measure can be used as a benchmark for the quantifications of the fund riskiness. Moreover, it can be effectively applied for comparison among different pension funds, in the spirit of achieving the comparability of results across different entities.

The paper is organized as follows: section 1 describes the risk sources formalization for a Defined Contribution Pension Fund. In section 2, the effect of financial and demographic risk on funding ratio uncertainty is investigated. Section 3 discusses the methodological and applicative perspectives of the quantile based risk measures. In the final section, a numerical evidence and concluding remarks are offered.

## 1. The risk sources formalization

Let us consider a Defined Contribution Pension Fund with an individual funding method, which pays off a capital resulting from a contribution accumulation process to the subscriber in case of predecease, disability or old age. In presence of a guarantee of yield, the liability $b_h$ undergone by the fund in year $h$, with respect to a generic subscriber is given by

$$b_h = \max\left\{W_h^A, W_h^{GAR}\right\}, \tag{1}$$

where $W_h^A$ is equal to the share of the equivalent assets constituting the Fund and $W_h^{GAR}$ denotes the minimum guaranteed benefit. In particular:

$$W_h^A = \sum_{s=0}^{h-1} n_s a_h = \sum_{s=0}^{h-1} \frac{c_s}{a_s} a_h ,$$

and

$$W_h^{GAR} = \sum_{s=0}^{h-1} n_s a_s (1+i)^{h-s} ,$$

where $n_s$ is the number of Fund's shares subscribed in year $s$, $a_h$ is the unit value of the share in year $h$, $c_s$ is the contribution paid to the fund in year $s$, $i$ is the minimum yield guaranteed.

Now, let us define $\{r_h; h = 0,1,2,...\}$ and $\{\mu_{x+h}; h = 0,1,2,...\}$ the random spot rate process and the mortality intensity process respectively, both of them measurable with respect to the filtrations $F^r$ and $F^\mu$. The above mentioned processes are defined on a unique probability space $\left(\Omega, \mathcal{F}^{r,\mu}, P\right)$ such that $\mathcal{F}^{r,\mu} = \mathcal{F}^r \cup \mathcal{F}^\mu$.

As the elimination of the active state can happen by death, disability, or old age, it is very important to specify the risk sources that influence the fund performance.

The valuation of the financial uncertainty involving the fund can be made assuming a two-factor diffusion process obtained by joining Cox-Ingersoll-Ross (CIR) model for the interest rate risk and Black-Scholes (BS) model for the stock market risk; the two sources of risk are correlated.

The interest rate dynamics is described by means of the diffusion process (Cox, Ingersoll, Ross, 1985):

$$dr_t = k(\theta - r_t)dt + \sigma_r \sqrt{r_t}\, dZ_t^r , \tag{2}$$

where $k$ is the mean reverting coefficient, $\theta$ is the long term period "normal" rate, $\sigma_r$ is the spot rate volatility, and $Z_t^r$ is a standard Brownian motion.

The diffusion process for the stock market dynamics is given by the stochastic differential equation (Black, Scholes, 1973):

$$dS_t = \mu_S S_t dt + \sigma_S S_t dZ_t^S , \tag{3}$$

where $\mu_S$ is the continuously compounded market rate, $\sigma_S$ is the volatility parameter, and $Z_t^S$ is a standard Brownian motion with the property.

$$Cov\left(dZ_t^r, dZ_t^S\right) = \varphi dt \quad \varphi \in R .$$

The demographic dynamics can be followed by means of a mean reverting Brownian Gompertz (MRBG) process (Milevsky, Promislow, 2001):

$$\mu_{x+h:h} = \mu_{x+h:0} \exp\left\{(\alpha + \beta x + \beta h)h + \sigma_x Y_h\right\},$$
$$\sigma_x, \mu_{x+h:0} > 0 , \tag{4}$$

where $\mu_{x+h:h}$ is the mortality intensity for a person attaining age $x+h$ in the future year $h$, $\alpha$ is a parameter which represents the rate of change in the mortality on the logarithmic scale, $\beta$ is an offset that reflects a rate of change that could differ with age, $\sigma_x$ is the standard deviation of the mortality intensity process and $\{Y_h\}$ is an Ornstein – Uhlenbeck process whose dynamics is given by

$$\begin{cases} dY_h = -bY_h dh + dX_h \\ Y_0 = 0 \end{cases},$$

where $b$ is the mean reverting coefficient and $\{X_h\}$ is a standard Brownian motion.

Obviously, if the mortality intensity dynamics is known, it is possible to calculate the survival probabilities:

$${}_s p_x = E\left( \exp\left\{ -\int_0^s \mu_{x+h:h} dt \right\} / \mathcal{F}_0^\mu \right),$$

where the operator $E$ denotes the expected value conditional on the informative structure at time $t$.

## 2. The effect of financial and demographic risks on funding ratio uncertainty

Here, we will consider a risk analysis consistent with the supervisory authority perspectives: in particular, we will take into account the financial and demographic risk drivers.

In solvency investigations, usual requirements for assessing the capability to meet future obligations imply a comparison between the random profile of the fund's assets and the random profile of the fund's liabilities.

In this context, the funding ratio at time $t$, $FR_t$, is defined as the market value of the assets at time $t$ ($A_t$), divided by the market value of the liabilities at time $t$ ($L_t$). It can be seen as a measure of solvency, that is a measure of the capability of assets to cover liabilities. Formally,

$$FR_t = \frac{\mathcal{A}_t}{\mathcal{L}_t}. \tag{5}$$

Referring to the liability value $L_t$, it follows

$$\mathcal{L}_t = E[L_t] = E\left[\sum_{j>t} N_j X_j v(t,j) \Big/ \mathcal{F}_t^{r,\mu}\right]. \tag{6}$$

In the previous, $N_j$ indicates the number of claims at time $j$, $X_j$ is the difference, at time $j$, between the fund's future obligations and the future contributions, $v(t,j)$ is the value at time $t$ of a monetary unit due at time $j$. The operator $E$ denotes the expected value conditional on the informative structure at time $t$.

For the asset value

$$\mathcal{A}_t = E[A_t] = E\left[\sum_{j<t} N_j X_j v^{-sign(t-j)}(t,j) \Big/ \mathcal{F}_t^{r,\mu}\right], \tag{7}$$

where $X_j$ in this case is the difference between the contributions collected and the benefit due and

$$sign(t-j) = \begin{cases} \dfrac{|t-j|}{t-j} & t \neq j \\ 1 & t = j \end{cases},$$

Measuring the impact of financial and demographic risks means quantifying its variability due to the randomness in the choice of the mortality evolution model and the financial evolution process with respect to a fixed mortality table and a fixed discount factor.

## 3. Quantile based risk measures: the expected shortfall

As well known, the expected shortfall (ES) is a "VaR-like" risk measure in the sense that it will reflect the quantiles of the funding ratio distribution. It is a risk measure retaining the benefits of VaR in terms of probabilistic contents while avoiding its limits. In particular, it satisfies the properties of coherence better than VaR, having the attraction of being sub additive. Moreover, the risk surface is convex and this ensures that a risk minimum is a unique one (Dowd, 2006).

Indicating by $FR(t)$, the stochastic funding ratio of the Defined Contribution Pension Fund at time $t$, the p-quantile of the funding ratio at a confidence level ($0<\alpha<1$) is the value defined by the following equation:

$$P\{FR_t < Q_t(\alpha)\} = 1-\alpha, \tag{8}$$

the average of the worst 1-α cases, that is the smallest values of the funding ratio is

$$ES_t = \frac{1}{1-\alpha}\int_\alpha^1 q_p dp,$$

where $q_p$ is the $p$-quantile of the funding ratio density function.

The ES is then the expected value of the worst 1-α values, that is the average of funding ratio values lower than the α-quantile:

$$ES_t = E[FR_t / FR_t < Q_t(\alpha)]. \tag{9}$$

Substantially, the VaR tells us the most we can expect to lose if a tail event does not occur; the ES tells us what we can expect to lose if a tail event does occur.

## The funding Ratio distribution: numerical evidences. Concluding remarks

Referring to a Defined Contribution Pension Plan we study the funding ratio distribution characteristics to asses the riskiness connected to the Fund.

To this end, we compute, at different evaluation dates $t$, the quantile $Q_t(\alpha)$ confidence level of the *Funding Ratio* distribution and the expected shortfall $ES_t(\alpha)$ relative to this quantile.

The computation of $Q_t(\alpha)$ and $ES_t(\alpha)$ requires the employment of a simulation technique to find the distribution of the Funding Ratio.

By means of the stochastic models of section 1 describing the evolution in time of interest rates (CIR model), stock market (BS model) and mortality (MRGB model), we simulate 10000 values of $FR_t$ for each evaluation date $t$.

The distribution of $FR_t$ can be approximated by the histogram of $FR_t$ values. Sorting the values into an increasing sequence from the worst to the better cases, so that $FR_t(j-1)<FR_t(j)$, the quantile $Q_t(\alpha)$ is estimated (Brigo, Mercurio, 2004). The Expected shortfall $ES_t(\alpha)$ is simulated on the basis of (9). According to the current risk based regulations we look at the $Q_t(99.5)$; this means that if the 99.5-th quantile is chosen as desired funding ratio level there is a chance of 0.5 in 100 that the Fund is insolvent. Therefore, the higher $Q_t(99.5)$ is, the lower the risk for the fund is. The same observations can be extended to $ES_t$.

In the following we show the results of the simulation procedure referring to a Defined Contribution Pension Fund in the case the age $x$ of entering the Fund for the generic subscriber is 30. The outgoing age $x+\xi$ is 60, being $\xi$ the maximum number of years during which the generic subscriber is a fund member before reaching the pension age.

Here is the input dataset:

- We assume for the CIR process the risk adjusted parameters (Cocozza, Di Lorenzo, Orlando, Sibillo, 2008): $k = 0.0452$, $\sigma = 0.0053$ and the initial value $r_0 = 0.0279$, estimated on the 3-month Italian T-bill January 1998 – January 2008.
- Referring to the time evolution of the reference Fund we pose $\mu_S = 0.03$, $\sigma_S = 0.20$.
- For the correlation coefficient $\varphi$ we adopt a slightly negative value ($\varphi = -0.06$) coherently with the literature for the Italian stock market.
- The survival probabilities are deduced by the Italian Data for the period of 1947-2002 using the MRBG process with the parameters $\alpha = -0.02$, $\beta = 0.0065$, $\sigma_x = 0.1$

The results are exposed in Table 1.

Table 1 shows the funding ratio distribution characteristics at different evaluation dates. In particular, Table 1 addresses the expected value of the funding ratio at time $t$, the standard deviation of the funding ratio relative to its expectation, the 99.5% quantile and the expected shortfall relative to this quantile.

Table 1 puts in evidence that, in our example, the expected value of the funding ratio is always higher than 1.

Table 1. Funding ratio distribution characteristics, x = 30, x+ ξ = 60

| | $E[FR_t]$ | $stDev[FR]_t / E[FR_t]]$ | $Q_t$ (99.5) | $ES_t$ (99.5) |
|---|---|---|---|---|
| $t = 5$ | 1.0444 | 0.0053 | 0.6315 | 0.5907 |
| $t = 10$ | 1.0491 | 0.0063 | 0.6302 | 0.5769 |
| $t = 15$ | 1.0499 | 0.0061 | 0.6212 | 0.5633 |
| $t = 20$ | 1.0514 | 0.0042 | 0.6193 | 0.5612 |
| $t = 25$ | 1.0535 | 0.0043 | 0.6178 | 0.5608 |

This result means that the adoption of a market based valuation system allows the Fund to release more capital for investment purposes. As well known, the current insurance practices set the minimum capital requirement as a fixed percentage of the mathematical provision. The results in Table 1 allow us to stress that the adoption of an internal model, as proposed by Solvency II guidelines, offers a more flexible system for assessing the Fund solvency.

On the basis of the simulation procedure, the expected funding ratio increases with $t$. This result depends on the adopted technical basis and mainly on the estimated term structure of interest rate.

Looking at the quantile values, we note that, even if the $E[FR_t]$ is higher than one, the fund is exposed to a considerable risk. In fact, for each $t$, $Q$ (0.995) value shows a remarkable increasing underfunding risk. As a consequence, the expected shortfall shows the same behavior with time and, coherently with its nature, it is lower than $Q$ (0.995) value for each $t$.

Therefore, our model allows for quantifying the risk exposure of the Fund in terms of deviation from the expected value of the funding ratio.

As one can see, if a tail event does occur, the asset values do not cover the liabilities being $ES_t$ (99.5) significantly lower than one for each $t$.

In the following we have studied the behavior of the risk exposure when we fix the outgoing age (x+ξ = 60) and vary the entering age into the Fund. Obviously, we will have a decreasing maturity date.

Tables 2 and 3 and Figure 1, clearly show that the risk borne out by the Fund decreases as the entering age increases. From an actuarial point of view, this is a natural consequence of the fact that the number of years of exposure to risk decreases.

Table 2. Funding ratio distribution characteristics, x = 35, x + ξ = 60

| | $Q_t$ (99.5) | $ES_t$ (99.5) |
|---|---|---|
| $t = 5$ | 0.7305 | 0.6917 |
| $t = 10$ | 0.7292 | 0.6732 |
| $t = 15$ | 0.7157 | 0.6645 |
| $t = 20$ | 0.7101 | 0.6575 |
| $t = 25$ | 0.7072 | 0.6548 |

Table 3. Funding ratio distribution characteristics, x = 40, x + ξ = 60

| | $Q_t$ (99.5) | $ES_t$ (99.5) |
|---|---|---|
| $t = 5$ | 0.8835 | 0.8471 |
| $t = 10$ | 0.8799 | 0.8241 |
| $t = 15$ | 0.8674 | 0.8139 |
| $t = 20$ | 0.8625 | 0.8064 |
| $t = 25$ | 0.8602 | 0.7958 |

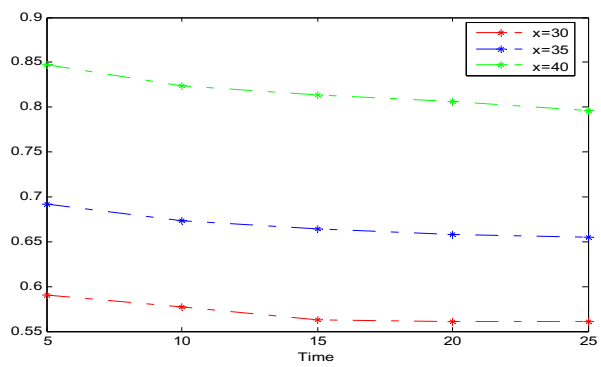


**Fig. 1. $ES_t$ (99.5) x = 30, x = 35, x = 40 and x + ξ = 60**

Moreover, we have considered the case in which the maximum number of years the generic subscriber is a fund member before reaching the pension age ξ is fixed and equal to 30.

As Tables 4 and 5 and Figure 2 show, the risk level decreases when entry age increases. Having fixed ξ, one of the possible causes is certainly the well known longevity phenomenon due to the improvements in mortality trends. In our case, the probability of reaching the pension age prevails over the probability of death and disability, thus causing a decreasing risk profile.

Table 4. Funding ratio distribution characteristics, x = 35, ξ = 30

| | $Q_t(99.5)$ | $ES_t(99.5)$ |
|---|---|---|
| *t = 5* | 0.7435 | 0.7047 |
| *t = 10* | 0.7322 | 0.6862 |
| *t = 15* | 0.7287 | 0.6775 |
| *t = 20* | 0.7231 | 0.6605 |
| *t = 25* | 0.7102 | 0.6578 |

Table 5. Funding ratio distribution characteristics, x = 40, ξ = 30

| | $Q_t(99.5)$ | $ES_t(99.5)$ |
|---|---|---|
| *t = 5* | 0.8885 | 0.8471 |
| *t = 10* | 0.8749 | 0.8291 |
| *t = 15* | 0.8624 | 0.8189 |
| *t = 20* | 0.8575 | 0.8094 |
| *t = 25* | 0.8602 | 0.7998 |

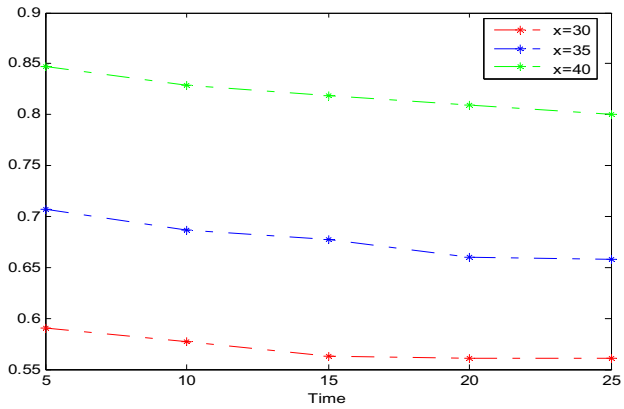


**Fig. 2. $ES_t$ (99.5) x = 30, x = 35, x = 40 and ξ = 30**

In conclusion, the main target of the paper was to analyze the solvency requirements for Defined Contribution Pension Fund by investigating the funding ratio via expected shortfall approach. With the adoption of market based funding requirements, the expected funding ratio is sufficiently high in terms of solvency, but, at the same time, the fund manager is exposed at a considerable level of risk that can be measured via a quantile approach. The suggested model allows to determine an absolute index of riskiness for the Fund. Moreover, it can also be applied for measuring the stochastic impact of the interest and mortality rates on the funding ratio distribution.

Further research on this subject will concern the quantification of the impact of the different risk sources on the global riskiness of the Fund.

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