"Solving an optimization problem of a profit calculation taking into account fixed costs in Excel"

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Abstract

Objective information for making a management decision is formed on the basis of the results of solving optimization problems of costs calculation. A contradiction in an analytical support of a decision making continues to exist, the content of which is a calculation of variable costs mostly in optimization models in the objective function only, but the category of fixed costs is ignored. Most at calculating production profit and each product sales z_j only variable costs, which are dependent on the production volume, are usually considered: $z_j = (c_j - q_j) x_j$, where c_j are selling prices, q_j are a product production costs; variable costs $q_j x_j$ are proportional to production volumes x_j. If fixed costs p_j are considered in addition to variable costs and they don't depend on the production volume, then following formula will be got to calculate the profit $z_j = (c_j - q_j) x_j + p_j$. But at $x_j = 0$ (when this type of a product is not produced) it should be $z_j = 0$, and not $z_j = p_j$, as it follows from the above-stated formula. Thus, this formula is not quite correct and should be replaced by the more complex one: $z_j = (c_j - q_j) x_j - p_j v_j$, where v_j are binary indicator variables that equal to unity ($v_j = 1$) if this type of a product is produced and zero ($v_j = 0$) otherwise. An existence of indicator variables means that the profit function is nonlinear.

The procedure of solving an optimization problem using Excel spreadsheet is demonstrated in the article. The advantage of the proposed procedure is the ability to solve the problem with parameters that allow you to find the limits of possible variations of the resources (and prices) for the purpose of increasing profits.

Keywords: an optimization problem, a calculation of costs, variable costs, fixed costs, Excel spreadsheet.
JEL Classification: C6, C8, D24.

Introduction

A problem of improvement of the current system of costs management remains actual in various modern models of the enterprise and its management. An effect of costs level depends on an objective forming of information for making a management decision by costs in the enterprise. As known this type of information is the most important in a management accounting. A classification of costs depending on an influence of changes in production volumes is divided into fixed and variable costs. And it is important to consider both types of costs to achieve enterprise goals. There are different views of economists about attributing costs to fixed or variable, but the particular contradiction in an analytical support of decision making is a fact that in most cases only variable costs are taken into consideration in the objective function of optimization models, but the category of fixed costs is ignored.

In standard optimization mathematical programming problems we should define the production output $x_1, x_2, \ldots, x_n$ at which the total profit reaches maximum. Most at calculating production profit and each product sales z_j only variable costs, which are dependent on the production volume, are usually considered: $z_j = (c_j - q_j) x_j$, where c_j are selling prices, q_j are a product production costs; variable costs $q_j x_j$ are proportional to production volumes x_j. If fixed costs p_j are considered in addition to variable costs and they don't depend on the production volume, then we will get the following formula to calculate the profit $z_j = (c_j - q_j) x_j + p_j$. But at $x_j = 0$ (when this type of a product is not produced) it should be $z_j = 0$, and not $z_j = p_j$, as it follows from the above-stated formula. Thus, this formula is not quite correct and should be replaced by the more complex one: $z_j = (c_j - q_j) x_j - p_j v_j$, where v_j are binary indicator variables that equal to unity ($v_j = 1$) if this type of a product is produced and zero ($v_j = 0$) otherwise. An existence of indicator variables means that the profit function is nonlinear.

When the volume production is reduced the profit goes down and it is zero ($z_j = 0$) at $x_j = p_j / (c_j - q_j)$; in a further reduction x_j "profit" z_j becomes negative, but it takes zero again at $x_j = 0$. This nonlinearity substantially complicates the problem of finding the optimal plan, even if the appropriate software is used, for example, the superstructure "Solver parameters" from Excel spreadsheet.

Some classes of nonlinear problems can be led to the linear form with additional constraints such as $x_j u_j = 0$, where one of two (mutually dual) unknown in the optimal plan must be equal to zero. For example, this is how to solve quadratic programming problems efficiently.

Problem statement

To solve our problem we also introduce variables u_j, which are equal to zero, if products are produced ($x_j > 0$), and equal to unity if products are not produced ($x_j = 0$). New variables are connected with "control variables" x_j with the help of relationships very similar to duality relationships: $x_j u_j = 0$. Intermediate values $0 < u_j < 1$ will be excluded at a profit maximization z. In fact, costs and a production profit of each product will be equal $p_j(1 - u_j) + q_j x_j$ and $z_j = (c_j - q_j)x_j + p_j u_j - p_j$, besides one of the two unknowns x_j, u_j and in these terms is zero. So, if $u_j > 0$ (then automatically $x_j = 0$) and u_j must be equal to its maximum value, that is unity, from the condition of maximum profit z_j.

Except relationships "duality" $x_j u_j = 0$, all the other conditions are of a linear type, therefore the problem can be solved with the help of a standard simplex method, which guarantees a fast convergence to the optimal solution and makes it possible to include parameters in reserves of resources and in selling prices for finished products.

Given the realities of the functioning of enterprises, there are bilateral constraints that impose on the production volumes $m_j \le x_j \le M_j$. Naturally, we are interested only in the case $m_j = 0$, where above mentioned problems appear (at $m_j > 0$ it is always $x_j > 0$, $u_j = 0$ and then the problem is a program of linear programming). At the suggestion of H. Taha constraints of unknowns from the top $x_j \le M_j$ are useful to write as $x_j \le M_j(1 - u_j)$, then at $u_j = 1$ it is automatically $x_j = 0$. Thus H. Taha believes that it is necessary to introduce conditions of binary variables u_j in an explicit form.

Problem estimation

Let's consider solving this problem on the example. Publishing House has received orders for the publication of three books A_1, A_2, A_3 which it is supposed to sell by the price $c_1 = 7.63$; $c_2 = 7.23$; $c_3 = 9.28$ UAH accordingly. The maximum demand for these books does not exceed $M_1 = M_2 = M_3 = 20$ thousand copies. Fixed costs for making artworks are $p_1 = 2270$, $p_2 = 1890$, $p_3 = 2820$ UAH. Variable costs to produce one copy of each book are equal to $q_1 = 1.8$; $q_2 = 1.5$; $q_3 = 2.1$ UAH. There must be $a_1 = 0.00032$; $a_2 = 0.00027$; $a_3 = 0.00035$ tones of paper for each book. There are $b_1 = 10$ tons of paper and $b_2 = 50$ thousand UAH in the account at all. We need to find the optimal plan x_1, x_2, x_3 of output of these books (in thousand copies).

Let's construct the mathematical model. The total expenditures of paper $a_1 x_1 + a_2 x_2 + a_3 x_3 = 0.32 x_1 + 0.27 x_2 + 0.35 x_3$ can not exceed its reserve $b_1 = 10$ tones:

$$0.32\, x_1 + 0.27\, x_2 + + 0.35\, x_3 \le 10.$$

Cash costs to produce each book subject to binary unknowns u_1, u_2, u_3 will be:

$$p_1(1 - u_1) + q_1 x_1 = 2.27 - 2.27\, u_1 + 1.8\, x_1,$$

$$p_2(1 - u_2) + q_2 x_2 = 1.89 - 1.89\, u_2 + 1.5\, x_2,$$

$$p_3(1 - u_3) + q_3 x_3 = 2.82 - 2.82\, u_3 + 2.1\, x_3$$

and should not exceed $b_2 = 50$ thousand UAH in total:

$$1.8 \cdot x_1 + 1.5 \cdot x_2 + 2.1 \cdot x_3 - 2.27\, u_1 - 1.89\, u_2 - 2.82\, u_3 \le 50 - (2.27 + 1.89 + 2.82) = 43.02.$$

The profit of the publication of each book is:

$$z_1 = (c_1 - q_1)x_1 - p_1(1 - u_1) = (7.63 - 1.8)x_1 + 2.27\, u_1 - 2.27,$$

$$z_2 = (c_2 - q_2)x_2 - p_2(1 - u_2) = (7.23 - 1.5)x_2 + 1.89\, u_2 - 1.89,$$

$$z_3 = (c_3 - q_3)x_3 - p_3(1 - u_3) = (9.28 - 2.1)x_3 + 2.82\, u_3 - 2.82.$$

The minimum number of copies, below which the production will not be profitable, equals:

$$m_1 = 2.27 / (7.63 - 1.8) = 0.389;$$

$$m_2 = 1.89 / (7.23 - 1.5) = 0.330;$$

$$m_3 = 2.82 / (9.28 - 2.1) = 0.393;$$

thousand copies.

We need to find the optimal plan of the publication, at which the total profit will be maximized: $z = z_1 + z_2 + z_3 \rightarrow \max$.

Constraints $u_1 \le 1$, $u_2 \le 1$, $u_3 \le 1$ are included in the number of basic constraints of the problem. Constraints $x_1 \le 20$, $x_2 \le 20$, $x_3 \le 20$ are replaced with more effective constraints by H. Taha: $x_1 \le 20(1 - u_1)$, $x_2 \le 20(1 - u_2)$, $x_3 \le 20(1 - u_3)$.

Non-negativity conditions of unknowns $x_1, x_2, x_3 \ge 0$, $u_1, u_2, u_3 \ge 0$ are taken into the consideration automatically in simplex method.

Let's write down constraints in the canonical form:

$$\begin{cases} 10.00 = 0.32\, x_1 + 0.27\, x_2 + 0.35\, x_3 + \mathit{6}_1, \\ 43.02 = 1.80 x_1 + 1.50\, x_2 + 2.10\, x_3 - \\ \quad 2.27\, u_1 - 1.89\, u_2 - 2.82\, u_3 + \mathit{6}_2, \\ 1 = u_1 + v_1, \\ 1 = u_2 + v_2, \\ 1 = u_3 + v_3, \\ 20 = x_1 + 20\, u_1 + w_1, \\ 20 = x_2 + 20\, u_2 + w_2, \\ 20 = x_3 + 20\, u_3 + w_3, \end{cases}$$

Remains of resources (balance unknowns of two first constraints) are marked as y_1, y_2. The balance unknowns of constraints $u_j \leq 1$ are marked by v_1, v_2, v_3; the balance unknowns of constraints $x_j \leq 20\,(1 - u_j)$ are marked by w_1, w_2, w_3.

Let's compose the objective function of the total profit of the enterprise as:

$$z = z_1 + z_2 + z_3 = -(2.27 + 1.89 + 2.82) + 5.83\,x_1 +$$

$$+ 5.73\,x_2\,7.18\,x_3 + 2.27\,u_1 + 1.89\,u_2 + 2.82\,u_3 =$$

$$= -6.98 + 5.83\,x_1 + 5.73\,x_2 + 7.18\,x_3 + 2.27\,u_1 +$$

$$+ 1.89\,u_2 + 2.82\,u_3 \rightarrow \max.$$

Let's fill the simplex table. We suppose that balance unknowns y_1, y_2, v_1, v_2, v_3, w_1, w_2, w_3 form the initial basis.

Table 1. Simplex table

№	B	b	x_1	u_1	x_2	u_2	x_3	u_3
1	y_1	10	0.32	0	0.27	0	0.35	0
2	y_2	43.02	1.8	-2.27	1.5	-1.89	2.1	-2.82
3	v_1	1	0	1	0	0	0	0
4	v_2	1	0	0	0	1	0	0
5	v_3	1	0	0	0	0	0	1
6	w_1	20	1	20	0	0	0	0
7	w_2	20	0	0	1	20	0	0
8	w_3	20	0	0	0	0	1	20
9	$-z_0$	6.98	5.83	2.27	5.73	1.89	7.18	2.82
10	$-t_1$	0	1	0	0	0	0	0
11	$-t_2$	0	0	0	1	0	0	0
12	$-t_3$	0	0	0	0	0	1	0

In order to investigate the effect of prices variations ($\Delta c_j = t_j$) we present the objective function as $z = z_0 + t_1\,x_1 + t_2\,x_2 + t_3\,x_3$, and write it in four rows in the table. In the row of the basic objective function z_0 the coefficients at unknowns are projections of the vector-gradient; now, they are all positive, thus, entering each of these unknowns into the basis leads to increasing the objective function.

Let's enter x_3 in the basis. After Gauss-Jordan transformation the table will be:

Table 2. Simplex table

№	B	b	x_1	u_1	x_2	u_2	x_3	u_3	w_3
13	y_1	3	0.32	0	0.27	0	0	-7	-0.35
14	y_2	1.02	1.8	-2.27	1.5	-1.89	0	-44.82	-2.1
15	v_1	1	0	1	0	0	0	0	0
16	v_2	1	0	0	0	1	0	0	0
17	v_3	1	0	0	0	0	0	1	0
18	w_1	20	1	20	0	0	0	0	0
19	w_2	20	0	0	1	20	0	0	0
20	w_3	20	0	0	0	0	1	20	1
21	$-z_0$	-136.6	5.83	2.27	5.73	1.89	0	-140.8	-7.18
22	$-t_1$	0	1	0	0	0	0	0	0
23	$-t_2$	0	03	0	1	0	0	0	0
24	$-t_3$	-20	0	0	0	0	0	-20	-1

In the result of this transformation the objective function z_0 is increased from -6.98 to +136.6. Unknown w_3 is not basic yet, so this column appears in the table. Let's enter x_2 in the basis, because there is a positive coefficient at this unknown in the row of the objective function z_0.

Table 3. Simplex table

№	B	b	x_1	u_1	x_2	u_2	x_3	u_3	y_2	w_3
25	y_1	2.816	-0.004	0.409	0	0.3402	0	1.0676	- 0.18	0.028
26	y_2	0.68	1.2	-1.513	1	-1.26	0	-29.88	0.667	-1.4
27	v_1	1	0	1	0	0	0	0	0	0
28	v_2	1	0	0	0	1	0	0	0	0
29	v_3	1	0	0	0	0	0	1	0	0
30	w_1	20	1	20	0	0	0	0	0	0
31	w_2	19.32	-1.2	1.513	0	21.26	0	29.88	-0.667	1.4
32	w_3	20	0	0	0	0	1	20	0	1
33	$-z_0$	-140.5	-1.046	10.941	0	9.110	0	30.432	-3.82	0.842
34	$-t_1$	0	1	0	0	0	0	0	0	0
35	$-t_2$	-0.68	-1.2	1.513	0	1.26	0	29.88	-0.667	1.4
36	$-t_3$	-20	0	0	0	0	0	-20	0	-1

The value of the objective function is increased from 136.6 to 140.5. Now, in the row of the objective function z_0 there are three positive elements, but we cannot enter unknowns u_2 and u_3 into the basis, because unknowns x_2 and x_3 related to them are entered in the basis. Let's enter w_3 into the basis again.

Table 4. Simplex table

№	B	b	x_1	u_1	x_2	u_2	x_3	u_3	y_2	w_2
37	y_1	2.43	0.02	0.378	0	-0.085	0	0.47	-0.167	-0.02
38	y_2	20	0	0	1	20	0	0	0	1
39	v_1	1	0	1	0	0	0	0	0	0

Table 4 (cont.). Simplex table

№	B	b	x_1	u_1	x_2	u_2	x_3	u_3	y_2	w_2
40	v_2	1	0	0	0	1	0	0	0	0
41	v_3	1	0	0	0	0	0	1	0	0
42	w_1	20	1	20	0	0	0	0	0	0
43	w_2	13.8	-0.857	1.081	0	15.186	0	21.343	-0.476	0.714
44	w_3	6.2	0.857	-1.081	0	-15.186	1	-1.343	0.4762	-0.714
45	$-z_0$	-152.1	-0.324	10.03	0	-3.677	0	12.462	-3.419	-0.601
46	$-t_1$	0	1	0	0	0	0	0	0	0
47	$-t_2$	-20	0	0	0	-20	0	0	0	-1
48	$-t_3$	-6.2	-0.857	1.081	0	15.186	0	1.343	-0.476	0.714

The value of the objective function is increased to 152.1. There are two positive coefficients in row z_0, but in the row of the objective function we cannot enter unknown u_3 with a positive coefficient in the row of the objective function into the basis, because there is already its paired unknown x_3. Let's enter u_1 into the basis. After this transformation we get the final table with the maximum value of the objective function $z_0 = 162.2$ thousand UAH.

Table 5. Simplex table

№	B	b	x_1	u_1	x_2	u_2	x_3	u_3	v_1	y_2	w_2
49	y_1	2.052	0.02	0	0	-0.085	0	0.47	-0.38	-0.17	-0.02
50	y_2	20	0	0	1	20	0	0	0	0	1
51	v_1	1	0	1	0	0	0	0	1	0	0
52	v_2	1	0	0	0	1	0	0	0	0	0
53	v_3	1	0	0	0	0	0	1	0	0	0
54	w_1	0	1	0	0	0	0	0	-20	0	0
55	w_2	12.719	-0.86	0	0	15.186	0	21.343	-1.08	-0.48	0.714
56	w_3	7.281	0.857	0	0	-15.17	1	-1.343	1.081	0.476	-0.71
57	$-z_0$	-162.2	-0.32	0	0	-3.677	0	12.462	-10.0	-3.42	-0.60
58	$-t_1$	0	1	0	0	0	0	0	0	0	0
59	$-t_2$	-20	0	0	0	-20	0	0	0	0	-1
60	$-t_3$	-7.281	-0.86	0	0	15.186	0	1.343	-1.08	-0.48	0.71

Now, there is only one positive coefficient at unknown u_3 in row z_0, but we cannot enter this unknown into the basis, because there is already its paired unknown x_3. All the other coefficients are negative, thus, the obtained optimal solution is: $x_1 = 0$; $x_2 = 20$; $x_3 = 7.281$ thousand copies. Cash reserves were spent completely ($y_2 = 0$), the remainder of the paper is $y_1 = 2.052$ tons.

If we lift constraints on the maximum number of copies ($x_j \leq 20$), then we need to publish only the second type of a book in the volume $x_2 = 32.07$ thousand copies, which provides the maximum possible profit $z_{max} = 182$ thousand UAH. Besides, this solution was received without constraints by H. Taha.

Binarity conditions of unknowns u_j, v_j were not given explicitly, but these unknowns were taken only two values 0 or 1 in all simplex-tables.

Let's consider the received optimal solution and construct the objective function using free unknowns, for this we combine four last rows of the table.

$$z = z_0 + t_1 x_1 + t_2 x_2 + t_3 x_3 =$$

$$= (162.2 + 20\,t_2 + 7.28\,t_3) - (0.32 - t_1 + 0.86\,t_3)\,x_1 - (3.68 + 20\,t_2 - 15.19\,t_3)\,u_2 + + (12.46 + 1.34\,t_3)\,u_3 - (10 + 1.08\,t_3)\,v_1 - (3.42 + 0.48\,t_3)\,y_2 - (0.60 + t_2 - 0.71\,t_3)\,w_2.$$

The obtained basic plan with price variations remains optimal, until all coefficients at free unknowns are positive (thus, for increasing the profit it is necessary to equate all free unknowns to their lowest value, i.e. zero).

Let's write optimality conditions:

$$0.32 - t_1 + 0.86\,t_3 \geq 0,$$

$$10 + 1.08\,t_3 \geq 0,$$

$$3.42 + 0.48\,t_3 \geq 0,$$

$$13.6 + 1.88\,t_2 \geq 0,$$

$$0.60 + t_2 - 0.71\,t_3 \geq 0.$$

Let's consider the effect of price variations for each type of books separately.

The first type of book will become profitable at increasing the price by $t_1 > 0.32$, that is, at the cost $c_1 = 7.63 + 0.32 = 7.95$ UAH and more; then the coefficient in row z_0 will change the sign and unknown x_1 must be entered into the basis. The third type of book is profitable to publish more copies, if its price is increased on $t_3 > 0.84$, that is, at $c_3 > 9.28 + 0.24 = 9.52$ UAH and more.

You can also write an optimal solution with the parameters in free terms (reserves of resources) from the last table. Let's denote variations of resources b_1, b_2 by s_1, s_2. There are columns of these parameters in the table in an implicit way, although we did not plan them specifically. Indeed, the columns of parameters s_1, s_2 are coincided with columns of remains of resources y_1, y_2, so we consider them in the general solution:

$$y_1 = 2.05 - 0.17 \, s_2,$$

$$x_2 = 20,$$

$$x_3 = 7.28 + 0.48 \, s_2,$$

$$z = (162.2 + 3.42 \, s_2) + 20 \, t_2 + (7.28 + 0.48 \, s_2) \, t_3.$$

The optimal solution does not depend on parameter s_1, naturally, because the first resource (paper) is in a surplus. At increasing the second resource (cash) to the value $s_1 = 2.05/0.17 = 12.06$ thousand UAH the output of books of the third type increases to $x_3 = 7.28 + 0.48 \cdot 12.06 = 13.07$ thousand copies, at the same time the profit reaches the value $z = 162.2 + 3.42 \cdot 12.06 = 203.445$ thousand UAH. Further profit growth are constrained by a shortage of paper yet.

Thus, we were able to get the full solution of this nonlinear problem by reducing it to the problem of linear programming with additional constraints such as $x_j \, u_j = 0$.

Now let's consider the solving of this problem with the help of the superstructure "Solver Parameters" of Excel spreadsheet. If the superstructure is installed, then Solver Parameters must be in menu of the Tools.

In Excel worksheet we put down the input data of this problem with explanations for understanding.

Table 6. Problem specification

	B	C	D	E	F	G	H
2	Calculation of fixed costs						
3							
4		A_1	A_2	A_3	Total costs	Reserve	Balance
5	The expenditure of paper per 1 thousand copies	0.32	0.27	0.35	0.94	10	9.06
6	Variable costs per 1 thousand.	1.8	1.5	2.1	12.38	50	37.62
7	Fixed costs, thos. UAH.	2.27	1.89	2.82			
8	Costs (c) 1 thousand copies	7.63	7.23	9.28			
9							
10		x_1	x_2	x_3			
11	Maximum demand, thousand copies	20	20	20			
12	Publication plan (x) thousand copies	1	1	1			
13	Binary variables v	1	1	1			
14	Products of M v	20	20	20			
15	Products of x (1 – v)	0	0	0			
16						Max	
17	Profit (per 1 thousand copies)	3.56	3.84	4.36	Total profit	11.76	

The publication plan x_j and binary variables v_j are taken as equal unity (this is the initial values to find the optimal solution).

In the column "Total costs" we write the formula:

Costs of paper = SUMPRODUCT (C5: E5; C12: E12)

Cash costs = SUMPRODUCT (C6: E7; C12: E13)

In column "Balance" we write the formula for a calculation of differences = Reserve – Total Costs.

In the row "Products Mv" we write the formula for a calculation of these products (M is marked by the maximum demand here).

The row of products x (1-v) is added for a visual control, these products always should be equal to zero, but in an explicit form these conditions are not used in the calculations.

In the row "Profit" we write the formula for a calculation of the profit from the publication of each type of books $z_j = (c_j - q_j) \, x_j - p_j \, v_j$, and the sum of these terms is calculated in the cell G17 (Total profit). The profit from the publication of each kind of books should not be negative (this condition is not available in an explicit form).

It is absolutely ready for a calculation. Let's set the tabular cursor in the cell "Total profit" and use the

superstructure "Solver Parameters". Panel of this superstructure will appear where you should indicate the range of control variables and formulate all constraints of this problem.

At first, we should fill all fields using the method by H. Taha.

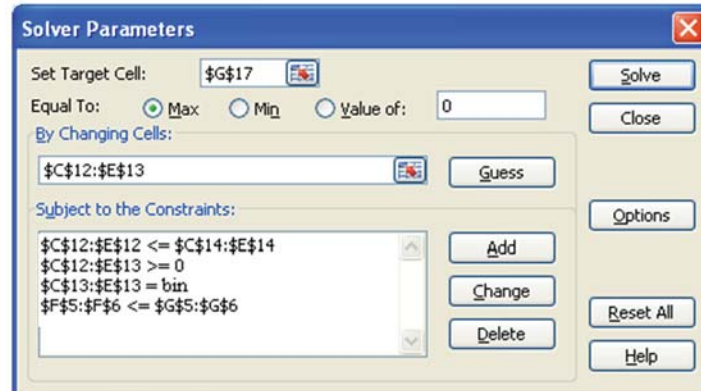The range "By changing cells" C12: E13 includes both unknown x_j and v_j.

Fig. 1. Window solver parameters

In the field "Subject to the constrains" we write the conditions:

$x_j <= M_j v_j$

$x_j >= 0$

$u_j = $ binary

Total Costs <= Reserve

Let's click on the button Solve and get the optimal solution.

Table 7. Decision (1)

	B	C	D	E	F	G	H
2	Calculation of fixed costs						
3							
4		A_1	A_2	A_3	Total costs	Reserve	Balance
5	The expenditure of paper per 1 thousand copies	0.32	0.27	0.35	7.948	10	2.052
6	Variable costs per 1 thousand.	1.8	1.5	2.1	50	50	0
7	Fixed costs, ths. UAH.	2.27	1.89	2.82			
8	Costs (*c*) 1 thousand copies	7.63	7.23	9.28			
9							
10		x_1	x_2	x_3			
11	Maximum demand, thousand copies	20	20	20			
12	Publication plan (*x*) thousand copies	0	20	7.281			
13	Binary variables *v*	0	1	1			
14	Products of M *v*	0	20	20			
15	Products of *x*(1 – *v*)	0	0	0			
16						Max	
17	Profit (per 1 thousand copies)	0	112.71	49.457	Total profit	162.167	

Let's visually satisfy that products $x(1-v)$ are equal to zero, that is, if the book is not published ($x_j = 0$), then its fixed costs are not included in the Total costs. Let's visually satisfy that all components of the profit from the publication each type of book are non-negative. Besides, both of these conditions can be formulated in an explicit form and included in a number of constraints of the problem.

Let's write the optimal solution. It was found that the first book is not profitable to publish ($x_1 = 0$), the second book should be published with the maximum edition ($x_2 = 20$), the edition of the third book $x_3 = 7.281$ thousands copies is limited by the shortage of resources. Cash costs were fully spent ($в_2 = 0$), the remainder of a paper equals $в_1 = 2.052$ tons. Besides the total profit is 162.167 thousands UAH.

Unfortunately, the superstructure "Solver Parameters" does not create a stability report at explicitly given conditions of binary or integer unknowns. Therefore we propose another way to fill the input fields. At first, we exclude binary

unknowns v_j from the number of control variables (we will change only basic unknowns x_j at an optimum search) and we will determine v_j with the help of logical expressions:

IF $x_j > p_j / (c_j - q_j)$ THEN $v_j = 1$ OTHERWISE $v_j = 0$.

In particular, the formula = IF (C12 > C7 / (C8-C6);1;0) is writing in the cell C13, then we copy into other cells D13 and E13.

Table 8. Decision (2)

	B	C	D	E	F	G	H
2	Calculation of fixed costs						
3							
4		A_1	A_2	A_3	Total costs	Supply	Remain
5	The expenditure of paper per 1 thousand copies	0.32	0.27	0.35	7.948	10	2.052
6	Variable costs per 1 thousand.	1.8	1.5	2.1	50	50	0
7	Fixed costs, thousand UAH.	2.27	1.89	2.82			
8	Costs (c) 1 thousand copies	7.63	7.23	9.28			
9							
10		x_1	x_2	x_3			
11	Maximal demand, thousand copies	20	20	20			
12	Publication plan (x) thousand copies	0	20	7.281			
13	Binary variables v	0	1	1			
14						Max	
15	Profit (per 1 thousand copies)	0	112.71	49.457	Total profit	162.167	

The input fields of superstructure we fill as:

By Changing cells – we indicate the range for x_j.

Subject to the Constraints $\quad x_j <= M_j$

$$x_j >= 0$$

Total Costs <= Supply

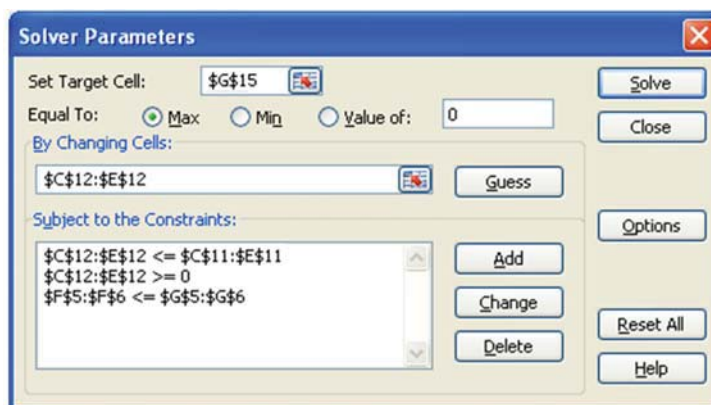We click on the button Run and get the optimal solution given above.

Fig. 2. Window solver parameters

Besides this solution, it is suggested to give three reports by results, by stability and by limits. For all the reports, the following procedure was accepted with references to specific cells of a spreadsheet, that is, the text on the left of the cell and the nearest text above are combined. For example, Total profit Max is a reference to the value of the objective function. For such report, it is desirable to use short (but very informative) texts on the left and above of the table cells so that their names are not too long. However, this is not considerable. Let's consider the Stability report:

Table 9. Report on stability

	Stability report		
By changing cells			
		Final	Normalized
Cells	Name	values	Gradient
C12	Publication plan (x) in thousand copies x_1	0	-0.32428576

Table 9 (cont). Report on stability

	Stability report		
D12	Publication plan (x) in thousand copies x_2	20	0.60142848
E12	Publication plan (x) in thousand copies x_3	7.280952436	0
Subject to the constraints			
		Final	Lagrange
Cells	Name	values	Multipliers
F5	The expenditure of paper per 1 thousand copies. Total costs	7.948333353	0
F6	Variable costs per 1 thousand copies. Total costs	50.00000012	3.419047693

In addition to the already known information, the stability report contains information about the normalized gradient and Lagrange multipliers.

The normalized gradient of the publication plan of the first book is negative, that is, the circulation of this book should be smallest; it is accepted equal to $x_1 = 0$ (don't publish (print) an unprofitable (money-losing) book). For the second book the normalized gradient is positive, that is, the circulation of this book should be biggest; it is accepted equal to $x_2 = 20$ (according to the maximum demand). For the third book the normalized gradient equals zero, that is, the circulation of this book is $x_3 = 7.948$ determined from the condition of the maximum profit.

Lagrange multipliers are shadow prices of resources; they show how increase the profit by increasing supplies of resources per unit. The shadow price of the first resource (a paper) equals

zero, because this resource is in plenty (7.948 were spent out of 10 tons). The second resource (money) was spent completely (fully) and its shadow price equals 3.419, which is interpreted as: if money increases by 1 thousand UAH (a credit, a loan) the profit increases by 3.419 thousand UAH.

Conclusion

It is obvious that an increase of a supply of only one resource at last can result to a depletion of supplies of other resources and a further profit growth will be controlled by the shortage of other resources. Optimization problems in many fields can be modeled and solved using Excel. The advantage of the suggested approach to the solution of the problem as linear is the possibility of solving the problem with parameters that allow you to find the limits of a possible variation of resources (and prices) for the purpose of increasing the profit.

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