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IMPROVING THE OPTION PRICING PERFORMANCE OF GARCH MODELS IN INEFFICIENT MARKET

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Abstract

Understanding the relation between option pricing and market efficiency is important. Indeed, emphasizing this relation generates new insights that are appropriate in practice. These insights give a better understanding of the current limitations of the option pricing and hedging methods. This article thus aims to improve the performance of the option pricing approach. To start, the relation between the option pricing methodology and the informational market efficiency was discussed. It is, therefore, useful, before proceeding to apply the standard risk-neutral approach, to check the efficiency assumption. New modified GARCH processes were used to model the dynamics of the asset returns in the option pricing framework. The new considered approaches allow describing the dynamic of returns when the market is inefficient. Using real data on CAC 40 index, the performance of different models as a function of maturity and moneyness was studied. The in-sample analysis, interested in the stability of the pricing models across time, showed that the new approach, developed under the affine GARCH process, is the most accurate. The study of the out-of-sample performance, which aims to evaluate the forecasting ability of different approaches, confirmed the results of the in-sample analysis. For the optional portfolio hedging, always the best hedging approach is that obtained under the affine GARCH model. After a regression study, it was found that the difference between theoretical and observed option values can be explained by factors, which are not taken into account in the proposed pricing formulae.

Keywords efficiency, bubbles, option pricing, hedging, performance, risk management

JEL Classification C12, C13, G14

INTRODUCTION

Improving the option pricing and hedging performance makes it essential to discuss the relation between the option pricing methodology and market informational efficiency. Such a relation has been omitted in the literature because, at the same time, academic researchers and practitioners in the financial field believe that these two subjects are independent.

Furthermore, the market is said efficient for a set of information I_t, available at a moment t, if and only if there are no asset price bubbles (Jarrow, 2013). One tests the market efficiency returns in this framework to test the existence of bubbles. In general, the speculative bubbles are observed when the asset price shows an explosive behavior for a short period, then the bubble burst, and the asset price returns to a much lower level.

The bubbles lead, generally, to positive feedback of the financial asset price on the corresponding return, or of the return on itself. However, it becomes useful to find an appropriate method for bub-

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ble detection, which can combine the dynamics of the bubbles with the GARCH option pricing models. To model the positive feedback on the asset returns, the FTS-GARCH model, developed by Corsi and Sornette (2014), suggests adding a new exogenous component in the conditional mean equation. Indeed, in the FTS-GARCH model, the return at moment t depends on the underlying asset price at the previous moment $t-1$.

As the main contribution in this paper, it is proposed to adapt the FTS-GARCH model to the option pricing methodology to improve its performance. One considers, in this framework, modified versions of the FTS-GARCH model considering non-symmetric GARCH processes. In fact, in its original form, the FTS-GARCH model assumes that the conditional variance corresponds to the simple GARCH model of Bollerslev (1986). To highlight, for the first time, the relation between the option pricing and the market efficiency in practice, real data on CAC 40 stock index are used. Indeed, there is no empirical work that has been performed to show this relation.

1. LITERATURE REVIEW

The efficient market assumption, for a set of information I_t, available at the moment t, suggests that at this moment, the asset price on the market reflects instantaneously all the information (Fama, 1970). According to this assumption, it is not possible to find investment strategies allowing having a return on the asset price because none has access to private information. In other words, it is not possible to find alpha-positive investment strategies based on the available set of information. Therefore, the informed investors act when the information becomes available to return the asset price to its correct value.

In the literature, there are distinguished three different forms for the market informational efficiency based on the available information. The first form, so-called low efficiency, corresponds to a set of information contained only the historical and current negotiated asset prices. The second form, so-called semi-strong efficiency, is that where the price reflects all the available public information. According to this form of efficiency, the information obtained from the balance sheets, the financial presses, and the fundamental analysis is taken into account. Finally, the third form of efficiency, so-called strong efficiency, considers that in addition to the public information, the asset price also reflects the private information. The validation tests of these assumptions accepted the low form of efficiency and rejected that of strong form. For the semi-strong efficiency, the evidence is mixed.

To test the market efficiency, the traditional approach considers a particular equilibrium model and starts by looking for a positive alpha. If the positive alpha exists, one may not know if it is due to the disability of the considered model or the market inefficiency (dilemma of the joint hypothesis). According to Camerer (1989), the joint hypothesis makes the evidence inconclusive. Recently, Jarrow and Larsson (2012) formulated the definition of market informational efficiency using financial mathematic tools. According to these authors and Jarrow (2013), the market is efficient for a set of information I_t if and only if there are no asset price bubbles. Therefore, testing the market efficiency returns, in this context, is to test for the existence of asset price bubbles. Jarrow (2016), based on the local martingale theory of bubbles, developed by Cox and Hobsen (2005), Heston, Loewenstein, and Willard (2007), Jarrow, Protter, and Shimbo (2007, 2010), among others, proposed three different methods that avoid the joint hypothesis problem to perform this test. Arshanapalli and Nelson (2016) presented an overview of econometric tools, used to test for stock price bubbles. Also, Wöckl (2019) gave a survey of theoretical bubble models and empirical bubble detection tests.

For the underlying asset return dynamic, the phenomenon of bubbles must be considered. In this framework, Corsi and Sornette (2014) used a mathematical process called FTS-GARCH model, in which one can integrate the positive feedback of the past asset prices on the current returns. This model allows combining the bubble dynamic with the standards of the financial industry to describe the volatility clustering phenomenon and assess the corresponding risk.

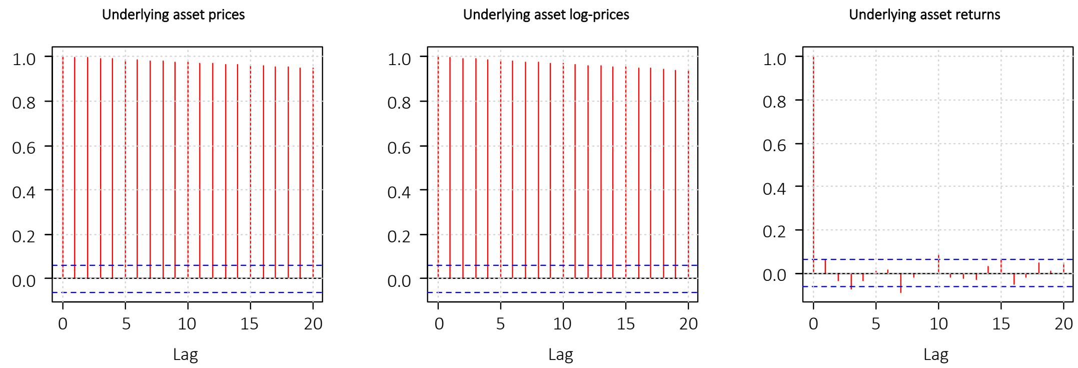

Figure 1. Autocorrelation functions of the CAC 40 index price, log-price and log-return (12/31/1987 – 12/31/2018)

This paper aims to adapt the original specification of the FTS-GARCH model in the option pricing framework to develop an appropriate tool to price options during the inefficient periods.

2. METHOD

Based on the theoretical results of Jarrow (2013), it is proposed to develop an appropriate tool allowing to price options in an inefficient market. The processes describing the underlying returns must take account of the bubble phenomenon. In this context, one can use a mathematical model in which the positive feedbacks of the asset past prices on the current returns are integrated. This model, developed by Corsi and Sornette (2014), allows combining the bubble dynamics with the standards of the financial industry to describe the clustering volatility phenomenon and evaluate the corresponding risk[1].

2.1. The FTS-GARCH model

The FTS-GARCH model is a standard GARCH process improved by adding a new component y_t in the conditional mean equation to describe the positive feedback. The equation of the conditional returns in the FTS-GARCH model framework is written as follows:

$$R_t = \ln\left(\frac{S_t}{S_{t-1}}\right) = \mu + \theta y_{t-1} + \sqrt{h_t} z_t. \qquad (1)$$

In this equation, the shock z_t is assumed to be $iid\ N(0\ ,\ 1)$ and h_t is the conditional variance of return on day t, which is known at the end of day $t-1$.

In its original form, the FTS-GARCH model considers that the variable y_{t-1} corresponds to the past price of the underlying asset $[y_{t-1} = S_{t-1}]$. However, the fact that the underlying asset price is not stationary can be considered as an undesirable property. For that, the asset price can be replaced by its logarithmic form $[\ln(S_{t-1})]$ or its corresponding return $[R_{t-1}]$. It is, therefore, useful to perform a stationarity test for three concerned variables. The following figure shows the autocorrelation function of each of them.

From the previous figure, only the log-return corresponds to a stationary series[2]. For this reason, it is proposed to use an improved version of the FTS-GARCH model, considering the past values of returns (R_{t-1}) as a new component in the conditional mean equation.

In the new model, the focus will be on parameter θ since it represents the nature and importance of the return feedback on itself. One considers then a statistical *t*-test for the following assumptions: $H_0: \theta \le 0$ against $H_1: \theta > 0$. The null assumption H_0 implies the absence of the feedback $(\theta = 0)$ or negative feedback (crash), contrariwise the alternative assumption implies the existence of a bubble.

1 The considered model is named Finite-Time Singularity GARCH (FTS-GARCH) model. The singularity phenomenon in the finite time occurs when a variable increases quickly and tends toward a finite value during a finite period.

2 This result can be confirmed by a unit root test (ADF) for 5% significance level. The corresponding p-values are 0.692, 0.941, and 0.001, respectively, for the index price, its logarithm, and return.

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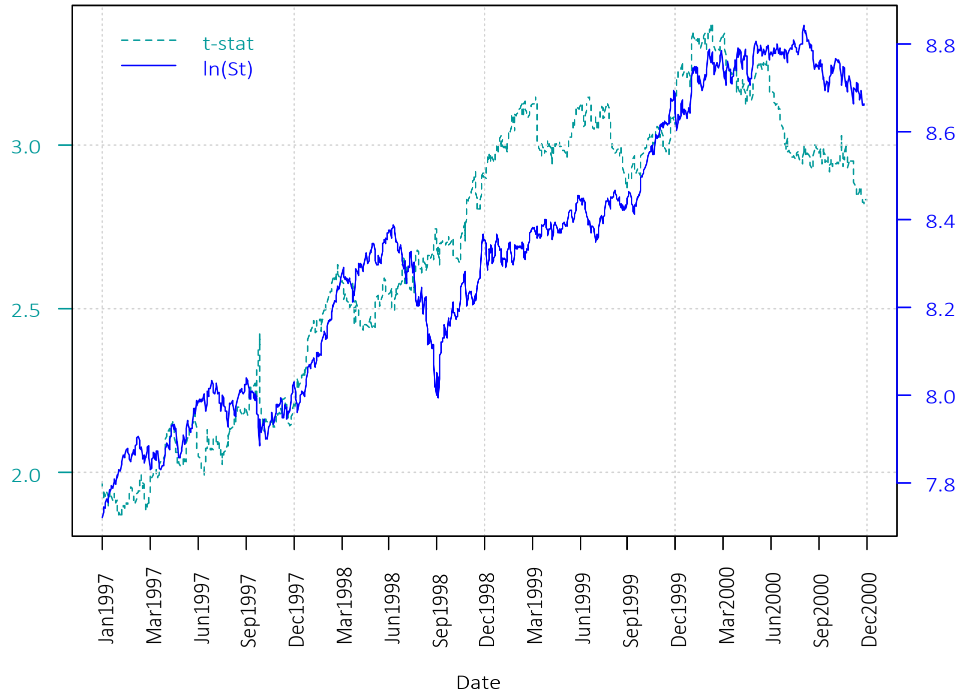

Figure 2. Statistical t-test of the parameter θ

To detect the bubbles in real time, one estimates the value of the parameter θ then one tests its statistical signification during the bubble period. One estimates, for instance, the FTS-GARCH model using samples of data ending by dates belonging to the period where the bubble is in development. Figure 2 shows that the first bubble, characterizing the annual price of CAC 40 index, has grown during the period 1997–2000. By estimating a simple GARCH model using samples ending on dates from this period, the following figure is obtained:

It is to be noted, from the previous figure, that when the bubble is in development, the t-statistic steadily increases and reaches its maximum value before the index price. The parameter θ becomes more significant at the peak of the bubble[3].

2.2. The FTS-GARCH option pricing model

In the GARCH option pricing model, two different specifications can be used to describe the return dynamic. The first one corresponds to the affine approach proposed by Heston and Nandi (2000), and the second one is the non-affine approach of Duan (1995). To adapt the original specification of the FTS-GARCH model in the option pricing framework, the constant parameter μ in equation (1) must be replaced by $r+\mu_t$, where r is the risk-free interest rate. The term μ_t is replaced by λh_t in the affine GARCH process and $\lambda\sqrt{h_t}-0.5h_t$ in the non-affine one, with λ is the constant price of risk. It should be noted that the conditional variance h_t can be written as:

Affine TFS-GARCH (AFTS):

$$h_t=\omega+\beta h_{t-1}+\alpha\left(z_{t-1}-\delta\sqrt{h_{t-1}}\right)^2. \tag{2}$$

Non-affine FTS-GARCH (NFTS):

$$h_t=\omega+\beta h_{t-1}+\alpha h_{t-1}\left(z_{t-1}-\delta\right)^2. \tag{3}$$

In these equations, the parameter δ captures the leverage effect. It captures the negative relation between shocks to returns and volatility, which results in a negative skewed distribution of returns. ω is a strictly positive parameter, α and β measure the one-period delay effect of return and conditional variance, respectively, on the instantaneous conditional variance, α and β are assumed to be positive.

3 To detect the asset price bubbles, one can also use the approaches developed by Jarrow (2016).

The option pricing demarche is generally performed in two steps. In the first step, the GARCH parameters are estimated under a physical probability measure using a sample of underlying asset returns. In the second step, the risk-neutral transformations of the estimated parameters are used to determine the option price[4].

2.3. The risk-neutral valuation

According to Jarrow (2013), the market is said efficient with regard to a set of information F if and only if there are no asset price bubbles. To detect the bubbles, the traditional economic approach requires building an equilibrium model for the asset price and testing a joint hypothesis of the absence of bubbles. Jarrow (2013) proved that, in the financial literature, the detection of the bubbles is difficult, even impossible, using the standard economic models. This author proved that a stochastic process could be adopted as a new alternative approach to test the existence of the asset price bubbles. He argued that this new approach allows avoiding the dilemma of the joint hypothesis.

To establish the local risk-neutral valuation relationship (LRNVR), one needs to know the mapping between the physical return shock (z_t) and the risk-neutral one (z_t^*). It is defined:

$$m_t = z_t^* - z_t. \tag{4}$$

To obtain the expression of m_t, one assumes that the conditional expectation of the risk-neutral returns, for each period t, is equal to the risk-free interest rate r. It can be written:

$$E_{t-1}^{Q}\left(\exp\left\{r + \mu_t + \sqrt{h_t}\left(z_t^* - m_t\right)\right\}\right) = \exp(r). \tag{5}$$

Under the risk-neutral probability measure, in the conditional volatility dynamic h_t, the term z_{t-1} will be replaced by $z_{t-1}^* - m_{t-1}$. The new return dynamics, under a risk-neutral probability distribution, will be used to simulate future prices of the underlying asset. These adjusted prices will be used to compute the option price using Monte Carlo simulation.

3. RESULTS AND DISCUSSION

This analysis aims to test the ability of these models to adjust the real data. If these models provide a good quality of adjustment, one turns to study their option pricing and hedging performance.

3.1. Analysis of the returns

To estimate the parameters for the used models, the method of maximum likelihood is used. Table 1 shows the parameter estimations of different models. There is used a large sample of data on the CAC 40 index returns between January 1988 and December 2012. The use of a long period of data allows increasing the accuracy of the parameter estimations. For the risk-free interest, an annual rate of 5% was considered.

Table 1. Estimations and properties (daily returns of CAC 40 from January 1988 to December 2012)

Parameter	Estimation	*t*-value	Estimation	*t*-value
	AFTS		NFTS	
λ	3.1729e-01	1.534e+00	3.3078e-03	7.000e+00
θ	2.9538e-03	2.972e-01	9.5099e-03	1.759e+00
ω	1.6805e-13	6.219e-07	3.2314e-06	7.632e+00
β	8.8200e-01	9.115e+02	8.7185e-01	9.484e+02
α	5.2744e-06	8.628e+00	6.3686e-02	1.070e+01
δ	1.2899e+02	9.799e+00	8.7053e-01	1.013e+01
Log*L*	18844.11		18932.40	
Persistence	0.9698		0.9838	
Empirical Skewness	−0. 3031		−0. 3528	
Empirical Kurtosis	4.9961		5.2796	
JB	1146.33		1499.98	

The positive value of θ confirms the existence of the asset price bubbles. The likelihood criterion promotes the non-affine model compared with the affine model. The non-affine model allows capturing the great persistence of the return volatility better than the affine model. The Jarque-Bera statistics, as well as the shape parameters (Skewness and Kurtosis) of the error distribution z_t, show that normal law cannot approximate these errors. The positive value of the parameter δ implies that the underlying return and its variance are negatively correlated. The return distribution, therefore, agrees on a negative asymmetry.

4 This methodology was adopted by several authors, such as Amin and Engle (1993), Härdle and Hafner (2000), and Christoffersen and Jacobs (2004).

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Table 2. Goodness-of-fit of the TFS-GARCH models

Statistic	AFTS	NFTS
S_n	1.209302 (p-value = 0.0166)	0.880497 (p-value = 0.0275)
T_n	2.066516 (p-value = 0.0106)	1.878010 (p-value = 0.0288)

To price options when the presence of bubbles characterizes the market, Jarrow (2013) proposes, first of all, to test the goodness-of-fit of the considered pricing model. If it is valid, it can be used to price the options and hedge the optional portfolios. For the GARCH models, the two more frequently used goodness-of-fit tests, to determine if the empirical distribution corresponds to the hypothetical probability distribution, are Kolmogorov-Smirnov and Cramer-von Mises tests. Ghoudi and Rémillard (2014) defined the empirical procedure that can calculate the statistics of Kolmogorov-Smirnov and Cramer-von Mises. By applying this procedure, it is obtained:

It is clear from the previous table that all models allow a good fit for the CAC 40 index return distribution. However, the p-value[5] is less than 3%, this allows to effectively approximate the error distribution by the standard normal law. Eventually, the requirement to use the above models in the option pricing methodology is satisfied.

3.2. Option pricing results

To evaluate the empirical performance of the option pricing models discussed above, the risk-neutral transformation is applied to price options using the estimation results in Table 1. Thereafter, the calculated prices will be compared to real data on the options with similar characteristics to judge the accuracy of different approaches. To develop this application, there are used observed option prices during the period from January 2010 to July 2013. The data concern European options on CAC 40 index[6]. Daily data on the option prices observed at the end of the trading day were collected. Hereinafter, only the data observed every Wednesday at the end of the day are used[7]. If Wednesday is a holiday, the observation of the nearest day is used. The sample is limited to options of maturity between 10 days and 6 months. Also, as it has been done by Hull and White (2017), the optional contracts whose delta parameter of Black and Scholes is lower than 0.05 or higher than 0.95 (for call options)[8] are excluded.

To examine the empirical performance of different models, a two-step approach will be followed. The first step consists of evaluating the in-sample performance, and the second step treats the out-of-sample performance. However, the data sample on options will, therefore, be divided into two sub-samples. The sub-sample **A** comprises the option prices observed between January 2010 and December 2012. It will be used exclusively to carry out the in-sample analysis. To carry out the out-of-sample analysis, there are used the data of the sub-sample **B** covering the period from January 2013 to July 2013. To study the performance of different approaches, the obtained results are compared with those provided by the reference models of Black and Scholes (1973) and Heston and Nandi (2000).

In this part of the analysis, the risk-neutral transformations of the likelihood estimations are used, obtained under the physical probability measure in Table 1, to determine the option values. To ascertain the stability of the obtained results, the relative pricing errors (RPE) are calculated as follows:

$$RPE = \frac{C_t - \hat{C}_t\left(\omega, \beta, \alpha, \delta^*\right)}{C_t}, \tag{9}$$

where C_t is the market option price observed at time t, $\hat{C}_t$ is the price calculated at the same time for the option having the same characteristics, using the risk-neutral transformation of the estimated parameters. Each year, one does the same calculation and one sees if the conclusions remain stable from one period to another. The following

5 The p-value corresponds to the probability of rejecting the null hypothesis of normality.

6 There is used the settlement price, taking into account the dividends, obtained at the end of day.

7 This type of filtering was used by several other studies like, for instance, Heston and Nandi (2000) and Christoffersen and Jacobs (2004).

8 For the put options, those whose delta of Black and Scholes is less than –0.95 or higher than –0.05 are excluded.

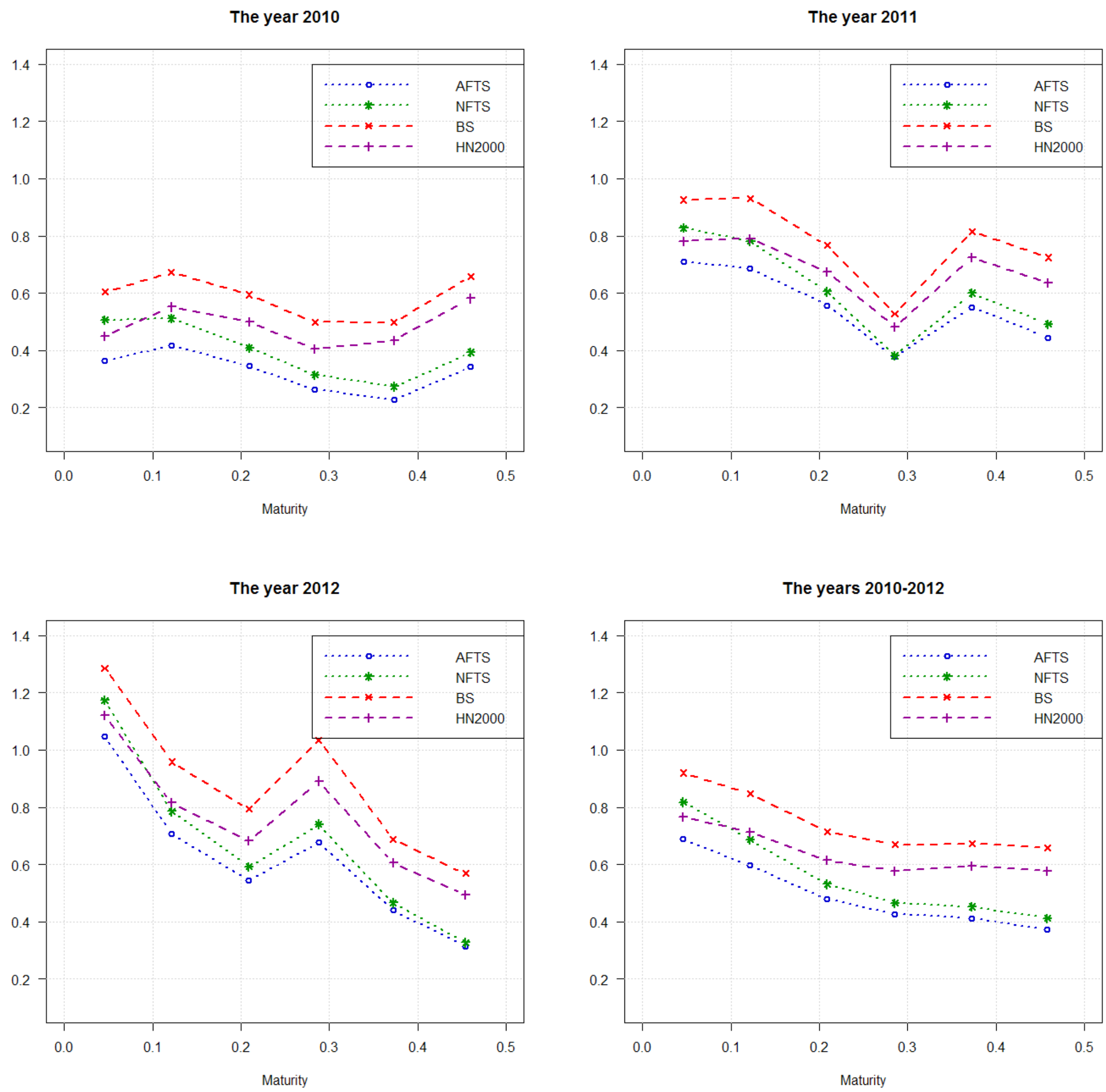

Figure 3. In-sample average absolute errors as a function of maturity

figure summarizes the average pricing error as function of the maturity for different period.

It is clear from Figure 3 that the AFTS model provides the best approximation for the option price. The reference formulae of Black and Scholes (1973) and Heston and Nandi (2000) are also dominated by the NFTS model, with superiority of Heston and Nandi (2000) compared with Black and Scholes (1973). The figure shows that the pricing errors tend to decline when maturity increases. The conclusions are stable, whatever the considered periods. The pricing errors as a function of the moneyness are represented in Figure 4.

The obtained conclusions are the same as those obtained for the evolution of the pricing errors with maturity. The new approach developed under the affine GARCH model (AFTS) is always the best. Also, the NFTS dominates the reference models of Black and Scholes (1973) and Heston and Nandi (2000).

The important test, concerning a pricing model, is its out-of-sample performance. It examines the accuracy of a pricing approach to forecast the future value of an option. One needs evaluating the difference between the forecasted and observed values of an option, having given maturity and strike price. In this context, the

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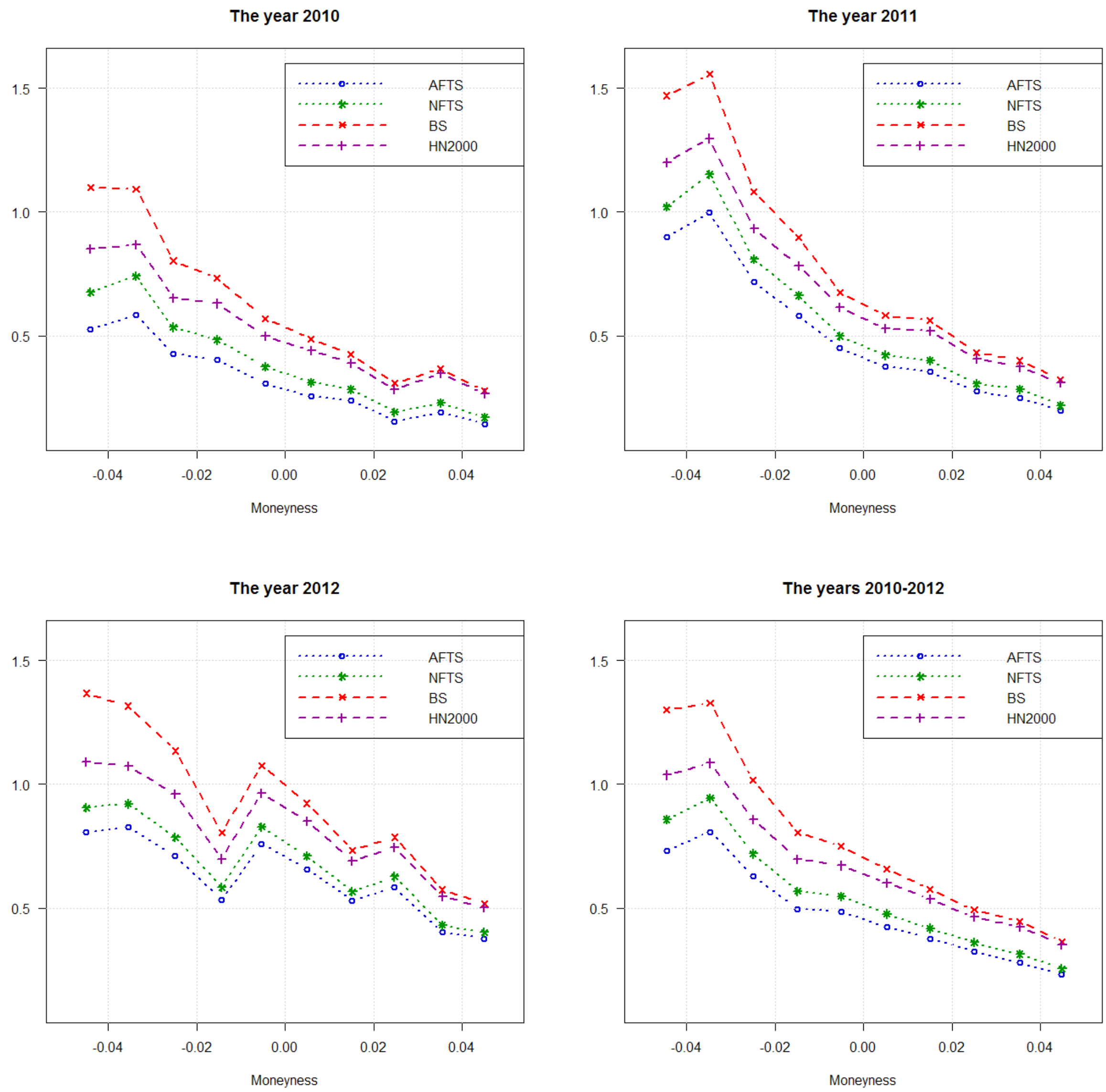

Figure 4. In-sample average absolute errors as a function of moneyness

average relative forecasted error as a function of maturity and moneyness for European call options is calculated. The studied period covers the first eight months of the year 2013. It is the sub-sample **B** described above. The following table summarizes the average relative forecasted errors for different pricing models as a function of maturity.

The AFTS model provides the most accurate option price for all maturities. The bad forecast is given by the model of Black and Scholes (1973). It is also important to note that all models overestimate the European call option value. The numbers in brackets refer to the standard deviations of the forecast errors. They allow concluding on the fluctuation of the errors around their average values.

Table 3. Out-of-sample forecast errors as a function of the maturity

Model	$\tau < 1$ month	1-2 months	2-3 months	3-6 months	All
AFTS	1.124 (1.45)	1.055 (1.00)	0.927 (0.67)	0.881 (0.55)	0.893 (0.80)
NFTS	1.352 (1.73)	1.188 (1.13)	1.014 (0.72)	0.940 (0.57)	0.988 (0.92)
BS	1.544 (2.04)	1.489 (1.44)	1.345 (0.96)	1.327 (0.77)	1.359 (1.12)
HN2000	1.237 (1.56)	1.247 (1.13)	1.163 (0.79)	1.181 (0.66)	1.186 (0.88)

The errors from the AFTS are more stable than those given the other pricing models. The analysis of the out-of-sample performance can be continued by calculating the forecast errors as a function of moneyness. Considering the five categories of moneyness, defined above, the following table is obtained:

Table 4. Out-of-sample forecast errors as a function of moneyness

Model	DOTM	OTM	ATM	ITM	DITM
AFTS	1.774 (1.28)	1.288 (0.93)	0.765 (0.34)	0.505 (0.23)	0.401 (0.20)
NFTS	1.981 (1.55)	1.430 (1.08)	0.843 (0.37)	0.554 (0.24)	0.440 (0.21)
BS	2.795 (1.83)	1.936 (1.19)	1.139 (0.42)	0.777 (0.35)	0.627 (0.33)
HN2000	2.266 (1.34)	1.649 (0.95)	1.028 (0.39)	0.727 (0.33)	0.601 (0.31)

Table 4 shows that the AFTS model provides the lowest forecast errors for all categories of moneyness. It is, therefore, the best pricing approach. The other approach proposed in this work (NFTS) is better than the reference models. When the moneyness increases, the forecast quality is improving for all models.

3.3. Option hedging results

To complete the empirical study, one is interested in the performance of different approaches to hedge the optional portfolios. To carry out this comparison, the methodology of Duan, Ritchken, and Sun (2007) is used. These authors provided the first tests to hedge options under GARCH. One considers a call option that will be hedged during n successive days. At the date t, the hedging error during the n days is given by:

$$\pi_t = \left(C_{t+n} - C_t\right) - \sum_{i=1}^{n} \Delta_{t+i-1}\left(S_{t+i} - S_{t+i-1}\right) - \\ - \sum_{i=1}^{n} r\left(C_t - \Delta_{t+i-1} S_{t+i-1}\right), \tag{10}$$

where Δ_t is the delta hedging ratio given by the pricing model at time t. According to Hull and White (2017), the delta parameter, calculated in the normal way, does not minimize the variance of changes in the optional portfolio value. This is due to the non-null correlation between the underlying asset returns and their variances. The delta parameter, corresponding to a minimum variance, can be given by the following relation:

$$\Delta_{MV} = \Delta_{BS} + \Lambda_{BS} \frac{\partial E\left(\sigma_{imp}\right)}{\partial S_t}, \tag{11}$$

where $E\left(\sigma_{imp}\right)$ is the expected implied volatility of the underlying asset, Δ_{BS} and Λ_{BS} are, respectively, the delta and vega parameters of Black and Scholes.

In the following, one will approximate $E\left(\sigma_{imp}\right)$ by $\bar{h}_t^{1/2}$, where:

$$\bar{h}_t = \frac{1}{\tau} \sum_{k=1}^{\tau} E_t\left(h_{t+k}\right) = \\ = \bar{h} + \frac{1}{\tau}\left(\frac{1-p^{\tau}}{1-p}\right)\left(h_{t+1} - \bar{h}\right), \tag{12}$$

where $\tau = T - t$, p and $\bar{h}$ are, respectively, the volatility persistence and the corresponding unconditional volatility.

Δ_{MV} depends on an element taking account of the underlying asset price variation and of another element taking account of the changes in the VTS. This element indicates the variation of the average expected volatility caused by the variation of the underlying asset price. However, for the GARCH models, the variations of the volatility subsequent to the variations of the underlying asset price are taken into account by second-order derivative. Therefore, the first-order derivative, $\partial\sqrt{h_t} / \partial S_t$, is null, and thereafter it is:

$$\Delta_{MV} \cong \Delta_{t,S}^{BS}\left(\tau, S_t, \sqrt{\bar{h}_t}\right). \tag{13}$$

To obtain the delta value, in the FTS-GARCH framework, just replace in the expression of Δ^{BS} the constant volatility by $\sqrt{h_t}$ corresponding to the FTS-GARCH model.

One tests the performance of the AFTS and NFTS models with regard to standard models such as those of Black and Scholes (1973) and Heston and Nandi (2000). The results, represented by the following figure, are obtained using real data on CAC 40 index options. The hedging errors as a function of moneyness for different categories of maturity are represented in Figure 5.

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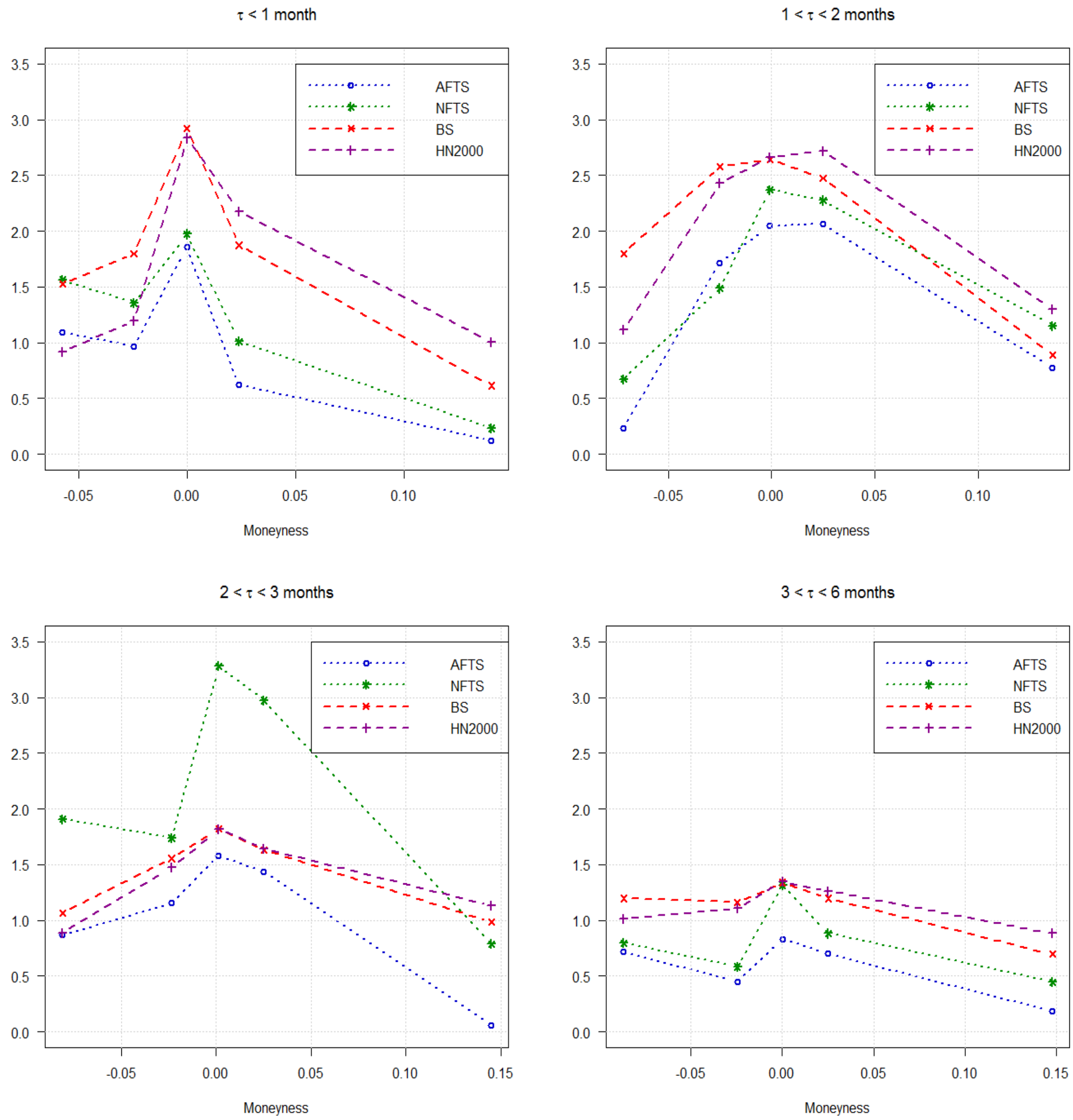

Figure 5. Hedging absolute errors as a function of moneyness

The AFTS model proposed in this article dominates the other approaches. This conclusion allows us to confirm the results obtained by the study of the pricing performance. One can retain the AFTS model as the best approach to price and hedge European call options.

CONCLUSION

The standard option pricing approaches stipulate the hypothesis of market efficiency. This assumption can be violated at certain times. However, if the existence of speculative bubbles characterizes the market, it is therefore inefficient. In this situation, the standard option pricing approaches will no longer be applied, and they are biased. To solve this problem, one must find a stochastic process able to describe the underlying return dynamic in the presence of bubbles properly.

In this work, the FTS-GARCH model of Corsi and Sornette (2014) was used that takes account of positive feedback of the underlying asset past prices on the current values of the corresponding returns. This model allows combining the dynamics of the bubbles with the standards of the financial industry to describe the volatility clustering phenomenon and evaluate the corresponding risk. To apply the new model in option pricing framework, it is, first of all, necessary to test its goodness-of-fit, to know whether or not the empirical distribution corresponds to the considered hypothetical one. In this respect, the two proposed tests are those of Kolmogorov-Smirnov and Cramer-von Mises. They proved that the proposed processes allow a good adjustment of the empirical distribution of returns.

The results obtained following the goodness-of-fit tests prove that the two pricing models, discussed in this article, can be used to calculate the option prices and manage the optional portfolio risks. Using real data on CAC 40 index, the performance of different models as a function of maturity and moneyness was studied. The in-sample analysis, interested in the stability of the pricing models across time, showed that the new approach (AFTS), developed under the affine GARCH process, is the most accurate. For all tested models, the pricing errors decrease with the moneyness, but they have not a stable pace with maturity. The in-sample analysis shows that the reference model of Black and Scholes is the least accurate.

The study of the out-of-sample pricing performance, which aims to evaluate the forecasting ability of different pricing approaches, confirmed the results of the in-sample analysis. The best forecasts of the European call option price are obtained using the AFTS process.

For the optional portfolio hedging, a static hedging strategy was tested. According to this strategy, always the best hedging approach is the AFTS model.

The above conclusions are limited to the studied empirical data and period. One can, therefore, apply the proposed approaches to other types of data and for different periods. It is also possible to establish a comparison between the proposed models and the stochastic volatility processes. Another relevant issue concerns the discussion of the relation between the market efficiency and the option pricing methodology in the stochastic volatility framework and for other types of options (the American options, for example).

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