"The theoretical surrender value in life insurance"

NUMBER OF REFERENCES	NUMBER OF FIGURES	NUMBER OF TABLES
0	0	0

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Abstract

In the context of the stochastic models for the management of life insurance portfolio, the authors explore, with simulation approach, the effects induced by the application of a particular method of calculation of the surrender value.

In the life insurance, the policyholder position is, at any moment, quantified by the mathematical reserve. In case the reserve amount results are positive, the insurance company can allow the contract surrender, consisting in an amount payment, called surrender value, commensurate with the mathematical reserve.

Generally, the insurance company enforces some restrictions in the surrender value determination, in order to avoid, first of all, that an amount is disbursed to the policyholder while, on the contrary, he results to be indebted to the Company. In this paper the authors will consider a surrender value calculation method based precisely on the profit recovery concept which shall be supplied by the contract in case it remains in the portfolio. Additionally, the authors shall analyze, by simulation approach, the effects caused by the enforcement of the surrender value calculation concept on a life portfolio profitability, and on the penalties extent enforced to the policyholders which cancel from the contract.

Keywords: surrender value, life insurance, internal risk model, stochastic simulation.

Introduction

In the life insurance relationship, the policyholder position is, at any moment, quantified by the mathematical reserve. In case the reserve amount results are positive, the insurance company can allow the contract surrender, consisting in an amount payment, called surrender value, commensurate with the mathematical reserve.

Generally speaking, the insurance company enforces some restrictions in the surrender value determination, in order to avoid, first of all, an amount is disbursed to the policyholder while, on the contrary, he results to be indebted to the company (because of acquisition expenses prepaid by the agents). For this purpose, the company acknowledges the surrender value to the policyholder, provided that the withdrawal doesn't take place during the contract first years during which, as well-known, the mathematical provision Zillmerised can be negative. A second restriction is usually adopted in order to reduce those contracts anti-selection effects that often cause mortality losses. For this reason, within the surrender value calculation formula is, generally, included a reduction factor which is the lowest in the whole life insurance, while it takes high values for those contracts for which the accident happening isn't sure and, therefore, a withdrawal can most likely hide an anti-selection phenomenon. Other surrender value restrictions are due to the penalty enforcement, also for the reason of the insurance company requirement to recover those future profits whose progressive formation is interrupted by the surrender.

With reference to this last requirement, we will consider a surrender value calculation method based precisely on the profit recovery concept which shall be supplied by the contract, in case it remains in the portfolio. Additionally, following the methodological formulation considered by Pitacco (1992), adapted to a stochastic model emblematic of a life company handling, we shall analyze, by simulation approach, the effects caused by the enforcement of the surrender value calculation concept on a life portfolio profitability, and on the penalties extent enforced to the policyholders which cancel from the contract. For such purposes, Section 1 reconstructs the insurance profit formation process for a life portfolio. Section 2 discusses the surrender value calculation model. Section 3 describes the policies generation features belonging to the Company insurance portfolio and assigns the standard parameters values utilized for the simulations. Section 4 comments the quantifications results (reported in Appendix) regarding the yearly profits time-discounted and the theoretical deductions performed with the surrender granting.

1. The insurance profit

The insurance profit of an insurance company operative in the life business originates from the handling of policies forming the portfolio, and it is originated by the deviation between predictions assumed in the premiums calculation and the real experience.

In defining the insurance profit regarding the portfolio, at this stage, we'll adopt the so-called generations approach already used by several writers. Such method consists in considering the

company portfolio (actual and future) like the amount formed by various generations, each of them formed by policies having "similar" features, with the exception of the amounts insured. Each generation is examined separately, so that the whole insurance portfolio result is determined year by year like the amount of the results attained for each generation.

It is assumed that the generation considered is formed by policies mutually independent, all of them having in common the following features: the insurance contract type, the back-term, the policyholder age at the stipulation date, the contract term, the premiums payment periods.

Omitting, to simplify, the reference indexes of each generation, a general expression of the generation insurance profit achieved during the financial period $(t, t + 1]$ can be expressed as:

$$\tilde{A}_{t+1} = \left(\tilde{W}_t' + \tilde{B}_{t+1} - \tilde{E}_{t+1} - \tilde{S}_{t+1}\right) \times \left(1 + \tilde{j}_{t+1}\right) - \left(\tilde{X}_{t+1} + \tilde{W}_{t+1}\right), \quad (1)$$

where:

$\tilde{W}_t' = v_t^b \tilde{w}_t \tilde{Q}_t$ is the full reserve revalued at the time t, achieved as product among the full reserve rate v_t^b, relevant to the generation insurance contract, the insured capitals total amount, $\tilde{w}_t$, and the revaluation index (of the reserves, capitals and premiums) at the time t, $\tilde{Q}_t = \prod_{h=1}^{t}\left(1 + \tilde{j}_h^v\right)$.

$\tilde{B}_{t+1} = b_{t+1}\left(\tilde{w}_t - \tilde{s}_{t+1}\right)\tilde{Q}_t$ is the expense-loaded premium paid by the policyholders at beginning of period (delays aren't predicted on the payment of the same and, therefore, the credits towards policyholders for premiums aren't considered); the amount is attained enforcing the premium rate b_{t+1}, relevant to the generation insurance contract, to the total capitals insured $\tilde{w}_t$ net of the capitals relevant to policies self-deleted (at the year beginning) for elapses $\tilde{s}_{t+1}$.

$$\tilde{E}_{t+1} = \left(\left(\alpha_{t+1} + \beta_{t+1}\right)\left(\tilde{w}_t - \tilde{s}_{t+1}\right)b_{t+1} + \tilde{\gamma}_{t+1}\tilde{w}_t\right) \cdot \tilde{Q}_t$$

is the insurance expenses amount, assuming that all relevant cash transactions take place at the financial period beginning. With reference to expenses "really" incurred by the company, reference is made to the second order technical bases. In particular the following hypotheses are assumed: 1) the commissions paid by the agents, recurring only in the policy first year are set equal to a deterministic rate α_{t+1} of the tariff premiums amount collected during the first year; 2) the collection expenses

incurred by the company every year when the premium payment, are equal to a deterministic rate β_{t+1} of the tariff premiums paid within the year; 3) the overhead and administration expenses (including also the fixed costs), are equal to a rate $\tilde{\gamma}_{t+1}$ of the insured capitals total amount in force at the considered financial period beginning. The rate $\tilde{\gamma}_{t+1}$ is contingent, because it was considered suitable to correlate the overhead expenses to the inflation index.

$\tilde{S}_{t+1}$ is the surrender values amount paid by the Company at the year beginning.

$\tilde{j}_{t+1}$ is the contingent interest rate attained by the technical resources (premiums and reserves) investments during the financial period.

$\tilde{X}_{t+1} = \tilde{x}_{t+1}\tilde{Q}_t$ is the accidents amount happened during the financial period, all of them paid as a possibility at the year end (therefore, there will not be available any reserve for amounts to be paid); it depends, such as for the reserves rates v_t^b and for the premiums b_{t+1}, from the generation insurance contract.

$\tilde{W}_{t+1} = v_{t+1}^b \tilde{w}_{t+1}\tilde{Q}_t$ is the full mathematical reserve final amount, in which the reserve revaluations acknowledged to the policyholders at the considered financial period end are included.

According to the Homans formula extended to a generation of policies, the yearly insurance profit can be subdivided into five components:

$$\tilde{A}_{t+1} = \tilde{A}_{t+1}^{(1)} + \tilde{A}_{t+1}^{(2)} + \tilde{A}_{t+1}^{(3)} + \tilde{A}_{t+1}^{(4)} + \tilde{A}_{t+1}^{(5)},$$

which derive:

♦ from the deviation among the payments for "predicted" accidents in comparison to the "real" ones (mortality profit):

$$\tilde{A}_{t+1}^{(1)} = \left(\tilde{W}_t' + \tilde{B}_{t+1} - \tilde{E}_{t+1}^* - \left(v_t^b - \gamma^*\right) \times \tilde{s}_{t+1}\right) \times \\ \times \left(1 + j^*\right) - \left(\tilde{X}_{t+1} + \tilde{W}_{t+1}\right); \quad (2)$$

having indicated by j^* the technical and financial basis of the first order and by $\tilde{E}_{t+1}^* = \left(\left(\alpha^* + \beta^*\right) \times b_{t+1} \times \left(\tilde{w}_t - \tilde{s}_{t+1}\right) + \gamma^* \times \tilde{w}_t\right) \times \tilde{Q}_t$, the expenses of the policies still in force at the time $t + 1$, according to expenses rates predicted in tariff for acquisition, α^*, for collection, β^*, and for handling, γ^*;

♦ from the full reserves surplus object of lapse in comparison to the working expenses anyway recurred, and to the surrender values amount

paid by the company (deletion profit for cancellations/surrenders):

$$\tilde{A}_{t+1}^{(2)} = \left(v_t^b - \gamma^* - \rho(t)\right) \times \left(1 + j^*\right) \times \tilde{s}_{t+1} \times \tilde{Q}_t; \quad (3)$$

being $\rho(t)$ the surrender price allowed in t, for insured amount unit;

♦ from the differencial among assumed and recurred expenses for the sole portion due to the expense rates deviation (expenses loading profit):

$$\tilde{A}_{t+1}^{(3)} = \left(1 + j^*\right) \times \left(\tilde{E}_{t+1}^* - \tilde{E}_{t+1}\right); \quad (4)$$

♦ from the differential between the yield realized and the one predicted in tariff enforced to the insurance investments volume (interest profit):

$$\tilde{A}_{t+1}^{(4)} = \left(\tilde{j}_{t+1} - j^*\right) \times \left(\tilde{W}_t' + \tilde{B}_{t+1} - \tilde{E}_{t+1}^* - \tilde{S}_{t+1}\right); \quad (5)$$

♦ from the profit remaining from the interaction between expenses loading and interest margin:

$$\tilde{A}_{t+1}^{(5)} = \left(\tilde{j}_{t+1} - j^*\right) \times \left(\tilde{E}_{t+1}^* - \tilde{E}_{t+1}\right). \quad (6)$$

The total amounts $\tilde{A}_{t+1}$ form the "installments" of the annuity from the generation. Having selected a suitable evaluation rate j, their actual value in t ($t = 0,1,...,n-1$), for a generation of contracts with term n years, is given by:

$$\tilde{U}_t = \sum_{h=t+1}^{n} \tilde{A}_h \times \left(1 + j\right)^{-(h-t)}. \quad (7)$$

A synthetic evaluation of the generation perspective profitability is supplied, first of all, by the predicted value $\bar{U}_t = E(\tilde{U}_t)$, which we can define "generation residual value predicted", but there appear to be expressive also other values typical of the distribution of $\tilde{U}_t$, defined in relation with the percent values, $U_{t;p}$, for which is valid $Prob\{\tilde{U}_t \le U_{t;p}\} = p$. In particular, the "generation probable residual value" $U_{t;0.5}$ fulfils the relation: $Prob\{\tilde{U}_t \le U_{t;0.5}\} = 0.5$.

2. The decision-making model for the surrender value

For any back-term t ($t = 0,1,...,n-1$), the distribution of probability of $\tilde{U}_t$ can be seen as function of different variables, both "exogenous" and "endogenous". For our purposes, it will be interesting to analyze the distribution of $\tilde{U}_t$ in function of the surrender value and of the deletion probabilities for lapse/surrender. Therefore, the decision-making variable is given here by the surrender value function.

The choice of the surrender value functional model requires the setting of a target, consistent with the insurance company politics. Such politics must conform themselves both with commercial requirements, and with the insurance handling profitability requirements. With reference to the profitability targets, it is clear that by means of the surrender value setting, the insurance company shall have influence on the insurance yearly profit associated with the contracts surrender. From the formula of $\tilde{A}_{t+1}^{(2)}$, it results that if the insurer should pay, as surrender value, the whole full reserve, from the contract surrendered, it shall not achieve any further profit. Then, a possible profits recovery (total or partial) requires the deduction of the full reserves object of lapse in function of the profits which could derive from the contracts, in case they, on the contrary, should remain in the portfolio. Therefore, the surrender values amount should be calculated as a difference between the full reserve and a profits rate that the contract surrendered could have generated.

For each period t ($t = 0,1,...,n-1$), a "prediction" of the surrender values total amount, calculated according to the approach based on the "profits" recovery concept, is supplied by the:

$$P(t;r) = max\left\{0; \bar{W}_t'^{(R)} - r \times U_t^{(R)}\right\}; \quad 0 \le r \le 1, \quad (8)$$

where: $\bar{W}_t'^{(R)}$ is the predicted value of the contracts surrendered full reserves, $U_t^{(R)}$ is the correspondent "value" (non-negative) of the profits time-discounted distribution that is intended to be recovered (totally or partially), according to the recovery rate r.

It is to be considered the surrender value reset in correspondence with the periods for which is $r \cdot U_t^{(R)} > \bar{W}_t'^{(R)}$, if, on the one hand, it prejudice the profits recovery possibility, on the other hand, this is in accordance with the insurance practice that, in particular, envisages a surrender value granting only upon a predetermined back-term elapsed time ("shortage period").

From the formula 8, it is deduced that the penalty enforced amount is given by the deduction enforced on the credit (full reserve) of the policy holders who lapse the contract; in relative terms, the deduction $D(t;r)$ is equal to:

$$D(t;r) = min\left\{1; \frac{r \times U_t^{(R)}}{\bar{W}_t'^{(R)}}\right\}; \quad 0 \le r \le 1. \quad (9)$$

Like "value" of the annual profits time-discounted distribution to be recovered from the withdrawing policyholders, $U_t^{(R)}$, it will be possible to adopt the expected value $\bar{U}_t^{(R)}$ or a particular percentage $U_{t;p}^{(R)}$, provided that it is naturally of positive sign.

3. The insurance portfolio

In order to examine the results of the surrender value calculation method based on the profits recovery concept, four portfolios formed by only one generation of policies, respectively, of endowment, of pure endowment, of deferred life annuity and of whole life insurance have been considered. The generations' features and the standard values assigned to the stochastic model parameters representative of the company handling are the following:

A) *generation features:*

♦ age at the contract stipulation $x = 35$ years;
♦ contract term $n = 30$ years, for the endowment and the pure endowment, $n = 35$ years, for the life annuity;
♦ yearly premiums payment period term $m = 30$ years, equal to the deferred period in the life annuity case.

B) *first order technical bases:*

♦ interest technical rate $j^* = 2\%$;
♦ mortality rates q^*: detected from the Italian RG-48 life table, for the male sex;
♦ for acquisition costs loading: the rate α^* multiplied by the first year tariff premium. In particular, the loading rate is considered equal to 4% for the endowment, the pure endowment and the whole life insurance, and equal to 3% for the life annuity;
♦ premiums collection costs loading: the rate $\beta^* = 5\%$ multiplied by the yearly tariff premiums;
♦ overhead and administration charges loading: the rate γ^*, multiplied every year by the insured capital (or income). In particular, it has been considered a loading rate equal to 0.4%;
♦ for the pure endowment and the whole life insurance, to 0.5%, for the endowment, and to 0.25%, for the deferred life annuity.

C) *generation starting size:* $l_x = 1,000$ units.

D) *insured amounts starting distribution* (expressed in monetary units):

♦ for the pure endowment, endowment and whole life insurance policies: $C = (30, 50, 80, 100, 150)$ with relevant equivalent frequencies

N = (35%, 25%, 20%, 15%, 5%). Therefore, the average capital insured is equal to 61.5 monetary units, while the insured amounts total is of 15340 currency units. The s.q.m., the skewness and the ratio of the insured capitals distribution variation are worth, respectively, 3.26, 0.96 and 0.53;
♦ for the deferred life annuity policies: C = (3, 5, 8, 10, 15) with relevant frequencies N = (35%, 25%, 20%, 15%, 5%). The insured instalments average amount is equal to 6.15 monetary units, while the insured amounts total is of 1534 currency units. The s.q.m., the skewness and the ratio of the insured capital distribution result, respectively, 3.26, 0.96 and 0.53.

E) *time horizon:* $T = 30$ years for the pure endowment and for the endowment policies, $T = 65$ years for the life annuity policies.

F) *stochasticity level:*

♦ the mortality presents a contingent trend. The deaths conditional number for each back-term follows a binomial distribution, with mortality yearly expected rates equivalent to those of the technical basis of second order which envisages a safety loading for mortality equal to 5%;
♦ the contract escapes present a stochastic trend. The withdrawals conditioned number per each back-term follows a binomial distribution, with yearly deletion probability for lapses/surrender given by the [1]

$$e^t = 0.01t \times e^{0.005t^2 - 0.15t}; \qquad (10)$$

♦ the yearly inflation rate presents a difference in comparison to its own average defined by a markov[2] stochastic process and, therefore:

$$\tilde{i}_{t+1} = \bar{i} + d \times \left(\tilde{i}_t - \bar{i} \right) + c \times \tilde{\varepsilon}_{t+1}. \qquad (11)$$

It is assumed that the inflation average rate is $\bar{i} = 2\%$, while the process "deviation" coefficient compared to the average value is $d = 0.25$, and the disturbance factor coefficient is $c = 0.001$. The process disturbance factors are, like hypothesis, contingent variables i.i.d. (with null average and unitary variance);

♦ following the Wilkie (1984) formulation, the inflation rate is considered the main explanatory factor of the financial yields trends. These last ones are described by a self-regressive stochastic process depending on the inflation itself and on contingent disturbances factors:

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[1] Cfr., Pitacco, E. (1992), p. 150.
[2] Cfr., Pentikainen, T. et al. (1989).

$$\tilde{j}_{t+1} = j_m + a_1 \times (\tilde{j}_t - j_m) + ((a_2 \times \tilde{i}_{t-1} + a_3 \times \tilde{i}_{t-2}) - \bar{i}) + \\ + c_1 \times \tilde{\varepsilon}_{t+1} \qquad (12)$$

where the disturbance factors coincide with the ones generated by the inflation process. The average interest rate is $j_m = 1,5 \times j^*$, while for the process other parameters: $a_1 = 0.25$, $a_2 = a_3 = 0.5$ and $c_1 = 0.001$;

♦ the overhead expenses are indexed, and an expenses safety loading rate $\lambda_j = 10\%$ is adopted;

♦ the profits retrocession to the policyholders, by means of the reserves revaluation at a rate consistent with the following calculation method is considered:

$$\tilde{j}_{t+1}^v = max\left(0; \frac{\eta \times \tilde{j}_{t+1} - j^*}{1 + j^*}\right), \qquad (13)$$

where η is the retrocession coefficient, considered equal to 85%;

♦ the rate chosen for the predicted profits evaluation, j, is considered equal to 8%.

During the analysis, some standard parameters have been modified in order to verify the sensitivity of the penalizations and distributions profiles of the time-discounted profits value.

4. A policies generation analysis

The evaluations have been performed by use of the stochastic simulation technique which allows to generate several alternative scenarios from which to derive immediately a general view of the different processes beneath the stochastic model of a life insurance company handling, in addition to supply information regarding the "sampling" distributions of the processes themselves main features.

The development of the courses simulated has been realized by the precise individual approach which required, for each financial period, the following planning steps:

a) lapses number simulation;
b) pulling out, at random, of a number of insured capitals equal to the lapses numbers among those in force at the financial period beginning considered, and aggregation of the relevant amount in order to generate the total amount of the contracts insured capitals lapsing;
c) insured capitals vector updating by cancellation of a number of policies equal to the ones chosen at random, in the previous step;
d) deaths number simulation;
e) pulling out, at random, of a number of insured capitals equal to the deaths number, among those still in force afterwards the renunciation

cancellations, and relevant amount aggregation in order to generate the total amount of the capitals paid because of death;
f) capitals vector updating by lapse of a number of policies equal to those lapsed because of death;
g) insured capitals vector alteration, according to the annual revaluation rate.

For each of the four portfolios, 10,000 courses simulated for a time horizon having size equivalent to the contract term (plus the possible postponement) have been generated.

The elaborations results, reported in form of graphics and tables in Appendix, can be summarized in this way.

Tables 1A, 1B, 1C and 1D (see Appendixes A-D) report the distributions main features of $\tilde{U}_t$, ($t = 0$, 5, 10, 15, 20, 25), when the escapes by the third parties ensured are excluded. The predicted time-discounted profits are of a level consistent with the saving content of the different insurance contracts considered and with the technical hypotheses considered. Observing the volatility measures values (s.q.m. and the variation coefficient), it is possible to conclude that the stochasticity level considered for the evaluations is not too high. The distribution skewness is almost always positive and close to zero.

Tables 2A, 2B, 2C and 2D (see Appendixes A-D) re-calculate the main features of the v.a. distributions $\tilde{U}_t$, ($t = 0$, 5, 10, 15, 20, 25), in presence of escapes by the policyholders, with yearly probability given by the e_t. The renunciations involve an abatement of the profits predicted and of the relevant s.q.m., while they produce a decrease of the variation coefficients during the first years, and their increase during the contract last years. The decrease of the variation coefficients during the first 10-15 years, where the highest number of surrenders lapses concentrates (despite the sequence of the yearly possibilities of lapses e_t), it can be explained with the lowest decrease of the time-discounted predicted profits in comparison to the relevant s.q.m., as a result of the profits recovered overall measure with the surrender values calculation method. On the contrary, their increase for higher back-term is due, precisely, to higher decrease of the portfolio size (which, like hypothesis, is closed) because of the lapses registered during the first years.

The skewness coefficients continue to assume moderate values; however, if Figures 5A-5D are considered, it can be detected that the surrenders presence (intermittent line) produces, generally, a skewness decrease in the time periods where the continuous curve (of the skewness indexes of the profits distributions $\tilde{U}_t$ in absence of surrenders) is

concave, and an increase during the periods, where the same curve is convex.

Figures 1A, 1B, 1C and 1D (see Appendixes A-D) reveal the percent values trends of the v.a. distributions $\tilde{U}_t$, (t = 0, 5, 10, 15, 20, 25), in absence of cancellations due to surrender. The shape of the percent values sheaf shows that the courses simulated bring a concave trend. Prime moment and median line of the v.a. distribution $\tilde{U}_t$ almost coincide. The bottom and top percent values are almost at the same distance from the average, because the skewness indexes result, as it was detected, frequently close to zero.

Figures 2A, 2B, 2C and 2D (see Appendixes A-D) show, for each generation, the full reserve average value trend and the average time-discounted profit profile of the generation (in absence of surrenders), and of the average time-discounted, so-called "minimum"[3]. Considering the generation contracts complex still in force at subsequent back-terms, t, the intersection of the full predicted reserve curve with the one of the average profits time-discounted, $\overline{U}_t$, in correspondence of the time t = 1, evidences that the surrender value given by the (8) can be granted only for $t \geq 2$. Therefore, assuming as typical value of the profits distribution to be recovered one of the percent values $U_{t;p}$ higher than the average profit value, a "shortage theoretical period" of longer term should be enforced.

The shortage period span for a surrender value grants is affected also by the hypotheses frame which define the analysis standard scenario. Figures 3A, 3B, 3C and 3D (see Appendixes A-D), for example, show that decreasing the evaluation rate, j, the time-discounted value of the yearly predicted profits increases and, consequently, the "shortage theoretical period" extends.

The prevailing saving content of the generations examined insurance contracts, makes the analyses results particularly sensible to the choice of the financial nature parameters. Figures 4A, 4B, 4C and 4D (see Appendixes A-D) describe the deductions trends D (t;1) for different hypotheses regarding the income average rate which characterizes the formula 11, considering like measure of the profits to be recovered, for the generation contracts complex still in force at the subsequent back-terms t, the predicted time-discounted value. As the rate j_m increases (from the standard 3% value up to 6%), it increases in remarkable amount the deductions in relative terms and extends the "shortage theoretical period", reaching 10 years, for the generation of mixed and deferred capital policies, up to 15 years for the deferred annuity generation and up to 18 years for the generation of lifetime policies.

Conclusion

The surrenders problem solution enforces the elaboration of a suitable approach methodology, which could be of remarkable importance, according to the operative scenarios, in the insurance profit formation.

In our paper, we examine the results of the surrender value calculation method based on the profits recovery concept, considering four portfolios formed by only one generation of policies, respectively, of endowment, of pure endowment, of deferred life annuity and of whole life insurance.

The computational model analyzed, for instance, allows to catch various aspects of the insurance portfolio handling and allows a tangible support to the decisions for the surrender conditions definition, whose effects must be evaluated even during the insurance product tariff formation.

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[3] The "minimum" average time-discounted profit corresponds to the predicted profit of a generation without disbursement due to lapse and to numerousness equivalent to the number of contracts which will elapsed exclusively because of death or expiration. Cfr., Pitacco, E. (1992), p. 146.

Appendix A. Analysis of a pure endowment policies generation

Table 1A. Distribution features of $\tilde{U}_t$ evidenced for the standard stochasticity level, excluding contract escapes

t	Average	s.q.m.	s.q.m./average	skew.
0	1885	150	0.079	0.053
5	2442	214	0.087	0.050
10	2887	283	0.098	0.031
15	3104	356	0.115	0.018
20	2916	408	0.140	0.068
25	2026	396	0.195	0.146

Table 2A. Distribution features of $\tilde{U}_t$ evidenced for the standard stochasticity level, with contract escapes

| t	Average	s.q.m.	s.q.m./average	skew.
0	1854	127	0.068	0.048
5	2274	181	0.080	0.038
10	2458	241	0.098	0.061
15	2520	301	0.119	0.078
20	2322	349	0.150	0.044
25	1608	343	0.213	0.123
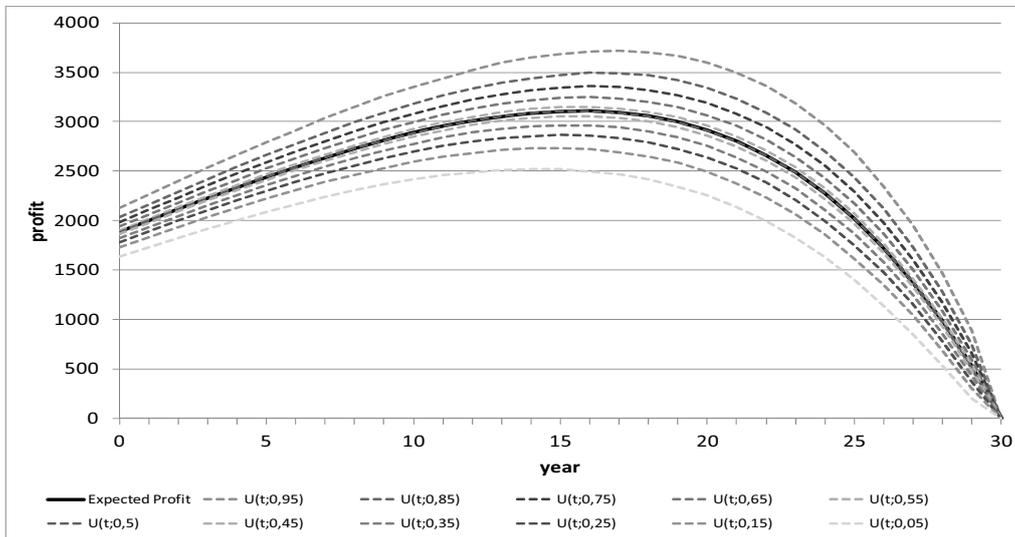
Fig. 1A. Average and percent values of the distributions of $\tilde{U}_t$
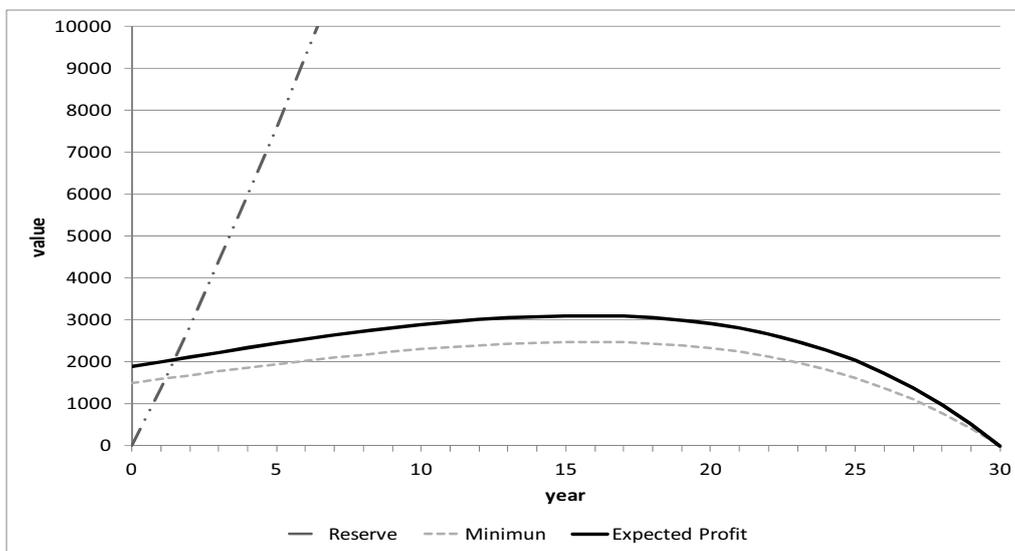

**Fig. 2A. Average values of the distributions of $\tilde{U}_t$ and of the generation full reserve**
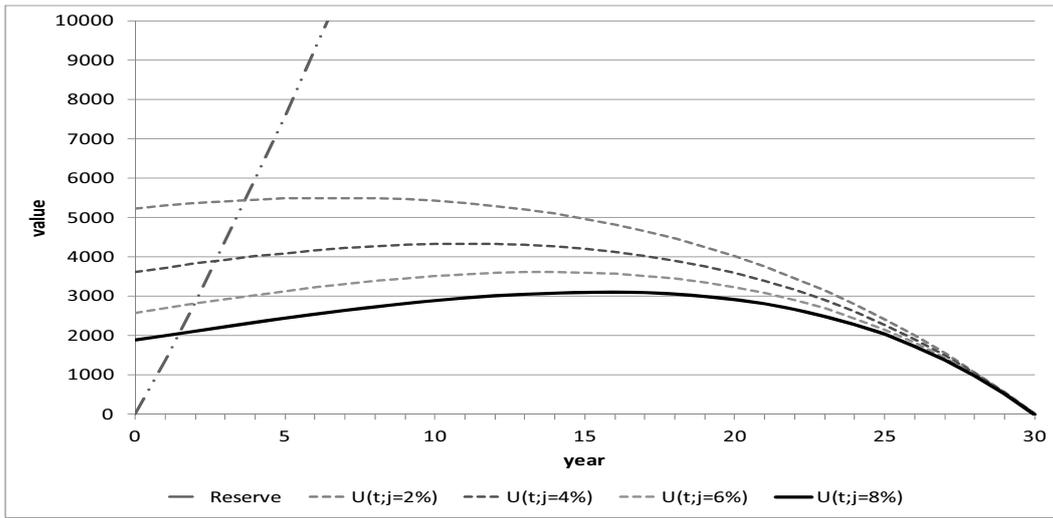
Fig. 3A. Average values of the distributions of $\tilde{U}_t$ for the evaluation rate j different values
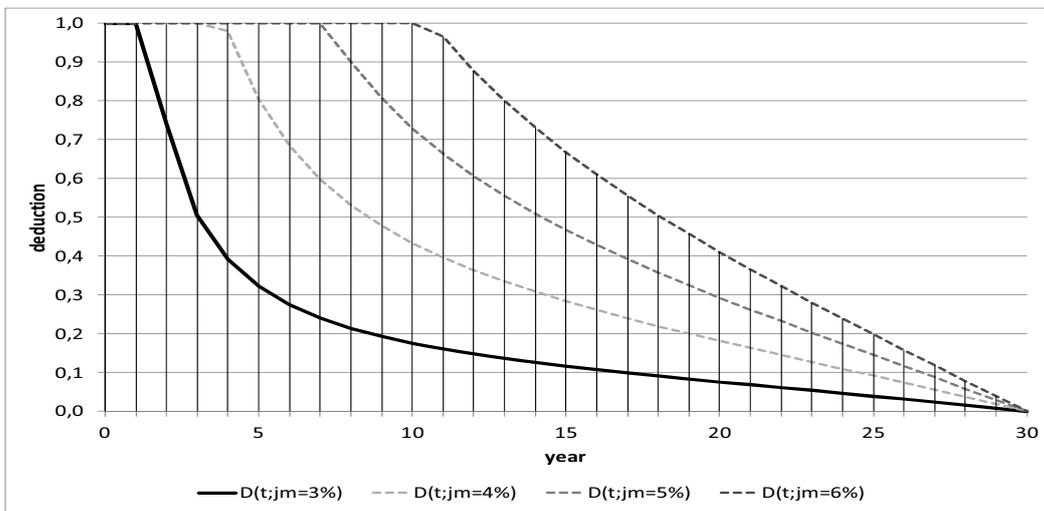
Fig. 4A. Comparative deduction D (t; 1) for different values of the yield average rate, j_m.
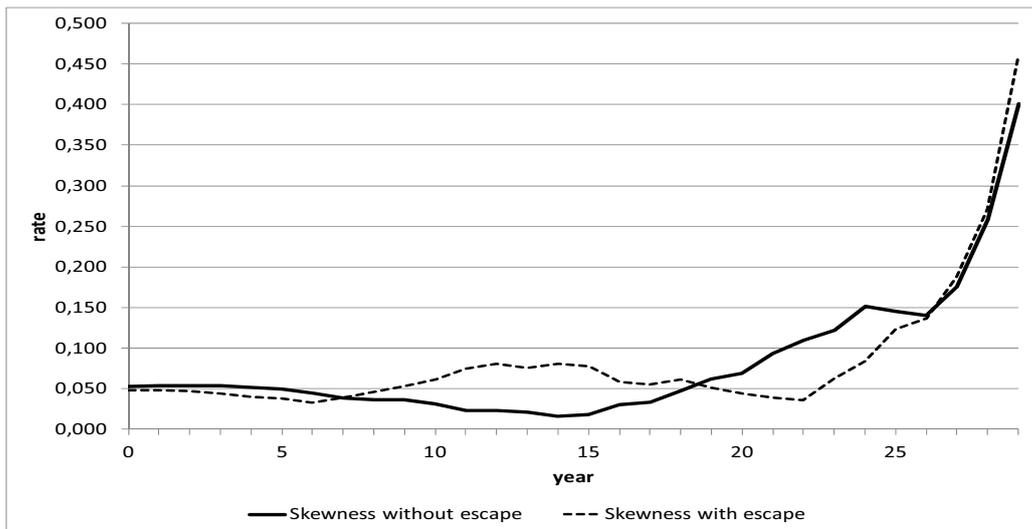
Fig. 5A. Skewness coefficients of the distributions of $\tilde{U}_t$ evaluated in presence and in absence of contract escapes

Appendix B. Analysis of a deferred life annuity policies generation

Table 1B. Distributions features of $\tilde{U}_t$ evidenced for the standard stochasticity level, excluding contract escapes

t	average	s.q.m.	s.q.m./average	skew.
0	3870	269	0.070	0.038
5	5239	387	0.074	0.045
10	6561	527	0.080	0.065
15	7700	687	0.089	0,090
20	8442	863	0.102	0.110
25	8453	1020	0.121	0.092

Table 2B. Distribution features of $\tilde{U}_t$ evidenced for the standard stochasticity level, with contract escapes

t	average	s.q.m.	s.q.m./average	skew.
0	3771	193	0.051	-0.001
5	4560	280	0,061	0,.26
10	4755	379	0.080	0.034
15	5073	490	0.097	0.056
20	5386	616	0.114	0.050
25	5348	739	0.138	0.073
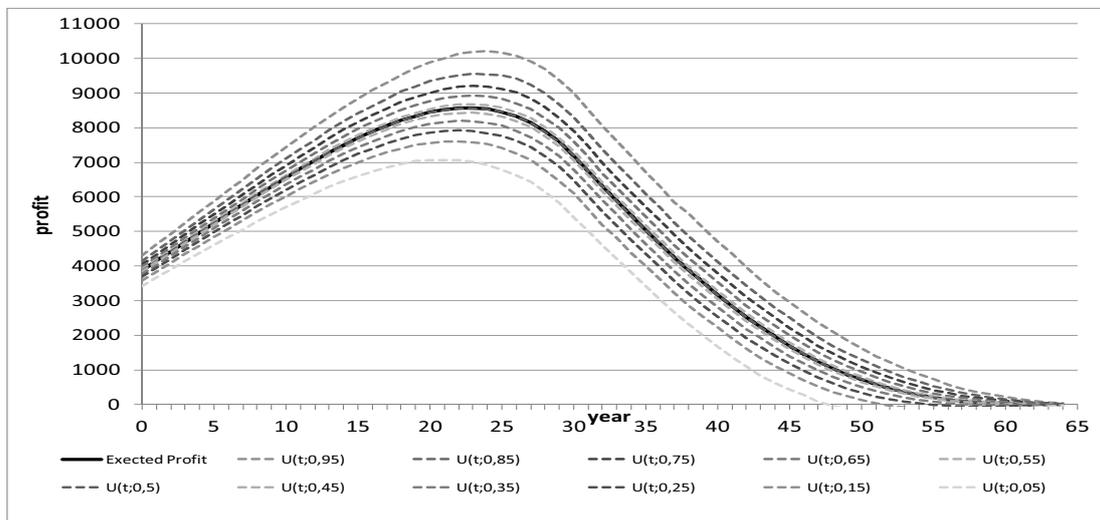
Fig. 1B. Average and percent values of the distributions of $\tilde{U}_t$
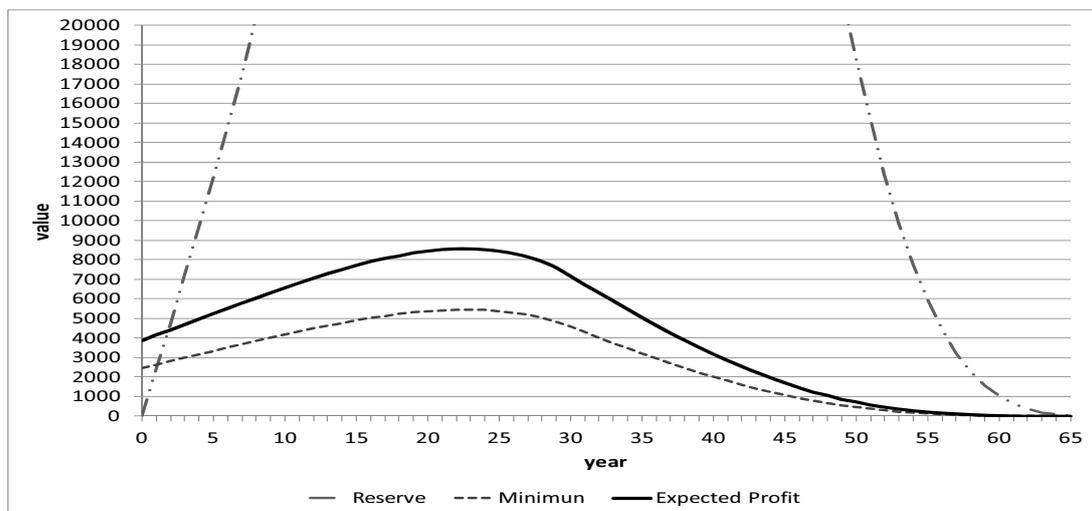
Fig. 2B. Average values of the distributions of $\tilde{U}_t$ and of the generation full reserve
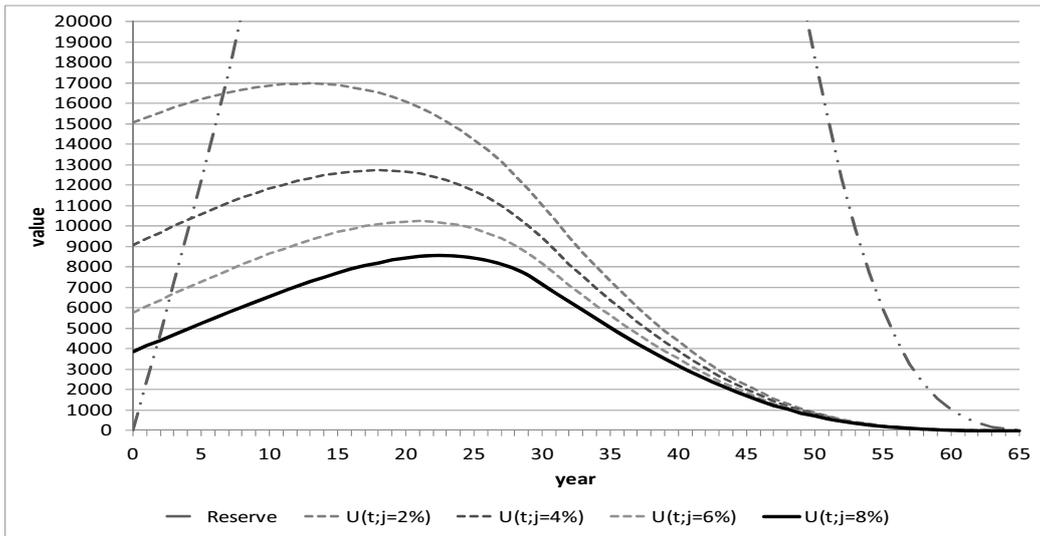
Fig. 3B. Average values of the distributions of $\tilde{U}_t$ for different evaluation rate j values
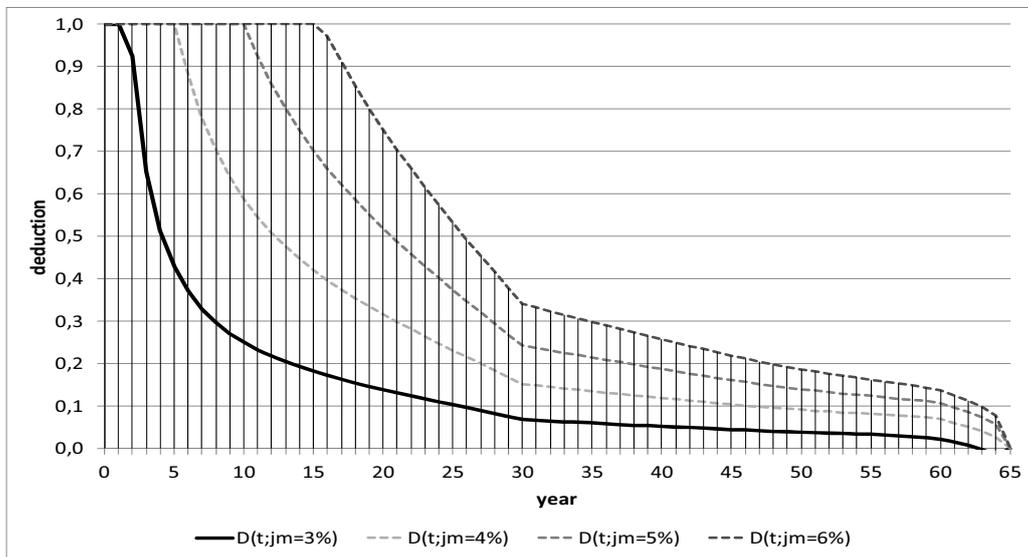

Fig. 4B. Comparative deduction D (t; 1) for different values of the yield average rate, j_{m}.
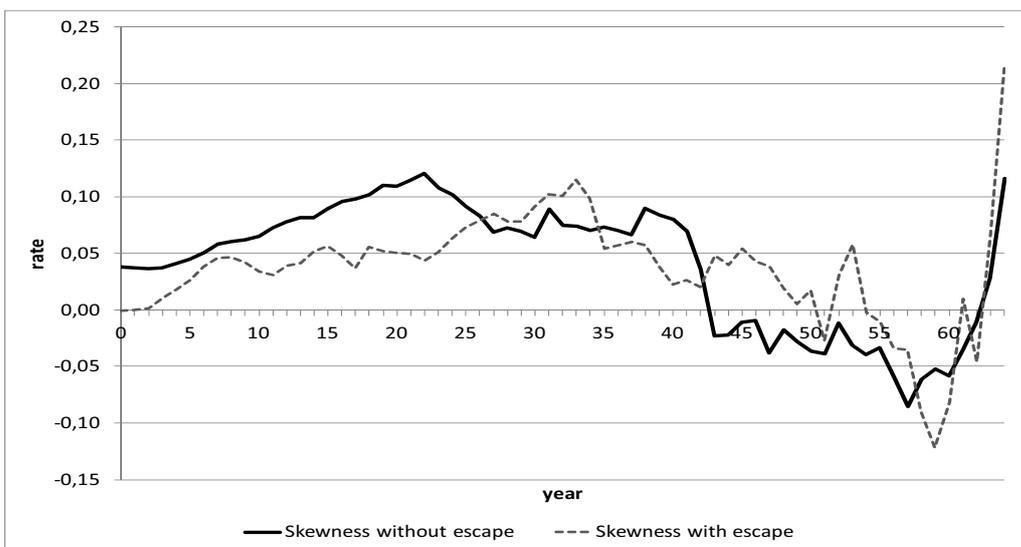
Fig. 5B. Skewness coefficients of the distributions of $\tilde{U}_t$ evaluated in presence and in absence of contracts escapes

Appendix C. Analysis of a endowment policies generation

Table 1C. Distributions features of $\tilde{U}_t$ evidenced for the standard stochasticity level, excluding contract escapes

t	average	s.q.m.	s.q.m./average	skew.
0	1871	203	0.109	-0.089
5	2397	245	0.102	0.001
10	2804	292	0.104	-0.008
15	2982	323	0.108	0.037
20	2763	335	0.121	0.025
25	1893	273	0.144	0.057

Table 2C. Distributions features of $\tilde{U}_t$ evidenced for the standard stochasticity level, with contract escapes

t	average	s.q.m.	s.q.m./average	skew.
0	1842	186	0.101	-0.221
5	2235	214	0.096	-0.115
10	2387	245	0.103	-0.015
15	2421	270	0.112	0.071
20	2207	276	0.125	0.066
25	1506	222	0.147	0.027
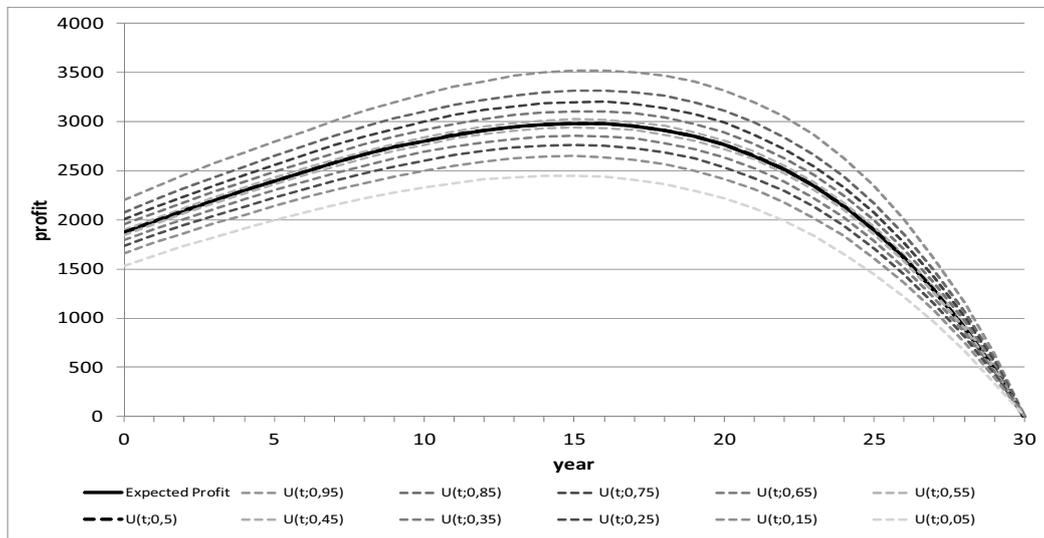
Fig. 1C. Average and percent values of the distributions of $\tilde{U}_t$
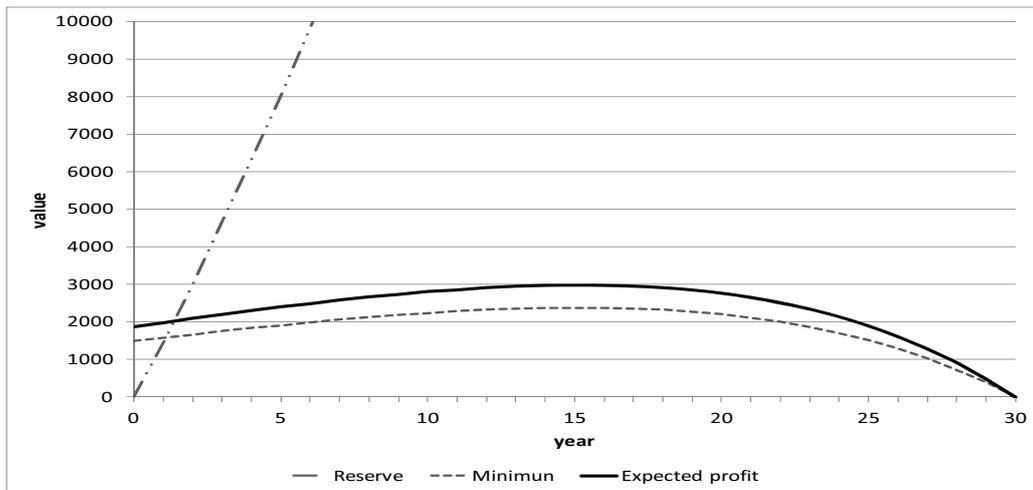
Fig. 2C. Average values of the distributions of $\tilde{U}_t$ and of the generation full reserve
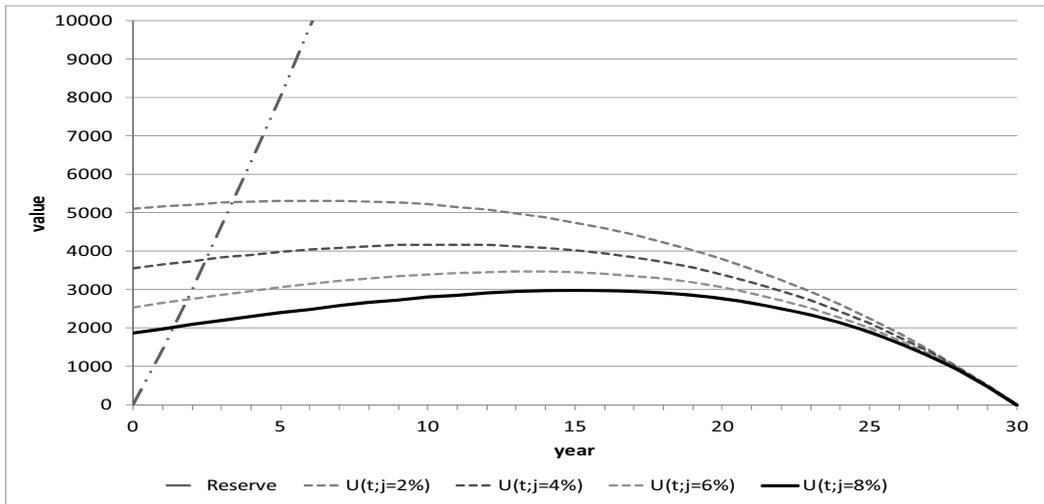
Fig. 3C. Average values of the distributions of $\tilde{U}_t$ for different evaluation rate j values
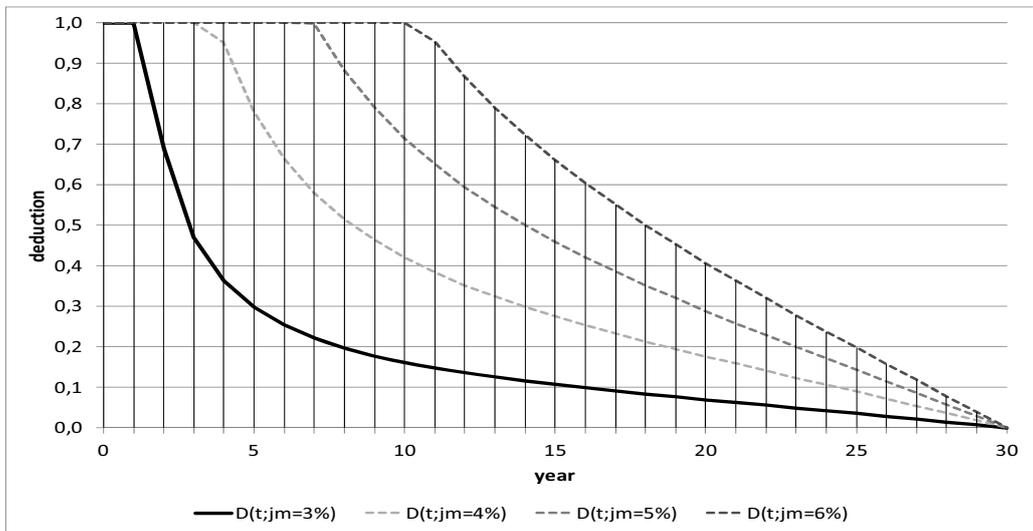
**Fig. 4C. Comparative deduction D (t; 1) for different values of the yield average rate, j_m**
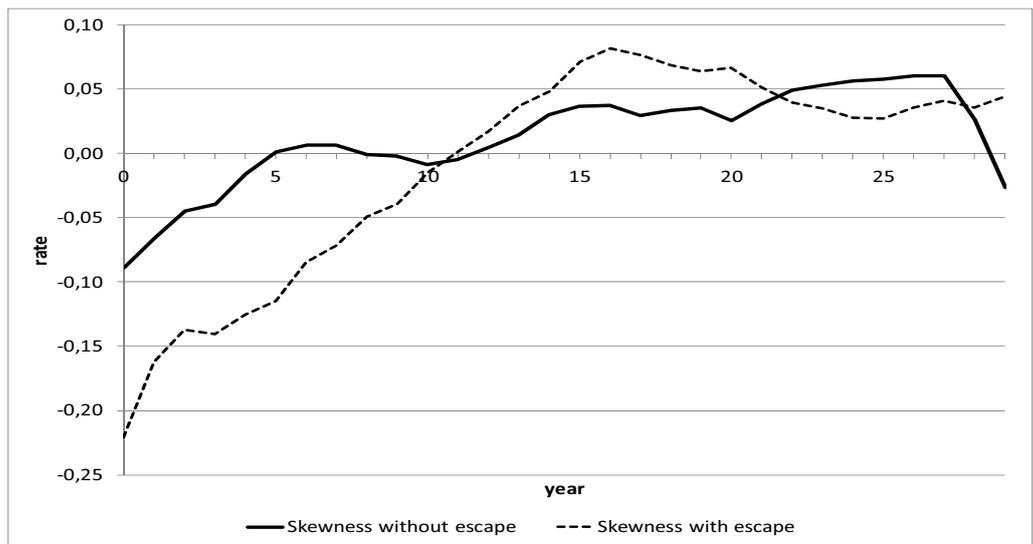
Fig. 5C. Skewness coefficients of the distributions of $\tilde{U}_t$ evaluated in presence and in absence of contract escapes

Appendix D. Analysis of a whole life insurance policies generation

Table 1D. Distribution features of $\tilde{U}_t$ evidenced for the standard stochasticity level, excluding contract escapes

t	average	s.q.m.	s.q.m./average	skew.
0	1740	201	0.115	-0.143
5	2288	241	0.106	-0.067
10	2821	288	0.102	-0.040
15	3297	330	0.100	-0.019
20	3650	378	0.103	0.008
25	3774	410	0.109	0.038

Table 2D. Distribution features of $\tilde{U}_t$ evidenced for the standard stochasticity level, with contract escapes

| t	average	s.q.m.	s.q.m./average	skew.
0	1708	184	0.108	-0.196
5	2129	213	0.100	-0.117
10	2401	250	0.104	-0.040
15	2677	283	0.106	-0.006
20	2919	314	0.108	-0.017
25	3007	339	0.113	-0.006
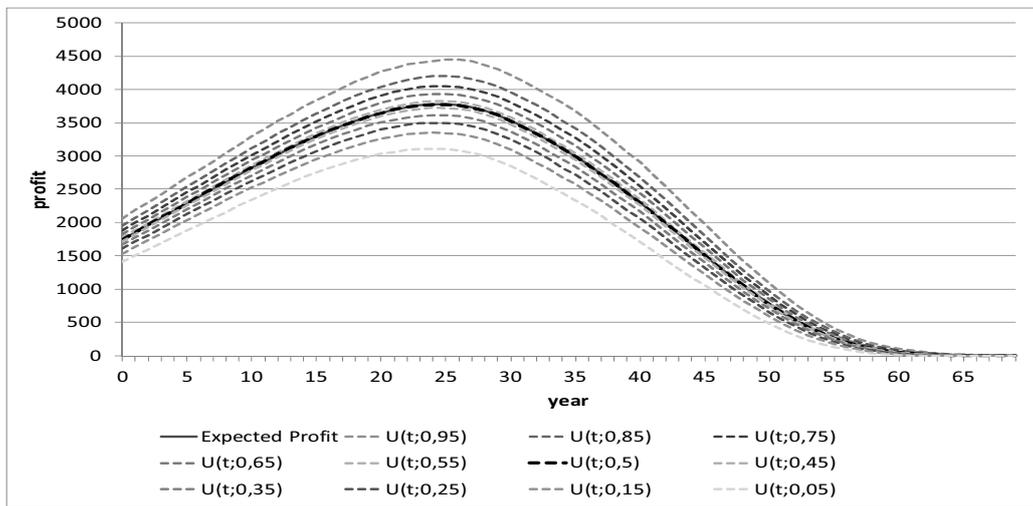
Fig. 1D. Average and percent values of the distributions of $\tilde{U}_t$
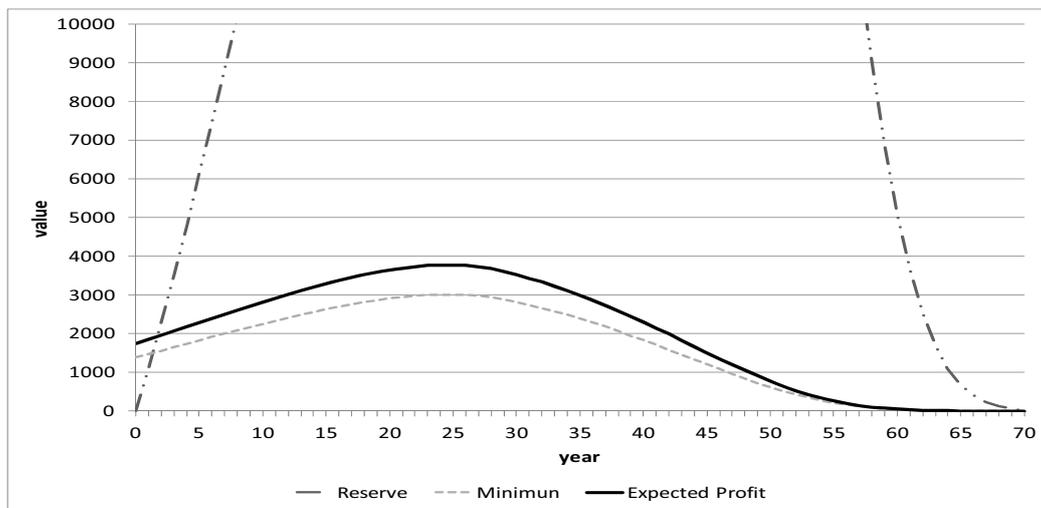

**Fig. 2D. Average values of the distributions of $\tilde{U}_t$ and of the generation full reserve**
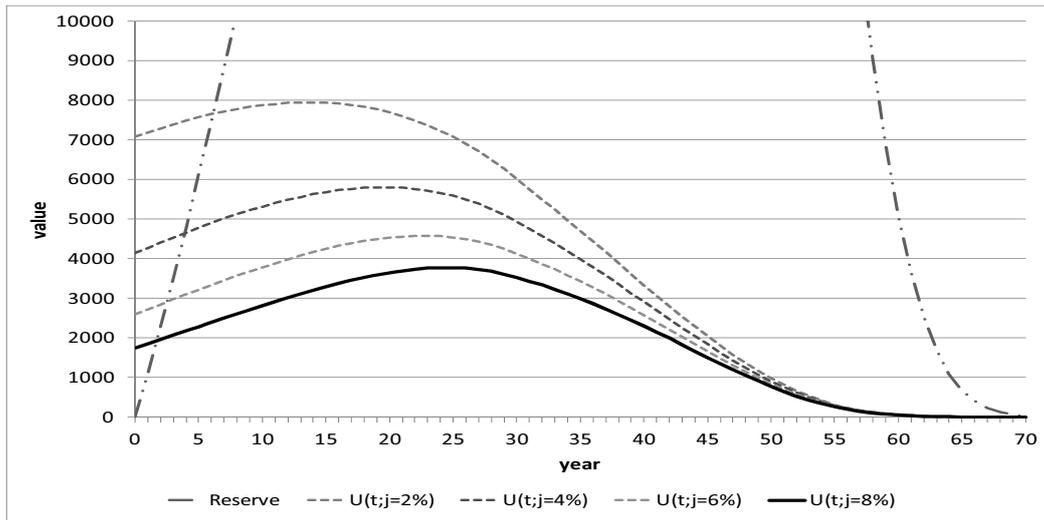
**Fig. 3D. Average values of the distributions of $\tilde{U}_t$ for the evaluation rate j different values**
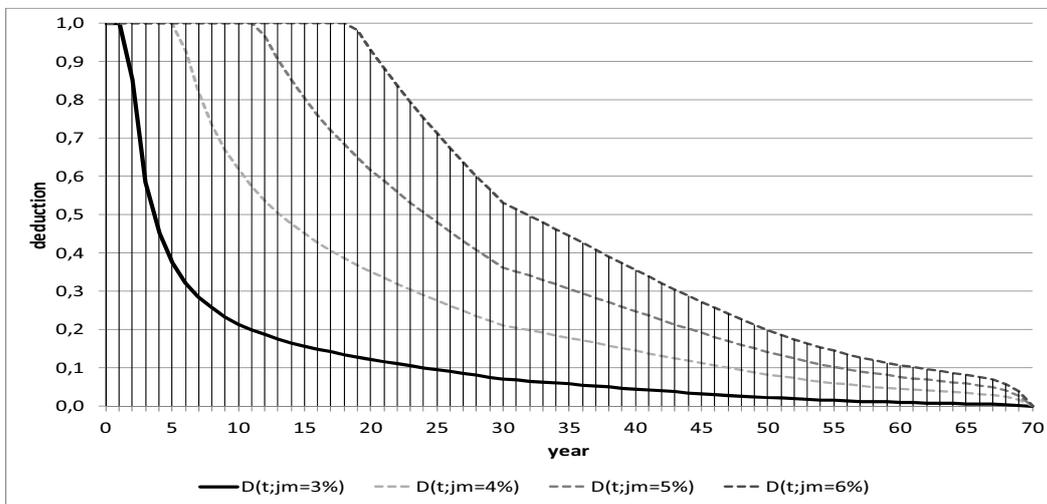
Fig. 4D. Comparative deduction D (t; 1) for different values of the yield average rate, j_m


Fig. 5D. Skewness coefficients of the distributions of $\tilde{U}_t$ evaluated in presence and in absence of contract escapes