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A Role of Lender Liability in Debt Contract[1]

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Abstract

We regard lender liability as a device which maintains the productivity of environmentally risky project by transferring legal liability for environmental damage from firms to banks. Our results show that banks are more likely to finance the risky project (1) with lender liability if the anticipated damage is large, and (2) with verifiable interim returns of the project if the damage is small. In the latter case, as the profitability of the alternative project increases, banks are more likely to finance the risky project without lender liability. These results imply that different policies should be required according to the characteristics of firms belonging to industries with environmental risk.

JEL Classification Code: G33, D82, K32.
Key words: lender liability, debt contract, incomplete contract.

1. Introduction

Environmental laws in most of the countries accept the polluter pay principle, which allocates to polluters legal liability for cleaning up environmentally contaminated sites. This principle can deter polluter-firms from choosing an action that results in future damages. However, a substantial rise in environmental concerns makes regulations stricter. This change of regulation causes the liability of polluters to be retroactive, and force polluters to make immediate payments of cleanup. Under such circumstances, the productivity of firms in industries with significant environmental risk may be eroded by cleanup costs; and the worst of it is, the firms may be in default. These concerns throw some new light on the role of *lender liability* under which lenders – in particular, banks – must pay for cleanup in place of their borrower-firms. In this paper, the role of lender liability is regarded as the one that forces the bank to pay for cleanup in order to mitigate the erosion of the productivity of its borrowing firms. The purpose of this paper is to consider how the information structure at the time of the payment of cleanup affects the asset allocation decision of the bank in the presence or the absence of lender liability.

We give CERCLA as a well-known example of lender liability, the 1980 Comprehensive Environmental Response, Compensation and Liability Act in the US: CERCLA specifies that lending banks are liable for cleanup costs due to environmental contamination generated by firms if the banks are found to participate actively in the management of the firms. This legislation is justified by the principle of *vicarious liability*; a bank is responsible for environmental damages because it can control the action choice of its borrower through loan contracts.[2] However, in spite of such legislation, the judicial interpretation of CERCLA is inconsistent with its judicial implementation[3]. In light of the court cases under CERCLA, banks are apt to be reluctant to lend to firms in industries with significant environmental risks rather than to control the borrowers' actions via monitoring activities and loan contract arrangements[4].

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[2] Segerson and Tietenberg (1992) introduce the principle of vicarious liability in the context of the malfeasance of employees. Since the firm can control the action choice of its employees via compensation schemes, it is more efficient to make the firm responsible for the malfeasance of its employees. This logic can be extended to the relationship between a lender bank and its borrower.

[3] The judicial inconsistency is seen in US v. Maryland Bank Trust in 1986 and in US v. Mirabile in 1985. See James (1988), Olexa (1991), Greenberg and Shaw (1992) and Evans (1994).

[4] Greenberg and Shaw (1992) indicate in their footnote 256 that lenders in the US changed their behavior in response to lender liability; in particular, they are more likely to decline to make loans to firms in industries with environmental risks. See also Schmidheiny, S. and F. J. L. Zorraquin (1996). They show that 62.5 percents of commercial banks in the US reject to lend firms belonging to industries with environmental risks, such as chemical industries.

Enhanced environmental liability also changes the borrower's behavior. Ringleb and Wiggins (1990) indicate in their empirical analysis that firms are likely to segregate latent hazardous sectors into small corporations in order to minimize the firms' liability exposure. These changes may be interpreted such that the lender liability rule based on vicarious liability does not work as intended. Furthermore, if such a tendency is strong, enhancing lender liability as well as the borrower's behavior towards legal liability for the damage may induce banks to decline to make loans to firms belonging to industries with environmental risks even though their productivities are sufficiently large to cover costs for recovering damage.

To investigate the effect of the enactment of the lender liability rule and the corresponding possible change in the behavior of the firm on the bank's investment decision, we consider the following situation. A bank determines to finance the project of a firm which belongs to an industry with significant environmental risk, or to invest in a project with deterministic returns. We refer to the former project as a risky project, while referring to the latter as a safety project. The risky project yields project returns in the interim period, but also generates environmental damage that needs immediate cleanup. The environmental agency cleans for the damage. However, the payment to the agency is made by the entrepreneur in the absence of lender liability, while in the presence of lender liability the payment is made by bank.

In the absence of lender liability, the entrepreneur must sacrifice the productivity of the project by liquidating a part of *ex ante* profitable assets for the cleanup payment. On the other hand, in the presence of lender liability, the profitability of his project is retained because he needs not liquidate his assets. Under the lender liability rule, the bank, as a sophisticated lender, can take steps to limit its exposure to lender liability by demanding insurance premiums about environmental risk. Specifically, the bank makes with the entrepreneur a loan contract that includes insurance premiums of providing indemnifications for the environmental damage.

Under these settings, we divide our analysis into two cases according as the interim returns are verifiable or not. Following Hart and Moore (1998), we suppose that the entrepreneur can divert the interim returns of the risky project to himself if the returns are unverifiable. This corresponds to the case in which the firm can segregate the environmentally risky project into a small firm by divesture without the bank's consent. Comparing the results obtained in the case of the verifiable interim returns with those in the case of the unverifiable interim returns we discuss which of the liability allocation or the verifiability of the interim returns is more important for the bank when choosing to finance the risky project in the initial period.

Our main results are summarized as follows. (1) The bank is more likely to finance the risky project in the presence of lender liability than in its absence if the project requires a large cleanup cost. (2) The bank is more likely to finance the risky project with the verifiable interim returns than that with the unverifiable interim returns if the project requires a small cleanup cost. In this case, as the interest rate of the safety project increases, the bank is more likely to finance the risky project in the absence of lender liability than in its presence.

These results suggest that in order to facilitate the investment in productive firms belonging to industries with high environmental risk, different policies are required according to the size of anticipated damage or the productivity of the alternative project.

More specifically, the lender liability rule is effective in inducing banks to lend firms belonging to industries which are anticipated to have large environmental damage. On the other hand, the information disclosure policy is effective in inducing banks to lend firms belonging to industries which are anticipated to have relatively small damage. Such information disclosure policy includes the one that prevent firms from segregating their environmental hazardous divisions into small firms.

The existing literature on lender liability mainly focuses on the allocation of legal liability between firms and banks in order to minimize damage. Pitchford (1995) first develops a formal model to analyze the optimal liability allocation between firms and banks in the moral hazard context. His welfare analysis shows that the partial lender liability minimizes the damage. On the other hand, Lewis and Sappington (2001) show that the liability must be fully allocated to the bank. Their analysis is based on the assumption that the firm can eliminate environmental damage by exerting its effort regardless of industries that the firm belongs to. In contrast to these analyses, we show that if the size of environmental damage depends on industries which firms belong to, the lender liability rule or the information disclosure policy should be used according to the anticipated size of damage.

The paper is organized as follows. Section 2 presents the model. Section 3 examines the case in which the interim returns of the risky project is verifiable. Section 4 examines the case in which the interim returns of the risky project is unverifiable. Section 5 derives the condition under which either the verifiability of the interim returns of the risky project or enactment of the lender liability rule affects more than the other the bank's investment decision.

2. The Model

We consider a bank that finances a project owned by an entrepreneur in a form of the debt contract. The project lasts for three periods (t=0,1,2), and is referred to as a *risky* project because it is profitable in the long-run but generates a loss in t=1. The bank has an alternative investment opportunity, which begins with t=0 and lasts until t=2.

The project is referred to as a *safety* project because it yields deterministic returns $r(\geq 1)$ per investing amount. We suppose that the market interest rate is zero.

The risky project

We follow the model introduced by Hart (1995) and Hart and Moore (1998), in which the entrepreneur of the project can liquidate assets partially in the midterm of the project.

The risky project owned by the entrepreneur requires an investment I in t=0 and yields interim project returns R together with an asset liquidation value L in t=1. However, this project also generates a loss D in t=1. The loss D is interpreted as a cleanup cost of environmentally hazardous materials which are generated by the firm in t=1. More specifically, the environmental agency cleans up the materials with a cost D, and either the entrepreneur or the bank must pay D to the agency according to the rule described in the subsequent analysis. We suppose that L is partially liquidable.

To the extent that a fraction of the asset remains unliquidated, the entrepreneur can continue the project until t=2. A fraction of the asset x ($0 < x \leq L$) yields a final value $\frac{x}{L}\widetilde{V}$ in t=2, where $\widetilde{V}$ is uniformly distributed in the interval $[0, V_H]$. The asset *is ex ante* profitable, that is, $E[V] \geq L$. Although both the entrepreneur and the bank know the expected value $E[V]$ and the distribution of $\widetilde{V}$ in t=1, they cannot observe the realized value V of $\widetilde{V}$ until t=2.

Uncertainty and information structure

We now impose assumptions on the uncertainty and the information structure of the model.

The values of R, L and D are random in t=0 and become certain in t=1. The value of V is stochastic in t=1 and becomes certain in t=2[1].

We assume that L and D are observable and verifiable in \$t=1\$ to a third party, such as a court, whereas R is observable but is not always verifiable. Following Hart and Moore (1998), we assume that if R is unverifiable, the entrepreneur can divert R for his private use but the bank cannot prevent him from doing so even though it can observe R.

We assume that the financial contract between the bank and the entrepreneur is the debt contract. Then, if R is verifiable in t=1, the bank can seize the value of the firm in t=1 even though the value of the firm is less than the contractual amount of repayment B in t=1. However,

[1] We follow the incomplete contract model with deterministic returns introduced by Hart (1995) and Hart and Moore (1998).

if R is unverifiable in t=1, the bank can only seize the value of the firm in t=2 if the value of the firm is less than the contractual amount of repayment B in t=2[1]. One might ask whether the bank can liquidate the firm to seize the value of the firm in t=1 even if R is unverifiable. However, as will be discussed in Section 4, the bank cannot recover its expenditure for the risky project by liquidating the firm if the entrepreneur diverts R. Although the bank lends more than I that is required to implement the risky project, we assume that the entrepreneur can divert the amount of funds exceeding I if R is unverifiable. Thus, the bank finances only the required investment amount of the risky project in t=0 and does not make any additional loans to the entrepreneur in the aftermath of the initial lending even though R is unverifiable.

Allocation of legal liability

Legal liability for bearing the burden D is allocated to the bank in the presence of the lender liability rule, whereas it is allocated to the entrepreneur in the absence of this rule. The enactment of the lender liability rule is determined exogenously before t=0. Under the lender liability rule, the bank which finances the risky project must pay D to the environmental agency. Thus, the total funds required by the risky project is $I + D$ under the lender liability rule. The bank is supposed to make the loan contract with provision of paying out D to the environmental agency in order to minimize its exposure on the liability. On the other hand, in the absence of the rule, the entrepreneur must pay D. We suppose that the entrepreneur is subject to limited liability and that D is a senior claim for the entrepreneur to the bank loan. The funds required to the bank by the risky project is then equal to I.

Technical assumptions

The technical assumptions in our model are summarized as follows:

Assumption 1: $0 < \max(L, R) < D < R + L$.

Assumption 2: . $I < R + L < I + D$

Assumption 3: $r \geq 1$.

Assumption 4: $E[V] \geq L$, where $E[V] = \frac{V_H}{2}$.

Assumption 1 shows that even though the entrepreneur's maximum payable amount in t=1, $R + L$, is large enough to cover D, neither L nor R is sufficient to do so. Assumption 2 shows that the sum of the interim returns and the liquidation value of assets exceeds the initial investment, but does not cover the sum of the initial investment and the loss. Assumption 3 indicates that the safety project yields the higher return rate than the market interest rate. This assumption allows us to describe the bank's problem as a choice between the risky and safety projects. Finally, Assumption 4 implies that the expected continuation value of the firm is greater than the liquidation value. Since we assume that a fraction of the asset x yields a final value $\frac{x}{L}\tilde{V}$, this assumption means that the entrepreneur who continues the risky project until t=2 has an incentive to minimize the asset liquidation when he must pay D in the absence of the lender liability rule.

Timing structure of the project

We describe the timing of events according as R is verifiable or not.

The case in which the interim returns of the risky project are verifiable

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[1] We can justify the assumption of the unverifiability of R in the context of the empirical findings introduced by Ringleb and Wiggins (1990) as follows. The entrepreneur can segregate a division of the firm into a small (private) subsidiary firm and transfer R to it. If laws do not prohibit the entrepreneur from doing so, we can assume that the bank cannot verify R.

In t=0, given the liability allocation, the bank determines whether to enter into a financial contract with the entrepreneur or to invest in the safety project.

If the bank finances the risky project, the project yields R with the liquidation value L, but generates the loss D in t=1. Since R is verifiable, the bank determines whether or not to terminate the risky project. If the bank chooses to terminate the risky project, all the assets of the firm are liquidated. Then, the game ends. On the other hand, if the bank chooses to allow the entrepreneur to continue the risky project, the game evolves differently according as the lender liability rule is enacted or not. In the absence of lender liability, the entrepreneur pays D to the environmental agency out of $R+L$ in t=1. As discussed in the preceding part of this section, the entrepreneur needs to minimize the asset liquidation for the payment D in order to maximize the expected value of the firm. In t=2, the final value is distributed according to the bank loan contract. Under the lender liability rule, the bank pays D to the environmental agency in t=1. Since all of the assets are left in the firm, the entrepreneur continues the project until t=2. In t=2, the final value of the firm $R+V$ is distributed according to the bank loan contract.

On the other hand, if the bank invests in the safety project in t=0, the bank receives deterministic returns r per investing amount in t=2.

The case in which the interim returns of the risky project are unverifiable

In t=0, given the liability allocation, the bank determines whether or not to finance the risky project. If the risky project is financed, it again yields R and L, but generates the loss D in t=1. What distinguishes the case with the unverifiable returns from that with the verifiable returns is that the banks cannot seize the full value of the firm in t=1 by terminating the project because the entrepreneur can divert R. As a result, the risky project is continues until t=2 in this case. In the absence of lender liability, the embezzlement of R by the entrepreneur results in the bankruptcy of the firm in t=1. This is because the liquidation value of the assets is insufficient to cover D under Assumption 1. Such bankruptcy corresponds to *strategic default*, which is introduced by Hart and Moore (1998). In fact, the entrepreneur is prevented from divering R and continues the project until t=2 if and only if his expected payoff under the bank loan contract exceeds R. On the other hand, in the presence of the lender liability rule, even though the entrepreneur diverts R in t=1, L is remained in the firm because the bank pays D to the environmental agency. The bank does not liquidate the firm because L is insufficient to recover its total expenditure $I+D$. In t=2, the final value of the firm is realized and is distributed according to the bank loan contract.

If the bank finances the safety project in t=0, it receives r per investing amount in t=2.

In the next section, we examine the case in which R is verifiable in t=1. In Section 4, we examine the case in which R is unverifiable in t=1.

3. Project Choice with the Verifiable Interim Returns of the Risky Project

In this section, we examine the case in which R is verifiable so that the bank can seize it in t=1. We first derive the condition under which the bank chooses to finance the risky project rather than the safety project in the absence of lender liability. We then obtain the corresponding condition in its presence. Finally, we investigate how the liability allocation affects the bank's choice of the project in t=0.

In the absence of lender liability

We solve the equilibrium of the game by backward induction. First, suppose that the bank finances the risky project in t=0. Then, we discuss the bank's decision in t=1 of whether or not to induce the entrepreneur to continue the project until t=2. We finally examine the condition under which the bank chooses to finance the risky project in t=0.

If the bank finances the risky project in t=0, the internal funds including the liquidation value of the assets in t=1 after the entrepreneur pays D to the environmental agency is

$R+L-D$ in the absence of lender liability. Since R is verifiable, it is the bank that determines whether to terminate the risky project in t=1 or to allow the entrepreneur to continue the project until t=2[1]. If the bank terminates the project in t=1, the bank gains $R+L-D$; and nothing is remained to the entrepreneur. On the other hand, if the bank does not terminate the project, the entrepreneur, as the residual claimant under the bank loan contract, pays out D from R in order to minimize the asset liquidation because Assumption 4 indicates that the asset makes it *ex ante* more profitable to remain in place than to be liquidated. Since $R<D<R+L$ from Assumption 1, the amount of the asset $D-R$ is liquidated and the final value of the firm becomes $\frac{R+L-D}{L}V$.

Given that the entrepreneur keeps the amount of the asset $R+L-D$ in place in t=1, the bank loan contract in this case is constructed as follows: if $\frac{R+L-D}{L}V \leq B$, the bank takes $\frac{R+L-D}{L}V$ and nothing is left to the entrepreneur; whereas if $\frac{R+L-D}{L}V > B$, the bank receives B and the entrepreneur obtains $\frac{R+L-D}{L}V - B$. Notice that the limited liability conditions for the bank and the entrepreneur imply

$$0 \leq B \leq \frac{(R+L-D)V_H}{L}. \tag{1}$$

Let π_B^{SN} and π_E^{SN} denote the expected payoffs of the bank and the entrepreneur in the absence of lender liability and in the case of verifiable R if the bank finances the risky project in t=0 and does not terminate it in t=1. Then,

$$\begin{aligned}\pi_B^{SN} &= \int_0^{\frac{LB}{R+L-D}} \frac{R+L-D}{L} V dF(V) + \int_{\frac{LB}{R+L-D}}^{V_H} B dF(V) \\ &= \frac{1}{V_H}\left[BV_H - \frac{LB^2}{2(R+L-D)}\right],\end{aligned} \tag{2}$$

$$\begin{aligned}\pi_E^{SN} &= \int_{\frac{LB}{R+L-D}}^{V_H} \left[\frac{R+L-D}{L}V - B\right] dF(V) \\ &= \frac{1}{V_H}\left[\frac{V_H(R+L-D)}{L} - B\right]^2.\end{aligned} \tag{3}$$

On the other hand, if the bank finances the risky project in t=0 but terminates the project in t=1, the expected payoffs of the bank and the entrepreneur are $R+L-D$ and 0, respectively, as discussed above.

Since if following from (1) and (3) that $\pi_E^{SN} \geq 0$ is always satisfied, the entrepreneur always prefers to continue the project until t=2. Thus, we can focus on the condition $\pi_B^{SN} \geq R+L-D$ under which the bank allows the entrepreneur to continue the project until

[1] Notice that the entrepreneur can terminate the project voluntarily by liquidating L for paying out D because of $L<D<R+L$ under Assumption 1. However, he never stops the project by himself because the bank can seize $R-(D-L)$ in t=1 as long as R is verifiable.

t=2 rather than to liquidate the firm in t=1. The range of the bank loan contract that satisfies $\pi_B^{SN} \geq R+L-D$ and (1) is provided by

$$B \in \left[\max\left(0, \frac{(R+L-D)V_H}{L} - \sqrt{\frac{V_H}{L}(\frac{V_H}{L}-2)}\right), \frac{(R+L-D)V_H}{L} \right].$$

Note that such $B \geq 0$ always exists if $\frac{V_H}{L} - 2 \geq 0$. Since $E[V] = \frac{V_H}{2}$, the condition $\frac{V_H}{L} - 2 \geq 0$ is equivalent to Assumption 4, $E[V] \geq L$. Thus, if Assumption 4 holds, the bank investing in the risky project always induces the entrepreneur to continue the project until t=2; and the entrepreneur accepts the loan contract offered by the bank.

Now, we examine the bank's investment decision in t=0. If the bank finances the safety project, the bank receives rI and the entrepreneur of the risky project gains nothing. This indicates that as long as $\pi_B^{SN} \geq rI$ and $\pi_E^{SN} \geq 0$ hold, the bank invests in the risky project by inducing the entrepreneur to enter into the loan contract with the bank in t=0. Since (1) and (3) ensure that $\pi_E^{SN} \geq 0$, we can focus our investigation on $\pi_B^{SN} \geq rI$. Solving $\pi_B^{SN} \geq rI$ with (1) and (2), we derive the range of B in which the bank finances the loan to the entrepreneur in t=0:

$$B \in \left[\frac{(R+L-D)V_H}{L} - \sqrt{\frac{(R+L-D)V_H}{L}\left(\frac{(R+L-D)V_H}{L} - 2rI\right)}, \frac{(R+L-D)V_H}{L} \right]. \quad (4)$$

Note that the lower limit of B is greater than 0 in this case. Since such $B \geq 0$ exists if and only if $\frac{(R+L-D)V_H}{L} - 2rI \geq 0$, the bank finances the risky project if and only if $\frac{(R+L-D)V_H}{L} - 2rI \geq 0$, that is,

$$\frac{V_H}{2} \geq \frac{rIL}{R+L-D}.$$

We now summarize these discussions in the following lemma:

Lemma 1. Suppose that R is verifiable. In the absence of the lender liability rule, the bank finances the risky project if and only if

$$E[V] = \frac{V_H}{2} \geq \Lambda_{SN},$$

where $\Lambda_{SN} \equiv \frac{rIL}{R+L-D}$.

In the presence of lender liability

We next examine the case in which the bank is restricted by the lender liability rule. If the bank finances the risky project in t=0, the project generates D as well as $R+L$ in t=1. Since the bank must incur D under the lender liability rule, the entrepreneur needs not liquidate a part of the asset. Again, it is the bank that decides whether to terminate the risky project and seize $R+L$ in t=1 or to allow the entrepreneur to continue the project until t=2. If the bank induces the entrepreneur to continue the risky project in t=1, the final value of the project in t=2 becomes $R+V$. The bank loan contract is then constructed as follows: if $R+V \leq B$, the bank takes $R+V$ and the entrepreneur receives nothing, whereas if $R+V>B$, the bank takes B and the entrepreneur receives $R+V-B$. Note that the limited liability constraints for the bank and the entrepreneur suggest

$$0 \le B \le R+V_H . \tag{5}$$

Given the bank loan contract form, we begin with checking the condition under which the bank chooses to allow the entrepreneur to continue the project in t=1. Let π_B^{SL} and π_E^{SL} denote the expected payoffs of the bank and the entrepreneur if the bank finances the risky project in t=0 and allows the entrepreneur to continue the project in t=1. Then, for (5),

$$\begin{aligned}\pi_B^{SL} &= \int_0^{B-R}(R+V)dF(V)+\int_{B-R}^{V_H} BdF(V)\\ &= \frac{1}{V_H}\left[BV_H-\frac{(B-R)^2}{2}\right],\end{aligned} \tag{6}$$

$$\begin{aligned}\pi_E^{SL} &= \int_{B-R}^{V_H}(R+V-B)dF(V)\\ &= \frac{1}{V_H}\left[\frac{V_H{}^2}{2}+(R-B)V_H+\frac{(R-B)^2}{2}\right]\\ &= \frac{1}{2V_H}\left[V_H+R-B\right]^2.\end{aligned} \tag{7}$$

On the other hand, if the bank finances the risky project in t=0 but terminates the project in t=1, the expected payoffs of the bank and the entrepreneur are $R+L$ and 0, respectively. Thus, the bank chooses to allow the entrepreneur to continue the risky project in t=1 if and only if $\pi_B^{SL} \ge R+L$ under (5). Since the entrepreneur always chooses to continue the project because (7) implies $\pi_E^{SL} \ge 0$ for (5), we only have to check the condition of $\pi_B^{SL} \ge R+L$. Indeed, given $R+V_H > \sqrt{V_H(V_H-2L)}$, the range of $B \in [0, R+V_H]$ that satisfies $\pi_B^{SL} \ge R+L$ is described by

$$B \in [R+V_H-\sqrt{V_H(V_H-2L)}, R+V_H]. \tag{8}$$

This condition holds if and only if $V_H(V_H-2L) \ge 0$, that is, $V_H \ge 2L$. Again, since this is equal to $E[V] \ge \frac{V_H}{2}$, Assumption 4 ensures that the bank financing the risky project in t=0 never terminates the project in t=1.

We now proceed to derive the condition under which the bank chooses to finance the risky project in t=0. Note that the bank's total expenditure on the risky project is $I+D$. Then, if the bank invests the same amount of funds in the safety project, the bank receives $r(I+D)$ while the entrepreneur does not receive anything in t=2. Thus, the bank chooses to finance the risky project and the entrepreneur accepts the loans from the bank if and only if $\pi_B^{SL} \ge r(I+D)$ and $\pi_E^{SL} \ge 0$ for $B \in [0, R+V_H]$. In fact, it is immediate from (7) that $\pi_E^{SL} \ge 0$ for any $B \in [0, R+V_H]$. Thus, we focus our analysis on $\pi_B^{SL} \ge r(I+D)$ \$\pi^{SL}_B under (5). Substituting (6) into $\pi_B^{SL} \ge r(I+D)$, we see that $\pi_B^{SL} \ge r(I+D)$ holds under (5) if and only if

$$B \in [R+V_H-\sqrt{V_H[V_H-2(r(I+D)-R)]}, R+V_H].$$

The condition above holds if and only if $V_H - 2[r(I+D) - R] \geq 0$, that is,

$$\frac{V_H}{2} \geq r(I+D) - R. \tag{9}$$

Given (9) with $E[V] = \frac{V_H}{2}$, we now obtain the following lemma.

Lemma 2. Suppose that R is verifiable. In the presence of the lender liability rule, the bank chooses to finance the risky project in t=0 if and only if

$$E[V] = \frac{V_H}{2} \geq \Lambda_{SL},$$

where $\Lambda_{SL} \equiv r(I+D) - R$.

Comparison

We now discuss how the liability allocation affects the bank's project choice in t=0 for each parametric configuration of (D, r).

Lemmas 1 and 2 show that if $E[V]$ is large enough to satisfy the inequalities presented in these lemmas, the bank chooses to finance the risky project regardless of the liability allocation determined by the rule. However, for not large enough $E[V]$, the bank's choice of the project depends on the liability allocation. If $E[V]$ satisfies only the inequality in Lemma 1, that is,

$$\Lambda_{SN} \leq E[V] < \Lambda_{SL}, \tag{10}$$

then the bank finances the safety project in the presence of lender liability while financing the risky project in its absence. On the other hand, if $E[V]$ satisfies

$$\Lambda_{SN} > E[V] \geq \Lambda_{SL}, \tag{11}$$

the bank finances the risky project in the presence of lender liability while financing the safety project in its absence. For simplicity, we say that the bank is *more likely to finance* the risky project in the absence (or presence) of lender liability than in its presence (absence) if (10) (or (11)) holds.

For simplicity, in addition to Assumptions 1-4, we will impose the following assumption:

Assumption 5: $I < \min(R, L)$.

Assumption 5 ensures that $\max(R, L) < R + L - I$. Combining Assumptions 1 and 5, we restrict the range of D to $R + L - I < D < +L$ in the following analysis.

Let Ω_v denote

$$\Omega_v \equiv \frac{R(R+L-D)}{D(R+L-D) - I(D-R)}.$$

Then, we obtain the following proposition.

Proposition 1. Suppose that R is verifiable.

(i) If $R + L - I < D < \frac{R + L - I + \sqrt{(R+L-I)^2 + 4IR}}{2}$, the bank's project choice depends on the interest rate of the safety project. More specifically, (i-1) If $\Omega_v > r$, the bank is more likely to finance the risky project in the presence of lender liability than in its absence.

(i-2) If $r \geq \Omega_v$, the bank is more likely to finance the risky project in the absence of lender liability than in its presence.

(ii) If $\frac{R+L-I+\sqrt{(R+L-I)^2+4IR}}{2} \leq D < R+L$, the bank is more likely to finance the risky project in the presence of lender liability than in its absence.

Proof: See the Appendix.

Proposition 1 shows that the enactment of the lender liability rule encourages the bank to finance the risky project except in the case of (i-2).

The intuition behind this proposition is given as follows. From Assumption 4, the asset of the risky project makes it more productive to be unliquidated than to be liquidated in t=1. However, in the absence of lender liability, the entrepreneur must liquidate the amount of the asset, $D-R$, for paying D to the environmental agency. The lender liability rule can prevent the asset liquidation by transferring legal liability for the damage from the entrepreneur to the bank. Thus, if D is large enough as in the case of (ii), the bank finds it more profitable to incur the cleanup cost in exchange for preventing the asset liquidation than to be exempted from the liability in exchange for allowing the entrepreneur to decrease the productivity of the project by the asset liquidation.

On the other hand, if D is small enough as in the case of (i), the lender liability rule cannot necessarily improve the productivity of the risky project: as r increases, the bank is more likely to finance the risky project in the absence of lender liability than in its presence. The reason is that the entrepreneur does not liquidate a large amount of the asset even in the absence of lender liability if D is small enough; whereas the bank's opportunity cost in investing the risky project is large in the presence of lender liability for fixed D if r is large enough[1].

4. Project Choice with the Unverifiable Interim Returns of the Risky Project

We proceed to examine the case in which R is observable but unverifiable. In this case, the entrepreneur of the risky project can divert R for his private use but the bank cannot prevent him from doing so. However, the termination of the risky project by the bank cannot be a credible threat to the entrepreneur because R is unverifiable.

Then, we derive the condition under which the bank finds it more profitable to finance the risky project rather than the safety project in t=0.

In the absence of lender liability

Suppose that the bank finances the risky project in t=1. If R is unverifiable, it is only the entrepreneur that decides whether or not to continue the project in t=1. That is, if the entrepreneur diverts R and pays D by liquidating the asset, the project is terminated because of $L < D$ from Assumption 1. On the other hand, if the entrepreneur chooses to continue the project until t=2, he pays D out of R as much as possible to minimize the asset liquidation because of Assumption 4. Given $R < D < R+L$ from Assumptions 1 and 2, $D-R$ of the asset is liquidated; and the asset remaining in place yields $\frac{R+L-D}{L}V$ as the final value of the project.

[1] Note that the bank chooses to finance the risky project in t=0 if $\pi_B^{SN} \geq rl$ in the absence of lender liability, and if $\pi_B^{SN} \geq r(l+D)$ in the presence of lender liability.

The bank loan contract is written as follows: if $\frac{R+L-D}{L}V \le B$, the bank takes $\frac{R+L-D}{L}V$ and the entrepreneur receives nothing; whereas if $\frac{R+L-D}{L}V > B$, the bank receives B and the entrepreneur gains $\frac{R+L-D}{L}V - B$. Note that the limited liability constraints for the bank and the entrepreneur lead to

$$0 \le B \le \frac{(R+L-D)V_H}{L}. \tag{12}$$

Let π_B^{AN} and π_E^{AN} denote the expected payoffs of the bank and the entrepreneur if the bank chooses to finance the risky project and the entrepreneur continues the project until t=2 in the absence of lender liability and in the case of unverifiable R. Since the bank loan contract in this case is the same as that in the absence of lender liability and in the case of verifiable R, we have

$$\pi_B^{AN} = \pi_B^{SN}, \tag{13}$$

$$\pi_E^{AN} = \pi_E^{SN}, \tag{14}$$

where π_B^{SN} and π_E^{SN} are given by (2) and (3). As long as $\pi_B^{AN} \ge rI$ and $\pi_E^{AN} \ge R$ hold under (12), the bank chooses in t=0 to finance the risky project rather than to invest in the safety project in t=1; and the entrepreneur enters into the bank loan contract.

Solving $\pi_B^{AN} \ge rI$ with (2) and (13), we derive the same range of B as (4), in which the bank is induced to finance the risky project in t=0. Similarly, solving $\pi_E^{AN} \ge R$ with (3) and (14), we obtain the range of B in which the entrepreneur is induced to continue the project until t=2:

$$0 \le B \le \frac{(R+L-D)V_H}{L} - \sqrt{\frac{2V_H(R+L-D)R}{L}}. \tag{15}$$

Thus, it follows from (4), (12), and (15) that $\pi_B^{AN} \ge rI$ and $\pi_E^{AN} \ge R$ hold under (12) if and only if

$$\sqrt{\frac{(R+L-D)V_H}{L}\left(\frac{(R+L-D)V_H}{L} - 2rI\right)} \ge \sqrt{\frac{2V_H(R+L-D)R}{L}}. \tag{16}$$

Such B exists if and only if $\frac{(R+L-D)V_H}{L} \ge 2rI$. Thus, the following lemma is derived.

Lemma 3. Suppose that R is unverifiable. In the absence of lender liability, the bank chooses to finance the risky project rather than the safe project if and only if

$$E[V] = \frac{V_H}{2} \ge \Lambda_{AN},$$

where $\Lambda_{AN} \equiv \frac{L(rI+R)}{R+L-D}$.

In the presence of lender liability

Suppose that the risky project is financed in t=0 under the lender liability rule. Since R is unverifiable, the bank can seize only L, which is less than its total expenditure, $I+D$, by Assumption 2. Thus, the bank has always an incentive to continue the project in t=1. In the presence of lender liability, the entrepreneur also has an incentive to continue the project until t=2 because none of the assets are liquidated even if he diverts R in t=1. The bank loan contract is then written as follows: if $V \leq B$, the bank takes V and the entrepreneur retains R; whereas if $V > B$, the bank receives B and the total gain of the entrepreneur becomes $R + V - B$. Notice that the entrepreneur's and the bank's limited liability constraints are satisfied as long as B is determined in the following range:

$$0 \leq B \leq V_H. \tag{17}$$

Let π_B^{AL} and π_E^{AL} denote the expected payoff of the bank and the entrepreneur if the bank finances the risky project and the entrepreneur continues the project until t=2 in the presence of lender liability and in the case of unverifiable R. Then, π_B^{AL} and π_E^{AL} are characterized as follows:

$$\begin{aligned}\pi_B^{AL} &= \int_0^B V dF(V) + \int_B^{V_H} B dF(V) \\ &= \frac{1}{V_H}\left[BV_H - \frac{B^2}{2}\right],\end{aligned} \tag{18}$$

$$\pi_E^{AL} = R + \int_B^{V_H} (V-B) dF(V). \tag{19}$$

Since both the bank and the entrepreneur have an incentive to continue the project until t=2, we now consider the condition under which the risky project is financed in t=0. Since $\pi_E^{AL} \geq R$ from (17), the entrepreneur always accepts the bank loan. Thus, we only have to investigate the condition under which the bank chooses to finance the risky project in t=0. In the presence of lender liability, the total amount funded by the bank to the entrepreneur's project is $I + D$. The amount of funds yields $r(I + D)$ if the bank chooses to invest in the safety project instead of the risky project in t=0. Thus, the bank chooses to finance the risky project if and only if

$$\pi_B^{AL} \geq r(I+D). \tag{20}$$

It is found from (18) and (20) with (17) that the bank finances the risky project in t=0 if and only if the bank loan B is set to be in the following range:

$$B \in [V_H - \sqrt{V_H(V_H - 2r(I+D))}, V_H]. \tag{21}$$

This leads to the following condition under which such a bank loan B exists:

$$V_H(V_H - 2r(I+D)) \geq 0 \text{, that is } \frac{V_H}{2} \geq r(I+D).$$

We thus obtain the following lemma.

Lemma 4. Suppose that R is unverifiable in the presence of lender liability. The bank finances the risky project in t=0 if and only if

$$E[V] = \frac{V_H}{2} \geq \Lambda_{AL},$$

where $\Lambda_{AL} \equiv r(I+D)$.

Comparison

We now examine how the allocation of legal liability affects the bank's choice of the project. Using the same logic in the preceding section, Lemmas 3 and 4 show that the bank chooses to finance the risky project regardless of the liability allocation if and only if

$$\max(\Lambda_{AN}, \Lambda_{AL}) \le E[V].$$

However, if $E[V]$ satisfies

$$\Lambda_{AN} \le (>) E[V] < (\ge) \Lambda_{AL},$$

the bank chooses to finance the safety (risky) project in the presence of lender liability.

Let Ω_u denote

$$\Omega_u \equiv \frac{LR}{D(R+L-D)-I(D-R)}$$

Then, we establish the following proposition.

Proposition 2. Suppose that *R* is unverifiable.

(i) If $R+L-I < D < \frac{R+L-I+\sqrt{(R+L-I)^2+4IR}}{2}$, the bank's project choice depends on the interest rate of the safety project. More specifically,

(i-1) If $\Omega_u > r$, the bank is more likely to finance the risky project in the presence of lender liability than in its absence.

(i-2) If $r \ge \Omega_u$, the bank is more likely to finance the risky project in the absence of lender liability than in its presence.

(ii) If $\frac{R+L-I+\sqrt{(R+L-I)^2+4IR}}{2} \le D < R+L$, the bank is more likely to finance the risky project in the presence of lender liability than in its absence.

Proof: See the Appendix.

The results of Proposition 2 are similar to those of Proposition 1. Hence, the intuition behind Proposition 2 is also similar to that of Proposition 1.

5. Which Factor Determines the Bank's Project Choice: Verifiability of the Interim Returns or the Liability Allocation Rule?

We now investigate which factor – the verifiability of the interim returns of the risky project or the enactment of the lender liability rule – determines the project choice made by the bank in t=0. For simplicity, we refer to the combination of the verifiability of R and the liability allocation of *D* as *ij*, where *i* = *S* (*A*) indicates that *R* is verifiable (unverifiable) and *j* = *N* (*L*) indicates that the entrepreneur (the bank) is liable for paying *D* to the environmental agency. As already shown in the preceding sections, the bank finances the risky project under the condition of ij as long as $E[V] \ge \Lambda_{il}$ (*ij* = *SN, SL, AN, AL*). This suggests that the bank is more likely to finance the risky project if Λ_{ij} is smaller.

To examine which factor is more likely to lead the bank to choose the risky project in t=0, we compare the results obtained in Propositions 1 and 2. Since the results of Propositions 1 and 2 are divided into two statements with the same parametric restrictions, we can directly compare each statement of (i) and (ii) of Proposition 1 with the corresponding statement of Proposition 2.

In the rest of this section, we discuss how *D* and *r* affect the critical point Λ_{ij}. More specifically, suppose that $\Lambda_{S\hat{j}} < \Lambda_{A\tilde{j}}$ ($\hat{j}, \tilde{j} = N, L$). If *E*[*V*] satisfies $\Lambda_{S\hat{j}} \le E[V] < \Lambda_{S\tilde{j}}$, irrespective of the liability allocation, the bank finances the risky project if *R* is verifiable; whereas it does not if *R* is unverifiable[1]. This indicates that the verifiability of *R* is the more important factor than the liability allocation when the bank chooses to finance the risky project in t=0. On the other hand, suppose that $\Lambda_{\hat{i}N} < (>) \Lambda_{\tilde{i}L}$ ($\hat{i}, \tilde{i} = A, S$). If *E* [*V*] satisfies $\Lambda_{\hat{i}N} \le (>) E[V] < (\ge) \Lambda_{\tilde{i}L}$, regardless of the verifiability of *R*, the bank finances the risky project in the absence (presence) of lender liability; while it finances the safety project in the presence (absence) of lender liability. In this case, the liability allocation rule is more important than the verifiability of *R* when the bank chooses to finance the risky project in t=0.

First, we deal with the case of $R + L - I < D < \dfrac{R + L - I + \sqrt{(R + L - I)^2 + 4IR}}{2}$.

Let Ω_w denote

$$\Omega_w \equiv \frac{R(R + 2L - D)}{D(R + L - D) - I(D - R)}.$$

Since $\Omega_w > \Omega_u > \Omega_v$ holds, we obtain the following proposition.

Proposition 3. Suppose that $R + L - I < D < \dfrac{R + L - I + \sqrt{(R + L - I)^2 + 4IR}}{2}$.

(i) If $\Omega_v > r \ge 1$, then $\Lambda_{SL} < \Lambda_{SN} < \Lambda_{AL} < \Lambda_{AN}$.

(ii) If $\Omega_u > r \ge \Omega_v$, then $\Lambda_{SN} \le \Lambda_{SL} < \Lambda_{AL} < \Lambda_{AN}$.

(iii) If $\Omega_w > r \ge \Omega_u$, then $\Lambda_{SN} \le \Lambda_{SL} < \Lambda_{AN} \le \Lambda_{AL}$.

(iv) If $r \ge \Omega_w$, then $\Lambda_{SN} < \Lambda_{AN} \le \Lambda_{SL} < \Lambda_{AL}$.

Proof: See the Appendix.

The statements from (i) to (iii) of Proposition 3 show that $\Lambda_{S\hat{j}} < \Lambda_{A\tilde{j}}$ ($\hat{j}, \tilde{j} = N, L$). This indicates that the verifiability of *R* is more important than the liability allocation when the bank chooses to finance the risky project. However, these statements also suggest that as *r* increases, the bank is more likely to finance the risky project in the absence of lender liability than in its presence. In fact, the statement (iv) shows that $\Lambda_{\hat{i}N} < (>) \Lambda_{\tilde{i}L}$ ($\hat{i}, \tilde{i} = A, S$). This indicates that if *D* is small but *r* is very large, the liability allocation rule is more important than the verifiability of *R* when the bank chooses to finance the risky project. More specifically, allocating legal liability for the damage to the entrepreneur induces the bank to finance the risky project.

The intuition behind this proposition is given as follows. Since *D* is small, the extent of the productivity improvement by the enactment of the lender liability rule is not significant. The bank finds it more profitable to prevent the entrepreneur from diverting *R* by verifying *R* directly rather than to incur the cleanup cost in exchange for stopping the asset liquidation. Thus, the verifiability of *R* is more effective in inducing the bank to choose the risky project than the liability allocation. However, as *r* increases, an increase in the opportunity cost of investing in the risky

[1] Since we do not obtain $\Lambda_{S\hat{j}} > E[V] \ge \Lambda_{A\tilde{j}}$ in the following analysis, we omit to explain this case.

project is larger in the presence of lender liability than in its absence. This implies that even though $E[V]$ is large enough to induce the bank to choose the risky project in the absence of lender liability, the bank may choose the safety project in the presence of lender liability. Indeed, if r is large enough to satisfy the case of (iv), the bank finds it more profitable to be exempted from lender liability irrespective of the verifiability of R than to be liable for the damage in exchange for preventing the asset liquidation.

We also discuss the policy implication of Proposition 3. To facilitate the investment in the risky project with relatively small $D \in \left(R+L-I, \dfrac{R+L-I+\sqrt{(R+L-I)^2+4IR}}{2} \right)$, the policy must be designed according to the productivity of the safety project. If r is not very large, the regulatory discussion should be focused on the question of how the entrepreneur is induced to verify R. If r is very large, the liability should be allocated to the entrepreneur.

We next proceed to compare the results of Proposition 1 (ii) with those of Proposition 2 (ii).

Proposition 4. Suppose that $\dfrac{R+L-I+\sqrt{(R+L-I)^2+4IR}}{2} \le D < R+L$. $\Lambda_{SL} < \Lambda_{AL} \le \Lambda_{SN} < \Lambda_{AN}$ is satisfied for any $r \ge 1$.

Proof: See the Appendix.

Proposition 4 shows that $\Lambda_{\hat{i}L} \le \Lambda_{\tilde{i}N}$ ($\hat{i}, \tilde{i} = S, A$). This indicates that the liability allocation is more important than the verifiability of R when the bank chooses to finance the risky project in t=0. Intuitively, if D is very large, the presence of lender liability improves the productivity of the risky project significantly because the bank can prevent the asset liquidation by incurring the cleanup cost. As a result, the bank finds it more profitable to finance the risky project in the presence of the lender liability rule than in its absence.

6. Discussion and Conclusion

The role of lender liability can be regarded as a device which mitigates the erosion of the productivity of the project caused by significant environmental risk. We investigate which liability allocation is more likely to induce the bank to finance the risky project of the entrepreneur in response to a change in the size of the cleanup cost of the environmental damage or the profitability of the alternative project (as denoted by the safety project). To examine how the information structure – that is, the verifiability of the interim returns, – affects the bank's financial decision and the liability allocation, we divide our analysis into two cases according as the interim returns of the risky project are verifiable or not.

Our main results are summarized as follows. First, we show that the bank is more likely to finance the risky project in the presence of lender liability than in its absence if the project requires a large cleanup cost. Second, we indicate that the bank is more likely to finance the risky project with the verifiable interim returns than that with the unverifiable interim returns if the project requires a relatively small cleanup cost. In this case, as the interest rate of the safety project increases, the bank is more likely to finance the risky project in the absence of lender liability than in its presence.

Our results suggest that in order to facilitate the investment in productive firms belonging to industries with high environmental risk, different policies must be used according to the size of the anticipated damage or the productivity of the alternative project. More specifically, the lender liability rule is effective in inducing banks to lend firms belonging to industries with large environmental damage. On the other hand, the information disclosure policy is effective in inducing banks to lend firms belonging to industries with relatively small damage. Such information disclosure policy includes the one that prevents firms from separating their environmental hazardous divisions into small firms.

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Appendix

Proof of Proposition 1

We define Δ_v as $\Delta_v \equiv \Lambda_{SL} - \Lambda_{SN}$.

Then,

$$\Delta_v = r(I+D) - R - \frac{rLI}{R+L-D}$$
$$= \frac{1}{R+L-D}\left[r[D(R+L-D) - I(D-R)] - R(R+L-D)\right]$$

$\Delta_v \geq (<)0$ indicates that the bank is more likely to finance the risky project in the absence (presence) of lender liability than in its presence (absence). To check $\Delta_v \geq 0$ or $\Delta_v < 0$, define

$$\psi(D) \equiv D(R+L-D) - I(D-R).$$

Solving $\psi(D) = 0$ yields $D = \dfrac{R+L-D \pm \sqrt{(R+L-I)^2 + 4IR}}{2}$. Since $R + L - I < D < R + L$ from Assumptions 2 and 5, we have

(i) $\psi(D) > 0$ holds if $R+L-I < D < \dfrac{R+L-I+\sqrt{(R+L-I)^2+4IR}}{2}$,

(ii) $\psi(D) \leq 0$ holds if $\dfrac{R+L-I+\sqrt{(R+L-I)^2+4IR}}{2} \leq D < R+L$.

In the case of (ii), we see $\Delta_v < 0$ because of $r \geq 1$ and $R + L - D > 0$ from Assumption 1.

We next consider the case of (i). Since $\psi(D) > 0$, we can define

$$\Omega_v \equiv \frac{R(R+L-D)}{\psi(D)}.$$

Furthermore, $\Omega_v > 1$ is derived from the following inequality:

$0 < \psi(D) \equiv D(R+L-D) - I(D-R) < D(R+L-D) - (R+L-D)(D-R) = R(R+L-D)$.

The second inequality is derived from $I > R + L - D$ of Assumption 2 and $D - R > 0$ from Assumption 1. Thus, (i) is divided into the following two cases:

(i-1) If $\Omega_v > r \geq 1$, then $\Delta_v < 0$,

(i-2) If $r \geq \Omega_v > 1$, then $\Delta_v \geq 0$.

Proof of Proposition 2

Following the similar logic of the proof of Proposition 1, we derive the results of this proposition.

Let us define $\Delta_u \equiv \Lambda_{AL} - \Lambda_{AN}$.

$$\Delta_u = r(I+D) - \frac{L(rI+R)}{R+L-D}$$
$$= \frac{1}{R+L-D}\left[r[D(R+L-D) - I(D-R)] - LR\right]$$

$\Delta_u \geq (<)0$ indicates that the bank is more likely to finance the risky project in the absence (presence) of lender liability than in its presence (absence).

Again, we can divide our discussion into the following two cases according as $\psi(D) > 0$ or $\psi(D) \leq 0$: which is defined in the proof of Proposition 1.

(i) $\psi(D) > 0$ holds if $R + L - I < D < \dfrac{R + L - I + \sqrt{(R + L - I)^2 + 4IR}}{2}$,

(ii) $\psi(D) \leq 0$ holds if $\dfrac{R + L - I + \sqrt{(R + L - I)^2 + 4IR}}{2} \leq D < R + L$.

In the case of (ii), it is clear that $\Delta_u < 0$.

In the case of (i), since $\psi(D) > 0$, we can define

$$\Omega_u \equiv \frac{LR}{\psi(D)}.$$

Furthermore, we have $\Omega_u > 1$ because of

$$0 < \psi(D) \equiv D(R + L - D) - I(D - R) < R(R + L - D) < LR.$$

The last inequality holds because of $R - D < 0$ from Assumption 1. Thus, this case is further divided into the following two cases:

(i-1) If $\Omega_u > r \geq 1$, then $\Delta_u < 0$,

(i-2) If $r \geq \Omega_u > 1$, then $\Delta_u \geq 0$.

Proof of Proposition 3

Suppose that D satisfies $R + L - I < D < \dfrac{R + L - I + \sqrt{(R + L - I)^2 + 4IR}}{2}$. By comparing Ω_v given in the proof of Proposition 1 and Ω_u given in the proof of Proposition 2, we have $\Omega_u > \Omega_v (> 1)$.

If $\Omega_u > \Omega_v > r$, we have $\Lambda_{SL} < \Lambda_{SN}$ from Proposition 1 (i-1) and $\Lambda_{AL} < \Lambda_{AN}$ from Proposition 2 (i-1). Using $\psi(D) > 0$ from $R + L - I < D < \dfrac{R + L - I + \sqrt{(R + L - I)^2 + 4IR}}{2}$ and $R + L - D > 0$ from Assumption 1, we have

$$\Lambda_{AL} - \Lambda_{SN} \equiv r(I + D) - \frac{rLI}{R + L - D}$$
$$= \frac{\psi(D)}{R + L - D} > 0.$$

Thus, we obtain the configuration of Λ_{ij} given in Proposition 3 (i).

If $\Omega_u > r \geq \Omega_v$, we have $\Lambda_{SN} \leq \Lambda_{SL}$ from Proposition 1 (i-2) and $\Lambda_{AL} < \Lambda_{AN}$ from Proposition 2 (i-1). Since $\Lambda_{SL} < \Lambda_{AL}$ holds because of

$\Lambda_{SL} \equiv r(I+D) - R < r(I+D) \equiv \Lambda_{AL}$, we have the configuration of Λ_{ij} given as in Proposition 3 (ii).

Lastly, if $r \geq \Omega_u > \Omega_v$, we have $\Lambda_{SN} \leq \Lambda_{SL}$ from Proposition 1 (i-2) and $\Lambda_{AN} \leq \Lambda_{AL}$ from Proposition 2 (i-2). Since $\Lambda_{AL} > \Lambda_{SL}$ and $\Lambda_{AN} > \Lambda_{SN}$ hold, we need to compare Λ_{SL} with Λ_{AN}.

$$\Lambda_{SL} - \Lambda_{AN} \equiv r(I+D) - R - \frac{L(rI+R)}{R+L-D}$$

$$= \frac{1}{R+L-D}\left[r[D(R+L-D) - I(D-R)] - R(R+2L-D)\right]$$

$$= \frac{1}{R+L-D}\left[r\psi(D) - R(R+2L-D)\right].$$

Let Ω_w denote

$$\Omega_w \equiv \frac{R(R+2L-D)}{\psi(D)}.$$

Since $\psi(D) > 0$ from the proofs of Proposition 1 (i) and Proposition 2, (i) $\Omega_w > \Omega_u$. Thus, $\Lambda_{SL} \geq (<) \Lambda_{AN}$ holds according as $r \geq (<) \Omega_w$. If $\Omega_w > r$, the configuration of Λ_{ij} is given as in Proposition 3 (iii). If $r \geq \Omega_w$, the configuration of Λ_{ij} is given as in Proposition 3 (iv).

Proof of Proposition 4

Suppose that D satisfies $\dfrac{R+L-I+\sqrt{(R+L-I)^2+4IR}}{2} \leq D < R+L$. It follows from Proposition 1 (ii) and Proposition 2 (ii) that $\Lambda_{SL} < \Lambda_{SN}$ and $\Lambda_{AL} < \Lambda_{AN}$. Since $\Lambda_{SL} < \Lambda_{AL}$ and $\Lambda_{SN} < \Lambda_{AN}$ always hold, we compare Λ_{AL} and Λ_{SN}.

$$\Lambda_{AL} - \Lambda_{SN} \equiv r(I+D) - \frac{rLI}{R+L-D}$$

$$= \frac{1}{R+L-D}\left[r[D(R+L-D) - I(D-R)]\right]$$

$$= \frac{r\psi}{R+L-D}.$$

Because of $\psi \leq 0$, we see $\Lambda_{AL} \leq \Lambda_{SN}$. Thus, we complete the proof of this proposition.