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Best estimate of the motor liability claims reserve under the direct reimbursement scheme

Abstract

In some European countries, a large subset of motor liability claims is managed under a direct reimbursement (DR) scheme. The first component that loss adjustors use to evaluate the claims reserve is given by the sum of the estimated provision for each outstanding claim (known as case reserves). Due to the heterogeneity of data available, the introduction of direct indemnity in third-party liability insurance has resulted in greater attention to the traditional statistical-actuarial methods used to control and/or verify the evaluations. This paper presents the first results of a study undertaken to define a new calculation method that enables quantifying the claims reserve using different hypotheses of the evolution of settlement mechanisms and in so doing offers greater transparency, objectivity as well as the opportunity to perform stress tests. A numerical application is presented comparing the results of the proposed model with those using traditional statistical methods.

Keywords: outstanding claims reserve, best estimate, direct reimbursement, multistate model.

Introduction

In literature, the "actuarial" calculation of the outstanding claims reserve has received particular attention with regard to third-party liability insurance due to the consistent payment delay (long tail) with respect to the attributable financial period.

However, in some countries, such as France, Italy, Spain and Portugal, insurers have agreed to participate in direct indemnifications schemes for certain types of collisions. In these schemes, once responsibility is ascertained, claimants are indemnified by their own insurance company that legally acts as an agent for the company insuring the responsible party.

The introduction of direct indemnity in third-party liability insurance (henceforth TPL) in Italy (Autorità Garante per la Concorrenza e il Mercato, 2006) has in recent years led to greater complexity in using statistical-actuarial methods to estimate the claims reserve due to the heterogeneity of data available for evaluations (claim handling procedure pre and post 2007) and the different dynamics underlying the risk factors that characterize and determine the overall business costs in relation to the claim handling procedures.

The Italian direct reimbursement (DR) scheme is somewhat similar to the IDA mechanism prevailing in France but is far more extensive.

Specifically, Presidential Decree 254 of 18 July 2006 and the direct reimbursement (DR) scheme between insurance companies under the so-called CARD agreement govern the limitations and rules for the application of DR, providing a new structure of the cost of claims based on the debtor and handler forfait (lump sum) mechanism. Four components constitute the new business costs: non-CARD claims, CARD claims, handler forfait and debtor forfait.

These four types of claims have different evolutionary profiles: consider for instance the settlement speed of CARD claims (by the policyholder's insurance company), which is faster than that of non-CARD claims (by the counterparty's insurance company). Hence, a situation arises whereby payments appear in a run-off triangle that appertain to generations of claims preceding the direct indemnity scheme and constitute different types of claims, which could undermine the reliability of an estimate based on this triangle.

This paper identifies a methodology that uses as input data the different hypotheses describing the evolution of the four types of claim handling procedures and quantifies the outstanding claims reserve separately, taking into account the characteristics of each type of claim.

The aim is to estimate the claims reserve with the methodological framework used to estimate cash flows resulting from a life insurance portfolio through estimating the number of claims 'handled' and the average cost for each distinct discriminating variable identified (for example, geographic area, type of claim handling procedure, type of vehicle).

The most frequently used methods collect input data in the well-known run-off triangle, evaluating the settlement effects and average costs but generally considering only two variables that result in the double-entry table (year of accident and claim duration). These aggregate claim loss reserving methods (see, e.g., Barnett and Zehnwirth, 2000;

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Blum and Otto, 1998; Brown and Gottlieb, 2007; Fisher et al., 1973; Institute of Actuaries, 1997; Mack, 1993; Quarg and Mack 2004; and Taylor, 1986) differ from individual models in actuarial literature and use two approaches: the theoretical, typically employing continuous models (Norberg, 1986, 1993 and 1999; Jewell, 1989; and Arjas, 1989) and the practical, using discrete models (Mahon, 2005; Murphy and McLennan, 2006; Larsen, 2007; and Taylor et al., 2006).

The proposed model falls into the category of individual claim loss reserving methods developed according to the discrete approach and entails calculating the 'termination' probability and the average costs, taking into account the information available for each claim (as will be specified later), hence not only year of accident and year of development.

To deal with this complexity, the statistical model proposed aims to:

- Manage the heterogeneity of data for different handling procedures under direct indemnity and consider the different dynamics underlying the risk factors.
- Take into account all the claim characteristics considered representative, hence not only as a function of accident year and development year.
- Render the reserves calculation more transparent and clear to third parties including a description of all the parameters and their benchmarks.

The dataset derives from a real insurance company and the numerical application enables comparing the results with those obtained with the application of traditional aggregate statistical methods.

Specifically, starting from the list of outstanding claims at the end of a particular financial year, including all the information available for each claim and indications of any partial payments, we group homogenous classes based on the characteristics considered representative to calculate the claims reserve.

Each characteristic therefore requires identifying the possible states of the claim and thereafter estimating the probability of transition between different states. To estimate the probability of transition between states, the model requires flow data, i.e., data that quantify the number and characteristics of claims that move from one state to another. To this end, the proposed model uses data relating to claims that were closed in the years preceding the evaluation date including all their characteristics and the cause of termination. Each claim is given an identification code according to the year of accident and enables following its history up to the time of final closure or at the evaluation date.

With regard to the states, we consider claims in the outstanding state (the claims reserve in the valuation year including reactivated claims), those in the closed state (either fully paid or settled without payment) and, finally, those in the outstanding state that have been partially paid.

The evaluated claims are those outstanding at a specific reporting date. Determining the claims reserve requires estimating for each year following this date the number of expected claims for each state and ascertaining the corresponding cash flow of expected payments. The total outstanding claims reserve is equal to the sum of future expected cash flows. The best estimate is calculated as the present value of all future expected cash flows.

The paper is organized as follows: Section 1 specifies the agreement between insurers that provides for the direct settlement of claims to policyholders and via forfait for compensation of payments made by the insurance company that reimbursed the damages, indicating the information and data used to evaluate the claims reserve. Section 2 explains the multistate model useful to estimate the probability of transition between states, describing the new claims reserve estimation model, the mathematical structure and advantages. Section 3 illustrates the numerical application in relation to the traditional methods to calculate the claims reserves, the results obtained with the proposed model and comparing these with the traditional models. The final Section concludes the paper.

1. Direct reimbursement and claims reserve in TPL insurance

1.1. CARD provisions. Direct reimbursement is the new indemnity insurance method introduced by Decree-Law n. 254/06 and in the case of road accident claims foresees that the injured party's insurance company reimburses the policyholder.

Direct reimbursement applies in the following cases:

- The parties involved must be identified by means of the agreed motor accident statement form (blue form in Italy).
- The accident must involve no more than two motor vehicles.
- The vehicles must have Italian license plates.
- The drivers must have taken out TPL insurance with an insurance company authorized to practice in Italy or with a foreign company that has acceded to direct reimbursement.
- Physical damage reported by the driver must be minor (up to 9% of permanent disability).

In cases of non-eligibility for compensation under direct reimbursement, the insured must request payment for damages from the other party's insurance company according to the traditional compensation method.

The new laws on direct reimbursement are intended to streamline the claim settlement and vehicle damage procedure, reduce the high costs of policies, ensure consumer protection and competition in the insurance industry.

The direct reimbursement implementation regulation, in force from 1 January 2007 for claims occurring from 1 February 2007, was amended by Presidential Decree n. 28 in February 2009 (published in the Official Gazette n. 77 of 2 April 2009) with the aim of ensuring complete fairness of the compensation mechanism between insurance companies and a flat-rate method based on a calculation that takes into account the categories of vehicles insured.

Reimbursement between insurance companies is based on average costs differentiated by types of vehicles insured for damage to property and injury to persons, and limited to property damage in no more than three homogeneous macro-areas. In particular, according to Presidential Decree n. 28/09, such reimbursement differentials are fixed and may be modified according to an Economic Development Ministerial decree based on actual development costs and experience gained, albeit limiting the number of changes and never for application periods of less than one year.

The real novelty that has had a strong impact on the administration of claims and on the rules to determine the profitability of the claim handling procedure is the reimbursement system regulated at the national level with clear criteria between the damaged vehicle policyholder's company and the insurance company of the policyholder responsible for the accident. Indeed, reimbursements are determined – as in other European countries – by way of awarding lump sums (forfait) for damage to vehicles, property transported and driver injury (if minor) according to geographic area and type of vehicle for claims incurred from 2010.

Similarly, forfait reimbursements are foreseen for damage to other passengers and their property, with the exception of particularly heavy injuries to passengers, which above a certain prefixed level provide, in addition to the forfait, full repayment of the amount in excess of this level.

To be noted is that since the claims reserve is estimated from a very consistent database with time series developed over a sufficiently long time horizon, these data should in principle be as homogeneous as possible. However, incorporating CARD and non-CARD claims leads to scarce homogeneity that hinders adequately considering the risk profile characterizing the two specific types of claims and theoretically requires different models to estimate the future costs according to type of claim. Since the insurer's burden resulting from the claim has to be evaluated and/or foreseen in addition to the cost of the claim itself, criteria are needed to consider the actual risk profile characterizing the portfolios of other insurance companies to accurately predict the average cost of claims resulting from their risks.

The introduction of direct reimbursements in TPL insurance in recent years has thus led to greater complexity in using statistical-actuarial methods to estimate the provision for outstanding claims. This, as previously mentioned, is due to the heterogeneity of available evaluation data (claim handling procedure pre and post 2007) and, above all, the different dynamics underlying the risk factors that characterize and determine the final costs of claim handling procedures.

The present work is framed within this perspective and aims to identify a reliable statistical method to support analytical evaluations and manage the multiplicity of historical data while considering the evolution of the measurement mechanisms (Fersini et al., 2010 and 2011).

1.2. Data and information analysis. The input data for the evaluation of the claims reserve always refers to the year of accident (or to the reporting year if prior to the year 2000) and to the reimbursement delay (development year). The information collected in the noted run-off triangle, typically reconstructed from the reporting forms required by the supervisory authority, may include:

- number and amount of outstanding claims;
- number and amount of claim payments;
- number of claims that have been settled without payment and therefore closed;
- number of reactivated claims.

With regard to the claim payment amounts, generally, once the evaluation has been completed, a square matrix is obtained where the elements of the upper triangle correspond to the known elements, since these relate to past information at the evaluation date, while those in the lower triangle are the estimated outstanding claim amounts.

Worth noting is that a claim can consist of several damage and/or injury components, each of which represents the primary insurance event and may thus pertain to several types of claim handling procedures, moving from one to another.

With the introduction of direct reimbursement in TPL, insurance companies have begun to collect claim information according to four types: CARD, non-CARD, debtor forfait and handler forfait, which we group into three types of claim handling procedures: non-CARD claims, debtor CARD claims and handler CARD claims. For a given insurance company, debtor CARD claims relate to accidents caused by policyholders for which the payout is the established debtor forfait payable to the insurance company that has reimbursed the damages.

The handler CARD claims procedure relates to the indemnity the insurer pays to policyholders for claims they have sustained (CARD claims) net of the forfait to be paid by the debtor company (handler forfait).

Finally, the non-CARD claim procedure collects information relating to claims falling outside the cases provided for in Presidential Decree 254 of 18 July 2006.

It follows that the cost of claims covered by the new direct reimbursement scheme will be given by:
CARD claims - handler forfait + debtor forfait where:

- CARD claims - handler forfait shows the effects of the indemnity mechanism of the direct reimbursement procedure. This is an expression of the ability or inability of the insurer to contain costs in terms of the forfait.
- Debtor forfait assigns to the insurer the cost of the damage caused by policyholders.

The sum of non-CARD claims and CARD claims denotes the cost of claims managed by the company. The difference between the handling forfait and the debtor forfait, when showing a positive amount, indicates that the frequency of claims in the company's portfolio is lower than the frequency of claims in the market.

Therefore, the new cost of business is made up of four components (non-CARD claims, CARD claims, handler forfait and debtor forfait) characterized by different evolutionary paths: consider, for instance, the settlement speed of CARD claims (by the policyholder's insurer) compared to non-CARD claims (by the counterparty's insurer).

Due to the scarcity of available data, a reverse breakdown of the historical series of payments pre 2007 according to the post 2007 logic is not possible. A situation thus arises where payments appear in a run-off triangle that belong to generations of claims preceding the coming into force of the direct reimbursement scheme, thereby constituting different types of claims that could undermine the reliability of estimates based on this triangle alone.

As mentioned in the introduction, the concept behind this work is to identify a methodology able to quantify the claims reserve taking into account the characteristics of each claim including the different hypotheses of the evolution of the three types of claim handling procedures.

The proposed model uses input data in relation to the analytical list of outstanding or active claims at the end of a given financial year including all available information for each claim (for reactivated claims, additional indications of the reactivation date as well as the accident date) and any partial payments.

Also required is the list of claims that have been 'terminated' in the 5 years preceding the evaluation date, including their characteristics and basis of closure (fully paid or settled without payment), and the entire registered history of claims not yet settled at the evaluation date.

Each claim is assigned an identification code in the year of accident that enables tracking the claim's history (in terms of partial payments, transition between types of claim handling procedures, etc.) up to the time of final closure.

We define the age of the claim reserve at the evaluation date as the period between the date of the accident and the evaluation date. This is expressed in years and provides information on how long the claim has been entered in reserve and thus in the outstanding state. In the case of previously closed claims (fully paid or settled without payment), the age at the evaluation date is the total number of years that have elapsed since the accident.

The claims assessed are those outstanding at the evaluation date – since they have not yet been fully paid or settled without payment – or have been reactivated.

2. A multistate model

2.1. Probabilistic structure. During its lifetime, the claim can assume one of the following states:

- Outstanding: claims that have been reported but not settled at the evaluation date including reactivated claims. In practice, these are claims for which no payment has been made.
- Fully paid: claims that have been settled with payment in full.
- Settled without payment: claims that are closed with no payment foreseen.

- Partially paid: claims for which a partial payment has been made.

Figure 1 is useful to identify a probabilistic model for the evaluations, illustrating for each of the three types of claim handling procedures – debtor CARD (DC), handler CARD (HC) and non-CARD (NC) – the four states in which the claim can be located at each point of time. The arrows indicate the possible transitions between states.

It is assumed that at the evaluation date, the number of claims in the outstanding and partially paid state is known.

Figure 1 clearly shows that a claim that at the evaluation date is in the outstanding state can move to one of the other three states: partially paid, fully paid or settled without payment. The claims in the partially paid state can move to the fully paid or settled without payment state. Finally, the claims in the fully paid state and those in the settled without payment state can move to the outstanding state in the event of reactivating the claim. Moreover, the theoretical approach summarized in Figure 1 also considers all possible transitions between claim handling procedure types.

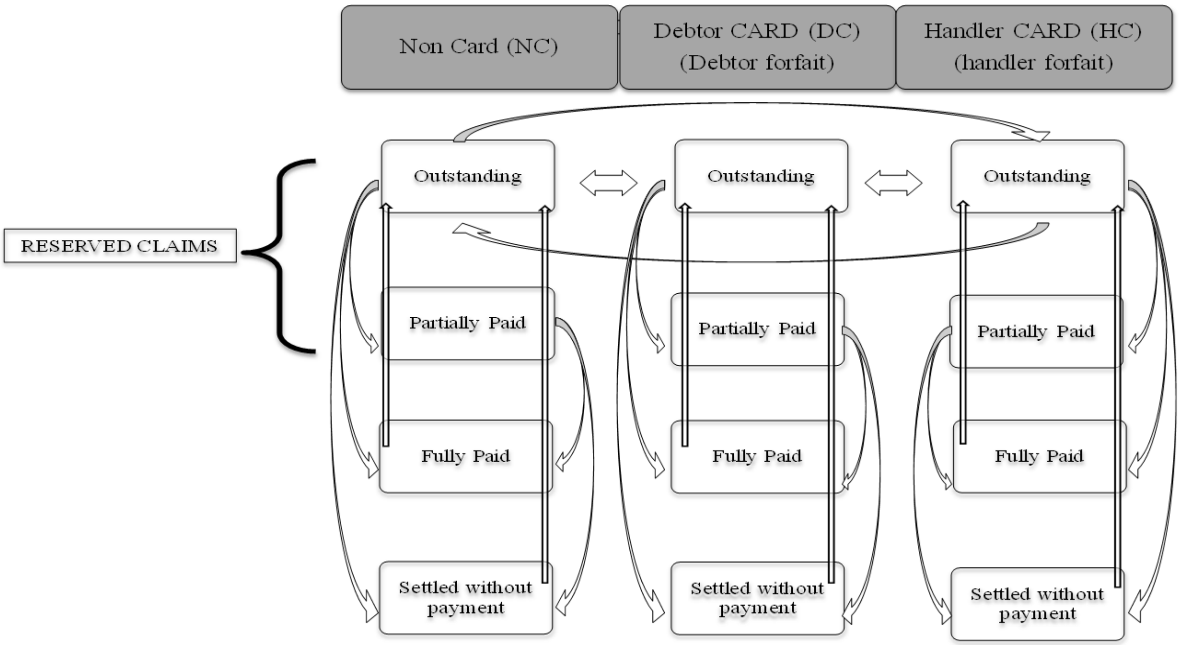


Fig. 1. Scheme of states

The probabilistic model is discrete in that the probability of transition between states is annual since the year is the base period of reference (Pitacco, 1995 and 1999).

We denote with "*p*" the probability of claims being outstanding ("survival" probability) and with "*q*" the probability of claims being closed ('deceased' probability).

Consider claim age *y* in the outstanding state in one of the three claim handling procedures *i*, where *i* = 1 for non-CARD (NC), *i* = 2 for handler CARD (HC) and *i* = 3 for debtor CARD (DC) at evaluation time *h* = 0 and defining the following annual probabilities:

$p_y^{A_iA_i}$ = probability of being in reserve in the "outstanding" state at age $y + 1$;

$p_y^{A_iP_i}$ = probability of being in reserve in the "partially paid" state at age $y + 1$;

$q_y^{A_iD_i}$ = probability of being closed as "fully paid" at age $y + 1$;

$q_y^{A_iS_i}$ = probability of being closed as "settled without payment" at age $y + 1$.

In case of a double exponent, the first denotes the initial state at age *y* (outstanding or partially paid) while the second denotes the state at year-end for survival probability and at death for closure probability.

In case of a single exponent, this denotes the initial state at age *y*.

Similarly, we can define the probabilistic structure of a claim of age *y* that is in the partially paid state for a given claim handling procedure *i* at the evaluation date *h* = 0:

$p_y^{P_iP_i}$ = probability of being in reserve in the "partially paid" state at the age $y + 1$;

$p_y^{P_iA_i}$ = probability zero since a claim in the "partially paid" state cannot move to the "outstanding" state;

$q_y^{P_iD_i}$ = probability of being closed as "fully paid" at age $y + 1$;

$q_y^{P_iS_i}$ = probability of being closed as "settled without payment" at age $y + 1$.

For a given claim handling procedure i, we can write the following relation:

$$p_y^{A_iA_i} + p_y^{A_iP_i} = p_y^{A_i}, \quad (1)$$

which represents the probability that the claim in handling procedure i is in reserve in $y + 1$ (thus outstanding or partially paid).

The following relationship is also clear:

$$p_y^{P_iP_i} + \cancel{p_y^{P_iA_i}} = p_y^{P_i},$$

which represents the probability that the claim in $y + 1$ is in the partially paid state.

Moreover:

$$q_y^{A_iD_i} + q_y^{A_iS_i} = q_y^{A_i} \quad (2)$$

$$q_y^{P_iD_i} + q_y^{P_iS_i} = q_y^{P_i} \quad (3)$$

respectively represent the probability that the claim is settled in $y + 1$ since it is in the outstanding state in y and the probability that the claim is settled in $y + 1$ since it is the partially paid state in y.

Finally:

$$p_y^{A_i} + q_y^{A_i} = 1, \quad (4)$$

$$p_y^{P_i} + q_y^{P_i} = 1, \quad (5)$$

Thus, by adding (1) and (2), we obtain for (4):

$$p_y^{A_iA_i} + p_y^{A_iP_i} + q_y^{A_iD_i} + q_y^{A_iS_i} = 1, \quad (6)$$

and:

$$p_y^{P_iP_i} + q_y^{P_iD_i} + q_y^{P_iS_i} = 1. \quad (7)$$

Consider now the reactivation possibility as presented in Figure 1 with the transition from settled without payment to outstanding, and from fully paid to outstanding.

The model excludes the possibility of repeated transitions to the same state during the year. We thus define the following probabilities:

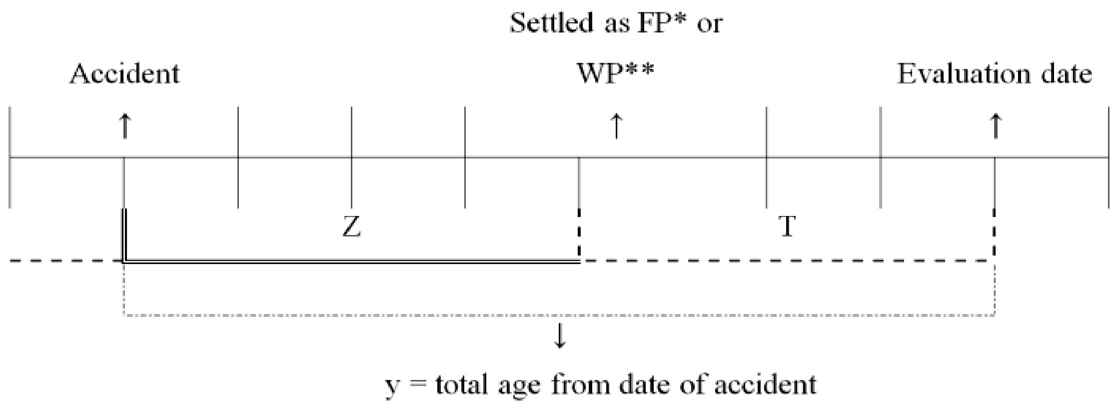
$p_y^{D_iA_i}$ = probably that in $y + 1$ the claim (settled without payment in y) is reactivated.

$p_y^{S_iA_i}$ = probability that in $y + 1$ the claim (settled without payment in y) is reactivated

The last two probabilities could be made to depend on the number of years spent in the reserve state (either outstanding or partially paid) before closure (either for final payment or settled without payment) and the number of years between closure and reactivation.

We define with z the number of years that have elapsed from the outstanding state to the settled state and with t the number of years in the settled state at the evaluation date.

Figure 2 shows all the variables on the same temporal axis.



Notes: FP*-fully paid, WP** without payment.

Fig. 2. Time diagram

We can redefine the reactivation probability as:

$p_{[t]+y-t}^{D_iA_i}$ = probability that a claim of age y has to be reactivated between y and y + 1 having spent t years in the fully paid state.

$p_{[t]+y-t}^{S_iA_i}$ = probability that a claim of age y has to be reactivated between y and y + 1 having spent t years in the settled without payment state.

This approach enables expressing a frequently registered phenomenon, namely, given a claim of a certain total age y, recently settled as fully paid or without payment, the reactivation probability is higher than that of a claim settled longer ago, thus:

$$p_{[t]+y-t}^{D_iA_i} > p_{[w]+y-w}^{D_iA_i} \qquad \text{if } t < w$$

$$p_{[t]+y-t}^{S_iA_i} > p_{[w]+y-w}^{S_iA_i} \qquad \text{if } t < w$$

To complete the possible transitions represented in Figure 1, we define the following probability of transition between claim handling procedures:

$p_y^{A_iAj}$ = probability that a claim age y activated in handling procedure i has to spend between y and y + 1 in handling procedure j, which is true for $i \neq j$, with i = 1, 2 and 3, and j = 1, 2, 3 corresponding to the three claim handling procedures.

Moreover, to be noted and as represented in Figure 1, the transition between claim handling procedures can only occur through the outstanding state.

If we consider the transition between claim handling procedures, we can rewrite (1) as:

$$p_y^{A_iA_i} + p_y^{A_iP_i} + \sum_{j \neq i} p_y^{A_iA_j} = p_y^{A_i}, \qquad (8)$$

which represents the probability that the outstanding claim in y in handling procedure i is in reserve in y + 1 regardless of whether it is outstanding in handling procedure i or in another handling procedure or partially paid in handling procedure i.

Moreover, given (8), we can rewrite (6) as:

$$p_y^{A_iA_i} + p_y^{A_iP_i} + \sum_{j \neq i} p_y^{A_iA_j} + q_y^{A_iD_i} + q_y^{A_iS_i} = 1. \qquad (9)$$

2.2. The model point approach and indications to estimate the probabilities. The model point approach, rather than analytically analyzing the claims being evaluated, enables grouping similar classes based on the following characteristics considered representative for the claims reserve calculation:

- year of accident/claim age (date of origin/age);
- development year (reserve age);
- geographic area of the policyholder facing a claim;
- claim handling procedure (NC, DC, HC);
- for handler CARD, specification of the type of claim: direct reimbursement scheme (the so-called "convenzione indennizzo diretto" or CID) and passenger injury or damage (the so-called "convenzione terzo trasportato" or CTT);
- type of vehicle (motorcycle, car and others. For the CID and CTT forfait, the distinction between major types of vehicles: "mopeds and motorcycles" and "vehicles other than motorcycles and mopeds");
- reserve amount (broken down by amount category in the case of very large amounts subject to different settlement mechanisms; a first differentiation is made by type of handling procedure in as much as higher amounts fall within non-CARD claims).

Traditional claims reserve estimation models entering the input data in the noted run-off triangle evaluate the settlement effect and the average costs taking into account only the two variables that result in the double-entry table (accident year and development year, sometimes only the latter).

The idea, in this case, is to calculate the termination probability and average costs taking into account all the information available for each claim, namely, not only year of accident and year of development.

Thus, from a methodological perspective, starting from the financial year h and considering the "average" assumptions, the total value of payments can be estimated for the financial year h + 1, h + 2, h + 3 and so forth, up to the complete termination of the claims reserve at time h (in the classic run-off triangle this is the amount of estimated payments on each of the diagonals).

Each selected claim characteristic requires identifying the states it can take to then estimate the probability of transition between the different states in which an event occurs.

In terms of estimating the probability of transition between states, the model requires flow data, i.e., data quantifying the number and characteristics of claims that move from one state to another.

The probabilistic framework presented in the previous section can be expressed taking into account all the characteristics considered representative, hence not only as a function of the age of the claim and the handling procedure.

The availability of statistical data affects the probability estimations used in the model; data interpolation or the use of approximate formulas is required when data is unavailable or non-homogenous.

2.3. Mathematical structure: expected numbers. We denote the evaluation date with $h = 0$. The evaluated claims are all those in reserve at evaluation date $h = 0$ for all generations (in this paper, the term claim is used in the sense of all damage and/or injury components included therein).

For simplification, we omit the indication of the type of handling procedure considered and the age of the claim. As such, the formulas do not contain the i index – albeit implicitly applying to each of the three claim handling procedures considered – nor do they contain the y index or the indications of all the characteristics considered representative.

Each h requires estimating the number of expected claims for each state (as defined above) and determining the corresponding expected cash flows.

The number of outstanding claims at time $h + 1$ is given by:

$$A_{h+1} = A_h - P^A_{h,h+1} - D^A_{h,h+1} - S^A_{h,h+1} + R^A_{h,h+1} - T^{OUT}_{h,h+1} + T^{IN}_{h,h+1}, \quad (10)$$

where, A_h = number of outstanding claims at time h, $P^A_{h,h+1}$ = number of outstanding claims at time h, partially paid between h and $h + 1$. $S^A_{h,\,h+1}$ = number of outstanding claims at time h settled without payment between h and $h + 1$. $D^A_{h,\,h+1}$ = number of outstanding claims at time h, fully paid between h and $h + 1$. $T^{OUT}_{h,\,h+1}$ = number of outstanding claims at time h, transferred to another handling procedure between h and $h + 1$. $T^{IN}_{h,\,h+1}$ = number of outstanding claims at time h, transferred from another handling procedure between h and $h + 1$.

$R_{h,\,h+1} = R^D_{h,h+1} + R^S_{h,h+1}$ = number of claims reactivated between time h and $h + 1$ from fully paid and settled without payment.

The number (cumulative) of partially paid claims at time $h + 1$ is equal to:

$$P_{h+1} = P_h - D^P_{h,h+1} - S^P_{h,h+1} + P^A_{h,h+1} = P_h - C^P_{h,h+1} + P^A_{h,h+1}, \quad (11)$$

where, P_h = cumulative number of partially paid claims at time h. $D^P_{h,\,h+1}$ = number of partially paid claims at time h, fully paid between h and $h + 1$. $S^P_{h,\,h+1}$ = number of partially paid claims at time h, settled without payment between h and $h + 1$. $C^P_{h,\,h+1}$ = number of partially paid claims at time h, fully paid or settled without payment between h and $h + 1$.

The number of reserved claims at time $h + 1$ is given by:

$$\begin{aligned} N_{h+1} &= A_{h+1} + P_{h+1} = (A_h + P_h) - (D^A_{h,h+1} + D^P_{h,h+1}) - (S^A_{h,h+1} + S^P_{h,h+1}) + (P^A_{h,h+1} - P^A_{h,h+1}) + R_{h,h+1} - \\ &- T^{OUT}_{h,h+1} + T^{IN}_{h,h+1} = (A_h + P_h) - (D^A_{h,h+1} + D^P_{h,h+1}) - (S^A_{h,h+1} + S^P_{h,h+1}) + R_{h,h+1} - T^{OUT}_{h,h+1} + T^{IN}_{h,h+1} = \\ &= N_h - D_{h,h+1} - S_{h,h+1} + R_{h,h+1} - T^{OUT}_{h,h+1} + T^{IN}_{h,h+1}, \end{aligned} \quad (12)$$

where N_h indicates the number of reserved claims at time h.

The cumulative number of fully paid claims at time $h + 1$ is equal to:

$$D_{h+1} = D_h + D^A_{h,h+1} + D^P_{h,h+1} - R^D_{h,h+1}, \quad (13)$$

where $R^S_{h,h+1}$ is equal to the number of claims reactivated between h and $h + 1$ from those fully paid at time h.

The cumulative number of claims settled without payment at time $h + 1$ is given by:

$$S_{h+1} = S_h + S^A_{h,h+1} + S^P_{h,h+1} - R^S_{h,h+1}, \quad (14)$$

where $R^S_{h,h+1}$ is equal to the number of claims reactivated between h and $h + 1$ of those settled without payment at time h.

The cumulative number of claims settled at time $h + 1$ is equal to:

$$\begin{aligned} C_{h+1} &= D_{h+1} + S_{h+1} = D_h + D^A_{h,h+1} + D^P_{h,h+1} - R^D_{h,h+1} + \\ &+ S_h + S^A_{h,h+1} + S^P_{h,h+1} - R^S_{h,h+1} = D_h + S_h + D_{h,h+1} + \\ &+ S_{h,h+1} - R_{h,h+1}. \end{aligned} \quad (15)$$

The cumulative claims settled at the end of each year are used to calculate the annual number of claims reactivated the following year.

2.4. Mathematical structure: numerical flows. Once the probabilistic framework and expected numerical consistencies have been identified, the expected numerical flows of the variables of interest can be determined, taking into account all claim characteristics deemed representative (and for simplicity not specified in the above formulae).

By aggregating the homogeneous characteristics for outstanding claims at time h, the number of fully paid claims between h and $h + 1$ from outstanding claims is given by:

$$D^{A_i}_{h,h+1} = \sum_{Z=1}^{m} \sum_{T=1}^{s} \ldots \sum_{y=0}^{w-h-1} {}^{Z,T,\ldots,y}A^i_h \; {}^{Z,T,\ldots}q^{A_iD_i}_{y+h}, \quad (16)$$

where:

${}^{Z,T,\ldots}q^{A_iD_i}_{y+h}$ is the annual probability of closure due to full payment between h and $h + 1$ in handling procedure i at the balance sheet date h

occurring in n previous generations, distinguished by age of claim $y + h$ and relevant characteristics (Z, T ....); A^i_h represents the number of outstanding claims in handling procedure i at time h; Z represents the type of vehicle; T represents the geographic area of the policyholder facing a claim; y represents the number of years at time $h = 0$ that have elapsed since the accident; $h = 0$ represents the date when the evaluation is carried out; w is the maximum age of a claim (maximum number of years for the payment of a claim).

Equation (16) determines the number of outgoing claims with full payment in a year between h and $h + 1$, which in h are in the outstanding state, grouped by homogenous characteristics. Having set as the evaluation date $h = 0$, whenever we apply (16), we calculate the number of outgoing claims between 0 and 1 when $h = 0$, between 1 and 2 when $h = 1$, and so forth. Equation (16) can therefore be applied to each $h = 0, 1, 2, ... w - 1$.

The sum of the results obtained for each h from 0 to $w - 1$ allows quantifying at time 0 the total amount of outgoing claims with full payment at time 0 up to the last time considered, which is that of claims occurring at time 0 aged $y = 0$.

Figure 3 shows the time axis for a given generation of claims identified by age y.

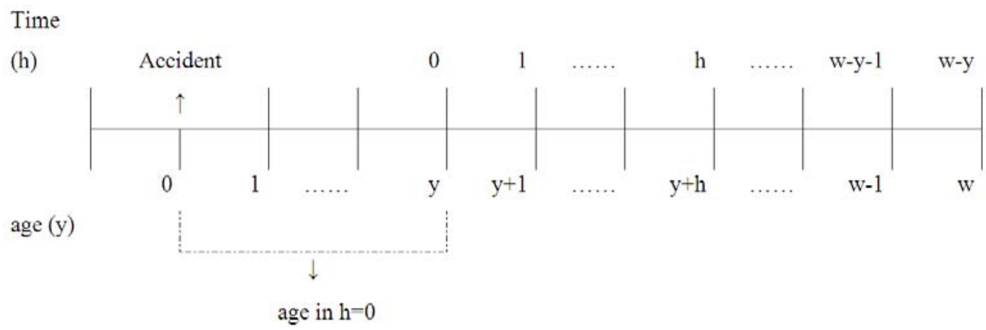


Fig. 3. Time axis of age y claims

The number of fully paid claims between h and h +1 of partially paid claims is given by:

$$D^{P_i}_{h,h+1} = \sum_{Z=1}^{m}\sum_{T=1}^{s} \dots \sum_{y=0}^{w-h-1} {}^{Z,T,\dots,y}P^i_h \; {}^{Z,T,\dots}q^{P_iD_i}_{y+h} \quad . \tag{17}$$

The number of partially paid claims between h and h +1 of outstanding claims is given by:

$$P^{A_i}_{h,h+1} = \sum_{Z=1}^{m}\sum_{T=1}^{s} \dots \sum_{y=0}^{w-h-1} {}^{Z,T,\dots,y}A^i_h \; {}^{Z,T,\dots}p^{A_iP_i}_{y+h} \quad . \tag{18}$$

The number of claims settled without payment between h and $h + 1$ of outstanding claims is given by:

$$S^{A_i}_{h,h+1} = \sum_{Z=1}^{m}\sum_{T=1}^{s} \dots \sum_{y=0}^{w-h-1} {}^{Z,T,\dots,y}A^i_h \; {}^{Z,T,\dots}q^{A_iS_i}_{y+h} \quad . \tag{19}$$

The number of claims settled without payment between h and $h + 1$ of partially paid claims is given by:

$$S^{P_i}_{h,h+1} = \sum_{Z=1}^{m}\sum_{T=1}^{s} \dots \sum_{y=0}^{w-h-1} {}^{Z,T,\dots,y}P^i_h \; {}^{Z,T,\dots}q^{P_iS_i}_{y+h}. \tag{20}$$

The number of reactivated claims between h and h +1 of fully paid claims is:

$$R^{D_i}_{h,h+1} = \sum_{Z=1}^{m}\sum_{T=1}^{s} \dots \sum_{y=1}^{w-h-1}\sum_{t=0}^{y-1} {}^{Z,T,\dots,y,t}D^i_h \; {}^{Z,T,\dots,t}p^{D_iA_i}_{[t+h]+y+h-(t+h)} \tag{21}$$

To note in this case is that variable y (age of claim) cannot assume value zero since the claim before being reactivated must remain closed for at least a year given that we exclude the possibility of closure and reactivation in the same year. Furthermore, the maximum value that t can assume (number of years during which the claim remained closed) is equal to $y - 1$, which corresponds to the claim being closed the year after the accident and remaining closed until the evaluation date (see Figure 2).

The number of claims reactivated between h and $h + 1$ of claims settled without payment is given by:

$$R^{S_i}_{h,h+1} = \sum_{Z=1}^{m}\sum_{T=1}^{s} \dots \sum_{y=1}^{w-h-1}\sum_{t=0}^{y-1} {}^{Z,T,\dots,y,t}D^i_h \; {}^{Z,T,\dots,t}p^{S_iA_i}_{[t+h]+y+h-(t+h)}. \tag{22}$$

Indicating with $T^{OUT,i}_{h,h+1}$ the sum of the number of outgoing claims from handling procedure i between h and $h + 1$ and with k the total number of claim handling procedures, we obtain:

$$T^{OUT,i}_{h,h+1} = \sum_{Z=1}^{m}\sum_{T=1}^{s} \dots \sum_{y=0}^{w-h-1} {}^{Z,T,\dots,y}A^i_h \sum_{\substack{j=1 \\ j\neq i}}^{k} {}^{Z,T,\dots}p^{A_iA_j}_{y+h}. \tag{23}$$

Indicating with $T_{h,h+1}^{IN,i}$ the sum of the number of "incoming" claims in handling procedure i between h and $h+1$, we obtain:

$$T_{h,h+1}^{IN,i} = \sum_{Z=1}^{m}\sum_{T=1}^{s} \cdots \sum_{y=0}^{w-h-1}\sum_{\substack{j=1\\ j\neq i}}^{k} {}^{Z,T,\ldots,y}A_h^j \; {}^{Z,T,\ldots}p_{y+h}^{A_j A_i}. \quad (24)$$

2.5. Mathematical structure: cash flows. Determining the total amount for final payment or partial payment of the claims reserve at the evaluation date requires introducing the concept of average costs broken down by geographic area, vehicle type, generation, type of claim handling procedure and so forth.

If we denote with ${}^{Z,T,\ldots}c_y^i$ the average cost broken down by geographic area, vehicle type, generation, claim handling procedure etc., the cost of final payments between h and $h+1$ h of outstanding claims at time h is given by:

$$F_{h,h+1}^{A_i D_i} = \sum_{Z=1}^{m}\sum_{T=1}^{s} \cdots \sum_{y=0}^{w-h-1} {}^{Z,T,\ldots,y}A_h^i \; {}^{Z,T,\ldots}q_{y+h}^{A_i D_i} \; {}^{Z,T,\ldots}c_{y+h}^i. \quad (25)$$

The cost of partial payments between h and $h+1$ of outstanding claims in h is given by:

$$F_{h,h+1}^{A_i P_i} = \sum_{Z=1}^{m}\sum_{T=1}^{s} \cdots \sum_{y=0}^{w-h-1} {}^{Z,T,\ldots,y}A_h^i \; {}^{Z,T,\ldots}q_{y+h}^{A_i P_i} \; {}^{Z,T,\ldots}c_{y+h}^i \; {}^{Z,T,\ldots}\lambda_{y+h}^i, \quad (26)$$

where ${}^{Z,T,\ldots}\lambda_{y+h}^i$ represents, for each homogeneous grouping, the percentage of the average cost once a claim has been partially paid.

The cost of the final payment between h and $h+1$ h for claims that at time t were in the partially paid state is given by:

$$F_{h,h+1}^{P_i D_i} = \sum_{Z=1}^{m}\sum_{T=1}^{s} \cdots \sum_{y=0}^{w-1} {}^{Z,T,\ldots,y}P_h^i \; {}^{Z,T,\ldots}q_{y+h}^{P_i D_i} \; {}^{Z,T,\ldots}c_{y+h}^i \left(1 - {}^{Z,T,\ldots}\lambda_{y+h}^i\right). \quad (27)$$

The $(1-{}^{Z,T,\ldots}\lambda_{y+h}^i)$ component is for each element the ones' complement of a percentage paid on the average cost once a claim has been partially paid to take into account that the remainder is fully paid with the second payment and thus closing the claim. The assumption is therefore that no more than two payments are made on the same claim.

2.6. Calculating the total claims reserve. The claims reserve at time $h = 0$ is represented by the sum of partial payments and full payments that will be made from time h up to the total termination of claims in reserve at the evaluation date.

Therefore, for each claim handling procedure we can write:

$$R_0^i = \sum_{h=0}^{w-1} F_{h,h+1}^{A_i D_i} + \sum_{h=0}^{w-1} F_{h,h+1}^{A_i P_i} + \sum_{h=0}^{w-1} F_{h,h+1}^{P_i D_i} = \sum_{h=0}^{w-1} F_{h,h+1}. \quad (28)$$

The total claims reserve at the final (non-discounted) cost at time $h = 0$ for all claim handling procedures is given by:

$$RT_0 = \sum_{i=1}^{k} R_0^i. \quad (29)$$

Consistently with the Solvency II regulations, the best estimate of the claims reserve can be defined as the present value at the evaluation date of the expected value of the claims reserve at final cost (GCAE, 2008; and IAA, 2009). We can use efficient market theory to obtain a formal definition of the fair value of a cash flow. Let $v(0, h+1) = [1 + i(0, h+1)]^{-(h+1)}$ be the market price at time 0 of a unitary zero coupon bond maturing in h and not affected by default risk. Thus, i (0, h +1) represents the risk-free rate.

If we denote future settlements with $F_{h,h+1}$, their expected present value (denoted with BE) can be calculated by simply including the discount rates as follows:

$$BE_0 = \sum_{h=0}^{w-1} F_{h,h+1} v(0, h+1). \quad (30)$$

In general, for any h:

$$BE_h = \sum_{t=h}^{w-1} F_{t,t+1} v(h, t+1). \quad (31)$$

If we assume uniform distribution of payments in the year, we can write (30) as follows:

$$BE_0 = \sum_{h=0}^{w-1} F_{h,h+1} v(0, h + \frac{1}{2}). \quad (32)$$

Likewise, (31) becomes:

$$BE_h = \sum_{h=t}^{w-1} F_{t,t+1} v(h, t + \frac{1}{2}). \quad (33)$$

2.7. Advantages of the model. The methodology proposed in this work has the following advantages:

- clarification of all hypotheses underlying the model;
- transparency;
- objectivity;
- opportunity to perform stress tests.

The subjectivity of traditional statistical methods to calculate the claims reserve is noted. Indeed, analyses generally undertaken in actuarial audits are based on the value allocated in the company's financial statements calculated with the inventory method and based on loss adjuster estimates. This does not imply that the evaluation is solely based on the budgetary reserve, but is certainly influenced by it and may affect the choice of method used or the time series considered to calculate the factors to be used in the projection of future costs.

Consider for instance the difficulties that the evaluator may encounter when independently providing a restricted range of acceptable results and the differences that could emerge if the value allocated in the financial statement is not known.

This is not the case in calculating the mathematical reserves in life insurance since the assumptions are explicit and above all representative of the real phenomena affecting the company's payments, thus making it is easier to verify the adequacy of the parameters used.

Therefore, the methodology, clarification of parameters and their benchmarks render the reserve calculation procedure more transparent and clear to third parties. In addition, clarifying the parameters also enables investigating the sensitivity of the claims reserve to the parameters used to identify those that are most significant for the evaluation and hence to be monitored and controlled. A sensitivity analyses can also provide important information on the performance of the claims reserve.

All this is not achievable with the statistical methods used in non-life insurance to calculate the claims reserve since stress tests and sensitivity analyses of parameters that describe the actual claim settlement mechanism are not always possible nor controlling for how one parameter can affect the value of the claims reserve and thereby verify the "danger" of the parameter itself (for example, the Chain Ladder method loses sight of the claim characteristics).

Furthermore, the hypotheses underlying claims are not known, such as the percentage of reactivated claims or those settled without payment, and parameters differentiated by vehicle, geographic area, etc. To also be noted is that factors calculated with triangulation methods may not be very significant since these can be based on a single year (as is the case for the last year of development of the triangle).

Clarification of the parameters can render the calculation of the claims reserve transparent. Moreover, reimbursement performance is viewed from a more realistic perspective by following the claim for a long enough period of time, using assumptions that can be very close to those used by the loss adjuster – or at least better reflect their performance – and thus incorporating the loss adjuster's experience.

3. Numerical application

3.1. Input data description and scope. The purpose of the following numerical application is to calculate the claims reserve of motor TPL based on the model described in the previous sections.

We use the Chain Ladder method to evaluate the differences with respect to a traditional reserve method. We first show the results obtained with a traditional model and thereafter those obtained with the proposed model. For simplification, the proposed model will be indicated with the acronym NM (new method) and the Chain Ladder method with CL.

Consider a medium-sized company engaged in Ministerial Segment 10 motor TPL insurance and the evaluation date of 31.12.2011.

The database for the CL method is constituted by the triangle of incremental amounts paid, including payments related to pre 2007 generations and the non-CARD, handler CARD and debtor CARD procedures for 2007 to 2011 generations.

The upper triangles of incremental payments and corresponding cumulative payments are reported respectively in Table 1 and Table 2 (the monetary amounts are expressed in thousands of euro).

Table 1. Incremental payments

Development year (y) / Accident year (g)	0	1	2	3	4	5	6	7	8	9	10
2001	26.800	28.609	10.863	5.328	3.826	2.688	3.502	1.979	941	1.027	2.688
2002	22.895	25.125	9.301	3.734	2.498	1.486	1.962	1.726	1.459	969	-
2003	23.047	24.650	7.966	4.111	1.969	3.643	2.220	1.088	1.466	-	-
2004	26.442	25.586	11.021	4.323	3.421	4.028	2.294	1.208	-	-	-
2005	26.370	24.734	10.200	7.011	3.872	2.347	1.429	-	-	-	-
2006	28.341	29.040	10.143	6.770	4.721	3.551	-	-	-	-	-
2007	30.406	33.854	14.144	5.679	3.761	-	-	-	-	-	-
2008	43.636	37.924	12.146	6.591	-	-	-	-	-	-	-
2009	44.920	38.464	11.285	-	-	-	-	-	-	-	-
2010	47.479	44.283	-	-	-	-	-	-	-	-	-
2011	48.112	-	-	-	-	-	-	-	-	-	-

Table 2. Cumulative payment

Development year (*y*) / Accident year (*g*)	0	1	2	3	4	5	6	7	8	9	10
2001	26.800	55.409	66.272	71.600	75.426	78.114	81.616	83.595	84.536	85.563	88.251
2002	22.895	48.020	57.321	61.055	63.553	65.039	67.002	68.728	70.187	71.156	
2003	23.047	47.697	55.663	59.774	61.743	65.386	67.606	68.693	70.160		
2004	26.442	52.029	63.050	67.372	70.793	74.821	77.116	78.324			
2005	26.370	51.104	61.304	68.315	72.187	74.535	75.963				
2006	28.341	57.381	67.524	74.294	79.015	82.566					
2007	30.406	64.260	78.405	84.084	87.845						
2008	43.636	81.560	93.705	100.297							
2009	44.920	83.384	94.669								
2010	47.479	91.762									
2011	48.112										

3.2. Chain Ladder method results. To estimate the claims reserve with the CL method, we first calculate the development factors. The results are shown in Table 3.

Table 3. Development factors f_h

h	1	2	3	4	5	6	7	8	9	10
f_h	1.975	1.179	1.080	1.049	1.042	1.032	1.020	1.017	1.013	1.031

These development factors are used to complete the bottom of the input data triangle and the results are shown in Table 4.

Table 4. Estimated cumulative payments

Development year (*y*) / Accident year (*g*)	0	1	2	3	4	5	6	7	8	9	10
2001											
2002											73.391
2003										71.065	73.297
2004									79.694	80.722	83.258
2005								77.517	78.873	79.891	82.400
2006							85.198	86.941	88.462	89.603	92.418
2007						91.532	94.450	96.382	98.068	99.333	102.453
2008					105.259	109.677	113.173	115.488	117.508	119.024	122.763
2009				102.258	107.317	111.821	115.385	117.746	119.806	121.351	125.163
2010			108.231	116.907	122.691	127.841	131.916	134.614	136.969	138.736	143.094
2011		95.014	112.066	121.050	127.039	132.371	136.590	139.384	141.823	143.652	148.165

Table 5 shows the estimated incremental payments.

Table 5. Estimated incremental payments

Development year (*y*) / Accident year (*g*)	0	1	2	3	4	5	6	7	8	9	10
2001											
2002											2.235
2003										905	2.233
2004									1.370	1.028	2.536
2005								1.554	1.356	1.017	2.510
2006							2.632	1.743	1.521	1.141	2.815
2007						3.687	2.917	1.932	1.686	1.265	3.121
2008					4.962	4.418	3.496	2.315	2.020	1.516	3.739
2009				7.589	5.059	4.505	3.564	2.361	2.060	1.545	3.812
2010			16.469	8.676	5.784	5.150	4.075	2.699	2.355	1.767	4.358
2011		46.901	17.053	8.983	5.989	5.332	4.219	2.794	2.438	1.829	4.513

We calculate the non-discounted claims reserve at the evaluation date (in compliance with Italian law) and the discounted reserve according to Solvency II. To discount the claims reserve, we use the risk-free rates estimated with the bootstrapping method starting from the Euro IRS yield curve as at 30.12.2011 and shown in Table 6.

Table 6. Risk free interest rates – Euro IRS 30.12.2011

t	$i(0, t)$
1	1.440%
2	1.320%
3	1.390%
4	1.560%
5	1.760%
6	1.930%
7	2.080%
8	2.200%
9	2.320%
10	2.400%
12	2.550%
15	2.680%
20	2.700%
25	2.640%
30	2.570%
40	2.560%
50	2.580%

Table 7 shows the values of the estimated discounted incremental payments to take proper account of the timing of payments.

Table 7. Discounted incremental payments

Development year (y) / Accident year (g)	0	1	2	3	4	5	6	7	8	9	10
2001											
2002											2.204
2003										892	2.175
2004									1.351	1.001	2.433
2005								1.532	1.321	976	2.359
2006							2.594	1.698	1.459	1.072	2.578
2007						3.635	2.842	1.854	1.584	1.159	2.779
2008					4.892	4.304	3.354	2.176	1.850	1.350	3.231
2009				7.481	4.928	4.322	3.349	2.162	1.834	1.335	3.194
2010			16.235	8.451	5.549	4.840	3.732	2.404	2.035	1.480	3.531
2011		46.236	16.611	8.619	5.628	4.884	3.758	2.415	2.043	1.482	3.542

The claims reserve values are reported by year of accident (Table 8) and by financial year (Table 9).

Table 8. Non-discounted and discounted CL claims reserve by accident year

Accident year	Non-discounted claims reserve	Discounted claims reserve
2002	2.235	2.204
2003	3.137	3.067
2004	4.934	4.785
2005	6.437	6.188
2006	9.851	9.402
2007	14.608	13.853
2008	22.466	21.157
2009	30.494	28.607
2010	51.332	48.258
2011	100.052	95.218

Table 9. Non-discounted and discounted CL claims reserve by financial year

Financial year	Non-discounted claims reserve	Discounted claims reserve
2012	88.305	87.051
2013	44.483	43.332
2014	29.774	28.566
2015	22.355	21.009
2016	17.868	16.366
2017	13.614	12.125
2018	10.434	9.016
2019	8.017	6.718
2020	6.188	5.014
2021	4.513	3.542

Table 10 shows the total non-discounted and discounted claims reserve.

Table 10. Total non-discounted and discounted claims reserve

Non-discounted claims reserve	245.549
Discounted claims reserve	232.739

To obtain an estimate of the final cost of the claims reserve taking into account the entire claims settlement process, this application uses an estimation model to determine the so-called tail factor.

To estimate the tail factor, the quantitative impact study 5 model is used (EIOPA, 2010). For further details, we refer the reader to the QIS5 document "Best-Estimate User Guide".

According to this model, the tail factor estimate is made through a log-linear regression analysis conducted on the development factors. In particular, the objective is to fit the factors resulting from the CL method based on the assumption that development factors follow an exponential path that decreases to the variation of the generic development year y.

Table 11 shows the values of the claims reserve by year of accident, corresponding to payments beyond the years covered by the CL model.

Table 11. Tail reserve by discounted and non-discounted generation

Accident year	Non-discounted tail reserve	Discounted tail reserve
2001	2.568	2.383
2002	2.135	1.942
2003	2.133	1.893
2004	2.423	2.094
2005	2.398	2.012
2006	2.689	2.189
2007	2.981	2.352
2008	3.572	2.729
2009	3.642	2.694
2010	4.164	2.983
2011	4.311	2.992

Table 12 shows the value of the total claims reserve also considering the tail.

Table 12. Total discounted and non-discounted claims reserve

Non-discounted CL claims reserve + non-discounted tail	278.563
Discounted CL claims reserve + discounted tail	259.003

3.3. CL application with differentiated analysis by claim handling procedure. The value of the claims reserve obtained in Table 12 does not however enable fully grasping the phenomena that affect the typical settlement dynamics of different claim handling procedures.

In light of these observations, a differentiated analysis is performed by claim handling procedure using the following input data:

- pre 2007 generation trapezium and 2007-2011 generation triangle for the Non-CARD (NC) procedure;
- 2007-2011 generation triangle for the Handler CARD (HC) procedure;
- 2007-2011 generation triangle for the Handler Forfait (HF) procedure;
- 2007-2011 generation triangle for the Debtor CARD (DC) procedure.

For the NC handling procedure, the DC method is applied separately to the 2001-2006 trapezium and to the 2007-2011 triangle.

For the 2007-2011 HC, HF and DC handling procedures, the value of the claims reserve is obtained by applying the DC method to the triangles referred to in points b), c) and d).

The CL method applied to the aforementioned triangles provides the estimate of payments to be made until the fourth year of development. However, given the lack of sufficiently representative data for the claim handling procedure analyzed, an analogy with those characterizing claims generated prior to CARD is deemed plausible since the effect is negligible in terms of improving the settlement speed for claims reimbursed after the fourth year of development. In line with this assumption, the link-ratios determined on the NC handling procedure trapezium in relation to the 2001-2006 generations are applied for 5-10 development years.

The results are shown in Table 13 broken down by claim handling procedure.

Table 13. Total claims reserve by non-discounted and discounted handling procedure

	Non-discounted claims reserve	Discounted claims reserve
NC 2001-2006	26.595	25.645
NC post 2007	98.523	93.506
HC 2007-2011	90.138	84.298
HF 2007-2011	62.299	57.777
DC 2007-2011	65.735	61.303
Total	218,692	206.974

Therefore, also considering the tail, calculated as described in section 3.2, we obtain the results shown in Table 14.

Table 14. Total claims reserve – non-discounted and discounted

Non-discounted CL claims reserve + non-discounted tail	251.706
Discounted CL claims reserve + non-discounted tail	233.239

Finally, since the input data available only allow using the CL method for a certain number of generations, the value of the claims reserve calculated by the insurance company for pre 2001 generations is assumed valid and equal to 16.561 thousand euro.

Table 15 shows the values of the total claims reserve including pre 2001 generations for the evaluations referred to in sections 3.2 and in 3.3. In particular, the total non-discounted reserve represents the provision for claims determined in line with Italian legislation, while the discounted reserve represents the best estimate (BE).

Table 15. Total claims reserve

	CL on total triangle	CL on differentiated triangles
Non-discounted CL claims reserve + Non-discounted tail + pre 2001	295.125	268.268
Discounted CL claims reserve + Discounted tail + Pre 2001	275.565	249.800

3.4. The proposed evaluation method: hypotheses and input data. This section sets out the hypotheses and input data used in the new method proposed. To avoid encumbering the application, we make approximations with respect to the extended model, which we in turn clarify.

Each claim is classified taking into account the year of accident, the year of development and the relative claim handling procedure at the evaluation date (NC, HC, HF and DC), hence the characteristics referred to in points a), b) and d) as described in section 2.2 (geographic area, type of vehicle, etc.) and disregarding the remaining characteristics.

With regard to the transition states, the following hypotheses apply:

- handler CARD claims can move to non-CARD;
- CARD claims once closed are not reactivated;
- reactivated claims are calculated based on the consistency of outstanding claims.

Furthermore:

- the overall number of claims are positioned at the beginning of the year;
- the payment amounts are positioned at the end of the year.

Table 16 shows the input data grouped by similar characteristics.

Table 16. Input data

Accident year	y	Claim handling procedure			
		NC	DC	HC	HF
2011	0	3.649	6.557	6.076	3.803
2010	1	2.211	2.342	1.671	700
2009	2	1.587	1.527	965	545
2008	3	1.081	408	555	343
2007	4	693	123	274	114
2006	5	618	0	0	0
2005	6	348	0	0	0
2004	7	285	0	0	0
2003	8	189	0	0	0
2002	9	151	0	0	0
2001	10	123	0	0	0
2000	11	100	0	0	0
N-12 and prec.	12	220	0	0	0

Table 16 shows the numerical consistency of the outstanding claims reserve at 31.12.2011 by claim handling procedure and age of accident.

As can be seen for the years prior to 2007, the claims reserve relating to the new CARD settlement procedure are equal to zero.

3.5. Results of the new method. To give an idea of the results obtained from the data elaboration, Table 17 (see Appendix) shows an example in relation to the NC handling procedure. In particular, this Table is the result of the relations set out in Sections 3.2. and 3.3. including the numerical evolution of the 11.255 outstanding claims in the NC handling procedure at 31.12.2011 for all generations.

Table 18 shows the corresponding evolution of the cash flows for each year of projection according to the application of the relations in Section 3.4. The last column of the table shows the estimated total discounted payments, the sum of which for all years is the value of the discounted claims reserve at 31.12.2011 for the non-CARD handling procedure alone.

Similarly, the model provides the same output for the other three claim handling procedures.

The discounted value of the claims reserve by handling procedure or age of claim is shown in Table 19. The last row of the Table shows the total by claim handling procedure. The value thus obtained for the total claims reserve in present value terms is €215.941, equal to the sum of the claims reserve relating to the first three handling procedures less the claims reserve relating to the handler forfait procedure. Furthermore, Table 20 shows the total claims reserve for the four distinct claim handling procedures by claim generation.

Table 19. Claims reserve by generation and claim handling procedure (discounted)

y	Claim handling procedure			
	NC	DC	HC	HF
0	29.539	15.108	14.053	8.646
1	26.726	6.241	4.075	1.703
2	26.943	4.468	2.560	1.529
3	21.130	1.084	1.460	878
4	15.380	353	740	300
5	14.995	29	-	-
6	9.136	16	-	-
7	8.070	14	-	-
8	5.761	9	-	-
9	4.885	7	-	-
10	4.210	6	-	-
11	3.612	5	-	-
12	8.373	11	-	-
Total	178.760	27.350	22.887	13.056

Table 20. Total claims reserve by generation (discounted)

y	Discounted claims reserve
0	50.054
1	35.338
2	32.442
3	22.795
4	16.173
5	15.024
6	9.153
7	8.084
8	5.770
9	4.892
10	4.216
11	3.617
12	8.383
Total	215.941

To compare the value of the claims reserve determined with the traditional CL method, the estimated amount for claims incurred but not yet reported (IBNR) must be added to the value of the claims reserve obtained with the proposed model. Applying the CL method, the value of the estimated total claims reserve already includes the estimate for IBNR claims since the run-off triangle of cumulative payments used as input also considers the payments made for these type of claims over the years. Conversely, the proposed model does not consider the estimation of the IBNR component but instead includes the "tail" evaluation as well as the evaluation of pre 2001 generations.

The estimation of late claims foresees, according to ISVAP Regulation n.16/08, a separate projection of the number of claims and the average cost of the generation based on historical analyses and on recent legislative changes. The value used is the company's estimate at 31.12.2011 equal to 18.245 euro.

Table 21. Total discounted claims reserve (including IBNR)

Discounted claims reserve + IBNR	234.176

The value in Table 21 thus obtained is comparable to the discounted values obtained with the traditional method as per Table 15.

Conclusions

In this work, an internal model is developed and adapted to the evaluation of the claims reserve for an insurance company operating in motor TPL insurance. An actual company is considered and the settlement mechanisms are analyzed following the introduction of the direct reimbursement scheme.

The first consideration concerns the entire process of determining the liability. The best estimate plus the risk margin should lead to a "prudent" liability evaluation, although recently the tendency has been towards "economic" rather than "prudent" evaluations.

The term prudent should therefore be understood as "standard" prudence, in the sense that the overall evaluation should be such that it can be compared in space (in the so-called global village) and in time in relation to several financial years.

Applying a "prudent" amount based on common assumptions to all the settlement mechanisms in every country in the world and at all times does not render the items comparable since future attainments may be very different for each company.

The results obtained show that the best estimate calculation is particularly sensitive to the method used and the hypotheses underlying it.

In this perspective (not using complex mathematical-statistical structures), the proposed model has the objective of evaluating the fair value of the claims reserve through a transparent calculation based on the loss adjuster's experience and the company's knowledge. This enables following the evolution of claims in the different handling procedures over a long time horizon and analyzing ex-post deviations of actual cash flows compared to the expected values while identifying the variables that lead to such differences.

The diversity of the results obtained with the different models compared to the proposed model clarifies the need for judiciousness in choosing a model to represent the reality.

In addition, the possibility of effecting a sensitivity analysis on the different parameters underlying the proposed model enables quantifying all the typical insurance risks not only related to the choice of model but also to an incorrect estimation of the basic parameters of the model.

The results obtained in Table 21 when compared to those in Table 15 demonstrate that an individual model thus constructed can capture the characteristics of a given line of business and a particular company with all its relative specificities in terms of the claim settlement phenomenon, which is fundamental to calculating the best estimate of the claims provision.

Our model may therefore provide the basis to construct and implement an internal model to calculate the claims provision and the necessary capital requirements according to the EIOPA Solvency II regulations.

Indeed, a subsequent aim of our study is to improve the stochastic model and the quantification of solvency capital requirements (SCR) and compare this to the standard formula used by EIOPA.

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Appendix

Table 17. Non-CARD claim handling procedure numerical evolution

Evaluation date	Initial no.	Initial no. of outstanding claims (net of partially paid)	Final no. of partially paid claims	No. of partially paid claims in the year	No. of fully paid settled claims (from open)	No. of fully paid settled claims (from partially paid)	No. of claims settled without payment (from open)	No. of claims settled without payment (from partially paid)	Reactivated in the year	Incoming from other handling procedures	Outgoing towards other handling procedures	Final no.	Final no. outstanding claims (net of partially paid)
31/12/2011												11.255.000	11.255.000
31/12/2012	11.255.00	11.255.00	0.00	956.68	3.126.64	0.00	1.626.14	0.00	394.05	8.43	60.14	6.844.57	5.887.89
31/12/2013	6.844.57	5.887.89	956.68	500.47	1.722.49	288.65	480.88	87.57	354.27	6.25	44.67	4.580.82	3.499.90
31/12/2014	4.580.82	3.499.90	1.080.92	297.49	921.77	290.84	175.44	58.49	272.08	5.36	17.50	3.394.22	2.365.14
31/12/2015	3.394.22	2.365.14	1.029.08	201.04	597.91	263.77	98.78	44.56	195.66	2.66	11.83	2.575.70	1.653.91
31/12/2016	2.575.70	1.653.91	921.79	140.58	401.98	224.75	61.68	35.02	146.24	1.92	8.27	1.992.16	1.189.55
31/12/2017	1.992.16	1.189.55	802.60	101.11	285.49	192.62	41.20	28.28	110.90	1.43	5.95	1.550.94	868.13
31/12/2018	1.550.94	868.13	682.81	73.79	208.35	163.87	27.17	21.72	84.01	1.07	4.34	1.210.57	639.56
31/12/2019	1.210.57	639.56	571.01	54.36	153.49	137.04	17.78	16.03	63.73	0.82	3.20	947.57	475.27
31/12/2020	947.57	475.27	472.30	40.40	114.07	113.35	11.46	11.39	47.87	0.62	2.38	743.43	355.47
31/12/2021	743.43	355.47	387.96	30.22	85.31	93.11	7.87	8.58	36.50	0.48	1.78	583.75	267.27
31/12/2022	583.75	267.27	316.48	22.72	64.14	75.96	5.23	6.19	27.50	0.37	1.34	458.77	201.71
31/12/2023	458.77	201.71	257.05	17.15	48.41	61.69	3.50	4.46	20.44	0.29	1.01	360.42	152.37
31/12/2024	360.42	152.37	208.05	12.95	36.57	49.93	2.29	3.12	15.26	0.22	0.76	283.23	115.28
31/12/2025	283.23	115.28	167.95	9.80	27.67	40.31	1.73	2.52	11.33	0.17	0.58	221.93	87.01
31/12/2026	221.93	87.01	134.92	7.40	20.88	32.38	1.31	2.02	8.88	0.13	0.44	173.91	66.00
31/12/2027	173.91	66.00	107.91	5.61	15.84	25.90	0.99	1.62	6.96	0.10	0.33	136.29	50.29
31/12/2028	136.29	50.29	86.00	4.27	12.07	20.64	0.75	1.29	5.45	0.08	0.25	106.81	38.47
31/12/2029	106.81	38.47	68.35	3.27	9.23	16.40	0.58	1.03	4.27	0.06	0.19	83.72	29.53
31/12/2030	83.72	29.53	54.19	2.51	7.09	13.01	0.44	0.81	3.35	0.05	0.15	65.62	22.74
31/12/2031	65.62	22.74	42.88	1.93	5.46	10.29	0.34	0.64	2.62	0.04	0.11	51.43	17.55
31/12/2032	51.43	17.55	33.88	1.49	4.21	8.13	0.26	0.51	2.06	0.03	0.09	40.31	13.58
31/12/2058	0.09	0.03	0.06	0.00	0.01	0.02	0.00	0.00	0.00	0.00	0.00	0.07	0.02
31/12/2059	0.07	0.02	0.05	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.06	0.02
31/12/2060	0.06	0.02	0.04	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.04	0.01
31/12/2061	0.04	0.01	0.03	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.03	0.01

Table 18. Non-CARD claim handling procedure cash flow evolution

Evaluation date	Partial payments in the year	Cash flows for full payments (from outstanding)	Cash flows for full payments (from partially paid)	TOTAL non-discounted cash flow	TOTAL discounted cash flow
31/12/2011					
31/12/2012	6.421.62	41.503.68	-	47.925.31	47.244.98
31/12/2013	3.706.68	24.352.49	1.916.05	29.975.22	29.199.72
31/12/2014	2.652.04	16.057.99	2.399.74	21.109.77	20.253.16
31/12/2015	2.058.76	12.070.09	2.570.09	16.698.94	15.693.62
31/12/2016	1.630.55	9.291.65	2.541.25	13.463.46	12.331.96
31/12/2017	1.312.69	7.412.85	2.468.09	11.193.64	9.969.62
31/12/2018	1.064.93	6.013.74	2.348.11	9.426.79	8.146.42
31/12/2019	867.70	4.899.97	2.181.84	7.949.52	6.660.96
31/12/2020	710.04	4.009.63	1.994.24	6.713.91	5.440.07
31/12/2021	582.45	3.289.10	1.801.33	5.672.88	4.452.58
31/12/2022	478.72	2.703.38	1.609.83	4.791.93	3.638.86
31/12/2023	393.82	2.223.91	1.427.47	4.045.19	2.966.65
31/12/2024	323.51	1.826.91	1.257.33	3.407.76	2.420.95
31/12/2025	265.66	1.500.19	1.101.37	2.867.22	1.971.10
31/12/2026	217.10	1.225.96	956.63	2.399.68	1.594.62
31/12/2027	177.84	1.004.24	825.30	2.007.38	1.298.08
31/12/2028	145.96	824.24	707.84	1.678.03	1.055.85
31/12/2029	119.98	677.52	603.97	1.401.46	857.96
31/12/2030	98.72	557.50	512.94	1.169.16	696.30
31/12/2031	81.26	458.88	433.65	973.79	564.13
31/12/2032	66.87	377.64	365.03	809.55	458.18
31/12/2033	54.95	310.28	305.69	670.91	371.09
31/12/2034	45.06	254.45	254.73	554.24	299.68
31/12/2035	36.83	207.98	211.05	455.87	241.04
31/12/2036	29.96	169.20	173.68	372.85	192.85
31/12/2037	24.28	137.11	142.12	303.52	153.73
31/12/2038	19.59	110.62	115.62	245.83	121.97
31/12/2039	15.82	89.33	94.02	199.17	96.84
31/12/2040	12.78	72.19	76.42	161.40	76.93
31/12/2041	10.34	58.38	62.10	130.82	61.16
31/12/2042	8.36	47.23	50.45	106.05	48.36
31/12/2043	6.77	38.23	40.98	85.98	38.24
31/12/2044	5.48	30.96	33.28	69.72	30.25
31/12/2045	4.44	25.08	27.02	56.54	23.93
31/12/2046	3.60	20.32	21.94	45.86	18.93
31/12/2047	2.92	16.47	17.81	37.19	14.98

Table 18 (cont.). Non-CARD claim handling procedure cash flow evolution

Evaluation date	Partial payments in the year	Cash flows for full payments (from outstanding)	Cash flows for full payments (from partially paid)	TOTAL non-discounted cash flow	TOTAL discounted cash flow
31/12/2048	2.36	13.35	14.46	30.17	11.85
31/12/2049	1.92	10.82	11.74	24.47	9.38
31/12/2050	1.67	9.43	10.24	21.34	7.98
31/12/2051	1.26	7.13	7.75	16.14	5.89
31/12/2052	1.02	5.75	6.24	13.01	4.62
31/12/2053	0.82	4.62	5.03	10.47	3.62
31/12/2054	0.66	3.72	4.04	8.41	2.83
31/12/2055	0.54	3.03	3.29	6.86	2.25
31/12/2056	0.43	2.40	2.61	5.44	1.74
31/12/2057	0.36	2.03	2.20	4.59	1.43
31/12/2058	0.27	1.54	1.67	3.47	1.05
31/12/2059	0.22	1.24	1.34	2.81	0.83
31/12/2060	0.17	0.98	1.06	2.21	0.63
31/12/2061	0.05	0.28	0.30	0.62	0.17