Step-by-Step Distribution of Investments
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1. Introduction

The main task of an investing organization is forming an optimum portfolio of investments. Further this set passes updating during activity of the organization both on structure and on parameters of investing the projects. As a result, providing optimum activity of the organization in the sphere of investment is reached.

The automatisation task in investing activity of the organization is connected with the development of a complex of mathematical models of management, including step-by-step decision-making methods. These methods are especially effective because they provide more flexible and adaptive activity of the organization.

In addition, the task is the reception of adequate information for preparing of administrative decisions. Thus expert, formal – logic, qualitative and quantitative methods are used for the analysis and estimations of alternative investment projects. In particular, it is necessary to analyze the amount of the organization investments, expense on each of investments, length of the period on each of investments, function and volume of return on income or of investment income (incomes returning on each of the investments), etc. The organization should take into account the presence of the volume limited from above onestep (i.e. "given out" in the beginning of the planning period) investments too.

Existing approaches to efficient investments management can be classified into the following groups:

- Modification of previous investing plans. This means the updating of optimum activity of the organization and updating the program of investment projects during the next period of time. Thus the preset constant step of definition of the moment of decision-making is accepted.
- Perfection of criterion functions for optimization of a portfolio of investments for a case dynamic multystep models of investment programs. Criterion functions thus have more complex character: they differ in the presence of growing number of restrictions, and also decision-making under conditions of uncertainty.
- Choice of a portfolio of investments by formalized criteria.
- Choice of a portfolio of investments by non-formalizable criteria group.

The capital attraction demands a correct and complex inspection of projects due to the fact that the capital attraction in the sphere of real investments is connected with a high level risk.

The method of development of the capital budget more often is used for an estimation of efficiency of capital investments in investment programs. It includes the quantitative and qualitative aspects providing:

- Search of variants of capital investments;
- Estimation and ranking of variants;
- Choice of better variants;
- The control for realization of capital investments and periodic decision-making.

Search of variants of capital investments, estimation and ranking of variants, the control for realization of capital investments on the part of the organization are the key procedures of management by investment projects and programs.

We shall consider the developed step-by-step decision-making model based upon the use of several optimization criteria.
2. Terminology

Enter the following designations:

a) For onestep investments:

\{U_i\} – set of investment projects (investments) \(U_i, 1 \leq i \leq N\);

\{U_i\}_1^k – set of the investment projects ordered by any criterion \(k, 1 \leq i \leq N\);

\(C\) – the volume of capital investments limited from above under investments;

\(N\) – quantity of investments \(U_i\);

\(C_i\) – volume (cost) of investment \(U_i\). We shall accept, that for each of investments \(U_i\) the investment of means is carried out at the beginning of period of the investment;

\(L_i\) – length of the period of investment \(U_i\);

\(D_i(t)\) – function of incomes return of investment \(U_i, 1 \leq i \leq N\); (time \(t\) in function \(D_i(t)\) is usual is broken on the periods with constant step, for example: month, year)

\[D_i = \sum_{t=0}^{L_i} D_i(t)\] – volume of incomes return;

b) For multystep investments:

\(C(t_j)\) – limited from above cost volume onestep (i.e. sold at the beginning of the period) investments at the moment \(t_j\);

\(C^*\) – the volume of the current liquid means of the organization limited from below;

\(\Delta t\) – a preset constant step of definition of the moment of decision-making \(t_j = t_{j-1} + \Delta t\);

\{U_i(t_j)\} – a set of investments \(U_i, 1 \leq i \leq N_j\), at the moment of decision-making \(t_j\);

\(N_j\) – total of possible investment projects at the \(t_j\);

\(k(T)\) – function of income expenditure to the beginning of investment \(t\), where \(T\) – length of an interval; differently \(C_t = C_{t+T} k(T)\), where \(C_t\) are expenses of the organization at the moment \(t\).

\(L_{ij}\) – length of the investment period \(U_i(t_j)\).

With a view of construction of the optimum plan of investments of the organization (step-by-step optimum strategy of the organization) it is necessary to use developed auxiliary distributive algorithms as described below.

3. Distributive investment algorithms

Algorithms offered below carry onestep character – to carry out distribution of means under investments at the fixed moment of time, i.e. in onestep a mode. Therefore with the purpose of simplification of the description we shall lower a time index \(j\), and also a symbol \(t_j\).

3.1. Distributive algorithm I on volume of income return \(D_i\)

This is required to make the decision on allocation of a quasi-optimum subgroup of investments which are necessary for financing on the basis of the task of the investments group of in a present situation of time \(t\) and restrictions from above on volume of capital investments, i.e. \(|U|\) and \(C\).

The task consists in the maximization of total return of incomes at restriction on the volume of capital investments \(C\). The same algorithm, which further will be designated as an algorithm \(I\), has the following step-by-step structure.

Stage 1. Order investment group during the reduction of value \(D_i\), that is we form group \(|U|^D\);

Stage 2. Since \(U1D\), to select \(W\) the first investments by order \(UiD\);

\[1 \leq i \leq W \leq N, \text{ for which:} \]

\[\sum_{i=1}^{W} C_i^D \leq C < \sum_{i=1}^{W+1} C_i^D. \] (1)
The first \( W \) investments are quasi-optimum, so they provide the decision of a task.

3.2. Distributive algorithm II under the relation of volume of income return to cost of the investment, i.e. to \( Y_i = D_i / C_i \)

The task is to maximize the total criterion \( Y \) by limiting the volume of \( C \).

The block diagram of Algorithm II is the following:

**Stage 1.** Order the group \( \{ U_i \} \) of investments by way of reduction of parameter \( Y \), by forming the group \( \{ U_i \}^Y \).

**Stage 2.** Select \( W \) the first investments since \( U_i^Y, 1 \leq i \leq W \leq N, U_i^Y \) for which

\[
\sum_{i=1}^{W-1} C_i^Y \leq \sum_{i=1}^{W} C_i^Y.
\]

The first \( W \) investments \( U_i^Y, 1 \leq i \leq W \), are the task decision.

3.3. Distributive algorithm III under the relation of total volume of incomes return for fixed period \( L \) to cost of the investment

The task consists in maximization of total criterion of relative by returning the incomes of period \( L \) at limiting of \( \mathcal{L} \).

Procedure of Algorithm III has the following form:

**Stage 1.** Define the total volume of the profit for each of the investment \( U_i \) during interval \( L \), that is \( D_i(L) = \sum_{t=0}^{L-1} D_i(t) \).

**Stage 2.** For each investment \( U_i \) we determine the parameter \( Y_i = D_i(L) / C_i \).

**Stages 3 and 4** coincide with stages 1 and 2 of algorithm II.

3.4. Distributive algorithm IV (combined), based on optimization by two criteria \( Y_i = D_i / C_i \) and \( L_i \)

Optimization task consists in a combination of two criteria at restriction on a financing volume. The appropriate algorithm has heuristic character.

Procedure of Algorithm IV has the following step-by-step type:

**Stage 1.** Order the set of investments \( \{ U_i \} \) by means of reduction of parameter \( Y \), that is it has the form \( \{ U_i \}^Y \);

**Stage 2.** Order set of investments \( \{ U_i \} \) in order to increase the parameter of length of the investment period \( L \) – it is formed the new ordered set of investments \( \{ U_i \}^L \);

**Stage 3.** Select in each of the ordered sets \( \{ U_i \}^Y \) and \( \{ U_i \}^L \) on \( M \) elements

\[
U_1^Y, U_2^Y, ..., U_M^Y, \text{ and } U_1^L, U_2^L, ..., U_M^L.
\]

The value of \( M \) is get by expert way (it grows with constant step \( \Delta M \), see stages 8-9);

**Stage 4.** Select a subgroup of investments including in both subsets (3) and (4). We shall name a number of crossed investments as a symbol \( Q \), and a subgroup – by a symbol \( \{ U_i \}^* \);

**Stage 5.** Order elements \( U_i^* \) by the first criterion \( Y_i = D_i / C_i \), so it is formed a subgroup \( \{ U_i \}^* \) from \( Q \) investments;

**Stage 6.** Check validity of inequality \( \sum_{j=1}^{Q} C_i^* > C \). If the inequality is real, we pass to the following stage. Otherwise we pass to stage 8;

**Stage 7.** Choice first \( W \) members of row \( U_i^*, 1 \leq i \leq W \), satisfying to an inequality
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\[
\sum_{i=1}^{w} C_i^* \leq C < \sum_{i=1}^{w+1} C_i^* .
\]

(5)

Investments \( U_i^* \), \( 1 \leq i \leq W \), \( U_i ^* \) represent the result of a task decision. Work of algorithm IV is finished.

**Stage 8.** Add the value of \( M \) up to value of \( M ^* \) according to formula

\[
M ^* = M + \Delta M .
\]

(6)

Here the size of \( \Delta M \) has constant character and also is defined by expert way.

**Stage 9.** Give to \( M ^* \Rightarrow M \) and transfer the control to **stage 3**.

The algorithm works until inequality \( \sum_{i=1}^{Q} C_i^* > C \) (**stage 6**) will not be realized.

**Notice.** A basis of Algorithm IV is the second criterion idea SIO, which is used in scheduling lays quite often. According to this idea, the more fast by feedback the investment has the advantage. So, in our opinion, the algorithm IV is more effective compared to distributive algorithms earlier described.

5. **Step-by-step complex algorithm of optimum investment activity**

The algorithms described above will be used in complex algorithm of investment decision-making offered below. Step-by-step procedure of complex algorithm is as follows:

**Stage 1.** Step-by-step transition to the next point \( t_j \) decision-making \( t_j + \Delta t \Rightarrow t_j \) is carried out.

**Stage 2.** In the next point \( t_j \) the possible set of potential investment projects \( \{ U_i(t_j) \} \), \( 1 \leq i \leq N_j \) is determined which the organization considers as alternative capital investments at realization.

**Stage 3.** For each of possible investments \( \{ U_i(t_j) \} \), we determine volumes of income return at the moment \( t_j + \Delta t , t_j + 2 \Delta t \), we shall designate the received functions of incomes return by symbols

\[
D_i(t_j, r \Delta t), \ 1 \leq i \leq N_j, \ 1 \leq r \leq R_i ,
\]

where parity takes place \( R_i \Delta t = L_i \).

**Stage 4.** Carry out the reduction of incomes at the moment \( t_j \) for each of investments

\[
U_i(t_j); \ C_i(t_j) = D_i(t_j, r \Delta t) K_r(r \Delta t).
\]

**Stage 5.** Summarize the given financial resources at the moment \( t_j \) for all investments and in all periods

\[
C^*(t_j) = \sum_{i=1}^{N_j} \sum_{r=1}^{R_i} [D_i(t_j, r \Delta t) K_r(r \Delta t)].
\]

**Stage 6.** Determine the amount of a cash in the organization at the moment \( t_j \), which we shall name a symbol \( C^{**}(t_j) \).

**Stage 7.** Determine the total volume of the current liquidity of the organization at the moment \( t_j \)

\[
C(t_j) = C^*(t_j) + C^{**}(t_j) .
\]

**Stage 8.** Check of inequality \( C(t_j) < C^* \) is carried out. If the inequality is fair, it is necessary to make the decision on a covering of shortage of liquid means of the organization, and management is transferred to the next stage of algorithm. In case of \( C^* \leq C(t_j) \) we pass to realization of **stage 10**.

**Stage 9.** The organization makes a unformulated decision for a covering of the current liquidity. Thus the value of \( C(t_j) \) is equated to size of \( C^* \).

**Stage 10.** The cash \( C^{**}(t_j) \) are redistributed between new investments according to procedure of the algorithm IV. Thus there is an equating
Stage 11. A realization of capital investments goes on after the allocation of an optimum set of investment projects; management is transferred to stage 1.

6. Conclusion

In our opinion, similar algorithms and procedures may be used in investment activity of the various credit organizations.

The offered algorithms will increase both income of investment activity and efficiency management of projects realization.

References