“Asset sales when resale price is uncertain”

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Asset sales when resale price is uncertain

Abstract

This paper develops an asset sale model where resale price follows a geometric Brownian motion. It derives a valuation formula for the value of the resale option and obtains the optimal option exercise rule. In addition, analytic measures of the probability of the option exercise are presented and some comparative static results are discussed. The model is shown to have applications to voluntary liquidations and mark-up pricing in mergers and acquisitions. It also throws light on option exercise behavior in real option models.

Keywords: asset sales, real options, first passage time.
JEL Classification: G31, G33.

Introduction

Sales of used assets are common occurrences for firms and industries. They are often involved in complex transactions such as voluntary liquidations, divestitures and restructuring (e.g., John and Ofek, 1995).

This paper adopts a real options framework to examine asset sales. Real options methods have been applied to examine investment decisions such as when to abandon a project (e.g., Dixit and Pindyck, 1994). In this paper, we treat asset sales as an American put option when sellers have the option to wait with selling an asset. An American put option gives the seller the right but not the obligation to sell an asset. This paper examines two particular aspects of an asset sale – valuation and timing of a sale.

Earlier literature considers valuation of an option but tends to overlook the timing of exercising of an option. For example, Merton (1973) derives a valuation formula for a perpetual American put option and obtains an option exercise rule with constant resale price, but does not consider when such an exercise should take place. This paper considers the valuation as well as the timing of an asset sale. Furthermore, it incorporates variable exercise price and therefore regards constant exercise price as a special case.

In their seminal model of investment under uncertainty, McDonald and Siegel (1986) suggest that their model can be reinterpreted as a model of scrapping from a call option perspective. As already stated this paper treats asset sales or “option to scrap” from a put option perspective instead. We consider a put option perspective as more appropriate because it provides more clarity and tractability particularly with respect to the timing of option exercise. In addition, this paper provides analytical measures for the valuation and the timing of a put option which existing literature lacks. For example, in a model of project abandonment, Myers and Majd (1990) rely on numerical method in valuing a put option and are not able to address the problem of the timing of abandoning the project.

The timing of a resale option is important for several reasons. First, existing research suggests the market does take into account the option element in equity valuations (Burgstahler and Dichev, 1997). Second, by modelling a resale option as an American put option, it provides insight into its exercising behavior. Third, empirical studies (Berger, Ofek and Swary, 1996) indicate that investors seem to value such options. Fourth, the optimal exercise of such options is important in maximizing share-holder value and should be treated as an integral part of corporate strategy. Lastly, complex corporate transactions such as voluntary liquidations often involve simultaneous and/or sequential sale of multiple assets. A study of a single asset sale is a necessary first step towards a better understanding of more complicated options. By addressing the issue of the timing of a single option exercise, we hope this paper moves one step forward towards the right direction.

The rest of this paper is organized as follows. Section one presents and solves a general model of asset sales when resale price is uncertain. The valuation formula contains Merton’s (1973) solution as a special case. We also derive a stochastic analogue for the optimal option exercise rule. Section two considers the density of the first passage for two-dimensional geometric Brownian motions to the optimal option exercise boundary. We derive analytical formulae for the expected time to option exercise and the variance of the time of exercise. In particular, we provide some economic intuitions regarding the necessary conditions for option exercise and no exercise. Section three suggests a number of applications of the model as well as directions for future research.

1. Option valuation with stochastic resale price

In his seminal paper on option pricing, Merton (1973) presents a valuation formula for a perpetual American put option with constant exercise price. A constant exercise price is appropriate for many financial contracts and greatly simplifies analysis, but may not be appropriate for real options due to a number of factors. First, many used asset markets are not active and competitive and resale price is often subject to negotiations among market participants. Second, unlike standardized financial options, real assets are normally specific and heterogeneous. Third, real assets tend to have low liquidity and marketability. For these reasons we model the resale price of a used asset as a stochastic process as suggested in Margrabe (1978).

Our model is specified as follows. Assume resale price, P, and discounted present profit, π, from the use of the asset follow geometric Brownian motions:

$$dP = \mu_s P dt + \sigma_s P dz_s, \tag{1}$$

$$d\pi = \mu_o \pi dt + \sigma_o \pi dz_o, \tag{2}$$

$$dz_s dz_o = \rho dt,$$

where μ_s and μ_o represent the mean growth rate of resale price and operating profit, respectively. σ_s and σ_o represent the standard deviation of resale price and operating profit, respectively. dz_s and dz_o are standard Wiener processes. ρ is the correlation coefficient between operating profit and resale price, and t is time.

Let F be the value of the resale option. F is clearly a function of the resale price, P, profit, π, and time, t. It can be shown that the value of the resale option satisfies the following equation (Myers and Majd, 1990):

$$\begin{aligned} &\frac{1}{2}\sigma_o \pi^2 F_{\pi\pi} + \rho\sigma_o\sigma_s \pi P F_{\pi P} + \frac{1}{2}\sigma_s^2 P^2 F_{PP} + \\ &+ r\pi F_\pi + rPF_P - rF + F_t = 0, \end{aligned} \tag{3}$$

with the following boundary conditions:

$$\begin{aligned} &a.\quad F(0,P,t) = P, \\ &b.\quad F(\infty,P,t) \to 0, \\ &c.\quad F(\pi,P,T) = \max[P(T) - \pi(T),0]. \end{aligned} \tag{4}$$

(4)a states that if operating profit is zero, the value of the resale option is simply the resale price. (4)b states that if operating profit is infinitely large, it never pays to sell the asset and the resale option is worth nothing. (4)c implies that at the end of the asset's physical life, the value of the resale option is the difference of its resale price and the operating profit, or zero, whichever is larger. The Appendix at the end shows that (3) can be transformed into a heat conduction equation:

$$\phi_1 + \frac{1}{2}\sigma_X^2 \phi_2 = 0, \tag{5}$$

with the following boundary conditions:

$$\begin{aligned} &\phi(0,t) \to \infty, \\ &\phi(\infty,t) = 0, \\ &\phi(Z,T) = \text{Max}[1 - e^{\frac{1}{2}\sigma_x^2 T + Z}, 0]. \end{aligned} \tag{6}$$

The above system has no closed-form solution due to the probability of early exercise. As Bunch and Johnson (2000) point out, only two American put options admit of analytic solutions: one with zero interest rate and the other with no maturity date and constant exercise price. The latter case was first solved by Merton (1973). For finite-lived American put options, no closed form solutions are known, so numerical methods must be employed.

For a particular case when an asset does not depreciate, its resale option can be treated as a perpetual American put option with stochastic exercise price. Margrabe (1978) considers a similar option and shows that for a perpetual put, the time derivative in equation (6) can be dropped. This leads to the following valuation equation:

$$\sigma_o \pi^2 F_{\pi\pi} + 2\rho\sigma_o\sigma_s \pi P F_{\pi P} + \sigma_s^2 P^2 F_{PP} = 0. \tag{7}$$

With boundary conditions:

$$\begin{aligned} &F(0, P, \infty) = P, \\ &F(\infty, P, \infty) \to 0, \\ &F(\pi, P, \infty) = \max(P - \pi, 0), \\ &\frac{\partial F}{\partial \pi^*} = -1. \end{aligned} \tag{8}$$

Now let $g = \pi/P$, by invoking Ito's lemma, one obtains:

$$dg = \mu g dt + \sigma g dz, \tag{9}$$

with

$$\mu = \mu_o - \mu_s - \sigma_o\sigma_s\rho + \sigma_s^2,$$

$$\sigma^2 = \sigma_o^2 - 2\sigma_o\sigma_s\rho + \sigma_s^2,$$

$$dz = \frac{\sigma_o dz_o - \sigma_s dz_s}{\sigma}.$$

From (9), it can be seen that the new variable, g, follows another geometric Brownian motion whose drift rate and variance depend on the drifts rates and variances of resale price and operating profits, as well as their correlation coefficients, as defined in (1) and (2).

Merton (1973) shows that for a perpetual American put option with constant exercise price, there is a critical stock price below which the option should be exercised. A nice feature of the optimal option exercise rule is that it does not depend on time. By analogy, there is a critical ratio of (1) and (2) for a perpetual American put option with random exercise price, such that if the ratio falls below a critical level the option will be exercised. Following this heuristic line of argument, one obtains the value of the option and its optimal exercise rule, as follows:

$$F = \frac{\pi}{1+\gamma}\left[\frac{(1+\gamma)\pi}{\gamma P}\right]^{-\gamma},$$
$$\gamma \equiv \frac{2r}{\sigma^2}, \tag{10}$$

$$g^* = \left(\frac{\pi}{P}\right)^* = \frac{2r}{2r+\sigma^2}. \tag{11}$$

Substituting (10) into (7), one can verify that (7) is satisfied. The option exercise rule is obtained by applying the boundary conditions and by maximizing the value of the resale option.

Although (11) is formally identical to equation (8.52) in Merton (1990, p. 300), there are two differences. First, resale price is random in our model whereas it is constant in Merton's model. Second, the variance in our model depends on variances of operating profit, resale price and their correlation coefficient whereas it depends on the variance of stock price alone in Merton. Of course, when exercise price is constant, (10) reduces to that of Merton. The same applies to the optimal option exercise boundary (11).

2. The timing of asset resale

Having derived the value of the resale option and the optimal exercise rule in the preceding section, we now consider whether and when the option will be exercised. For this purpose, we employ the first passage time concept in stochastic processes, known as optimal stopping problem. Rhys et al. (2002, p. 438) point out the idea of the first passage time is straightforward. Given a particular random variable, one asks if the variable will ever reach a particular level and if so when. The time, T, at which the random variable first reaches a boundary is itself a random variable known as the "first passage time" of that variable to that boundary. The distribution of this random variable T and in particular its moments are obviously useful to real option model builders who are not only interested in the optimal action boundary, but also the length of time that the agent has to wait before taking action.

In finance literature, the first passage time has been used to describe growth-optimum portfolio policy (Merton, 1990, p. 169, 191) and option valuation (Black and Cox, 1976). In the latter case, it has been solved for standard Brownian motion with zero drift in Bachelier (1900), standard Brownian motion with positive drift in Yaksick (1996) for American call option, and geometric Brownian motion for call and put options in Shackleton and Wojakowski (2002). Following Rhys et al. (2002), we obtain density of the first passage for random variable g from an initial value g_o to a critical value g^* as follows:

$$P(\tau > t) = \varphi(d_1) - e^{\frac{2\mu g^*}{\delta^2}}\varphi(d_2),$$
$$d_1 = \frac{g^* - \mu t}{\sigma\sqrt{t}}, \tag{12}$$
$$d_2 = \frac{-g^* - \mu t}{\sigma\sqrt{t}},$$

$$f(t) = \frac{g^*}{\sigma\sqrt{2\pi t^3}} e^{-\frac{\left[g^* - \mu t\right]^2}{2\delta^2 t}}, \tag{13}$$

The expectation and variance of T, are:

$$E[T_g] = \frac{\log\frac{g^*}{g_0}}{\mu - \frac{1}{2}\sigma^2}, \tag{14}$$

$$v(T_g) = \frac{\sigma^2 \log\left(\frac{g^*}{g_0}\right)}{\left(\mu - \frac{1}{2}\sigma^2\right)^3}. \tag{15}$$

It is useful to explore (14) and (15) and gain further insights into the factors that affect the expected first passage time T in particular, as it affords useful economic interpretations.

Equation (14) has three values:

First, if $E(Tg) = 0$, the nominator must equal zero, i.e. $g^* = g_o$. The intuition is that if g starts at g^*, the asset will be sold immediately. This explains why markets for certain assets are active and there is a high trading volume. For durable physical assets, this is less likely because significant transactions costs are involved.

Second, when $E(Tg) > 0$, which means $2(\mu_o - \mu_s) < \sigma_o^2 - \sigma_s^2$ or $2\mu_o + \sigma_s^2 < \sigma_o^2 + 2\mu_s$, the asset will eventually be sold. This probably happens to many assets particularly during economic downturns when asset owners face considerable uncertainty. It is also likely to happen to venture capitalists.

Third, when $E(Tg) \to \infty$, the resale option will never be exercised. Although the resale option is valuable, the asset may never be sold within a finite period of time. To illustrate, consider the case of a constant resale price. There are now two possibilities. If operating profit has a downward trend, the asset will be sold eventually. Even if operating profit grows over time, there is still the possibility that the asset will be sold at a certain point in time. This may occur in a growing industry in which there is great profit potential but the market is so volatile that some owners may find it profitable to sell their assets to others instead of continuing to use the asset. If expected first passage time is infinite, the growth effect and volatility effect completely offsets each other. This means that the resale option is certain to be exercised, but the average time to the exercise point is infinitely long.

The above analysis may throw light on models of investment under uncertainty at the micro-level, particularly relating to uncertainty-investment sign in theoretical models and empirical tests. For example, theoretical models developed in Caballero (1991) and Sarkar (2000) and empirical tests carried out in Harchaoui and Lasserre (2001) and Moel and Tufano (2002). As far as empirical tests are concerned, there are a number of factors at work. One factor is that the critical action level derived from the theory is not observable. In studying firm-level investment in Ghana, Pottillo (1998, pp. 523-524) made the following insightful remarks:

The firm allows the marginal revenue product to fluctuate stochastically, and invests only when the marginal revenue product of capital hits an optimally derived trigger. It can be shown that the trigger is increasing in the standard deviation of the demand process. However, average investment during a given period depends on how soon and how often the marginal revenue product of capital reaches the trigger. Although greater uncertainty raises the trigger, a more volatile process may hit the trigger more often. Thus, the net effect on short-run investment depends on the balance of these factors.

Firm level data on investment is typically collected for a fixed time period. Total investment within a given period depends on the frequency and volume of individual investments. In real options models, this aggregation problem has not been resolved despite an early attempt by Caballero and Pindyck (1996). Our model does give rise to implementable predictions with respect to the average time to selling the asset. This is a sub-class of survival models. However, incorporating frequency and magnitude of individual investments within a given period of time remains a challenge.

3. Some obervations

In this paper, we contribute to the real options literature by explicitly introducing the timing of exercising a resale option when resale price follows a random process. By employing the optimal stopping approach, we are able to obtain analytic measures of option exercise timing. This enables us to gain insights into factors that affect option exercise in practice, and we consider a few scenarios for the expected time to option exercise. By looking at the expected first passage to the optimal option exercise rule, we discover a number of interesting model predictions. In particular, although such options have a positive value, it is not at all certain that the optimal resale boundary will be reached within a given period of time. The probability of reaching the boundary may be certain, but the expected time may be infinite. Casual observations suggest that these predictions make economic sense.

Our model may throw light on a number of problems in corporate finance. First, it provides one rationale for corporate voluntary liquidation. This is easy to understand, because voluntary liquidation is analogous to the exercise of an American put option. When liquidation value and the value of going-concern are uncertain, corporations must at any point in time decide when and whether it is best to close down the business. One of our model predictions is that liquidation value must exceed the going-concern value by a certain percentage. Of course, liquidating firms in practice normally have other features apart from performance variability. The other features include low growth opportunities and market to book values, high insider ownership and liquidity, as Fleming and Moon (1995) and Mehran, Nogler and Schwartz (1998) show. Companies seem to be aware of this "shut-down" option, which is reflected by significant average stock returns when liquidation decision is announced (Erwin and McConnell, 1997). This indicates that voluntary liquidation may be an important rational and value-enhancing corporate decision.

Second, our model provides insight into take-over premium widely documented in mergers and acquisitions as in Walking and Edmister (1985) and Schwert (1996). Various theories have been put forward to explain this phenomenon. Roll (1986) hypothesizes that bidding firms are too optimistic and overestimate potential benefits, and end up paying too much. Our model shows that this so-called over-payment may simply reflect the value of the option to liquidate the firm. That this option value is taken into account by an acquirer is not surprising. It also means that an incumbent firm has to be compensated for the loss of this valuable liquidation. In practice, this excess payment may be distributed among various stakeholders such as directors, managers, employees, equity-holders and debtors.

Our model also provides an alternative interpretation for discount sales for consumer goods. To see this, suppose a supermarket has unsold goods on the shelves. If they face a financial constraint, they must sell current goods in order to replenish it with new goods. Revenues from the sale of current goods and new goods are all uncertain. However, if the shop keeper believes that demand for current goods is weak and demand for new goods is strong, it will make sense to sell current goods at a discount so that the proceeds may be used to order new goods. The discount in this case reflects the option value of being able to order new goods by quickly selling current goods. An in-depth study of this issue, however, is best left for future research.

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Appendix

Let $H(X,t) = \frac{F(\pi,P,t)}{P}$ and $X = \frac{\pi}{P}$, equations (3) and (4) now become

$$\frac{1}{2}\sigma_x^2 X^2 H_{XX} + H_t = 0 \text{ with } \sigma_x^2 \equiv \sigma_p^2 - 2\rho\sigma_p\sigma_o + \sigma_o^2$$

a. $H(X=0,t)=1$

b. $\lim_{X\to\infty} H(X,t) = 0$

c. $H(X,t=T) = \max[1-X(T),0]$

Make another change of variable, $\varphi(Z,t) \equiv \frac{H(X,t)}{X}$, $Z = \log X - \frac{1}{2}\sigma_x^2 t$

We obtain

$$\frac{1}{X}\phi_1 = \frac{G_X}{X} - \frac{G}{X^2}; \;\; \phi_1 = G_X - \frac{G}{X}$$

$$\frac{1}{X}\phi_{11} = G_{XX} - G_X + \frac{G}{X}; \;\; \phi_{11} = XG_{XX} - G_X + \frac{G}{X}$$

$$\phi_2 = \frac{G_t}{X} + \frac{1}{2}\sigma_X^2[G_X - \frac{G}{X}] = \frac{G_t}{X} + \frac{1}{2}\sigma_X^2[XG_{XX} - \phi_{11}]$$

$$\phi_2 + \frac{1}{2}\sigma_X^2\phi_{11} = \frac{1}{X}[G_t + \frac{1}{2}\sigma_X^2 X^2 G_{XX}]$$

Finally, using the identity $G_t + \frac{1}{2}\sigma_X^2 X^2 G_{XX} = 0$, we obtain equations (5) and (6).