“Non-linear predictability of stock market returns: Comparative evidence from Japan and the US”

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Non-linear predictability of stock market returns: comparative evidence from Japan and the US

Abstract

Using smooth transition regression model analysis, we examine the non-linear predictability of Japanese and US stock market returns by a set of macroeconomic variables between 1981 and 2012. The theoretical basis for investigating non-linear behavior in stock returns can be based on the interaction between noise traders and arbitrageurs or behavioral finance theories of non-linear risk aversion. As heterogeneity in investors’ beliefs gives reason to suspect a smooth transition between extremes, rather than abrupt, a smooth transition regression model is estimated. Our findings support differences in non-linearity of stock returns in Japan and the US that might be linked to different share-ownership of the Japanese stock market compared to the US. In addition, differences in the legal system might have some influence over our findings as well. The US results also suggest greater heterogeneity in the relationship between stock returns and macro variables in the US data relative to the Japanese data. The reasons behind the differences in our results, both between countries and between regimes are probably due to the different economic conditions faced by Japan and the US over our sample, to the possible existence of bubbles in the data and to investor behavior consistent with ‘behavioral finance’ theories of investor behavior.

Keywords: stock market return, smooth transition regression model, forecasting, behavioral finance, Japan.


Introduction

A very significant literature now exists that investigates the relationship between stock returns (or excess stock returns) and a broad range of macroeconomic and financial variables, across a number of different stock markets and over a number of different time horizons. Important recent work in this area includes Campbell and Yogo (2006), Ang and Bekaert (2007), Cochrane (2008), Campbell and Thompson (2008) and Welch and Goyal (2008). Many researchers have found a range of variables to be correlated to returns. These include stock valuation variables (e.g., Ball, 1978; Campbell and Shiller 1988a, b; Fama and French 1988, 1989; Fama, 1990; Campbell and Hamao, 1992; Cochrane, 1992; Hodrick, 1992; Campbell and Viceira, 2002; Cochrane, 2008; Ang and Baeckert, 2007; Campbell and Thompson, 2008). In addition a number of researchers have found evidence that stock returns may be predicted by interest rate movements (e.g., Fama and Schwert, 1977; Keim and Stambaugh, 1986; Campbell, 1987; Fama and French, 1989; Hodrick, 1992; Peseran and Timmerman, 1995). Many authors have also investigated the relationship between stock returns and broader macroeconomic (or business conditions) variables, such as inflation, GDP, industrial output and the money supply (e.g., Urich and Wachtel, 1981; Fama, 1981; Chen, Roll and Ross, 1986; Fama and French, 1989; Fama, 1990; Schwert, 1990; Peseran and Timmerman, 1995; Black, Fraser and MacDonald, 1997; McMillan, 2003).

While these papers have been based on linear relationships, recent research has also found evidence of non-linear relationships between macroeconomic or financial variables and stock market returns. One method of examining these relationships is through the application of smooth transition regression (STR) model. The STR model is a regime switching model that allows a smooth (non-linear) transition from one regime to another. This model may be viewed as emerging from the work of Quandt (1958) and Bacon and Watts (1971) on switching regression models (see, Terasvirta, 1994 for further discussion) and has been developed by Chan and Tong (1986). The univariate generalization of the STR model, the smooth transition autoregressive model (STAR), has also been extensively applied in this field. (e.g., McMillan, 2001a, 2003; Franses and van Dijk, 2000; Aslanidis et al., 2003).

In this paper we make use of the STR model as it has a number of appealing features. First, it allows a smooth (non-linear) transition between regimes. Under the Markov switching model changes in regime are abrupt, implying traders act simultaneously. A smooth transition between regimes allows the more appealing outcome that different traders within the market act at different points in time. Second, unlike standard Markov switching models, where regime changes are unobservable and governed by a Markov process, the STR model allows switching behavior to depend upon observable variables. Third, it may be possible to model different types of market behavior, determined by the magnitude of returns.

1 The STR methodology has also been used to investigate a number of other issues.
1. Motivation for non-linear models in finance

One theoretical explanation of non-linear behavior in stock markets is based on the idea of interaction between different types of trader, for example, between noise and informed traders. In this framework the market activity of noise traders (e.g., Kyle, 1985; Black, 1986) will push stock prices away from equilibrium values. At some point in this process informed traders find it profitable to enter the market and arbitrage prices back towards equilibrium. Thus noise trading may dominate informed trading, at returns close to equilibrium, that is within upper and lower thresholds around equilibrium, and informed traders enter the market when returns ‘stray’ beyond these thresholds.

One explanation for the existence of thresholds relies on trading costs such as bid-ask spreads and transactions costs (e.g., Cootner, 1962), that is, informed traders only enter the market when the profits from doing so outweigh the transaction costs incurred. A second rationale draws on the behavioral finance literature (e.g., DeLong et al., 1990; He and Modest, 1995; Hong and Stein, 1999; Shleifer, 2000). Here it is assumed noise traders do not trade randomly but, for example, extrapolate trends (momentum traders) or are prone to herd behavior. It is also assumed that there are only imperfect arbitrage opportunities within financial markets. Under these conditions arbitrage is not riskless, and in particular, informed traders face the risk of ‘mis-price-deepening’ (e.g., Shleifer, 2000); that is, at any point in time, returns may be forced further from equilibrium by the activities of noise traders. An informed trader may suffer losses in the short to medium term if he or she attempts to profit from an arbitrage opportunity. Therefore informed traders may require a relatively large difference between actual and equilibrium returns, before they trade in order to compensate for this risk. A third possible reason for the existence of non-linear adjustment of this type, was developed in the context of foreign exchange markets, and suggests that small deviations from equilibrium may be considered unimportant by informed traders, whereas large deviations are more likely to be corrected. This kind of interaction between different types of investors can be modelled by a LSTR model.

The behavioral finance literature also suggests a number of reasons why investor behavior may depend upon the movement of stock market returns, for example whether they are rising or falling. Under Prospect theory (e.g., Kahneman and Tversky, 1979), it is assumed investor utility is dependent upon gains and losses, as opposed to final wealth, and that agents are loss averse. This means investors are less likely to trade, in order not to realize a loss, when markets are falling than when they are rising. Barberis et al. (2001), have extended this idea by suggesting that an investor’s willingness to trade rises with the previous profitability of trading (e.g., Thaler and Johnson, 1990). They suggest that capital gains realized on a rising stock market may then lead to investors taking on more risk when markets rise relative to when they fall. Lopez (1987) develops a framework where emotions, particularly fear and hope, strongly influence investor behavior, and risk tolerance. Since these emotions are likely to be strongly influenced by the state of the market, this suggests another reason why investor behavior might differ across rising and falling markets. As will be discussed in more detail later, asymmetric investor behavior in positive and negative return regimes can be modelled by a LSTR1 model.

In this paper we investigate whether the STR model can be used to explain the behavior of the US and Japanese stock markets. Both markets have experienced periods of rapidly rising and falling prices, which may result in non-linear behavior in returns that may be picked up by the STR model. In addition, the unprecedented downturn during the 1990s in Japan may result in significant differences between the US and Japanese markets. Differences in share ownership across the two countries may also impact upon our results. It is well known that there are substantial differences between the US and Japanese financial systems. In particular, the Japanese economy is relatively dependent upon indirect finance, and this (partly through the keiretsu system) has had an impact upon patterns of share ownership in Japan relative to the US. Share ownership amongst households and individuals is low relative to the US, while share ownership within the financial and the domestic corporate sector is relatively high (e.g., Prowse, 1992; Yonezawa and Miyake, 1998; Bogle, 2005; Altunbas et al., 2007). In addition, differences in legal systems might also have some influence over our results. The US may be characterized as having a common law legal system, and Japan as a civil law legal system. As a result Japan tends to have lower levels of investor protection and less securities disclosure regulation (e.g., Roe, 2006). These differences between the two countries might lead to differences in investor behavior.

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1 See e.g., Obstfeld and Taylor (1997), Coakley and Fuertes (2001).
2 See e.g., McMillan (2007, 2009) for a discussion of these issues.

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3 For more on the role of the banking system in the Japanese economy, and its role in Japan’s sustained downturn of the 1990s, see e.g. Hutchinson, Ito and Westermann (2006).
4 Roe (2006) suggests that it may be easier to manipulate stock markets in civil law countries such as Japan.
that may be picked up in our non-linear model. One possibility, if informational asymmetries exist between private and institutional investors, for example if institutions are able to collect and process more information than individuals, is that private investors may be more noise driven relative to institutional investors. The more balanced share ownership structure in the US might also give us a reason to suspect smoother transitions between regimes.

2. Data and empirical analysis

For the empirical analysis, we start by including a relatively large number of variables in our regressions. These have been chosen because they commonly appear in return predictability studies (see discussion in the introduction) and because they are theoretically plausible (for example might be expected to influence expected dividends and/or the real discount rate within a standard present value framework). We make use of the following variables: the real stock price 1 as our dependent variable, the dividend yield, real industrial production, inflation 2, real M2, real ten year yield 3, real 3 month rate 4, real retail sales, the OECD G7 leading indicator 5, the yen/dollar exchange rate, the corporate risk spread 6, and our other explanatory variables is low. See, e.g., Lambrick (2006) for further discussion.

For the risk and term spreads in the US and Japan, we transform the data into logarithmic time series.

Unit root tests were applied to the US and Japanese data. All variables were found to be I(1) except for the real 10 year yield 6 in Japan and the term spread in the US. These variables were found to be stationary. Where necessary, the variables are transformed into stationary series to be used in our modelling below.

As the term spread shows perfect colinearity with the long and short interest rates, we test their significance on the stock market separately and then apply the more significant one for the stock market model.

3. Methodology

Our modelling strategy is to specify a linear VAR model for both the Japanese and the US stock market data, with the real return as the dependent variable and a number of lags (based upon the Akaike criterion) of our macro variables. We then sequentially eliminate insignificant lags in order to arrive at parsimonious models and compare the performance of these models to appropriate non-linear smooth transition regression (STR) models (discussed below).

The standard smooth transition regression (STR) model with a logistic transition function, often referred to as the LSTR model (e.g., Lütkepohl and Krätzig, 2004) has the form:

\[ y_t = \varphi' z_t + \theta' z_t G(y_t, c, s_t) + \mu_t, \]

\[ \mu_t \sim iid \left(0, \sigma^2ight), \quad (1) \]

where \( \varphi \) and \( \theta \) are parameter vectors, \( z_t = (w_t', x_t')' \) is an \((m+1) \times 1\) vector of explanatory variables with \( w_t = (1, y_{t-1}, \ldots, y_{t-p})' \) and \( x_t = (x_{t1}, \ldots, x_{tk})' \).

\[ G(y_t, c, s_t) = \left(1 + \exp\{-\gamma \sum_{k=1}^{K} (s_k - c_k)\}\right)^{-1}, \gamma > 0 \quad (2) \]

is the logistic transition function. \( G(y_t, c, s_t) \) depends upon the transition variable \( s_t \), the slope parameter \( \gamma \), which measures the speed of transition between regimes and the vector of location parameters \( c \). For \( K = 1 \) (the LSTR1 model), \( G(y_t, c, s_t) \) is a smooth monotonically increasing function of \( s_t \), lying between 0 and 1 and centred on one location parameter \( c_1 \). Thus, where \( s_t \) is a measure of stock market return, this model is able to capture a smooth

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1 For the US this is the S&P 500 index, for Japan it is the Nikkei 225.
2 We make use of CPI for both the US and Japan.
3 This is the T-Bond yield in the US and the 10 year bond yield from the Bank of Japan in Japan.
4 This is the T-Bill rate in the US and the 3 month rate from Eurostat in Japan.
5 Theoretically (the marginal utility of) consumption is related to asset prices (e.g., Cochrane, 2005). However, consumption data is not available with monthly frequency. We follow Burgstaller (2002) and use retail sales as a proxy variable.
6 The OECD G7 leading indicator is a weighted composite variable, made up of a number of macroeconomic time series from the G7 economies designed to predict short-term movement in G7 industrial production. It includes variables such as durable goods orders, consumer sentiment or weekly hours of work. Although there is some overlap in the variables used to calculate this leading indicator and the variables we use in our regression analysis the correlation between it and our other explanatory variables is low. See, e.g., Lambrick (2006) for further discussion.
7 This is the corporate BBB risk spread over 10 year government bonds in the US and the corporate bond yield series from the Bank of Japan for Japan.
transition between two regimes that may be characterized by asymmetric investor behavior between market returns of different magnitude, such as rising and falling markets (e.g., Maddala, 1977). For $K = 2$ (the LSTR2 model), $G(y, c, s)$ is symmetric around the midpoint of $c_1$ and $c_2$, and allows the capture of a smooth transition between an inner regime (between $c_1$ and $c_2$) and an outer regime. This allows us to model the case where investor behavior is similar for very large and very small changes to returns (outer regime), such as under fundamentals trading but a different behavior exists for between these values (inner regime), such as under noise trading. See Öcal and Osborn (2000) and van Dijk and Franses (1999) for further detail.

Our modelling strategy consists of four steps as described by Teräsvirta (1994, 1998) and Eitrheim and Teräsvirta (1996). Our first step is to specify an appropriate linear relationship by estimating a parsimonious VAR model. Linearity is then tested against the non-linear alternatives of an LSTR1 or LSTR2 model. One complication is that the transition function is not identified under the null hypothesis. This problem may be overcome by approximating the transition function by a (third order) Taylor expansion around $\gamma = 0$, which results in:

$$y_t = \alpha_0 z_t + \alpha_1 z_t s_t + \alpha_2 z_t s_t^2 + \alpha_3 z_t s_t^3 + \mu_t'.$$  

Under the null hypothesis$^1$ of linearity $H_0$:

$$H_{04} : \alpha_3 = 0$$

$$H_{03} : \alpha_2 = 0 | \alpha_3 = 0$$

$$H_{02} : \alpha_1 = 0 | \alpha_2 = \alpha_3 = 0.$$

These three hypotheses are usually tested using a series of F-tests. The (heuristic) decision rule is to select the LSTR2 model if $H_{03}$ is most strongly rejected, otherwise select the LSTR1 model (e.g., Terasvirta, 1994). The third step is to estimate the parameters of our preferred model using conditional maximum likelihood. Estimation starts with finding initial values for $\gamma$ and $c$ by a grid search that minimizes the residual sum squared (RSS) for a range of $\gamma$ and $c$. Once acceptable starting values have been found, the unknown parameters of the STR model can be estimated by using a form of the Newton-Raphson algorithm to maximize the conditional maximum likelihood function$^2$. Finally, the non-linear model is evaluated by comparing remaining autocorrelation and the sum of squared residuals with the linear model in (IS) and out of sample (OOS). For the OOS forecasts only data available at the time of the prediction is used. The linear and non-linear regression coefficients are re-estimated recursively and applied for the OOS forecast (for a discussion, see, e.g., Welch and Goyal, 2008; Lundbergh and Teräsvirta, 2004). Furthermore, we follow Welch and Goyal (2008) and show the IS and OOS predictive performance of the linear and non-linear models on a monthly forecasting horizon. This is achieved by plotting the cumulative squared prediction errors of the NULL minus the cumulative squared prediction error of the linear and non-linear models (ALTERNATIVE). The NULL is the prevailing mean stock return.

4. Results

First of all we specify our linear VAR using the Japanese data. Sequentially eliminating insignificant lags results in a parsimonious model where the real return is influenced positively by the first lag of the OECD G7 leading indicator, the fifth lag of the Yen-US Dollar exchange rate and the sixth lag of the ten year yield$^3$.

The results of our test for non-linearity of the Japanese data are presented in Table 1. From the table we can see that the F-test (H01 of equation 3) is rejected at usual significance levels. The strongest rejection of our further sequence of hypothesis tests suggests the LSTR1 should be selected, the mean average error (MAE) and in sample root mean squared error (RSME) statistics also suggest the LSTR1 model best fits the data (see Table 2). From Table 2 we can also see that the location parameter for the stock market return ($c_1$) for the LSTR1 model is -3.15%. Thus the lower regime from our model may be characterized as a bear market regime (returns below -3.15%), and an upper regime where returns fall by less or are positive (including periods of quickly growing returns such as the bubble economy boom).

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$^1$ Note that under this null $\mu_t' = \mu_t$ from equation (1)

$^2$ We use the JMulti software to optimize the non linear model. Further information can be found at www.jmulti.com.

$^3$ We identify the same predictive variables if we run univariate regressions using each of our macro/finance variables. This also holds for the US data.
Table 1. Testing linearity against STR in Japan

<table>
<thead>
<tr>
<th>Variables in AR part:</th>
<th>CONST OECD_log_d1(t-1)</th>
<th>YENUSD_log_d1(t-5)</th>
<th>r10yr(t-6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample range:</td>
<td>[1981 M1, 2004 M6]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>p-values of F-tests:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>transition variable</td>
<td>F</td>
<td>F4</td>
<td>F3</td>
</tr>
<tr>
<td>RNKY_log_d1(t-1)</td>
<td>0.010144</td>
<td>0.42045</td>
<td>0.032406</td>
</tr>
</tbody>
</table>

In the lower (bear market) regime of Table 2, in common with our linear model, the first lag of the OECD G7 leading indicator and the yen-dollar exchange rate both show a positive effect on stock market returns. However, the ten year yield is no longer significant in explaining stock market returns. For the upper regime, above -3.15%, the OECD G7 leading indicator and the exchange rate variables are no longer significant, however the sixth lag of the real 10 year yield interest rate has a positive impact on real stock market returns. For completeness we also estimate a LSTR2 model for the Japanese data. The inner regime is found to exist for returns between 3.71% and 5.48% (see Table 2). For the inner regime we find that our results are quantitatively similar to the linear case (see Table 2). Returns are positively influenced by the first lag of the OECD G7 leading indicator, the fifth lag of the Yen-US Dollar exchange rate and the sixth lag of the 10 year yield. In the outer regime (returns above 5.48% and below 3.71%) the OECD G7 leading indicator and the 10 year yield both have a negative impact upon return whereas the exchange rate variable becomes insignificant.

Table 2. Linear, LSTR1 and LSTR2 regression estimation results for the Japanese stock market

<table>
<thead>
<tr>
<th>Japan 1981M1-2012M5</th>
<th>Linear</th>
<th>LSTR1 (lower)</th>
<th>LSTR1 (upper)</th>
<th>LSTR2 (inner)</th>
<th>LSTR2 (outer)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.408 (0.241)</td>
<td>0.0052 (0.0070)</td>
<td>0.4512 (0.2704)</td>
<td>-1.5794 (0.6952)</td>
<td>12.275 (0.7480)</td>
</tr>
<tr>
<td>OECD(-1)</td>
<td>3.258 (0.861)</td>
<td>3.84210 (0.9139)</td>
<td>Not significant</td>
<td>13.9466 (4.1268)</td>
<td>-11.4479 (4.2217)</td>
</tr>
<tr>
<td>YENUSD(-5)</td>
<td>0.205 (0.122)</td>
<td>0.2007 (0.1224)</td>
<td>Not significant</td>
<td>0.1941 (0.1199)</td>
<td>Not significant</td>
</tr>
<tr>
<td>R10YR(-6)</td>
<td>0.388 (0.234)</td>
<td>Not significant</td>
<td>0.4199 (0.2614)</td>
<td>1.4859 (0.6714)</td>
<td>-1.2075 (0.7225)</td>
</tr>
</tbody>
</table>

| Y                    | 41.1393 (70.852) | 353.0031 (801.0118) |
| c1                   | -0.0315 (0.0066) | 0.0371 (0.0018) |
| c2                   | 0.0548 (0.0039) |
| AIC                  | -5.674 | -5.6663 | -5.6993* |
| SC                   | -5.620* | -5.5725 | -5.5575 |
| R-sq                 | 0.0818 | 0.0965 | 0.1226* |
| o S.D. of resid      | 0.0582 | 0.0581 | 0.0571* |
| Skewness             | -0.1498* | -0.1636 | -0.2037 |
| Kurtosis             | 3.7639* | 3.9542 | 3.8677 |
| Jarque-Bera          | 7.8941* (0.0193) | 11.3629 (0.0034) | 10.8910 (0.0043) |
| ARCH-LM(6)           | 22.2874* (0.0011) | 23.9265 (0.0005) | 26.8333 (0.0002) |
| LM Godfrey(6)        | 0.2304 (0.3241) | 0.4123* |
| In Sample 1981M1-2004M6 | RMSE 0.0582 | 0.0573* | 0.0760 |
| MAE                  | 0.0454 | 0.0438* | 0.0582 |

Out of Sample 2004M7-2012M5

| RMSE | 0.0591* | 0.0596 | 0.0966 |
| MAE  | 0.0445* | 0.0453 | 0.0629 |

Notes: (Std. Dev.), {p-value}, * denotes preferred model.
The positive coefficient on the first lag of the OECD G7 leading indicator for our linear model is in line with Brown and Otsuki (1990) and Mukherjee and Naka (1995) who find that as an export oriented country, the Japanese economy and stock market depend to a large degree on ‘world’ market conditions. However, our finding that this relationship is significant only for the lower regime of our LSTR1 model, suggests this positive relationship is partly driven by bear markets, where the impact of falling stock prices coincides with downturns in the world economy. The results of our LSTR2 model suggest this relationship is also positive when returns lie between 3.7% and 5.5%, but negative when returns lie outwith this band. This result for the outer regime is likely to be strongly influenced by the sustained downturn in the Japanese economy during nineties and early part of the twentieth century.

The positive coefficient on the fifth lag of the yen-dollar exchange rate in our linear model is perhaps surprising, given our definition of the exchange rate, as standard economic theory suggests exchange changes negatively influence the real economy and therefore should also negatively impact upon the stock price. However, this may be picking up portfolio effects, if expected changes in the stock price are positively influencing exchange rates (e.g., Giovannini and Jorian, 1987; Roll, 1992). The finding that this result holds in the lower but not the upper regime of our LSTR1 model and for the inner and not the outer regime of our LSTR2 model, suggests this result is strongly influenced both by growth of and significant falls in the NIKKEI.

The positive relationship with the sixth lag of the 10 year real interest rate, in the linear model and in the lower regime of the LSTR1 model, also presents a surprise, but should be seen from the backdrop of the Japanese ‘bubble economy’ of the late eighties and the subsequent deflation and probable liquidity trap, after the bursting of this bubble, during the 1990s and early 2000s. One interpretation of this result is that it is supportive of the view that falling interest rates during the downturn in the Japanese economy, and subsequent narrowing of spreads between lending and borrowing rates in the commercial banking sector, from around 1991, lead to a fall in (new) bank lending. Markets might expect this to reduce future economic activity and factor this into stock prices (e.g., Krugman, 1998; Goyal and McKinnon, 2003). However, the significance of the positive coefficients for the upper regime of the LSTR1 model and the inner regime of the LSTR2 model suggest this result holds most strongly when returns are rising. The coefficient for the outer regime of the LSTR2 model suggests real interest rates have the expected negative impact for stock returns greater less than 3.71% and more than 5.48%.

We should also mention that on the basis of the AIC criterion, the LSTR2 model is preferred to LSTR1 (see Table 2). The diagnostic tests indicate remaining ARCH effects as we do not account for regime switching volatility, while remaining autocorrelation is not an issue. The best out of sample fit, measured by RMSE and MAE, is actually achieved by the linear model, very closely followed by the LSTR1 model.

For the US data the AIC criterion suggests 12 lags in the linear VAR model. Results are provided in Table 3. We follow the same method as we did for the Japanese data and find real US returns over the period to be positively related to both the first lag of the OECD G7 leading indicator and the sixth lag of the risk spread, and to be negatively related to the twelfth lag of M2.

Table 3. Linear, LSTR1 and LSTR2 regression estimation results for the US stock market

<table>
<thead>
<tr>
<th>Variable</th>
<th>Linear</th>
<th>LSTR1 (Part1)</th>
<th>LSTR1 (Part2)</th>
<th>LSTR2 (Part1)</th>
<th>LSTR2 (Part2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OECD(-1)</td>
<td>1.066</td>
<td>1.222</td>
<td>Not significant</td>
<td>Not significant</td>
<td>1.5512</td>
</tr>
<tr>
<td>Risk spread(-6)</td>
<td>0.031</td>
<td>0.1350</td>
<td>Not significant</td>
<td>Not significant</td>
<td>0.0570</td>
</tr>
<tr>
<td>M2(-12)</td>
<td>-1.346</td>
<td>-1.713</td>
<td>Not significant</td>
<td>-1.3448</td>
<td>Not significant</td>
</tr>
<tr>
<td>Y</td>
<td>140.297</td>
<td>1.2980</td>
<td>(377.0777)</td>
<td>(1.6192)</td>
<td></td>
</tr>
<tr>
<td>c_1</td>
<td>-0.0219</td>
<td>0.0400</td>
<td>(0.0008)</td>
<td>(0.0000)</td>
<td></td>
</tr>
<tr>
<td>c_2</td>
<td>0.0400</td>
<td>0.0400</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td></td>
</tr>
</tbody>
</table>
Table 3 (cont.). Linear, LSTR1 and LSTR2 regression estimation results for the US stock market

<table>
<thead>
<tr>
<th>US 1981M1-2012M5</th>
<th>Variable</th>
<th>Linear</th>
<th>LSTR1</th>
<th>LSTR2</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIC</td>
<td>-6.2117</td>
<td>-6.2428*</td>
<td>-6.2018</td>
<td></td>
</tr>
<tr>
<td>SC</td>
<td>-6.1583*</td>
<td>-6.1359</td>
<td>-6.0949</td>
<td></td>
</tr>
<tr>
<td>R-sq</td>
<td></td>
<td>0.0343</td>
<td>0.0912*</td>
<td>0.0531</td>
</tr>
<tr>
<td>σ S.D. of resid</td>
<td>0.0441</td>
<td>0.0435*</td>
<td>0.0444</td>
<td></td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.7638</td>
<td>-0.6394*</td>
<td>-0.7731</td>
<td></td>
</tr>
<tr>
<td>Kurtosis</td>
<td>6.0423</td>
<td>4.8994*</td>
<td>5.9912</td>
<td></td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>129.8968</td>
<td>58.7651*</td>
<td>127.0786</td>
<td>0.00</td>
</tr>
<tr>
<td>ARCH-LM(12)</td>
<td>5.6291</td>
<td>5.0511</td>
<td>4.6076*</td>
<td></td>
</tr>
<tr>
<td>LM Godfrey(6)</td>
<td>0.7683*</td>
<td>0.6087</td>
<td>0.6766</td>
<td></td>
</tr>
</tbody>
</table>

In Sample 1981M1-2004M6

| RMSE            | 0.0441  | 0.0428* | 0.0436  |       |
| MAE             | 0.0331  | 0.0328* | 0.0329  |       |

Out of Sample 2004M7-2012M5

| RMSE            | 0.0459  | 0.0454* | 0.0473  |       |
| MAE             | 0.0339  | 0.0335* | 0.0340  |       |

Note: (Std. Dev.), {p-value}, * denotes preferred model.

Table 4 presents our test of linearity of our model using the US data (H01). We reject the null of linearity and find LSTR to be our preferred non-linear model\(^1\), and find the threshold stock market return (c) to be -2.19%. As with the Japanese data, the lower regime from our model maybe characterized as a bear market regime (returns below -2.19%), where results are quantitatively similar to those we found for the linear model. In the upper regime only the risk spread influences real returns but with an unexpected negative coefficient. As with the Japanese data we also estimate a LSTR2 model for the US market. In this case we find the inner regime collapses to a point at 4.00%, this is a special case of the LSTR2 model where both regimes are symmetric around a single threshold and this has often be modelled as an exponential smooth transition model (ESTR1) (e.g., Lütkepohl and Krätzig, 2004). Here the outer regime shows a positive impact of the OECD G7 leading indicator and the risk spread while the middle regime shows a negative effect of money supply M2.

The findings for our linear model and for the lower regime in our LSTR1 model support the view that the US stock market responds positively to the first lag of the OECD leading indicator and to the sixth lag of the risk spread and negatively to the twelve lag of M2. These results are intuitively plausible as they suggest US stock returns (as was the case for Japan) are positively related to ‘world’ economic conditions. The finding that the money supply negatively influences real returns is also in line with previous research (e.g., McMillan, 2001b; Humpe and MacMillan, 2009), and is usually interpreted as picking up the impact of future inflation upon stock prices. The positive relationship between real returns and the risk spread is also as expected from theory (e.g., Fama and French, 2002; Black et al., 1997; Schwert, 1990).

Table 4. Testing linearity against STR in the US

<table>
<thead>
<tr>
<th>TESTING LINEARITY AGAINST STR IN THE US</th>
<th>Variables in AR part:</th>
<th>CONST OECD log d1(t-1) risk spread_d1(t-6) M2_log_d1(t-12)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample range:</td>
<td>[1981 M1, 2004 M6]</td>
<td></td>
</tr>
<tr>
<td>p-values of F-tests:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>transition variable</td>
<td>F</td>
<td>F4</td>
</tr>
<tr>
<td>RSPX log_d1(t-1)</td>
<td>0.031514</td>
<td>0.40899</td>
</tr>
</tbody>
</table>

An unexpected result is the negative impact of the risk spread on returns in the upper regime of our LSTR1 model. This suggests that for returns greater than -2.19% the risk spread has a negative impact upon returns. This is probably picking up the impact of episodes of a falling risk spread during periods of rising stock prices during the late eighties, mid nineties and from 2002 in our sample, and is supportive of evidence that the business cycle is negatively correlated with the risk spread (e.g., Zhang, 2002).

\(^1\) The Akaike criterion for the LSTR1 model confirms it as our preferred model, it also has the best in sample fit, and best out of sample fit as measured by RMSE and MAE. The diagnostic tests do not indicate remaining ARCH effects or autocorrelation.
In Japan, as can be seen from Table 2, the smoothing parameter, $\gamma$ for the LSTR1 model is 41.14, for the US data $\gamma$, for the same model, is 140.3. Therefore for both countries the transition functions are relatively steep (this is confirmed by Figure 1 and 3), which is indicative of the detection of distinct upper and lower regimes. For the LSTR2 model $\gamma$ is 353.0 for Japan and is 1.28 in the US. This implies switching between recognisable inner and outer regimes in Japan (Figure 2). In the case of the US data, the transition function is much smoother (Figure 4) and the data are not separated distinctly into regimes. Although Figure 4 suggests a distinct outer regime, it also suggests much trading takes place ‘between’ regimes, that is there is a slow adjustment between outer and inner regime. This may reflect the greater heterogeneity of share ownership in the US. Finally, the plots of the cumulative squared prediction errors (SSE) in Japan and the US (Figure 5 and 6) further support our findings. As we plot the SSE of the prevailing mean stock return (NULL) minus the estimated model (ALTERNATIVE), a positive and raising line indicates better predictive performance of the estimated model versus the naive mean stock return model (e.g., Welch and Goyal, 2008). In Japan (Figure 5), the linear and LSTR1 model appear to persistently outperform the naive model. However, the LSTR1 model does not perform better than the linear model in and out of sample. The US models perform better than the naive model, but are not persistently outperforming. The LSTR1 model does best in and out of sample compared to the naive, linear and LSTR2 model. In particular around the 1987 stock market crash, the LSTR1 model seems to do better than the other models.

**Crossplot G of LSTR1 in the Japan**

![Crossplot G of LSTR1 in the Japan](image1.png)

**Fig. 1. Transition function for LSTR1 model in Japan**

**Crossplot G of LSTR2 in the Japan**

![Crossplot G of LSTR2 in the Japan](image2.png)

**Fig. 2. Transition function for LSTR2 model in Japan**
**Fig. 3. Transition function for LSTR1 model in the US**

**Fig. 4. Transition function for LSTR2 model in the US**

**In and out of sample performance in Japan**

**Fig. 5. Cumulative SSE differences for Japan**
In and out of sample performance in US

Fig. 6. Cumulative SSE differences for US

**Conclusion**

This paper has investigated the non-linear behavior of stock market returns between 1981 and 2012 using both Japanese and US data. Using a general to specific approach in a linear VAR model we find different macroeconomic and financial variables drive stock market returns across the two countries. By applying the Logistic Smooth Transition Model to our data we also find strong evidence of a non-linear relationship between stock returns and these variables, for both sets of data we find the LSTR1 to be the preferred model (although out of sample goodness of fit measures select the linear model for the Japanese data). Applying the LSTR1 model allows us to separate the relationship between stock returns and our macro/finance variables into an upper and lower regime. We find that the relationship between stock returns and the macro/finance variables in our model differ across regimes in both countries. That is the relationship between stock returns and macro/finance variables is dependent upon the ‘state’ of the market. We also applied an LSTR2 model to our data and found some evidence of an inner and outer regime in the Japanese data. Applying the LSTR2 model to the US data provided evidence of an outer regime but not strong evidence for an inner regime. The US results also suggested greater heterogeneity in the relationship between stock returns and macro/finance variables in the US data relative to the Japanese data. The reasons behind the differences in our results, both between countries and between regimes in our LSTR models are difficult to determine. They are probably due to the different economic conditions faced by Japan and the US over our sample, to the possible existence of bubbles in the data and to investor behavior consistent with ‘behavioral finance’ theories of investor behavior.

**References**


