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A capital structure model (CSM) with tax rate changes

Abstract

Perpetuity gain to leverage (G_L) research originates in Modigliani and Miller (1963) and was extended by Miller (1977) to incorporate personal taxes. This research analyzes G_L when issuing debt to retire unlevered equity. Hull (2007, 2010, 2012) extended this research by developing the Capital Structure Model (CSM) that demonstrates how the costs of borrowing affect G_L and the optimal debt-equity ratio (ODE). The CSM research, like the mainline capital structure research originating in Modigliani and Miller (1958, 1963), has a shortcoming: it fails to address the effect on G_L and ODE when a leverage change alters corporate and personal tax rates. To overcome this shortcoming, we derive new CSM equations that allow tax rates to be dependent on leverage. The directions of how tax rates change with leverage are based on arguments we supply. Through the derivational process, we discover a new “ α ” variable to add to the prior “ α ” variable discovered by Miller and his predecessors. One of our new equations uses the original Hull (2007) CSM framework that assumes an unlevered situation, no growth and discount rates that change with leverage. Another new equation adds complexity by using the more recent CSM framework of Hull (2012) for a levered firm with growth and a wealth transfer. We use two of our new equations to illustrate the role of changing tax rates. Our illustrations suggest managers cannot ignore even relatively small changes in tax rates. In particular, they cannot ignore the new “ α ” variable we discover.

Keywords: capital structure model (CSM), gain to leverage, changes in tax rates.

JEL Classification: G32, C02.

Introduction

Capital structure research, including perpetuity gain to leverage (G_L) research originating in Modigliani and Miller (1963), implicitly operates under the assumption tax rates do not change when leverage changes. But is this the case? Is it possible there could be wealth-impacting shifts in tax rates when firms undergo debt-for-equity and equity-for-debt increments? For example, consider the situation where there is a large capital structure change in which sizeable contingents of equity and debt owners in different tax brackets enter and exit the firm’s ownership structure. Or consider a capital structure alteration undertaken in response to a change in tax laws be it either the corporate tax rate or personal tax rates. For the latter case, a firm could experience shifts in its investor clienteles so that the after-personal tax view of a firm’s value is significantly modified by a leverage change. Of further consideration, there could be large differences in tax rate sensitivities to leverage changes based on firm size making it imperative that G_L formulas take into account such sensitivities.

The challenge of incorporating changes in tax rates within a G_L equation motivates this study. This study uses the U.S.A. as its reference point making the general findings and conclusions applicable as most tax systems around the world, like the U.S.A., provide substantial incentives for debt over equity. In this paper, we use the framework of the Capital Structure Model (CSM) of Hull (2007; 2010; 2012) to derive G_L equations where tax rates are allowed to be dependent on the capital structure choice. With

these equations in place, we are able to theoretically explore the “what if” scenario of how a change in corporate and personal tax rates affect G_L and the optimal debt-equity ratio (ODE). In the process, we extend the CSM research by deriving a series of CSM equations enabling us to tackle the following research question:

“How will a shift in tax rates resulting from a leverage change affect firm value and thus influence the managerial decision concerning how much leverage is needed to achieve its maximum G_L and ODE ?”

In answering our research question, this paper is designed to integrate a number of other corporate finance topics besides changes in corporate and personal taxes. These topics include (1) unlevered versus levered situation; (2) non-growth versus growth including the growth concepts introduced by Hull (2010), namely, the *minimum unlevered equity growth rate* (that tells us when growth is profitable) and the break-through concept of the *equilibrium levered equity growth rate* (that reveals the simultaneous influence of the plowback-payout and debt-equity choices on G_L); and, (3) agency considerations, such as the shift in risk among security holders as studied by Jensen and Meckling (1976) and Masulis (1980), asset substitution as contained in Jensen and Meckling and Leland (1998), underinvestment as examined by Myers (1977) and Gay and Nam (1998), and, the association between an optimal leverage ratio and wealth effects as investigated by Leland (1998) and Hull (1999).

This paper offers a number of contributions to the capital structure research. First, we review the literature pointing out that prior research has failed to consider the impact of tax rates changing with

leverage. Second, we develop arguments showing how tax rates are expected to change with leverage. Third, we derive new CSM equations that allow tax rates to be dependent on a firm's debt-equity choice. Fourth, we discover a new " α " variable that is found in the 2nd component of CSM equations. Thus, we find that changes in tax rates affect more than just the well-known " α " variable associated with Miller (1977). Fifth, our new CSM equations cover a variety of situations faced by financial manager such as unlevered versus levered, growth versus nongrowth, wealth transfers versus non-wealth transfers, and debt-for-equity increment versus equity-for-debt increment. Sixth, we illustrate that relatively small changes in tax rates caused by a leverage change can have substantial impacts on G_L and ODE . The new " α " variable we discover proves to be the key tax rate change variable.

The remainder of the paper is as follows. Section 1 reviews the mainline capital structure research. Section 2 overviews the CSM research and identifies the same shortcoming found in the review of the mainline research. Section 3 discusses how tax rates are expected to change with leverage. Section 4 derives new G_L equations where tax rates are dependent on leverage. Section 5 provides illustrations demonstrating how relatively small changes in tax rates can exercise a substantial influence over G_L and ODE . The paper ends by offering conclusions.

1. Mainline capital structure research

This section reviews the mainline capital structure research including the perpetuity G_L domain of inquiry. We call attention to a shortcoming of this research, which is its failure to consider how G_L and ODE can be influenced by changes in tax rates as leverage changes.

1.1. Earlier capital structure research. Along with the short-lived Modigliani and Miller (1958) theory of capital structure irrelevance, mainline capital structure research owes its foundation to the seminal perpetuity gain to leverage (G_L) research originating with Modigliani and Miller (1963), referred to henceforth as MM. Four simplifying conditions used by MM in deriving G_L include corporate taxes, no personal taxes, no growth, and an unlevered situation. Of importance to this paper, not only are personal taxes not considered but no discussion is made of tax rates changing when a firm's leverage changes. The bare bone MM conditions yielded a tax shield model for a debt-for-equity increment:

$$G_L = T_C D, \quad (1)$$

where T_C is the effective corporate tax rate (includes federal, regional and municipal taxes) and D is perpetual riskless debt. Without personal taxes and

with a riskless perpetual interest payment (I), $D = \frac{I}{r_F}$, where r_F is the cost of riskless debt.

MM (1963) noted a tax advantage for debt does not mean firms should issue all debt. For example, they suggested debt may be too expensive when personal taxes are considered. This personal tax consideration was subsequently explored by researchers (Farrar and Selwyn, 1967; Myers, 1967; Stapleton, 1972; Stiglitz, 1973) who analyzed its importance on the debt-equity choice. As noted by Myers, a key point of Farrar and Selwyn's argument was all-equity financing (or even negative leverage) is possible given both corporate and personal taxes. This personal tax rate research culminated with Miller (1977) when he extended (1) to get:

$$G_L = (1 - \alpha)D, \quad (2)$$

where $\alpha = \frac{(1 - T_E)(1 - T_C)}{(1 - T_D)}$, T_E and T_D are the respective equity and debt personal tax rates, and now $D = \frac{(1 - T_D)I}{r_D}$, where financial distress costs are possible so that $r_D > r_F$ holds. On the firm level, Miller advocated financial distress costs could be disregarded as they are negligible and personal tax on debt is high compared to equity such that $(1 - T_D) \approx (1 - T_E)(1 - T_C)$ causing $\alpha \approx 1$ and implying $G_L \approx 0$. Thus, for Miller, incorporating personal taxes restored an earlier conclusion by Modigliani and Miller (1958) that firm value lies in its operating assets and not its choice of financing. Equation (2), like (1), contains no allowance for the possibility of tax rates changing with leverage so as to impact the maximum G_L or ODE .

By focusing only on a tax effect, MM were criticized by scholars (Baxter, 1967; Kraus and Litzenberger, 1973; Jensen and Meckling, 1976) who argued for substantial valuation effects from bankruptcy and agency costs. Miller (1977) had cited Warner (1977) when arguing there is no evidence debt-related costs have any real impact on firm value for large, publicly-held corporations. However, subsequent researchers (Altman, 1984; Jensen, 1986; Cutler and Summers, 1988; Fischer, Heinkel and Zechner, 1989; Kayhan and Titman, 2007) offered findings contrary to Miller's arguments thus questioning the adequacy of "tax-only" G_L equations.

1.2. Post-Miller tax research. Post-Miller researchers have studied the role of taxes. For example, Auerbach (1983) corroborated previous findings showing that the tax bracket of equity investors is negatively related to dividends paid. Gordon and MacKie-Mason (1994) found investors in low tax brackets have an incentive to own noncorporate firms (e.g., firms tending to be small

and entrepreneurial) that generate substantial taxable income. For these investors, their income is taxed at their low personal tax rates rather than at higher corporate tax rates.

Graham (1999) discovered debt usage is significantly correlated with tax rates. He also found a specification that adjusted tax benefits for a personal tax penalty statistically dominated a specification that did not. Finally, he distinctly identified not only the positive effect of corporate taxes on debt usage but also the negative effect of personal taxes on debt usage. Using data from U.S. Statistics of Income, Corporate Income Tax Returns, Gordon and Lee (2001) held personal tax rates fixed and found a cut of 10% in the corporate tax rate could reduce debt financing by 3.5%.

Overesch and Voeller (2010) provided a literature review on corporate and personal taxation before examining European firms from 23 countries. They offered evidence that corporate tax rates and personal equity tax rates have a significant positive impact on leverage, while personal debt tax rates have a negative influence. In addition, they found the capital structures of smaller companies react more heavily to higher tax benefits of debt. Faccio and Xu (2014) used a sample of 184 changes in corporate tax rates and 298 changes in personal tax rates to pinpoint a multitude of tax reforms affecting statutory tax rates across OECD countries from 1981 through 2009. They discovered the market highly values tax benefits. Panier, Pérez-González and Villanueva (2014) studied novel tax provision (the notional interest deduction) introduced in Belgium in 2006. They found this policy change affects tax rates causing a statistically significant change in the use of equity. By issuing more equity and thus changing its *ODE*, firm value from an investor's perspective was maximized.

1.3. Shortcoming of mainline capital structure research. Despite the recognition by extant research that corporate and personal tax rates influence firm value and the leverage choice, this research has failed to directly isolate the influence caused by corporate and personal tax rates changing due to a leverage change. In this paper, we attempt to overcome this shortcoming and in the process provide added direction to capital structure theory while laying groundwork to better guide and explain managerial behavior.

In their review of the capital structure literature, Graham and Leary (2011) commented that researchers have explained only part of the observed behavior. They add that researchers have also studied the wrong models and issues while improperly measuring key variables. Given this

woeful judgment of extant capital structure research, it is imperative that researchers attempt new investigative and inventive avenues of exploration. Such exploratory research should include examining the impact of leverage changes on corporate and personal tax rates to determine if something new and inventive can be uncovered.

2. Capital Structure Model (CSM) research

This section looks at the CSM research. We find the same shortcoming within the CSM research that is present in mainline research. Namely, we discover the failure to consider how G_L and *ODE* can be dependent on changes in tax rates caused by a leverage change.

2.1. Overview of CSM research. Seeking a fresh approach with measurable variables aimed at guiding managerial decision-making, Hull (2007, 2010, 2012) builds on the Miller perpetuity G_L research given in (2) by developing the Capital Structure Model (CSM). Keeping the MM and Miller unlevered and non-growth conditions, Hull (2007) derived a CSM equation incorporating discount rates dependent on the leverage change. This equation is:

$$G_L^{D \rightarrow E} = \left[1 - \frac{\alpha r_D}{r_L} \right] D - \left[1 - \frac{r_U}{r_L} \right] E_U, \quad (3)$$

where $D \rightarrow E$ indicates debt-for-equity increment, r_U and r_L are the unlevered and levered equity rates, E_U is unlevered equity value, the 1st component captures a positive tax-agency effect, and the 2nd component represents financial distress costs (captured by increasing r_L values as debt increases) such that this component's negativity can offset the positive 1st component as debt increases.

Hull (2010) extended (3) by incorporating growth and in the process revealed how the role of the plowback-payout decision affects the leverage decision. The CSM growth equation is:

$$G_L^{D \rightarrow E} = \left[1 - \frac{\alpha r_D}{r_{Lg}} \right] D - \left[1 - \frac{r_{Ug}}{r_{Lg}} \right] E_U, \quad (4)$$

where r_{Ug} and r_{Lg} are the growth-adjusted discount rates on unlevered and levered equity, $r_{Ug} = r_U - g_U$ with r_U and g_U the borrowing and growth rates for unlevered equity, and $r_{Lg} = r_L - g_L$ with r_L and g_L the borrowing and growth rates for levered equity. Hull showed that while g_U depends on its plowback-payout decision, g_L depends on both the plowback-payout and debt-equity decisions. The introduction of growth can alter the two components in (4) so that they differ in sign from their corresponding expected positive and negative components in (3).

This alteration tends to occur for a high growth firm past its critical point, which is a point where the plowback ratio (PBR) using internal equity equals T_C . Hull argues firms can lose value if they cannot sustain a PBR of at least T_C when financing with internal equity. The argument is based on the fact firms are taxed the first time on internal funds used for growth and then are taxed the second time on the earnings that the growth generates for dividends. External equity does not have this double taxation because it is not a source of corporate taxable income until it generates earnings payable as dividends. Using external equity, the firm would have to sustain a PBR equal to the marginal flotation costs of external equity instead of T_C .

Hull (2012) incorporated a levered situation within the CSM framework and derived G_L equations including those showing how a wealth transfer (linked to a shift in risk between debt and equity) impacts firm value. For a levered situation, we have to distinguish between a variable that takes two values. Hull does this by using subscripts. In essence, the subscript “1” refers to a variable’s less levered value and “2” to its more levered value. Thus, for a debt-for-equity increment, “1” denotes before the increment and “2” signifies after the increment. For an equity-for-debt increment, “1” is after and “2” is before.

Hull’s levered situation equations focus on how the cost of current debt (r_{D_1}) might change. For a debt-for-equity increment, the three outcomes for r_{D_1} are a decrease, no change, and an increase. Hull states an increase in r_{D_1} is the most likely outcome and it can be caused by the new debt (D_2) being senior to the outstanding debt (D_1) but can also result from the claims of D_1 being diluted by D_2 . For this outcome, Hull shows:

$$G_{L_2}^{D \rightarrow E} = \left[1 - \frac{\alpha r_{D_2}}{r_{L_2}} \right] D_2 - \left[1 - \frac{r_{L_1}}{r_{L_2}} \right] E_{L_1} - \left[1 - \frac{r_{D_1}}{r_{D_1 \uparrow}} \right] D_1, \quad (5)$$

where the “2” in $G_{L_2}^{D \rightarrow E}$ indicates there was at least one prior leverage change, r_{D_1} is the discount rate on the earlier debt (D_1); r_{D_2} is the discount rate on the new debt (D_2); $r_{D_1 \uparrow}$ is r_{D_1} after its risk shifts upward by issuing D_2 ; E_{L_1} is levered equity value prior to the new debt-for-equity increment; r_{L_1} is the growth-adjusted levered equity discount rate prior to the increment with $r_{L_1} = r_{L_1} - g_{L_1}$ where r_{L_1} and g_{L_1} are equity’s discount and growth rates prior to the increment; and, r_{L_2} is the growth-adjusted levered equity discount rate after the increment with $r_{L_2} = r_{L_2} - g_{L_2}$ where r_{L_2} and g_{L_2} are equity’s discount and growth rates after the

increment. The last component is negative and identical to the fall in value for D_1 caused when its discount rate increases from r_{D_1} to $r_{D_1 \uparrow}$.

The CSM equations support trade-off theory (DeAngelo and Masulis, 1980; Hackbarth, Hennessy, and Leland, 2007; Berk, Stanton, and Zechner, 2010; Korteweg, 2010; Van Binsbergen, Graham and Yang, 2010). They can also help overcome the practical problems of measuring the numerous tax, bankruptcy and agency effects. In his recent CSM pedagogical paper, Hull (2014) states: “*The CSM research makes the measurement task manageable through its development of equations that require managers to simply estimate tax, borrowing, and growth rates.*”

2.2. Shortcoming of CSM research. Despite the recognition by CSM research that tax rates impact G_L and ODE , this research has the same shortcoming as mainline capital structure research: it has failed to isolate the influence caused by corporate and personal tax rates changing when leverage changes. The theoretical and empirical support for the effect of taxes on capital structure indicates that the dependence of tax rates on leverage is an unexplored avenue for future research. For example, tax rate researchers (Graham, 1999; Overesch and Voeller, 2010; Faccio and Xu, 2014) document the positive effect of higher corporate tax rates and lower equity tax rates on debt and the negative effect of higher personal tax rates on debt income. With the effect of tax rates documented, the next step involves investigating how a change in leverage will influence firm value by changing tax rates.

3. Arguments for how tax rates are expected to change with leverage

In this section, we argue Miller’s α should increase with debt. This directional relation is predictable once we consider the arguments for how the three tax rates should change with debt.

3.1. There is little theory on how tax rates change with leverage. The Miller (1977) and the CSM (2007, 2010, 2012) derivations implicitly assume a leverage change does not change tax rates. The fact the relation between tax rates and leverage is rarely (if ever) discussed leads one to ask why? Perhaps it is because there is no accepted theoretical concept associated with how tax rates change with leverage such as is found for discount rates where the notion of default risk premium explains the positive relation. Consequently, it is taken for granted tax rates are independent of leverage, which is possible if firms can maintain large positive taxable earnings (making T_C stable) and can attract investors within personal tax clienteles (making T_D and T_E stable). We will now explore this possibility and argue that the tax rates and leverage are not independent.

3.2. Arguments for an increasing α and its impact on Miller's G_L . Despite being devoid of an accepted theoretical concept for how tax rates change with leverage, the research is not entirely silent on the subject. For example, Graham (2000) represents a line of research that argues a firm is less likely to use the entire debt tax shield as debt increases. This serves to lower T_C as leverage increases. There is also a supply and demand argument for how tax rates should change with leverage. To illustrate on the personal tax level, consider the situation where a firm undertakes a period of leveraging up and investors attempt to lower their taxable income. Such a firm would initially bring in those debt investors with lower tax rates. However, if the supply of investors is limited, continued debt issuance would eventually bring in investors with increasingly higher tax rates. Similarly, for the retirement of equity brought about by a debt-for-equity increment, the principle of self-interested behavior would predict the firm would first lose those equity investors with higher tax rates thus causing a lower T_E .

Further consider a period where many companies tend to take on more leverage. The increase in T_D could be a widespread phenomenon. The end result would be a clientele of debt investors with a larger effective T_D . Since an overall leveraging up period means equity is falling, the end result would be a clientele of equity investors with a smaller effective T_E .

While one may not know the exact strength of the expected changes in tax rates caused by a debt-for-equity increment, the above arguments imply the net influence on α would be increasing. This is as seen below:

$$T_E \downarrow, T_C \downarrow \text{ and } T_D \uparrow \rightarrow \frac{(1-T_E) \uparrow (1-T_C) \uparrow}{(1-T_D) \downarrow} \rightarrow \alpha \uparrow$$

due to three upward forces on α .

From the above depiction of α increasing with a debt-for-equity increment, we expect Miller's G_L as given in (2) to fall as firms undergo greater debt-for-equity increments. As α increases and approaches 1 this means $G_L = (1-\alpha)D$ approaches zero. For greater debt levels and trivial bankruptcy and agency costs, we get Miller's result of $G_L \approx 0$ due simply to the interplay of corporate and personal taxes. However, $G_L = 0$ could not hold for any debt level as Miller advocates but would occur only at that debt level where tax rates change causing $\alpha = 1$.

3.3. The impact of α within the CSM framework. The 1st component of (3) will behave like Miller's G_L equation with decreasing values as α increases

with more and more debt-for-equity increments. For Miller, if $\alpha > 1$ then G_L is negative. For (3), if α becomes greater than $\frac{r_D}{r_L}$ then the 1st component can be negative. For equations (4) and (5), α has to become greater than $\frac{r_D}{r_{Lg}}$ and $\frac{r_{D2}}{r_{Lg2}}$ for negativity to result. Since a growth-adjusted discount rate on levered equity is smaller than an equity rate without growth, it is less likely α in itself could become large enough to reverse a positive G_L to a negative G_L . As will be seen in the next section when we derive this paper's new CSM equations, changes in tax rates will affect more than one CSM component thus adding a further dimension to consider.

In conclusion, if tax rates are dependent on the leverage choice, then G_L formulations should try to incorporate this dependency so as to not overlook the potential impact when these tax rates change with leverage. While this section focused on the increase in α caused by a debt-for-equity increment, we would conversely expect α to fall for an equity-for-debt increment.

4. G_L equations with changing tax rates

In this section, we will broaden the perpetuity G_L research using the CSM framework. We will do this by deriving new G_L equations when tax rates are dependent on the debt-equity choice.

4.1. Suppose tax rates change for Hull (2007) for a debt-for-equity increment. For our first CSM extension, we begin with (3) given by Hull (2007). This equation assumes a debt-for-equity exchange for an unlevered situation with no growth and with discount rates that increase with leverage. The only modification of (3) is that we now allow tax rates on corporate earnings and investor income to be dependent on leverage. The change in T_E would be dependent on the proportions of capital gains or dividends that might be altered when there is a leverage change. Because the firm is unlevered with no outstanding debt, we have no direct change in T_D for this extension but, given our arguments in Section 3, we still expect T_D be an increasing function of debt.

We label the two applicable tax rates prior to the debt-for-equity increment as T_{C1} and T_{E1} and afterwards as T_{C2} and T_{E2} . This is consistent with the labeling used earlier where the subscript "1" refers to the value for a less levered situation and "2" for a more levered situation. Proceeding in an algebraic fashion similar to Hull (2007), we show in Appendix A that:

$$G_L^{D \rightarrow E} = \left[1 - \frac{\alpha_1 r_D}{r_L} \right] D - \left[1 - \frac{\alpha_2 r_U}{r_L} \right] E_U, \quad (6)$$

where $\alpha_1 = \frac{(1-T_{E2})(1-T_{C2})}{(1-T_D)}$ and

$$\alpha_2 = \frac{(1-T_{E2})(1-T_{C2})}{(1-T_{E1})(1-T_{C1})}$$

While the proof is not formally shown, we could derive an equation similar to (6) using growth. The procedure is like that shown in Appendix A except we substitute growth-adjusted discount rates of r_{Lg} and r_{Ug} for r_U and r_L . This equation is:

$$G_L^{D \rightarrow E} = \left[1 - \frac{\alpha_1 r_D}{r_{Lg}} \right] D - \left[1 - \frac{\alpha_2 r_{Ug}}{r_{Lg}} \right] E_U \quad (7)$$

4.2. Suppose tax rates change for Hull (2007) for an equity-for-debt increment. Following the Hull (2007) procedure for an equity-for-debt increment but allowing tax rates to change, we can get an equation that is the inverse of (6). Appendix B derives this equation for a firm becoming unlevered and shows:

$$G_L^{E \rightarrow D} = \left[1 - \frac{\alpha_2 r_U}{r_L} \right] E_U - \left[1 - \frac{\alpha_1 r_D}{r_L} \right] D, \quad (8)$$

where the 1st component is positive and more than offsets the negativity of the 2nd component if $G_L > 0$. Similarly, for a growth situation, we could get an equation like (8) except we substitute growth-adjusted discount rates for unlevered and levered equity rates.

4.3. Suppose tax rates change for Hull (2012). For our next CSM extension, we begin with the G_L equation of Hull (2012) given in (5) that assumes a debt-for-equity exchange for a levered situation with changing growth-adjusted discount rates and a wealth transfer. Once again, we will modify our chosen CSM equation to allow tax rates to be dependent on leverage. For a levered situation, we also now have a change in T_D .

As before, we will use the subscript “1” to label the tax rates for the less levered situation and so have T_{C1} , T_{E1} and T_{D1} . For the more levered situation, we again use the subscript “2” and so have T_{C2} , T_{E2} and T_{D2} . Proceeding in an algebraic fashion similar to Hull (2012), Appendix C shows:

$$G_{L2}^{D \rightarrow E} = \left[1 - \frac{\alpha_1 r_{D2}}{r_{Lg2}} \right] D_2 - \left[1 - \frac{\alpha_2 r_{Lg1}}{r_{Lg2}} \right] E_{L1} - \left[1 - \frac{r_{D1}}{r_{D1\uparrow}} \right] D_1, \quad (9)$$

where the “2” in $G_{L2}^{D \rightarrow E}$ indicates at least one prior leverage change, $\alpha_1 = \frac{(1-T_{E2})(1-T_{C2})}{(1-T_{D2})}$, r_{D2} is the

cost of the new debt, r_{Lg2} is the growth-adjusted levered equity discount rate after the increment with $r_{Lg2} = r_{L2} - g_{L2}$ where r_{L2} and g_{L2} are equity’s discount and growth rates after the increment, $D_2 = \frac{(1-T_{D2})I_2}{r_{D2}}$ with I_2 the interest paid on new debt, $\alpha_2 = \frac{(1-T_{E2})(1-T_{C2})}{(1-T_{E1})(1-T_{C1})}$, $r_{Lg1} = r_{L1} - g_{L1}$, where r_{L1} and g_{L1} are equity’s discount and growth rates prior to the increment, $E_{L1} = \frac{(1-T_{E1})(1-T_{C1})(C-I_1)}{r_{Lg1}}$, r_{D1} is the cost of the old debt, $r_{D1\uparrow}$ captures both r_{D1} and its increase due to its upward shift in risk, $D_1 = \frac{(1-T_{D1})I_1}{r_{D1}}$ with I_1 the interest paid on old debt, and α_1 in (9) is now slightly altered from α_1 in (6) and (7) because T_{D2} has replaced T_D given the tax rate on debt income can now change from its earlier levered value. The change from T_D to T_{D2} also changes D_2 as the new debt investors pay more taxes when T_D increases.

Suppose tax rates change in the manner argued so that $T_{C1} > T_{C2}$, $T_{E1} > T_{E2}$ and $T_{D2} > T_{D1}$. These latter three relations imply $(1-T_{C2}) > (1-T_{C1})$, $(1-T_{E2}) > (1-T_{E1})$ and $(1-T_{D1}) > (1-T_{D2})$. The increase in α_1 is depicted below in a manner done earlier in Section 3.

$$T_{E2\downarrow}, T_{C2\downarrow} \text{ and } T_{D2\uparrow} \rightarrow \frac{(1-T_{E2})\uparrow(1-T_{C2})\uparrow}{(1-T_{D2})\downarrow} \rightarrow \alpha_1 \uparrow$$

due to three upward forces on α_1 .

A greater α_1 serves to make a positive 1st component of (9) smaller if the cost of debt is less than the growth-adjusted rate. For the nongrowth situation, the cost of debt is always less than the cost of equity but, for a growth-adjusted levered equity rate as found in (9), Hull (2010) has shown this is not always the case especially for higher leverage ratios where the cost of debt can be greater than a growth-adjusted levered equity rate. We have a similar situation with the 2nd component (as described below) where growth leads to uncertainty on how a change in tax rates influences the component.

Since $\alpha_2 > 1$ given $(1-T_{C2}) > (1-T_{C1})$ and $(1-T_{E2}) > (1-T_{E1})$, the effect of α_2 on the 2nd component of (9) will be determined by the fraction $\frac{r_{Lg1}}{r_{Lg2}}$. As can be seen in the pedagogical exercise by Hull (2011), this fraction can go from being less than one to being greater than one if high enough debt levels are reached. If $\frac{r_{Lg1}}{r_{Lg2}} < 1$ and $\alpha_2 > 1$ then the 2nd component will be less negative (or even become

positive) but if $\frac{r_{Lg1}}{r_{Lg2}} > 1$ then the 2nd component will become more positive (or even go from negative to positive). Once again, we see changes in tax rates caused by a leverage can influence G_L and ODE .

Besides (6), (7), (8) and (9), there are other prior CSM equations that could be derived to incorporate tax rate changes. The same additions to these other prior CSM equation would result in that α_1 and α_2 would also be added to their 1st and 2nd components.

5. G_L for a levered situation with growth, wealth transfers and changing tax rates

In this section, we illustrate how a change in tax rates can have significant consequences in terms of G_L and ODE . Our illustrations will focus on the new equations of (6) and (9) that represent both a simple case and a complicated case.

5.1. Illustration of a change in tax rates using equation (6). We will first illustrate the role of changing tax rates for a simple case by using (6). For this illustration, we will use the values from the pedagogical exercise of Hull (2008) when using (3). We do this because (6) is like (3) except (6) allows for changing tax rates. Thus, we will be able to compare results with and without changing tax rates. Our illustration will allow relatively small changes of 5% in tax rates for each debt-for-equity choice. Given our arguments in Section 3, this means T_C and T_E will decrease 5% and T_D will increase 5% as a greater debt-for-equity choice is chosen.

Following Hull (2008), the unlevered equity value (E_U) is \$10B (B = billions) and the firm can choose from nine debt-for-equity choices that lead to issuing from \$1B to \$9B in new debt to retire E_U . Hull fixed tax rates at $T_C = 0.30$, $T_E = 0.05$, and $T_D = 0.15$ for all choices. These rates are considered

reasonable for firms located within the U.S.A. given that corporations are profitable and will be taxed towards the higher end of their effective tax rate, and equity owners have preferential tax treatment on their earnings paying a lower tax rate than debt owners. These tax rates generate $\alpha_1 = 0.78235$. Since tax rates do not change in (3), α_2 would equal 1 by definition were it to be found in (3). For his given cash flows, discount rates and tax rates, Hull showed (3) yields a maximum G_L of \$1.333B achieved by issuing \$5B in new debt to retire \$5B of E_U . This \$5B choice yields an ODE of 0.79. The question now is will this maximum G_L of \$1.333B and ODE of 0.79 change if we allow tax rates to be modified while maintaining the same values for all other variables found in the Hull (2008) pedagogical exercise?

When using (6), we have Hull’s three tax rate values of $T_C = 0.30$, $T_E = 0.05$, and $T_D = 0.15$ occur at his \$5B debt-for-equity choice as this is the middle choice (and also happens to be where the maximum G_L is achieved). To achieve this while allowing a 5% decrease in T_C and T_E for each \$1B additional debt, we have to set the following initial unlevered values of $T_C = 0.3877$ and $T_E = 0.0646$. The value for T_D is dependent on the leverage choice even though at the time of the debt-for-equity increment there is no debt and thus no T_D per se. Because T_D has no unlevered value, we set it at $T_D = 0.1234$ for the \$1B debt choice and increase it by 5% to achieve Hull’s rate of 0.15 at the \$5B choice. The values for T_C range from 38.77% for the unlevered choice to 24.44% for the \$9B choice. The corresponding values for T_E range from 6.46% to 4.07% and for T_D from 12.34% to 18.23%. The values for α_1 range from an unlevered value of 0.6490 to 0.8865 for the \$9B choice, while α_2 values range from 1.0352 for the \$1B choice to 1.0196 for the \$9B choice.



Fig. 1. Optimal G_L increase when tax rates change using equation (6)

Given these tax rates along with Hull’s discount rates and cash flows, Figure 1 compares the results between (3) and (6) with G_L along the vertical axis and the nine debt-for-equity increment choices along the horizontal axis. All values in the figure are in billions of dollars (\$B). This figure shows the maximum G_L of \$1.333B for (3) changes to \$1.589B for (6). This is an increase in value of nearly 20%. Thus, using only a 5% change in tax rates in the predicted directions, we find there is almost a four times greater percentage change in the maximum G_L compared to tax rate changes. The optimal debt-for-equity choice is now to retire \$4B in equity instead of \$5B. This causes a fall in ODE from 0.79 to 0.53, which is a drop of over 33%. Thus, for every 5% change in tax rates, there is nearly a seven times greater percentage change in ODE .

Figure 2 plots the difference between values for the first two components separately and combined. Each difference is computed by subtracting (3) from (6). The difference in the combined components is the same as the difference in G_L . For example, for the first choice of \$1B debt-for-equity, Figure 2 reports a difference of \$0.0482B computed when the 1st component in (3) is subtracted from the 1st component in (6). This difference represents the gain in the 1st component when tax rates are allowed to change. The gain is the result of $\frac{r_D}{r_L}$ being multiplied by $\alpha_1 = 0.64900$ instead of 0.78235 where the latter value is used by (3) for all debt-for-equity choices. Beginning with the \$6B choice α_1 will be greater than 0.78235 causing the difference in 1st components to be less. This difference is shown in Figure 2 to be $-\$0.0801B$.

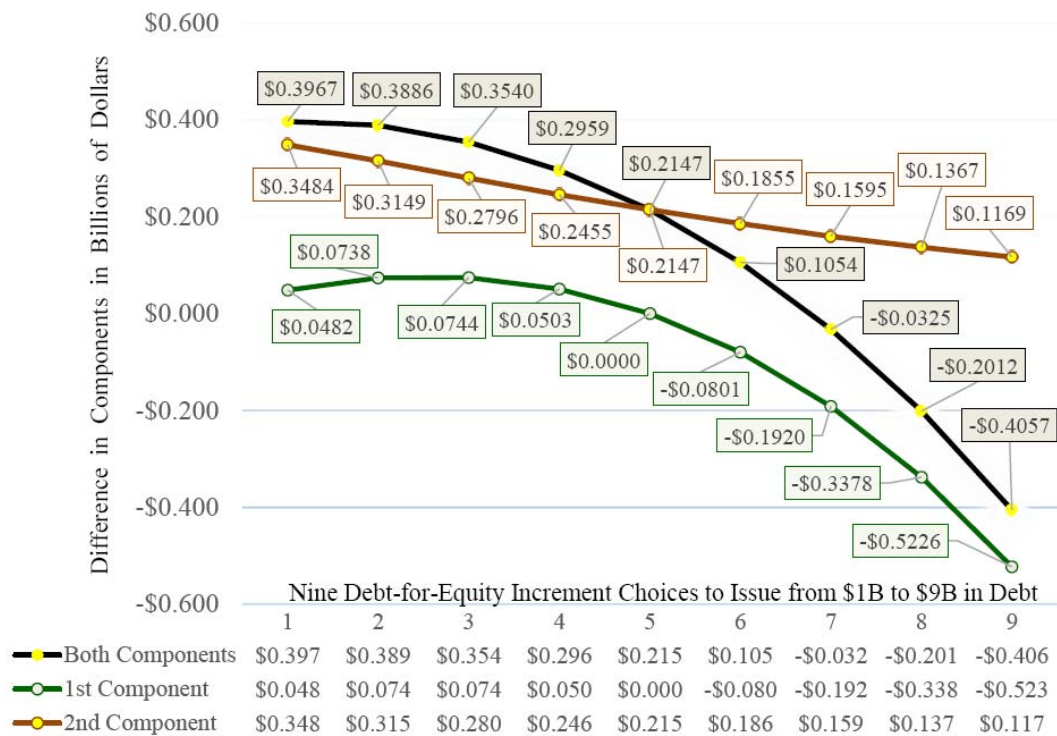


Fig. 2. Values when equation (3) is subtracted from equation (6)

The difference between the 2nd components for the \$1B choice is \$0.3484B and is much greater than the difference of \$0.0484B found for the 1st component. As seen in Figure 2, this trend of the 2nd components dominating continues until the \$7B choice is reached at which point the absolute value of the difference $-\$0.1920B$ between the 1st components is greater than the difference of \$0.1595B for the 2nd components. The total difference of both components (which is the same as the total difference in the two G_L values) is \$0.3967B for the \$1B choice.

This positive differences in G_L between (6) and (3) falls gradually but remains positive until the \$7B choice where we get $-\$0.0325B$.

The 2nd component explains nearly 90% of the difference in G_L for the \$1B choice. For the \$5B choice, we see the 2nd component explains the total difference in G_L of \$0.215B. This is because we set the after-leverage tax rates in (6) to be the same as that used in (3) for \$5B choice causing the 1st components to be equal. This is not the case for α_2 in the 2nd component of (6) because α_2 includes tax

rates before and after the leverage with $\alpha_2 > 1$. As explained earlier, $\alpha_2 = 1$ for (3) since by definition all tax rates used in (3) have the same values before and after the leverage change.

As can be seen in comparing the two figures, the superior G_L values occurring in Figure 1 for the first six debt-for-equity choices using (6) are largely (or in some cases totally) explained by the 2nd component. While the 2nd component is very instrumental in determining G_L and ODE, this 2nd component is missing from Miller while α_2 is missing from prior CSM equations. Whereas α_1 makes the 1st component in (6) less positive than the 1st component in (3) for choices from \$0.6B to \$0.9B, α_2 always makes the 2nd component in (6) less negative (or of greater value) than the 2nd component in (3). For the \$1B choice, the 2nd component in (6) is actually not less negative than (3) but it is positive. This is because r_L is still low with less financial risk and so the fraction $\frac{r_U}{r_L}$ is

close to 1 so that $\frac{\alpha_2 r_U}{r_L} > 1$. Given the above, we can see the overall effect on G_L from changes in tax rates would have to be positive for at least the debt choices from \$1B to \$5B.

For the tax rates used in (3) and (6), there is a lower tax rate on equity compared to debt. This is not only for the fixed rates we use from Hull's pedagogical exercise, but also the way the rates change over time. The difference of 5.29% between T_D and T_E for the \$1B choice increases to 14.16% for the \$9B choice. In essence, as we bring in more debt investors, we assume their tax rates increase with leverage due to finite supply of investors. By losing equity investors over time, we would tend to lose those with higher tax rates. For other scenarios (such as less differential between equity and debt tax rates) different results will occur. For example, suppose we have tax rates change so T_E and T_D are always equal. Fixing tax rates as such, along with the other MM-Miller derivational assumptions, would bring us back to the situation of a firm issuing unlimited debt. The particular scenario we have chosen for our illustration (where $T_D > T_E$ for all debt issuance choices) is widely accepted as factual in the U.S.A. and adds support to our illustration's main point which is: changes in tax rates (even relatively smaller changes) can profoundly alter the maximum G_L and ODE.

5.2. Illustration of a change in tax rates using equation (9). Equations (5) and (9) share in the same derivational assumptions except (9) allows tax rates to change when the debt-equity choice changes. In comparing these two equations, we will focus on a comparison of key variables that α_1 and

α_2 can influence. This comparison will serve to illustrate that changing tax rates can impact the maximum G_L and ODE when we have a levered situation with constant growth and a wealth transfer.

Let us begin by considering the following values: $T_{C1} = 0.30$, $T_{C2} = 0.25$, $T_{E1} = 0.10$, $T_{E2} = 0.05$, $T_{D1} = 0.15$, and $T_{D2} = 0.20$. Using these values in (9) gives

$$\alpha_1 = \frac{(1-T_{E2})(1-T_{C2})}{(1-T_{D2})} = \frac{(1-0.05)(1-0.25)}{(1-0.20)} = 0.8906$$

and

$$\alpha_2 = \frac{(1-T_{E2})(1-T_{C2})}{(1-T_{E1})(1-T_{C1})} = \frac{(1-0.05)(1-0.25)}{(1-0.10)(1-0.30)} = 1.1310.$$

Since α_1 is computed in terms of tax rates for the more levered situation, let us compute α for the less levered situation or what it would be without any change in tax rates such as given by (5). Doing this gives:

$$\alpha = \frac{(1-T_{E1})(1-T_{C1})}{(1-T_{D1})} = \frac{(1-0.10)(1-0.30)}{(1-0.15)} = 0.7412.$$

Multiplying $\frac{r_{D2}}{r_{Lg2}}$ by 0.8906 instead of 0.7412 will make this 1st component less positive as long as $\frac{\alpha_1 r_{D2}}{r_{Lg2}} < 1$ holds. It can even cause this component

to become negative if $\frac{\alpha_1 r_{D2}}{r_{Lg2}} > 1$ results from

allowing an increase from $\alpha = 0.7412$ to $\alpha_1 = 0.8906$. Suppose this component is negative without the change from 0.7412 to 0.8906. If so, then multiplying by 0.8906 instead of 0.7412 would cause it to become more negative.

Similarly to what was just portrayed for α_1 , the same can hold when $\alpha_2 > 1$ replaces $\alpha_2 = 1$ where the latter is a situation where there are no changes in tax rates. In other words, the 2nd component of $- \left[1 - \frac{\alpha_2 r_{Lg1}}{r_{Lg2}} \right] E_{L1}$ can become less negative, can go from negative to positive, or can become more positive depending on the sign and value of $\frac{r_{Lg1}}{r_{Lg2}}$.

Conclusion

This paper treads new ground by incorporating changes in tax rates within the CSM's perpetuity G_L framework. In the process, we offer important contributions to the capital structure research. First, we scan the tax rate literature and find very little has been done in terms of analyzing how a change in tax rates might be an important consideration when computing a firm's maximum G_L and ODE. Second, we provide arguments showing the direction that tax rates will change with leverage.

Third, we derive new CSM equations for the gains to leverage by allowing tax rates to be dependent on the debt-equity choice. We begin by developing G_L equations for a leverage change using a simplified CSM framework for an unlevered firm with no growth and no wealth transfers but with changes allowed for discount rates and tax rates. We derive G_L equations with these assumptions for both a debt-for-equity increment and also for an equity-for-debt increment. We also derive a more sophisticated G_L equations for a leverage change consisting of a levered firm with growth and wealth transfers where discount, growth and tax rates all change with leverage.

Fourth, we show the change in tax rates affects two components of a CSM equation in terms of an " α " variable. We call these α_1 and α_2 where α_1 is different from Miller's " α " by changing with leverage. The variable α_2 is a new discovery and found in the 2nd component of CSM equations. We demonstrate that α_2 can have an even greater influence than α_1 in determining a maximum G_L and *ODE*. Fifth, our new equations cover a variety of situations for the financial manager such as unlevered versus levered, growth versus nongrowth, wealth transfers versus non-wealth transfers, and debt-for-equity increment versus equity-for-debt increment. Sixth, we use our newly derived CSM equations to illustrate how small changes in tax rates

can cause substantial changes in the capital structure decision-making process.

The CSM offers a vigorous set of G_L equations that can supply useful insight on understanding and solving capital structure problems. This set of equations can help us understand the effects of changing discount rates, growth rates and tax rates as well as the influence from shifts in risk when a firm undergoes a capital structure change. While the CSM research is still relatively new and in need of critical analysis to find shortcomings, this paper has attempted to overcome one shortcoming of this research, namely, the impact of tax rates changing with leverage. By investigating this shortcoming, we have been able to extend Hull (2007, 2010, 2012) by offering new CSM equations. It is the authorship's hope this paper can further stimulate exposure to the CSM so it can be enriched by scrutiny from corporate finance researchers and practitioners around the world.

Future G_L research can extend this paper by further exploring the theoretical implications, practical applications, and pedagogical exercises inherent in this paper's CSM extension. In particular, a practical paper with teaching implications along the lines of Hull (2005, 2008, 2011, 2014), but with changes in taxes incorporated, can be developed. This future paper and its exercises and applications would expand on the illustrations given in Section 5.

References

1. Altman, Edward (1984). A Further Empirical Investigation of the Bankruptcy Cost Question, *Journal of Finance*, 39 (4), pp. 1067-1087.
2. Auerbach, Alan J. (1983). Stockholder Tax Rates and Firm Attributes, *Journal of Public Economics*, 21 (2), pp. 107-127.
3. Baxter, Nevins (1967). Leverage Risk of Ruin and the Cost of Capital, *Journal of Finance*, 22 (3), pp. 395-403.
4. Berk, Jonathan, Richard Stanton and Josef Zechner (2010). Human Capital, Bankruptcy and Capital Structure, *Journal of Finance*, 65 (3), pp. 891-926.
5. Cutler, David and Lawrence Summers (1988). The Costs of Conflict Resolution and Financial Distress: Evidence from the Texaco-Pennzoil Litigation, *Rand Journal of Economics*, 19 (2), pp. 157-172.
6. DeAngelo, Harry and Ronald Masulis (1980). Optimal Capital Structure under Corporate and Personal Taxes, *Journal of Financial Economics*, 8 (1), pp. 3-29.
7. Farrar, Donald E. and Lee L. Selwyn (1967). Taxes, Corporate Financial Policy, and Returns to Investors, *National Tax Journal*, 20 (4), pp. 444-454.
8. Faccio, Mara and Jin Xu (2014). Taxes, Capital Structure Choices and Firm Value, Working Paper, Purdue University, Terre Haute, Indiana, USA.
9. Fischer, Edwin O., Robert Heinkel and Josef Zechner (1989). Optimal Dynamic Capital Structure Choice: Theory and Tests, *Journal of Finance*, 44 (1), pp. 19-40.
10. Gay, Gerald D. and Jouahn Nam (1998). The Underinvestment Problem and Corporate Derivatives Use, *Financial Management*, 27 (4), pp. 53-69.
11. Gordon, Roger and Jeffrey K. MacKie-Mason (1994). Tax Distortions to the Choice of Organizational Form, *Journal of Public Economics*, 55 (2), pp. 279-306.
12. Gordon, Roger, Young Lee (2001). Do taxes affect corporate debt policy? Evidence from U.S. Corporate Tax Return Data, *Journal of Public Economics*, 82 (2), pp. 195-224.
13. Graham, John (1999). Do Personal Taxes affect Corporate Financing Decisions? *Journal of Public Economics*, 73 (2), pp. 147-185.
14. Graham, John R. (2000). How Big Are the Tax Benefits of Debt? *Journal of Finance*, 55 (5), pp. 1901-1941.
15. Graham, John R. and Mark T. Leary (2011). A Review of Empirical Capital Structure Research and Directions for the Future, *Annual Review of Financial Economics*, 3 (December), pp. 309-345.

16. Hackbarth, Dirk, Christopher Hennessy and Hayne Leland (2007). Can the Trade-off Theory Explain Debt Structure? *Review of Financial Studies*, 20 (5), pp. 1389-1428.
17. Hull, Robert M. (1999). Leverage Ratios, Industry Norms, and Stock Price Reaction: An Empirical Investigation of Stock-for-Debt Transactions, *Financial Management*, 28 (2), pp. 32-45.
18. Hull, Robert M. (2005). Firm Value and the Debt-Equity Choice, *Regional Business Review*, 24 (May), pp. 50-75.
19. Hull, Robert M. (2007). A Capital Structure Model, *Investment Management and Financial Innovations*, 4 (7), pp. 8-24.
20. Hull, Robert M. (2008). Capital Structure Decision-Making: A Pedagogical Application, *Journal of Financial Education*, 34 (Fall), pp. 88-111.
21. Hull, Robert M. (2010). A Capital Structure Model with Growth, *Investment Management and Financial Innovation*, 7 (4), pp. 26-40.
22. Hull, Robert M. (2011). Debt-Equity Decision-Making with and without Growth, *Managerial Finance*, 37 (8), pp. 765-787.
23. Hull, Robert M. (2012). A Capital Structure Model with Wealth Transfers, *Investment Management and Financial Innovation*, 9 (3), pp. 19-32.
24. Hull, Robert M. (2014). Debt-Equity Decision-Making with Wealth Transfers, *Managerial Finance*, Forthcoming.
25. Jensen, Michael C. and William H. Meckling (1976). Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure, *Journal of Financial Economics*, 3 (4), pp. 305-360.
26. Jensen, Michael C. (1986). Agency Costs of Free Cash Flow, Corporate Finance, and Takeovers, *American Economics Review*, 76 (2), pp. 323-329.
27. Kayhan, Ayla and Sheridan Titman (2007). Firms' Histories and their Capital Structure, *Journal of Financial Economics*, 83 (1), pp. 1-32.
28. Korteweg, Arthur (2010). The Net Benefits of Leverage, *Journal of Finance*, 65 (6), pp. 2137-2170.
29. Kraus, Alan and Robert H. Litztenberger (1973). A State-Preference Model of Optimal Financial Leverage, *Journal of Finance*, 28 (4), pp. 911-922.
30. Leland, Hayne (1998). Agency Costs, Risk Management and Capital Structure, *Journal of Finance*, 53 (4), pp. 1213-1243.
31. Masulis, Ronald (1980). Effects of Capital Structure Change on Security Prices: A Study of Exchange Offers, *Journal of Financial Economics*, 8 (2), pp. 139-178.
32. Miller, Merton (1977). Debt and Taxes, *Journal of Finance*, 32 (2), pp. 261-275.
33. Modigliani, Fred and Merton Miller (1958). The Cost of Capital, Corporation Finance and the Theory of Investment, *American Economics Review*, 48 (3), pp. 261-297.
34. Modigliani, Fred and Merton Miller (1963). Corporate Income Taxes and Cost of Capital: A Correction, *American Economics Review*, 53 (3), pp. 433-443.
35. Myers, Stewart (1967). Taxes, Corporate Financial Policy and the Return to Investors: Comment, *National Tax Journal*, 20 (4), pp. 455-462.
36. Myers, Stewart (1977). Determinants of Corporate Borrowing, *Journal of Financial Economics*, 5 (2), pp. 147-175.
37. Overesch, Michael and Dennis Voeller (2010). Impact of Personal and Corporate Taxation on Capital Structure Choices, *FinanzArchiv: Public Finance Analysis*, 66 (3), pp. 263-294.
38. Panier, Frederic, Francisco Pérez-González and Pablo Villanueva (2014). Capital Structure and Taxes: What Happens When You Also Subsidize Equity? Working Paper, Stanford University, Palo Alto, California, USA.
39. Stapleton, R.C. (1972). Taxes, the Cost of Capital and the Theory of Investment, *The Economic Journal*, 82, pp. 1273-92.
40. Stiglitz, Joseph (1973). Taxation, Corporate Financial Policy and the Cost of Capital, *Journal of Public Economics*, 2 (1), pp. 1-34.
41. Van Binsbergen, Jules, John R. Graham, and Jie Yang (2010). The Cost of Debt, *Journal of Finance*, 65 (6), pp. 2089-2136.
42. Warner, J. (1977). Bankruptcy Costs: Some Evidence, *Journal of Finance*, 32 (2), pp. 337-348.

Appendix A. Proof of equation (6)

Proof of equation (6) for the situation of an unlevered firm undergoing a debt-for-equity increment with no growth and tax rates dependent on the leverage choice. We have the following definitions for V_U and V_L . $V_U = E_U = \frac{(1-T_{E_1})(1-T_{C_1})C}{r_U}$, where T_{E_1} is unlevered equity's tax rate, T_{C_1} is the unlevered corporate tax rate, C is the perpetual before-tax cash flow to unlevered equity, and r_U is the unlevered cost of equity. $V_L = E_L + D$ with $E_L = \frac{(1-T_{E_2})(1-T_{C_2})(C-I)}{r_L}$ where T_{E_2} is levered equity's tax rate, T_{C_2} is the levered corporate tax rate, I is the perpetual interest payment, and r_L is the cost of levered equity; and $D = \frac{(1-T_D)I}{r_D}$, where T_D is debt's personal tax rate and r_D is the cost of debt. Noting $V_U = E_U$, $V_L = E_L + D$ and substituting in the definition for E_L , the equation of $G_L = V_L - V_U$ gives:

$$G_{L_2}^{D \rightarrow E} = \frac{(1-T_{E_2})(1-T_{C_2})(C-I)}{r_L} + D - E_U.$$

Multiplying out the 1st component and rearranging:

$$G_{L_2}^{D \rightarrow E} = D - \frac{(1-T_{E_2})(1-T_{C_2})I}{r_L} - E_U + \frac{(1-T_{E_2})(1-T_{C_2})C}{r_L}.$$

Multiplying the 2nd component by $\frac{(1-T_D)r_D}{(1-T_D)r_D} = 1$ to get $-\left(\frac{(1-T_{E_2})(1-T_{C_2})r_D}{(1-T_D)r_L}\right)\frac{(1-T_D)I}{r_D}$ which is

$$-\left(\frac{(1-T_{E_2})(1-T_{C_2})r_D}{(1-T_D)r_L}\right)D \text{ and factoring out } D:$$

$$G_{L_2}^{D \rightarrow E} = \left[1 - \frac{(1-T_{E_2})(1-T_{C_2})r_D}{(1-T_D)r_L}\right]D - E_U + \frac{(1-T_{E_2})(1-T_{C_2})C}{r_L}.$$

Multiplying the 3rd component by $\frac{(1-T_{E_1})(1-T_{C_1})r_U}{(1-T_{E_1})(1-T_{C_1})r_U} = 1$ to get $\left(\frac{(1-T_{E_2})(1-T_{C_2})r_U}{(1-T_{E_1})(1-T_{C_1})r_L}\right)\frac{(1-T_{E_1})(1-T_{C_1})C}{r_U}$ which is

$$\left(\frac{(1-T_{E_2})(1-T_{C_2})r_U}{(1-T_{E_1})(1-T_{C_1})r_L}\right)E_U \text{ and factoring out } E_U:$$

$$G_{L_2}^{D \rightarrow E} = \left[1 - \frac{(1-T_{E_2})(1-T_{C_2})r_D}{(1-T_D)r_L}\right]D - \left[1 - \frac{(1-T_{E_2})(1-T_{C_2})r_U}{(1-T_{E_1})(1-T_{C_1})r_L}\right]E_U.$$

Setting $\alpha_1 = \frac{(1-T_{E_2})(1-T_{C_2})}{(1-T_D)}$ and $\alpha_2 = \frac{(1-T_{E_2})(1-T_{C_2})}{(1-T_{E_1})(1-T_{C_1})}$, we get:

$$G_{L_2}^{D \rightarrow E} = \left[1 - \frac{\alpha_1 r_D}{r_L}\right]D - \left[1 - \frac{\alpha_2 r_U}{r_L}\right]E_U. \quad (6)$$

Appendix B. Proof of equation (7)

Proof of equation (7) for the situation of a levered firm undergoing an equity-for-debt increment with no growth and discount rates and tax rates dependent on the leverage choice. We have the following definitions for V_U and V_L . $V_U = E_U = \frac{(1-T_{E_1})(1-T_{C_1})C}{r_U}$ where T_{E_1} is unlevered equity's tax rate, T_{C_1} is the unlevered corporate tax rate, C is the perpetual before-tax cash flow to unlevered equity, and r_U is the unlevered cost of equity. $V_L = E_L + D$ with $E_L = \frac{(1-T_{E_2})(1-T_{C_2})(C-I)}{r_L}$ where T_{E_2} is levered equity's tax rate, T_{C_2} is the levered corporate tax rate, I is the perpetual interest payment, and r_L is the cost of levered equity; and $D = \frac{(1-T_D)I}{r_D}$, where T_D is debt's personal tax rate and r_D is the cost of debt. Noting $V_U = E_U$, $V_L = E_L + D$ and substituting in the definition for E_L , the equation of $G_L^{E \rightarrow D} = V_U - V_L$ gives:

$$G_L^{E \rightarrow D} = E_U - \frac{(1-T_{E_2})(1-T_{C_2})(C-I)}{r_L} - D.$$

Multiplying out the 2nd component and rearranging:

$$G_L^{E \rightarrow D} = E_U - \frac{(1-T_{E_2})(1-T_{C_2})C}{r_L} - D + \frac{(1-T_{E_2})(1-T_{C_2})I}{r_L}.$$

Multiplying the 2nd component by $\frac{(1-T_{E_1})(1-T_{C_1})r_U}{(1-T_{E_1})(1-T_{C_1})r_U} = 1$ to get $\left(\frac{(1-T_{E_2})(1-T_{C_2})r_U}{(1-T_{E_1})(1-T_{C_1})r_L}\right)\frac{(1-T_{E_1})(1-T_{C_1})C}{r_U}$ which is

$$\left(\frac{(1-T_{E_2})(1-T_{C_2})r_U}{(1-T_{E_1})(1-T_{C_1})r_L}\right)E_U \text{ and factoring out } E_U:$$

$$G_L^{E \rightarrow D} = \left[1 - \frac{(1-T_{E_2})(1-T_{C_2})r_U}{(1-T_{E_1})(1-T_{C_1})r_L}\right]E_U - D + \frac{(1-T_{E_2})(1-T_{C_2})I}{r_L}.$$

Multiplying the last component by $\frac{(1-T_D)r_D}{(1-T_D)r_D} = 1$ to get $-\left(\frac{(1-T_{E2})(1-T_{C2})r_D}{(1-T_D)r_L}\right)\frac{(1-T_D)I}{r_D}$ which is $-\left(\frac{(1-T_{E2})(1-T_{C2})r_D}{(1-T_D)r_L}\right)D$ and factoring out D :

$$G_{L^E \rightarrow D} = \left[1 - \frac{(1-T_{E2})(1-T_{C2})r_U}{(1-T_{E1})(1-T_{C1})r_L}\right]E_U - \left[1 - \frac{(1-T_{E2})(1-T_{C2})r_D}{(1-T_D)r_L}\right]D.$$

Setting $\alpha_2 = \frac{(1-T_{E2})(1-T_{C2})}{(1-T_{E1})(1-T_{C1})}$ and $\alpha_1 = \frac{(1-T_{E2})(1-T_{C2})}{(1-T_D)}$, we get:

$$G_{L^E \rightarrow D} = \left[1 - \frac{\alpha_2 r_U}{r_L}\right]E_U - \left[1 - \frac{\alpha_1 r_D}{r_L}\right]D \quad (7)$$

where $\left[1 - \frac{\alpha_2 r_U}{r_L}\right]E_U > 0$ and $-\left[1 - \frac{\alpha_1 r_D}{r_L}\right]D < 0$.

Appendix C. Proof of equation (9)

Proof of equation (9) for the situation of a levered firm undergoing a debt-for-equity increment with discount rates, growth rates and tax rates dependent on the leverage choice and a wealth transfer due to risk shifting such that r_{D1} becomes $r_{D1\uparrow}$ causing the prior debt of D_1 to fall to $D_{1\downarrow}$. We have the following definitions for V_{L1} and V_{L2} . $V_{L1} = E_{L1} + D_1$ with $E_{L1} = \frac{(1-T_{E1})(1-T_{C1})(C-I_1)}{r_{Lg1}}$ where T_{E1} and T_{C1} are the equity and corporate tax rates before the debt-for-equity increment, C is the perpetual before-tax cash flow to unlevered equity, I_1 is the perpetual interest payment prior to the increment, and r_{Lg1} is the growth-adjusted levered equity discount rate prior to the increment with $r_{Lg1} = r_{L1} - g_{L1}$ where r_{L1} and g_{L1} are equity's discount and growth rates prior to the increment; and $D_1 = \frac{(1-T_{D1})I_1}{r_{D1}}$ where T_{D1} and r_{D1} are debt's personal tax rate and the cost of borrowing with both prior to the increment.

$V_{L2} = E_{L2} + D_{1\downarrow} + D_2$ with $E_{L2} = \frac{(1-T_{E2})(1-T_{C2})(C-I_1-I_2)}{r_{Lg2}}$ where T_{E2} and T_{C2} are the equity and corporate tax rates after the increment, I_2 is the perpetual interest payment on the new debt, and r_{Lg2} is the growth-adjusted levered equity discount rate after the increment with $r_{Lg2} = r_{L2} - g_{L2}$, where r_{L2} and g_{L2} are equity's discount and growth rates after the increment; $D_{1\downarrow} = \frac{(1-T_{D1})I_1}{r_{D1\uparrow}}$ where $r_{D1\uparrow}$ captures both r_{D1} and its increase due to upward shift in risk;

and $D_2 = \frac{(1-T_{D2})I_2}{r_{D2}}$ where T_{D2} and r_{D2} are debt's personal tax rate and the cost of borrowing with both after the increment. For this derivation, we assume D_2 has more senior claims than D_1 . This causes $r_{D1\uparrow} > r_{D1} > r_{D2}$ to hold due to dilution of the claims of D_1 such that D_1 falls to $D_{1\downarrow}$. Noting $V_{L1} = E_{L1} + D_1$, $V_{L2} = E_{L2} + D_{1\downarrow} + D_2$ and substituting in our definitions for E_{L2} , $D_{1\downarrow}$ and D_2 into the equation of $G_{L_2^D \rightarrow E} = V_{L2} - V_{L1}$ gives:

$$G_{L_2^D \rightarrow E} = \frac{(1-T_{E2})(1-T_{C2})(C-I_1-I_2)}{r_{Lg2}} + \frac{(1-T_{D1})I_1}{r_{D1\uparrow}} + \frac{(1-T_{D2})I_2}{r_{D2}} - E_{L1} - D_1.$$

Multiplying out the 1st component and rearranging:

$$G_{L_2^D \rightarrow E} = \frac{(1-T_{D2})I_2}{r_{D2}} - \frac{(1-T_{E2})(1-T_{C2})I_2}{r_{Lg2}} - E_{L1} + \frac{(1-T_{E2})(1-T_{C2})(C-I_1)}{r_{Lg2}} - D_1 + \frac{(1-T_{D1})I_1}{r_{D1\uparrow}}.$$

Recognizing the 1st component is D_2 , multiplying the 2nd component by $\frac{(1-T_{D2})r_{D2}}{(1-T_{D2})r_{D2}} = 1$ to get

$-\left(\frac{(1-T_{E2})(1-T_{C2})r_{D2}}{(1-T_{D2})r_{Lg2}}\right)\frac{(1-T_{D2})I_2}{r_{D2}}$ which is $-\left(\frac{(1-T_{E2})(1-T_{C2})r_{D2}}{(1-T_{D2})r_{Lg2}}\right)D_2$ and factoring out D_2 :

$$G_{L_2^D \rightarrow E} = \left[1 - \frac{(1-T_{E2})(1-T_{C2})r_{D2}}{(1-T_{D2})r_{Lg2}}\right]D_2 - E_{L2} + \frac{(1-T_{E2})(1-T_{C2})(C-I_1)}{r_{Lg2}} - D_1 + \frac{(1-T_{D1})I_1}{r_{D1\uparrow}}.$$

Multiplying the 3rd component by $\frac{(1-T_{E_1})(1-T_{C_1})r_{Lg1}}{(1-T_{E_1})(1-T_{C_1})r_{Lg1}} = 1$ to get $\left(\frac{(1-T_{E_2})(1-T_{C_2})r_{Lg1}}{(1-T_{E_1})(1-T_{C_1})r_{Lg2}}\right)\frac{(1-T_{E_1})(1-T_{C_1})(C-I_1)}{r_{Lg1}} =$
 $= \left(\frac{(1-T_{E_2})(1-T_{C_2})r_{Lg1}}{(1-T_{E_1})(1-T_{C_1})r_{Lg2}}\right)E_{L_1}$ and factoring out E_{L_1} :

$$G_{L_2}^{D \rightarrow E} = \left[1 - \frac{(1-T_{E_2})(1-T_{C_2})r_{D_2}}{(1-T_{D_2})r_{Lg2}}\right]D_2 - \left[1 - \frac{(1-T_{E_2})(1-T_{C_2})r_{Lg1}}{(1-T_{E_1})(1-T_{C_1})r_{Lg2}}\right]E_{L_1} - D_1 + \frac{(1-T_{D_1})I_1}{r_{D_1\uparrow}}.$$

Multiplying the last component by $\frac{r_{D_1}}{r_{D_1}} = 1$ to get $\left(\frac{r_{D_1}}{r_{D_1\uparrow}}\right)\frac{(1-T_{D_1})I_1}{r_{D_1}}$ which is $\left(\frac{r_{D_1}}{r_{D_1\uparrow}}\right)D_1$ and factoring out D_1 :

$$G_{L_2}^{D \rightarrow E} = \left[1 - \frac{(1-T_{E_2})(1-T_{C_2})r_{D_2}}{(1-T_{D_2})r_{Lg2}}\right]D_2 - \left[1 - \frac{(1-T_{E_2})(1-T_{C_2})r_{Lg1}}{(1-T_{E_1})(1-T_{C_1})r_{Lg2}}\right]E_{L_1} - \left[1 - \frac{r_{D_1}}{r_{D_1\uparrow}}\right]D_1.$$

Setting $\alpha_1 = \frac{(1-T_{E_2})(1-T_{C_2})}{(1-T_{D_2})}$ and $\alpha_2 = \frac{(1-T_{E_2})(1-T_{C_2})}{(1-T_{E_1})(1-T_{C_1})}$, we get:

$$G_{L_2}^{D \rightarrow E} = \left[1 - \frac{\alpha_1 r_{D_2}}{r_{Lg2}}\right]D_2 - \left[1 - \frac{\alpha_2 r_{Lg1}}{r_{Lg2}}\right]E_{L_1} - \left[1 - \frac{r_{D_1}}{r_{D_1\uparrow}}\right]D_1. \quad (9)$$