“Toward waste management contracts”

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Abstract

This paper deals with the cost of treatment of final waste, that is waste which cannot, in the absence of recycling opportunities, be reduced by any appropriate tax scheme. The authors propose a new way to handle this waste based on Waste Management Contracts (WMC) which largely involve households in the cost reduction process. Within a set of feasible (i.e. budget budget-balancing, incentive-compatible and acceptable) contracts, the study characterizes the optimal WMC and compare this system with a more standard one based on a combination of advance and per-bag disposal fees.

Keywords: waste management, disposal fee policy, household effort, contracts.
JEL Classification: Q28, H21.

Introduction

Over the last few years, environmentalists and policy makers have focused increasing attention on the question of waste management (see for instance Jenkins (1993), Dinan (1993) or Fullerton and Kinnanan (1995)), and it is now widely recognized that consumption is generating an increasing amount of garbage, the treatment of which induces a growing social cost. From that point of view, it is becoming obvious that households, acting as citizens, should participate in waste management programs, or at least have sufficient incentive to do so. Their contributions can take two forms: they can reduce the amount of waste generated by their consumption and, perhaps more generally, make a targeted effort to reduce waste treatment costs.

To the best of our knowledge, most of the literature focuses on reducing the amount of waste. These policies for green design often combine upstream and downstream tax schemes to modify the design of the product or to encourage recycling[1]. Although these policies are more or less efficient depending on the commodity under consideration (see Palmer et al., 1997; or Jenkins et al. (2003), it is widely acknowledged that a two-part taxation mechanism based on a generalized deposit-refund system (see Palmer and Walls, 1997; or Fullerton and Wolverton, 2000) can implement the socially optimal waste reduction policy. But the transmission of these incentives depends on the existence of markets for recyclables, and on the ability to use these goods in a reversed production system. So even if this potential exists, it is difficult to imagine that no residual waste would remain or that its collection and destruction would be cost-free.

Our paper essentially deals with this final waste, which is typically buried in landfills or incinerated. This activity, which is performed by a public or private agency, is usually costly for society. From that point of view, our object is to design a contract that involves households more deeply in the reduction of their waste treatment costs.

One well-known option is to implement a per-bag pricing policy. This particular contract provides some incentive to reduce the amount of waste and thus its total treatment cost. But the nature of the effort provided by households is not directly observable. It could, for instance, be dedicated to illegal dumping (see Fullerton and Kinnanan, 1995, 1996), which increases the social cost of waste. In other words, the implementation of a disposal fee policy (DF for short) requires a costly incentive scheme to prevent midnight dumping (see Choe and Fraser, 1999; 2001) and therefore gives rise to inefficiency. One can even argue that this DF cannot be too high, otherwise the incentive for illegal dumping is very strong and it can only be restrained by expensive monitoring. In other words, it is not sure that an acceptable DF will cover the waste treatment cost. That is why these per-bag pricing systems are often combined with an advance disposal fee (ADF for short) which consists in charging at least part of the waste treatment cost when the household buys the good. This is why the per-bag pricing policies are often based on a monitoring technology combined with a *two-part tariff:* a DF to provide some incentive to reduce the amount of waste and an ADF to cover the waste treatment cost[2].

Our objective is to propose an alternative to this *two-part tariff* in the form of a waste management contract (WMC for short). This contract does not rely on a disposal fee that might induce illegal dumping but nevertheless (1) motivates the households to provide a specific effort to reduce the cost of final waste treatment and (2) ensures that the agency in charge of waste treatment covers the cost of its activity. The mechanism behind this contract is quite simple. Households pre-pay the waste

[1] For different applications the reader is referred to Fullerton and Wu (1998), Walls and Palmer (2001) or Calcott and Walls (2000, 2005).

[2] For several discussions concerning ADF versus DF or for a comparison of two-part tariff and two-part taxation schemes, the reader is referred to Shinkuma (2007; 2003).

treatment cost as in a standard full ADF system, and they are not charged for the disposal of their waste. But they have the opportunity to freely sign a contract specifying a set of activities that reduce the waste treatment cost and defining a monetary compensation for performing these activities. A monitoring mechanism must nevertheless be introduced to prevent infringements to the contract. But this one is quite different from the one associated with a DF system. It does not penalize illegal dumping, but only infringements of the contract. Consequently, only the people who have signed the contract need to be monitored, rather than the whole population, as in a DF system.

From this point of view, this contract gives households the opportunity freely to act for the preservation of the environment by helping to reduce waste treatment costs and to obtain, if they accept, financial compensation that is covered by the gains made in the waste treatment process. This may seem rather abstract, at first glance. But policy makers can easily identify specific activities in the pretreatment stage prior to disposal and decentralize these processes to the household level. They include, for instance, sorting the waste into different bins, destroying certain kinds of material, composting and so on. The idea is simply to give to the waste producers (households) the opportunity to perform some pretreatment activities or, if they do not want to do so, to pay for it. This is, for instance, the reasoning behind the new regulations that came into force in the UK in October 2007 (for more detail the reader is referred to *Treatment of non-hazardous wastes for landfill. Your waste – your responsibility*, Environment Agency UK, 2007).

In this paper, however, we shall not enter into the specific nature of these waste management contracts. For the policy maker, the important thing is that it should reduce the final waste treatment cost by a certain proportion. This is why we assume that such a contract specifies: (1) a target proportion of cost reduction, (2) a payment in compensation and (3) a probability of being controlled associated with a fine for infringements.

We restrict the set of all contracts in the following way. We first require that the public agency in charge of waste destruction works under an *ex post-budget balancing constraint*; i.e., the effective waste treatment cost and the monitoring cost must be covered by the ADF net of the payment to contractors and augmented by any fines collected. This means that we do not want to levy a specific (lump sum) tax to cover the costs of this agency. Secondly, we do not want to consider contracts that nobody wants to sign. This is why we also introduce a *participation constraint* whereby at least some agents are prepared to accept this contract without cheating. Finally, and since the efforts provided by the households are not observable, we also restrict our attention to WMC that are enforceable, in the sense that they must satisfy an *incentive compatible constraint* ensuring that each contractor provides the required effort.

These contracts are implemented in a model constructed along the lines of Choe and Fraser (1999). However, we depart from their paper in several respects. First, we do not assume that the policy maker has any opportunity to modify the waste content of a good by taxing the producers. Thus, we only deal with final waste produced by the consumers. We also introduce agents who are heterogeneous with respect to their willingness to pay for the commodity and their ability to provide an effort. These characteristics are distributed over a continuum of agents, but no particular assumptions are imposed on this distribution.

In this setting, we characterize the set of WMC that satisfy the budget, participation and incentive-compatibility constraints. In fact, we show that we can impose restrictions on the required waste treatment rate and on the probability of control that are equivalent to the above conditions. In a second step, this characterization gives us the opportunity to study the welfare properties of these contracts. We show that within this set of constraints, it is always in the interest of the waste disposal agency to raise the required waste treatment rate as high as possible and to lower the probability of control in order to limit the monitoring cost. The result is essentially linked to the entry of "poorer and environment friendlier" consumers into the market. A higher waste treatment rate requires a higher subsidy, and this typically gives poorer consumers the opportunity to buy the goods, as long as they are not too inconvenienced by the effort required. In a third step, these preliminary results allow us to characterize an optimal WMC, from a welfare maximization point of view. We show that an optimal WMC always reduces the waste management cost, independently from the monitoring cost, and that the optimal policy follows from a trade-off between the welfare gain from a rise of the required waste treatment rate and the associated increase in monitoring costs. The optimal policy is also related to the average cost of the effort in the population. We finally compare our WMC to a two-part tariff policy consisting in a combination of ADF and DF. We show that a WMC is more efficient when the total amount of fees (ADF plus DF) collected in a two-part tariff policy is not smaller than the waste treatment cost.

The paper proceeds as follows: the next section defines our basic assumptions and describes a WMC. In section 2, we present the restrictions on the set of these contracts imposed by the incentive, participation and budget-balancing constraints. In section 3, we associate a level of welfare with each feasible contract and give some basic properties of this function. Section 4 is devoted to construction of the optimal contract. In section 5 we compare a WMC to a two-part tariff. Finally, the last section contains some concluding remarks. The proofs of the different results are relegated to an Appendix.

1. The basic assumptions and the WMC

We consider a commodity produced by a representative firm and sold to a continuum of consumers. Consumption produces final waste, i.e., waste that cannot be reduced by any kind of market mechanism like a deposit-refund system or recycled, even partially. For simplicity, we assume that one unit of consumption generates one unit of waste. Its destruction is not free. We denote by c the destruction cost of one unit of waste.

Since we mainly focus on consumer behavior, we also largely simplify the behavior of the representative firm by assuming that: (1) the commodity is sold on a competitive market; (2) there is no way to reduce the intrinsic waste content at the production level; and (3) the unit production cost is zero. This means that the competitive price p reflects the part of the waste treatment cost that is prepaid by the household and which, in our WMC, is given by the unit treatment cost c.

1.1. The demand side. We introduce a continuum of heterogeneous consumers[1] who decide whether or not to buy the good, i.e. $x \in \{0,1\}$ and suffer the disutility of the effort $e \in [0,1]$ dedicated to reducing the waste treatment cost. They share the same utility function $u(x,e,\mu) = \alpha x - \theta e - \mu$, where μ denotes monetary spending. They are nevertheless heterogeneous with respect to their willingness to pay $\alpha \in [0, A]$ and their marginal cost $\theta \in [0, \Theta]$ of the waste treatment effort. However, to ensure that at least one agent is able to consume when the waste management cost is prepaid, we assume that $A > c$, and, in the same vein, we say that $\Theta > c$ otherwise all consumers are willing to provide an effort.

[1] The selection of a discrete choice model may perhaps seem restrictive. But, in most of the literature, the authors essentially choose a representative agent economy with continuum of choices. We are convinced that our approach is equally general, since no restrictions are put on the distribution of characteristics. One can even argue that the aggregate behavior is less restrictive than that induced by a single agent optimization problem.

The distribution of these two characteristics across the population is summarized by a probability distribution over $[0, A] \times [0, \Theta]$ whose cumulated distribution function (c.d.f. for short) is denoted $F(\alpha, \theta)$. This is assumed to be absolutely continuous with a strictly positive density $f(\alpha,\theta) := \partial^2_{\alpha,\theta} F(\alpha,\theta) > 0$. Moreover, we denote by $f(\alpha,\cdot) := \int_0^{\Theta} f(\alpha,\theta) d\theta$ the marginal density of α and by $f(a/\theta) := \dfrac{f(\alpha,\theta)}{f(\cdot,\theta)}$ the conditional density of α given θ. A symmetric interpretation holds for $f(\cdot,\theta)$ and $f(\theta/\alpha)$.

Now remember that the effort made by households reduces the waste treatment cost. We measure the outcome of this activity by the proportion $r(e)$ by which the unit waste treatment cost is reduced[2]. However, we assume that there is an upper bound $\overline{r} \le 1$ to this proportion and that this relation is linear, i.e. $r(e) = \overline{r} \cdot e$ for $e \in [0,1]$. The largest waste management cost reduction is obtained when the intensity of the effort is maximal. Consequently, we can say that $e(r) = \dfrac{r}{\overline{r}}$ denotes the level of effort required to reduce the waste management cost by a proportion of $r \in [0, \overline{r}]$.

1.2. The Waste Management Contract. The mechanism behind the WMC is the following. We first implement a full ADF system by including the waste treatment cost in the price of the commodity and by charging nothing for waste disposal. But we also give each household the opportunity to sign a contract whereby they help to reduce the waste treatment cost. If they accept, they have to provide some effort in order to reach an assigned target on which they are randomly controlled. In return, they receive a payment for performing this activity. For example, they could be delivered several different bins and undertake to sort their waste into them, being paid in proportion to the amount of waste they sort. This contract therefore (1) delegates some costly waste treatment and/or destruction activities from the waste disposal agency to the consumers and thus (2) frees some resources (since the waste treatment cost is paid in advance) which can be used to pay the participating households.

[2] This assumption fits our discrete choice model particularly well, since a consumer buys at most one unit of good. Otherwise one would have to take into account the amount of good consumed by the household.

So, if we want to model this idea in the simplest way, we can say that the waste treatment agency (1) selects a target $r \in \left[0, \overline{r}\right]$ defining the proportion by which the waste treatment is reduced, (2) specifies, in a contract, a set of tasks to achieve this reduction, and (3) proposes a payment s per unit of waste transformed[1]. A monitoring system is required to ensure that the terms of the contract are fulfilled, i.e., to check whether target r is reached. So let us denote by π the probability of a participating household being controlled and let us introduce a cost $m(\pi)$ per control. We assume that this cost is increasing and convex (i.e., $m^{'}(\pi) > 0$ and $m''(\pi) > 0 \neq$) and that the absence of monitoring is free (i.e., $m(0) = 0$) while perfect monitoring is very expensive (i.e., $m(1) > c$). If this target is not met, the offender pays a fine, which should not be too disproportionate to the fault. Given that each household only consumes one unit of good, generating a waste treatment cost of c , it seems reasonable to set the fine at that level.

To summarize, we say that a WMC is described by the triple (r, s, π) consisting of a cost reduction target r , a subsidy s , and a probability of control π associated with a fine of c for offenders. So let us now move to the behavior of households when such a contract is on offer. This will provide some information on their choice and will help us to restrict the set of contracts to those that exclude cheating, induce voluntary participation and satisfy the budget-balancing constraint of the waste disposal agency.

1.3. The choice of a consumer. In our discrete choice model, if a household of type (α, θ) buys nothing, its utility is zero. If it buys the good, it can refuse (utility indexed by 0) the waste reduction contract and stay in a standard ADF. In that case, it pre-pays the cost c of waste disposal and has no incentive to make an effort. Under our zero marginal production cost assumption, it pays $p = c$ for the good and its utility is given by:

$$u_0^{(\alpha,\theta)} = \alpha - c .$$

If the household accepts the contract, it receives a subsidy of s, but it always has the choice (indexed by e or $\overline{e}$) whether or not to respect the terms of the contract. In the former case, it makes the required effort by delivering the transformed waste and its utility is given by:

$$u_e^{(\alpha,\theta)}(r,s) = \left(\alpha - c\right) + s - \theta \frac{r}{\overline{r}} .$$

Otherwise, it makes no effort but runs the risk of being caught with probability π and being fined. Since there is no infringement cost[2], contrary to the DF literature based on illegal dumping costs, it obtains:

$$u_{\overline{e}}^{(\alpha,\theta)}(s,\pi) = \left(\alpha - c\right) + s - \pi c$$

Consequently, we can say that the best strategy of a household of type (α, θ) is the one that gives it the highest payoff, i.e., which satisfies:

$$\max\left\{0, u_0^{(\alpha,\theta)}, u_e^{(\alpha,\theta)}(r,s), u_{\overline{e}}^{(\alpha,\theta)}(s,\pi)\right\} .$$

2. The set of feasible contracts

Now that we know how a household of type (α, θ) behaves with respect to consuming the good, accepting the WMC and making the necessary effort, we can restrict the set of contracts to those that share some desirable properties. We want to ensure that: (1) all agents who accept the contract have sufficient incentive to meet the required cost reduction target, (2) at least some agents are willing to participate in the program, and (3) the waste treatment agency can cover both the waste management costs and the monitoring costs.

The first condition can be defined quite easily. We simply require that cheating is for everybody a dominated choice.

Definition 1. *The Incentive Constraint (IC) is satisfied if and only if:* $\forall \left(\alpha, \theta\right) \in \left[0, A\right] \times \left[0, \Theta\right],$

$$\max\left\{u_e^{(\alpha,\theta)}(r,s), u_0^{(\alpha,\theta)}, 0\right\} \geq u_{\overline{e}}^{(\alpha,\theta)}(s,\pi). \qquad \text{(IC)}$$

In order to define the participation constraint, we need to define what we mean by "some agents" accept the contract. Since we work with a continuum of agents, we simply require that there must be a subset E of agents with non-zero measure that accept the contract and perform the effort. So if P(E) denotes the proportion of these households, we say that:

Definition 2. *The Participation Constraint (PC) is verified if and only if:* $\exists E \subset \left[0, A\right] \times \left[0, \Theta\right]$, *and* $P(E) > 0$ *such that:*

[1] We also implicitly assume that each household only transforms its own waste. In a more general setting, one can imagine that the environmental friendly consumers, i.e. with a low θ , would wish to transform the waste of their non-participating neighbors or even that the WMC system induces a market for waste. But all these effects go in the same direction; they increase the amount of waste treated.

[2] It is always possible to introduce such a cost into the model but this does not really change the results. It gives the waste treatment agency the possibility to decrease the subsidy, to require greater effort and/or to reduce the probability of control without breaking the incentive constraint.

$$\forall(\alpha,\theta)\in E,\quad u_e^{(\alpha,\theta)}(r,s) = \max\left\{0, u_0^{(\alpha,\theta)}, u_e^{(\alpha,\theta)}(r,s), u_{\bar{e}}^{(\alpha,\theta)}(r,\pi)\right\}. \quad \text{(PC)}$$

The construction of the budget constraint requires some additional notation. Let us denote by $P(0)$ the proportion of households that buy the good but refuse the contract, and by $P(\bar{E})$ those who accept the contract but cheat[1]. In our discrete choice setting, we can therefore say that the advance disposal fees collected per unit of waste are given by $\left[(P(\bar{E})+P(E)+P(0))\cdot c\right]$, while the waste treatment cost is described by

$$\left[(P(\bar{E})+P(0))\cdot c+P(E)\cdot\left(1-r\right)\cdot c\right],$$

since a proportion $P(\bar{E})$ of the households do not respect the contract. The subsidies paid to the agents are $\left[(P(\bar{E})+P(E))\cdot s\right]$. Finally, concerning the monitoring activity, the controls, at a rate of π, only apply to the population of contracting households. They therefore cost $\left[\pi\cdot(P(\bar{E})+P(E))\cdot m(\pi)\right]$ but pay back fines of $\left[\pi\cdot P(\bar{E})\cdot c\right]$. The budget constraint is therefore given by:

$$\left[(P(\bar{E})+P(E)+P(0))\cdot c\right]+\left[\pi\cdot P(\bar{E})\cdot c\right]\geq \left[(P(\bar{E})+P(0))\cdot c+P(E)\cdot\left(1-r\right)\cdot c\right] +\left[(P(\bar{E})+P(E))\cdot s\right]+\left[\pi\cdot(P(\bar{E})+P(E))\cdot m(\pi)\right]$$

and, after some rearrangements, we obtain:

$$P(E)\cdot\left(s-r\cdot c+\pi\cdot m(\pi)\right) +P(\bar{E})\cdot\left(s-\pi\cdot c+\pi\cdot m(\pi)\right)\leq 0.$$

But we can go a step further. Let us remember that the subsidy s acts, for each household, like a discount on the price of the good. Consequently, whatever π and r are, any waste treatment agency seeking a contract to maximize the total surplus always exhausts this constraint. This is why we can say:

Definition 3. *The Balancing Budget Constraint (BBC) is satisfied if and only if:*

$$P(E)\left(s-r\cdot c+\pi\cdot m(\pi)\right)+ + P(\bar{E})\left(s-\pi\cdot c+\pi\cdot m(\pi)\right)=0 \quad \text{(BBC)}$$

It is now important to identify the set of contracts that satisfy these three conditions. If IC and PC are verified, we know that $P(E)>0$ and $P(\bar{E})=0$ respectively. This means, under (BBC), that the subsidy $s=r\cdot c-\pi\cdot m(\pi)$. We can go a step further. At least from an intuitive point of view, we can imagine that if the subsidy is negative, nobody will want to participate in the waste management program. On the other hand, if it is too high, especially if it is higher than the expected cost of the fine, i.e., $s>\pi\cdot c$, it will be in the interest of the household to accept the contract and cheat. This clearly imposes upper and lower bounds on the subsidy. We can also say that the lower bound must be strictly positive (except for the case in which $r=\pi=0$), otherwise it would be impossible to find an open set[2] E on which PC is satisfied. This is why we can say that:

Lemma 1. *If IC, PC, and BBC are satisfied then (1) $s=r\cdot c-\pi\cdot m(\pi)$, (2) $r\cdot c-\pi\cdot m(\pi)>0$ except for $r=\pi=0$ and (3) $r\cdot c-\pi\cdot m(\pi)\leq\pi\cdot c$.*

But, what is more interesting for us is that these conditions are not only necessary but also sufficient. In fact we can say that:

Proposition 1. *The set of feasible contracts (i.e. satisfying BBC, PC, and IC) is fully characterized by the three above conditions. In other words, the subsidy is given by: $s=r\cdot c-\pi\cdot m(\pi)$ and the required rate r of reduction of the waste treatment cost and the probability π of control belong to:*

$$\mathcal{F}\left\{(r,\pi)\in[0,\bar{r}]\times[0,1]:\frac{\pi\cdot m(\pi)}{c}<r\leq\frac{\pi\cdot\left(m(\pi)+c\right)}{c}\right\}\cup\{(0,0)\}$$

Finally, and since we have restricted our attention to feasible contracts, we also observe that $u_e^{(\alpha,\theta)}(r,s)$ and $u_{\bar{e}}^{(\alpha,\theta)}(s,\pi)$ can be written respectively as:

$$\begin{cases} u_e^{(\alpha,\theta)}(r,\pi)=\left(\alpha-c\right)+\left(r\cdot c-\pi\cdot m(\pi)\right)-\theta\dfrac{r}{\bar{r}} \\ u_{\bar{e}}^{(\alpha,\theta)}(r,\pi)=\left(\alpha-c\right)+\left(r\cdot c-\pi\cdot m(\pi)\right)-\pi c \end{cases}.$$

3. The surplus and its basic properties

If we assume that there is a public agency in charge of the destruction of this final waste, it will propose a feasible contract that maximizes the surplus of the consumers. So let us now derive the surplus associated with each feasible contract, i.e., for each $(r,\pi)\in\mathcal{F}$ and $s=r\cdot c-\pi\cdot m(\pi)$. We also restrict our attention to non-trivial feasible contracts, i.e.,with $(r,\pi)\neq(0,0)$ since a trivial WMC is a full ADF system in which the average surplus is, trivially, given by $\int_0^{\Theta}\int_c^{A}u_0^{(\alpha,\theta)}dF$.

To perform this computation, it is important to distinguish between households who buy the commodity and accept the contract and those who refuse it.

[1] Of course, these probabilities depend of all the parameters of the model. We omit them in order to simplify the notation.

[2] Since our distribution of probability is absolutely continuous, only sets containing an open set have non-zero probability.

This first set of households is given by:

$$C(r,\pi) = \left\{(\alpha,\theta) \in [0,A]\times[0,\Theta] : \underset{(1)}{u_e^{(\alpha,\theta)}(r,\pi) \geq u_0^{(\alpha,\theta)}} \underset{(2)}{\geq 0}\right\}. \quad (1)$$

A simple computation shows that condition (1) is equivalent to $\theta\frac{r}{\overline{r}} \leq c \cdot r - \pi \cdot m(\pi)$ or, in other words, that:

$$\theta \leq \theta(r,\pi) := \overline{r}\left(c - \frac{\pi \cdot m(\pi)}{r}\right).$$

But if $\theta \leq \theta(r,\pi)$, condition (2) becomes:

$$\alpha \geq \alpha(r,\pi,\theta) := \theta\frac{r}{\overline{r}} + \pi \cdot m(\pi) + (1-r)\cdot c.$$

Moreover, it is immediate that $\alpha(r,\pi,\theta) \in [\pi \cdot m(\pi) + (1-r)\cdot c, c] \subset [0,A]$ and $\theta(r,\pi) \in [0,\Theta]$ since $c < \Theta$ and $\overline{r} \leq 1$. We can therefore simplify the expression (1) and say that:

$$C(r,\pi) = \{(\alpha,\theta) \in [0,A]\times[0,\Theta]:$$
$$0 \leq \theta \leq \theta(r,\pi)$$
$$\text{and } \alpha \geq \alpha(r,\pi,\theta)\}.$$

It follows that the surplus computed on the population that buys the good and accepts the contract is given by[1]:

$$S_C(r,\pi) = \int_{C(r,\pi)} u_e^{(\alpha,\theta)}(r,\pi)dF =$$
$$= \int_0^{\theta(r,\pi)}\left(\int_{\alpha(r,\pi,\theta)}^A u_e^{(\alpha,\theta)}(r,\pi) f(\alpha/\theta)d\alpha\right) f(\cdot,\theta)d\theta.$$

Let us now move to the surplus of the set of households who buy the good but do not accept the contract. This set is described by[2]:

$$\overline{C}(r,\pi) = \left\{(\alpha,\theta) \in [0,A]\times[0,\Theta] : \underset{(1)}{u_0^{(\alpha,\theta)} \geq u_e^{(\alpha,\theta)}(r,\pi)} \underset{(2)}{\geq 0}\right\}.$$

By (1) we can say that $\theta \geq \theta(r,\pi)$ and by (2) that $\alpha \geq c$. Their surplus therefore corresponds to:

$$S_{\overline{C}}(r,\pi) = \int_{\overline{C}(r,\pi)} u_0^{(\alpha,\theta)}dF$$
$$= \int_{\theta(r,\pi)}^{\Theta}\left(\int_c^A u_0^{(\alpha,\theta)}(r,\pi) f(\alpha/\theta)d\alpha\right) f(\cdot,\theta)d\theta.$$

It follows that the total surplus is simply given by:

$$S(r,\pi) := S_C(r,\pi) + S_{\overline{C}}(r,\pi)$$

$$= \int_0^{\theta(r,\pi)}\left(\int_{\alpha(r,\pi,\theta)}^A u_e^{(\alpha,\theta)}(r,\pi) f(\alpha/\theta)d\alpha\right) f(\cdot,\theta)d\theta$$

$$\int_{\theta(r,\pi)}^{\Theta}\left(\int_c^A u_0^{(\alpha,\theta)}(r,\pi) f(\alpha/\theta)d\alpha\right) f(\cdot,\theta)d\theta.$$

We can also observe that:

Remark 1. *Since the total surplus under a full ADF system* (when $(r,\pi) = (0,0)$) *is given by* $\int_0^{\Theta}\int_c^A u_0^{(\alpha,\theta)}dF$, *we can say* $S(r,\pi) \geq \int_0^{\Theta}\int_c^A u_0^{(\alpha,\theta)}dF$, *or in other words, that a full ADF system is always weakly dominated by a non-trivial WMC. Moreover, if the budget-balancing subsidy* $s = c\cdot r - \pi \cdot m(\pi)$ *is strictly positive then this inequality holds strictly.*

It is also interesting to see how this surplus reacts to a change in either the probability of control or the required waste treatment rate. This will us give some insights into the nature of the surplus maximizing WMC. So, and from an intuitive point of view, if the monitoring probability π increases, the budget-balancing subsidy $s = c\cdot r - \pi\cdot c(\pi)$ automatically decreases. It follows that the welfare of the consumers who have adopted the waste management contract decreases, and therefore the total surplus also decreases. The effect of a change in the required waste treatment rate r is, however, less obvious. On the one hand, an increase in r contributes to a higher subsidy s. This provides more incentive to accept the contract and gives new consumers the opportunity to enter the market. On the other hand, it also implies that consumers who accept the contract must provide a greater effort. We nevertheless show that the first effect always dominates the second one:

Proposition 2. *Let us denote by* $P(r,\pi) := \int_0^{\theta(r,\pi)}\int_{\alpha(r,\pi,\theta)}^A dF$ *the proportion of households who accept the waste management contract and by* $\overline{\Theta}(r,\pi) := \int_0^{\theta(r,\pi)}\int_{\alpha(r,\pi,\theta)}^A \theta\frac{dF}{P(r,\pi)}$ *their average marginal disutility of effort. We observe that:*

1. $\forall(r,\pi) \in \mathcal{F}$ *and* $(r,\pi) \neq (0,0)$, $\partial_\pi S(r,\pi) = -(\pi m'(\pi) + m(\pi))P(r,\pi) < 0$, *i.e., when the probability of control increases, the households' surplus decreases.*
2. $\forall(r,\pi) \in \mathcal{F}$ *and* $(r,\pi) \neq (0,0)$, $\partial_r S(r,\pi) = \left(c - \frac{\overline{\Theta}(r,\pi)}{\overline{r}}\right)P(r,\pi) > 0$ *i.e., the surplus increases with the required waste treatment rate r, since for all households who accept the contract, the subsidy* $c\cdot r - \pi\cdot m(\pi)$ *is always greater than the monetary value of the effort*

[1] To prevent any confusion, note that this quantity is not the average surplus of the consumers who buy the good and execute the contract. If the reader is interested in this quantity, he must divide this surplus by the probability of being in this set, that is $P[C(r,\pi)]$.

[2] Since we have assumed that our measure is absolutely continuous, we decide by convention to only use weak inequalities.

$\theta\frac{r}{\bar{r}}$. This implies that $c > \frac{\theta}{\bar{r}}$ for these households, and therefore $c > \frac{\bar{\Theta}(r,\pi)}{\bar{r}}$.

4. The optimal WMC

Let us now try to characterize the optimal WMC. As usual in a second-best situation, the social planner seeks to implement the feasible contract that maximizes the total surplus within the set $\mathcal{F}$ of feasible contracts satisfying PC, IC and BBC. In other words, he chooses:

$$\left(r^*,\pi^*\right) \in \underset{(r,\pi)\in\mathcal{F}}{\arg\max} \underbrace{\int_{C(r,\pi)} u_e^{(\alpha,\theta)}(r,\pi)dF + \int_{\bar{C}(r,\pi)} u_0^{(\alpha,\theta)}dF}_{S(r,\pi)}. \quad (2)$$

By the earlier definition of feasible contracts (see proposition 1) and the fact that $S(r,\pi)$ is increasing in *r* (see proposition 2), we can immediately say that the upper constraint on *r* will by binding and therefore reduce the set $\mathcal{F}$ to:

$$\mathcal{F}' = \left\{(r,\pi) \in [0,\bar{r}]\times[0,1] : r = \min\left\{\frac{\pi\cdot\left(m(\pi)+c\right)}{c}, \bar{r}\right\}\right\}.$$

This is a closed subset of the compact set $[0,\bar{r}]\times[0,1]$, hence a compact set. Since $S(r,\pi)$ is also a continuous function, we can say without ambiguity that:

Lemma 2. *There always exists an optimal WMC that solves the above program.*

But it could be interesting to go a step further and specify the different properties of this optimal WMC. So let us first focuson the constraint $\mathcal{F}'$. Since the surplus is decreasing in π, an optimal WMC can never be such that $\frac{\pi\cdot\left(m(\pi^*)+c\right)}{c} > \bar{r}$, otherwise it would be possible to reduce the monitoring cost without modifying the waste reduction target *r*. We can therefore "forget" the $\min$ in the definition of $\mathcal{F}'$. This observation, combined with the fact that the monitoring costs are large when everybody is controlled, i.e. $m(1) > c$, tells us that:

Proposition 3. *The following properties hold:*

1. *The probability of control is bounded from above by $\pi_{\sup} < 1$, which solves $\frac{\pi_{\sup}\cdot\left(m(\pi_{\sup})+c\right)}{c} = \bar{r}$.*
2. *The optimal strategy corresponds to a situation in which the subsidy is equal to the cost of a cheating strategy, i.e., $s = r\cdot c - \pi\cdot m(\pi) = \pi\cdot c$.*
3. *With each $r \in [0,\bar{r}]$, we can associate a unique probability of control $\pi(r)$ with the property that the subsidy corresponds to the cost of cheating.*

But if there exists, by (3), a unique $\pi(r)$ that solves $r\cdot c - \pi\cdot m(\pi) = \pi\cdot c$, we can replace our constrained optimization problem by an unconstrained one simply by replacing the probability of control by $\pi(r)$ and thus solve:

$$r^* \in \underset{r\in[0,\bar{r}]}{\arg\max} \underbrace{\int_{C(r,\pi)} u_e^{(\alpha,\theta)}(r,\pi(r))dF + \int_{\bar{C}(r,\pi)} u_0^{(\alpha,\theta)}dF}_{S(r,\pi(r))}.$$

A standard examination of this optimization problem brings us to the conclusion that:

Proposition 4. *At an optimal WMC$\left(r^*,\pi^*\right)$, it can be observed that:*

1. *Both the target r^* and the probability of control π^* are strictly positive.*
2. *A household that accepts the contract receives a strictly positive subsidy of: $s^* = r\cdot c - \pi^*\cdot m(\pi^*) > 0$.*
3. *The following marginal condition is satisfied:*

$$c - (\pi^*\cdot m'(\pi^*) + m(\pi^*))\cdot\left.\frac{d\pi}{dr}\right|_{r^*} \geq \frac{\bar{\Theta}\left(r^*,\pi^*\right)}{\bar{r}}$$

with equality when $r^* < \bar{r}$.

The last first-order condition is quite intuitive. It tells us that on average, for the household accepting the WMC, the marginal benefit of an increase in the target r must be equal to the marginal cost induced by the increase in effort. To be more precise, the left-handside of the last equation is nothing other than $\frac{ds(r,\pi(r))}{dr}$, that is the marginal increase in the subsidy paid to households that accept the WMC. If we now recall that $\bar{\Theta}\left(r^*,\pi^*\right) := \int_0^{\theta(r,\pi)}\int_{\alpha(r,\pi,\theta)}^{A} \theta\frac{dF}{P(r,\pi)}$ is the average marginal disutility of effort of the households accepting the WMC, so that $\frac{\bar{\Theta}\left(r^*,\pi^*\right)}{\bar{r}}$ stands for the average marginal cost of an increase in the rate *r* of waste treatment.

Moreover, since the target r^* is strictly positive, we also know that the basic ADF system (which, by construction, belongs to the set of feasible WMC) is never selected. We can therefore say that a WMC dominates a pure ADF. But is this WMC better than a two-part tariff (TPT) that combines ADF and DF?

5. A WMC versus a two-part tariff

The answer to this question is less obvious, because these two mechanisms are quite different and do not share the same properties. In fact, a WMC directly controls the level of effort by imposing a target on the reduction of waste treatment costs, while the DF system seeks to motivate households to reduce waste through a per-bag price system. This is not without consequence, since:

- ♦ The monitoring activity is very different. In the first case, the controller only looks for infringements to the contract within the population who signed up to it, while in a TPT system, the whole population must be monitored to prevent illegal dumping. We can therefore expect the WMC to save money.
- ♦ The utility allocation can be rather different. In a TPT, if the sum total of ADF plus DF is lower than the waste treatment cost, a household making no effort benefits from the efforts made by the other households, since its total waste treatment charge is reduced. In a WMC, on the other hand, all households that refuse the contract pay their real waste treatment cost, except when the IC constraint is not satisfied. So in this case, if we want to compare the two systems, we must forget the IC constraint.

Before comparing the two systems, let us first quickly recall the main properties of a TPT. It is characterized by a triple (a, d, π) specifying the ADF included in the price of the commodity, denoted by a, the DF charged for each unit of waste, denoted by d, and the probability π that a household will be controlled for illegal dumping. The monitoring cost per control $m(\pi)$ is assumed to be the same[1]. We also keep the same utility function for a household of type (α, θ), given by $u(x, e, \mu) = \alpha x - \theta e - \mu$, with $x = \{0,1\}$ and $e \in [0,1]$. But the household's effort, and hence its level of waste reduction, are now endogenous. A waste production function is therefore required. To be consistent with the cost reduction function introduced in section 2, we define this function by $\min\{x - \overline{r} \cdot e, 0\}$. Since in this case, the households can illegally dump part of their waste, let us denote that by f. Consequently, the household cost of consumption is given by:

$$C(x,e,f) = a \cdot x + \left((1-f) \cdot d + f \cdot c \cdot \pi\right) \cdot \min\{x - \overline{r} \cdot e, 0\}$$

with $x = \{0,1\}$, $e \in [0,1]$, and $f \in [0,1]$

[1] These activities are quite different, since under a WMC the controller looks for an infringement to the contract, while in a TPT he controls for illegal dumping. But if we want to compare both systems, at least formally, such an assumption is unavoidable.

and each household chooses (x, e, f) so as to maximize their utility, taking into account their spending $\mu = C(x, e, f)$.

Since the households minimize their spending, it is immediate that the DF must verify $d \le c \cdot \pi$, otherwise they will dump their waste illegally, i.e., choose $f = 1$. Moreover, as the waste disposal agency also has an interest in minimizing its monitoring costs, it sets the probability of control at $\pi = \frac{d}{c}$.

This is why we focus on TPT given by $(a, d, \frac{d}{c})$. In this case, any household minimizing its consumption costs sets its effort to $e = 1$ or $e = 0$ according to whether $\theta \ge d \cdot \overline{r}$ or $\theta < d \cdot \overline{r}$ respectively, and decides to buy the good whenever its utility is greater than zero. Thus, we can say that:

Lemma 3. *If (a, d, π) is a TPT, the following statements are true:*

1. *When $\pi = \frac{d}{c}$, this TPT prevents illegal dumping and minimizes the monitoring cost for a given couple (a, d).*
2. *The indirect expected utility of a household of type $(\alpha, \theta) \in [0, A] \times [0, \Theta]$ is given by*

$$V_{TPT}^{(\alpha,\theta)} = \max\left\{0, \alpha - (a+d) + \max\left\{d \cdot \overline{r} - \theta, 0\right\}\right\}.$$

3. *If $P_{TPT}(a, d)$ denotes the proportion of the effective buyers who perform no waste reduction effort, the budget constraint per effective buyer is satisfied when*

$$a + d \ge c + \frac{d}{c} m\left(\frac{d}{c}\right) - \overline{r}\left(c - d\right)\left(1 - P_{TPT}(a, d)\right).$$

Let us now start the comparison. If we quickly look at the indirect utility of a consumer under a TPT, we can say that:

Remark 2. *If $(a+d) \ge c + d \cdot \overline{r}$, i.e., the advance fee and the disposal fee are "high", any pure ADF system in which the fee is equal to the waste treatment cost allocates (weakly) higher utilities to the agents because $V_{TPT}^{(\alpha,\theta)} \le \max\{0, \alpha - c\}$. Since an optimal WMC (strictly) dominates this pure ADF (see remark 1), it also dominates a TPT.*

So let us restrict our attention to TPTs with the property that $(a+d) < c + d \cdot \overline{r}$. As stated above, it is now important to know whether or not the total waste treatment charge ($a + d$) is greater than the waste treatment cost c.

In the first case, we can easily conclude that our optimal WMC obtained in section 4 always dominates a TPT. The argument is as follows. We take any TPT characterized by $(a, d, \frac{d}{c})$ and associate with it a WMC given by $(r, s, \pi) = (\overline{r}, c-(a+d)+\overline{r}d, \overline{r}\frac{d}{c})$, i.e., with the property that the cost reduction target is $\overline{r}$, the subsidy is given by $s = c-(a+d)+\overline{r}d$, and the probability of control is $\pi = \overline{r}\frac{d}{c}$. This allows us to compare the outcomes under these two rules. We show that (1) the indirect utility obtained with this WMC is, for each household (α, θ), greater than the utility obtained with the TPT and (2) the WMC saves money by reducing the monitoring costs, since the probability of control $\pi_{WCM} = \overline{r}\frac{d}{c} < \pi_{TPT} = \frac{d}{c}$. So if this WMC also satisfies (IC), (PC) and (BBC), it is obvious that our optimal WMC defined in section 4 dominates the TPT.

However, if the total waste treatment charge ($a + d$) paid by a household making no effort is strictly smaller than the waste treatment cost c, it is impossible to "replicate" the utility allocation of a TPT by an incentive-compatible WMC because, by construction, incentive compatibility ensures that households making no effort have to pay their real waste treatment cost. We can nevertheless show that in this case, the WMC that we have associated with a TPT allocatesthe same utility to the households, satisfies (PC) and (BBC) and again saves money compared with the TPT. So even if we cannot show that our optimal WMC dominates a TPT because some households benefit from the effort of others, we can at least exhibit a WMC that performs the utility allocation and is less costly. To sum up:

Proposition 5. *Let $(a, d, \frac{d}{c})$ be a TPT that satisfies the budget constraint and let $(\overline{r}, c-(a+d)+\overline{r}d, \overline{r}\frac{d}{c})$ be the associated WMC. We can say:*

1. *This WMC weakly dominates the TPT in terms of welfare, since it allocates to each household a level of utility at least equal to that obtained under the TPT system.*
2. *If $a + d \geq c$, the associated WMC verifies (PC) and (IC) and more than satisfies the budget constraint. We can therefore claim that an optimal WMC strictly dominates this TPT.*
3. *If $a + d < c$, the WMC associated with a TPT allocates the utilities in the same way. This contract does not verify (IC) but it does satisfy (PC) and saves money compared with the TPT.*

Conclusion

In the paper, we have addressed the question of reducing the cost of treating the final waste produced by households. By final waste, we mean the residual waste for which no further recycling is possible, even stimulated by a suitable taxation scheme. Even if it cannot be totally destroyed, this waste often requires additional costly treatment before being, say, reintroduced into our environment. Since these costs are borne by society, and especially by consumers, the idea of this paper was to study a mechanism that involves these agents in reducing the treatment cost (instead of reducing the amount of waste) by providing some voluntary effort. More precisely, we introduced what we call a Waste Management Contract (WMC). In this setting, households are charged an advance disposal fee that covers the waste treatment cost, but they can accept a WMC specifying a set of cost-reducing activities which they can perform and for which they earn a subsidy. This contract is also coupled with a monitoring scheme to discourage infringement. In this context, we first identified the set of feasible contracts, i.e., those satisfying incentive, participation and budget-balancing constraints, then we characterized an optimal contract from a welfare point of view, and finally we compared this kind of system to a more standard one combining an advance disposal fee and an "end-of-life" disposal fee.

However, this paper remains particular in several respects. First, even if our argument requires no specific assumptions on the distribution of the characteristics of agents, remaining quite general from that point of view, we have assumed that (1) the effect of the effort on the cost reduction rate is linear, and (2) that the preferences of each agent remain linear. It could perhaps be interesting to relax these assumptions by introducing a more general relation between the effort and its effect on the waste treatment cost, or even to depart from our discrete choice setting.

From a less technical point of view, the reader has probably noticed that we have focused on waste management policies addressing the treatment of "end-of-pipe" pollution. Recyclables and incentives to reduce the waste content of a good are not explicitly taken into consideration. This would require a more global model, and is not without consequences, especially if households have to allocate a limited effort between the cost reducing activities stipulated by a WMC and standard recycling behavior motivated by a deposit-refund system. We leave this point for future work.

Finally, we have also assumed, as usual in this literature, that the market for the good is competitive. If this assumption is relaxed, the optimal design of the contract must take into account not only the waste management issue but also its effects on market power. In this case, we are typically in the situation where one instrument, the WMC, needs to regulate two inefficiencies: the imperfect observability of the effort and the existence of rent-seeking behavior due to imperfect competition.

References

1. Calcott, P. and M. Walls (2000). Can Downstream Waste Disposal Policies Encourage Upstream "Design for Environment"? *American Economic Review*, 90, pp. 233-237.
2. Calcott, P. and M. Walls (2005). Waste Recycling and Design for Environment: Role for Market and Policy Instruments, *Resource and Energy Economics*, 27, pp. 287-305.
3. Choe, C. and I. Fraser (1999). An Economic Analysis of Household Waste Management, *Journal of Environmental Economics and Management*, 38, pp. 234-246.
4. Choe, C. and I. Fraser (2001). On the Flexibility of Optimal Policies for Green Design, *Environmental and Resource Economics*, 18, pp. 367-371.
5. Dinan, T.M. (1993). Economic Efficiency Effects of Alternative Policies for Reducing Waste Disposal, *Journal of Environmental Economics and Management*, 25, pp. 242-256.
6. Environment Agency UK (2007). *Treatment of non-hazardous waste for landfill: your waste-our responsibility*, www.environment-agency.gov.uk.
7. Fullerton, D. and T.C. Kinnaman (1995). Garbage, Recycling and Illicit Burning or Dumping, *Journal of Environmental Economics and Management*, 29, pp. 78-91.
8. Fullerton, D. and T.C. Kinnaman (1996). Household Responses to Pricing Garbage by the Bag, *American Economic Review*, 86, pp. 971-984.
9. Fullerton, D. and W. Wu (1998). Policies for Green Design, *Journal of Environmental Economics and Management*, 36, pp. 131-148.
10. Fullerton, D. and A. Wolverton (2000). Two Generalizations of a Deposit-Refund System, *American Economic Review*, 90, pp. 238-242.
11. Jenkins, R.R., Robin R. (1993). The Economics of Solid Waste Reduction, Hants, Edward Elgar Publishing Limited.
12. Jenkins, R.R., S.A. Martinez, K. Palmer and M.J. Podolsky (2003). The Determinant of Household Recycling: a Material-Specific Analysis of Recycling Program Features and Unit Pricing, *Journal of Environmental Economics and Management*, 45, pp. 294-318.
13. Palmer, K. and M. Walls (1997). Optimal policies for Solid Waste Disposal Taxes, Subsidies and Standards, *Journal of Public Economics*, 65, pp. 193-205.
14. Palmer, K., H. Sigman and M. Walls (1997). The Cost of Reducing Municipal Solid Waste, *Journal of Environmental Economics and Management*, 33, pp. 128-150.
15. Shinkuma, T. (2003). On the Second-Best Policy of Household's Waste Recycling, *Environmental and Resource Economics*, 24, pp. 77-95.
16. Shinkuma, T. (2007). Reconsideration of an Advance Disposal Fee Policy for End-of-Pipe Durable Goods, *Journal of Environmental Economics and Management*, 53, pp. 110-121.
17. Walls, M. and K. Palmer (2001). Upstream Pollution, Downstream Waste Disposal and the Design of Comprehensive Environmental Policies, *Journal of Environmental Economics and Management*, 41, pp. 94-108.

Appendix

Proof of lemma 1.

Step 1: BBC, IC, and PC $\Rightarrow (i)\ \ s = r \cdot c - \pi \cdot m(\pi)$.

Since IC and PC respectively entail that $P(E) > 0$, i.e., at least some households accept the contract, and $P(\overline{E}) = 0$, i.e., nobody cheats, the result directly follows from the definition of BBC.

Step 2: BBC, IC, PC $\Rightarrow (ii)\ \ r \cdot c - \pi \cdot m(\pi) > 0$ except for $r = \pi = 0$.

Let us first verify that ((i) and non (ii) $\Rightarrow non(PC)$. In fact if (i) is true, $u_e^{(\alpha,\theta)}(r,s)$ and $u_{\overline{e}}^{(\alpha,\theta)}(s,\pi)$ can be respectively written as:

$$\begin{cases} u_e^{(\alpha,\theta)}(r,\pi) = (\alpha - c) + (r \cdot c - \pi \cdot m(\pi)) - \theta \dfrac{r}{\overline{r}} \\ u_{\overline{e}}^{(\alpha,\theta)}(r,\pi) = (\alpha - c) + (r \cdot c - \pi \cdot m(\pi)) - \pi c \end{cases}. \tag{3}$$

So if $r \cdot c - \pi \cdot m(\pi) \le 0$ and $(r,\pi) \ne (0,0)$ then $\forall(\alpha,\theta) \in [0,A] \times]0,\Theta]$, $u_0^{(\alpha,\theta)} = \alpha - c > u_e^{(\alpha,\theta)}(r,\pi)$. Since this implies that $\max\{0, u_0^{(\alpha,\theta)}, u_{\overline{e}}^{(\alpha,\theta)}(r,\pi)\} > u_e^{(\alpha,\theta)}(r,s)$, PC can only be verified for a subset of $[0,A] \times \{0\}$: a set that contains no open subsets. But our probability distribution is absolutely continuous (i.e. only sets containing open sets has a strictly positive probability), it follows that PC cannot be true. But this preliminary observation leads us also to the conclusion (by contraposition) that $PC \Rightarrow (non(i)$ or(ii)). But by step 1, $(BBC, IC, PC) \Rightarrow (i)$, hence we can say that $BBC, IC, PC \Rightarrow (ii)$.

Step 3: BBC, IC, and PC$\Rightarrow$ (iii) $r\cdot c-\pi\cdot m(\pi)\le \pi\cdot c$.

As in step 2, if we show that ((i) and non (iii))$\Rightarrow non(IC)$ our result is obtained. It therefore remains to find a household (α,θ) with the property that $u_{\bar{e}}^{(\alpha,\theta)}(r,\pi)> max\left\{u_e^{(\alpha,\theta)}(r,\pi),u_0^{(\alpha,\theta)},0\right\}$. So let us set $(\alpha,\theta)=(A,\Theta)$. Since (i) is true we can use (3) and because $A>c$ and $non(iii)$, we observe that $u_{\bar{e}}^{(A,\Theta)}(r,\pi)>u_0^{(A,\Theta)}>0$. It therefore remains to verify that $u_{\bar{e}}^{(A,\Theta)}(r,\pi)>u_e^{(A,\Theta)}(r,\pi)$. So let observe that:

$$u_{\bar{e}}^{(A,\Theta)}(r,\pi)=\left(A-c\right)+\left(r\cdot c-\pi\cdot m(\pi)\right)-\pi\cdot c$$

$$>\left(A-c\right)+\left(r\cdot c-\pi\cdot m(\pi)\right)-r\cdot c \quad \left\{\begin{array}{l}\text{since } non(iii) \text{ and } \pi\cdot m(\pi)>0\\ \text{imply that } \ r\cdot c>\pi\cdot c\end{array}\right\}$$

$$=\left(A-c\right)+\left(r\cdot c-\pi\cdot m(\pi)\right)-\frac{r}{\bar{r}}\cdot\left(\bar{r}\cdot c\right)>\left(A-c\right)+\left(r\cdot c-\pi\cdot m(\pi)\right)-\frac{r}{\bar{r}}\cdot\Theta \quad \text{since } \Theta>c \text{ and } \bar{r}\le 1$$

$$=u_e^{(A,\Theta)}(r,\pi)$$

Proof of proposition 1.

Remark: In this proof, (i), (ii) and (iii) referee to the property exhibited in lemma 1.

Step 1: $\left((i)\text{and}(iii)\right)\Rightarrow$ IC.

Let us first notice that under (i), $u_{\bar{e}}^{(\alpha,\theta)}(r,\pi)$ is defined by (3). So if (iii) holds, we have:

$$\forall\alpha\in\left[0,A\right] \quad u_0^{(\alpha,\theta)}=\alpha-c\ge\alpha-c+r\cdot c-\pi\cdot m(\pi)-\pi\cdot c=u_{\bar{e}}^{(\alpha,\theta)}(r,\pi).$$

It follows, by the definition of a maximum, that:

$$\forall\left(\alpha,\theta\right)\in\left[0,A\right]\times\left[0,\Theta\right], \qquad \max\left\{u_e^{(\alpha,\theta)}(r,s),u_0^{(\alpha,\theta)},0\right\}\ge u_{\bar{e}}^{(\alpha,\theta)}(s,\pi). \tag{IC}$$

Step 2: $\left((i)\text{ and }(iii)\right)\Rightarrow$ BBC.

By step 1, we know that IC is true. It follows that $P(\bar{E})=0$ and since (i) is verified we can write that:

$$P(E)\left(s-(1-r)c+\pi\cdot m(\pi)\right)+P(\bar{E})\left(s-\pi\cdot c+\pi\cdot m(\pi)\right)=0. \tag{BBC}$$

Step 3: $\left((i),(ii)\text{ and }(iii)\right)\Rightarrow$ PC.

Let us first observe that under (i), $u_e^{(\alpha,\theta)}(r,\pi)$ and $u_{\bar{e}}^{(\alpha,\theta)}(r,\pi)$ are given by (3). Moreover by (ii) we typically have to sub-case one in which $r=\pi=0$ and one in which $s=\left(r\cdot c-\pi\cdot m(\pi)\right)>0$ and $(r,\pi)\ne(0,0)$, in the former case, (PC)) is obviously satisfied since

$$\forall\left(\alpha,\theta\right)\in\left[0,A\right]\times\left[0,\Theta\right] \quad u_0^{(\alpha,\theta)}=u_e^{(\alpha,\theta)}(r,\pi)=u_{\bar{e}}^{(\alpha,\theta)}(r,\pi).$$

Now let us move to the second case. Since $A>c$, we can say that $\forall\left(\alpha,\theta\right)\in E=\left]c,A\right[\times\left]0,\frac{\bar{r}}{r}s\right[$

$$u_e^{(\alpha,\theta)}(r,\pi)=\left(\alpha-c\right)+s-\theta\frac{r}{\bar{r}}>\alpha-c=u_0^{(\alpha,\theta)}>0.$$

But we also know that $\left((i)\text{ and }(iii)\right)\Rightarrow$ IC, or in other words that:

$$\forall\left(\alpha,\theta\right)\in\left[0,A\right]\times\left[0,\Theta\right], \qquad \max\left\{u_e^{(\alpha,\theta)}(r,s),u_0^{(\alpha,\theta)},0\right\}\ge u_{\bar{e}}^{(\alpha,\theta)}(s,\pi).$$

By using the previous equation we conclude that:

$$\forall\left(\alpha,\theta\right)\in E=\left]c,A\right[\times\left]0,\frac{\bar{r}}{r}s\right[,u_e^{(\alpha,\theta)}(r,s)>\max\left\{u_{\bar{e}}^{(\alpha,\theta)}(s,\pi),u_0^{(\alpha,\theta)},0\right\}.$$

There exists therefore a non-zero subset of agents who adopt the contract.

Proof of proposition 2.

This computation is a tedious exercise since $S(r,\pi)=S_C(r,\pi)+S_{\bar{C}}(r,\pi)$ with

$$\begin{cases} S_C(r,\pi) = \int_0^{\theta(r,\pi)} \left(\int_{\alpha(r,\pi,\theta)}^{A} u_e^{(\alpha,\theta)}(r,\pi) f(\alpha/\theta) d\alpha \right) f(\cdot,\theta) d\theta \\ S_{\bar{C}}(r,\pi) = \int_{\theta(r,\pi)}^{\Theta} \left(\int_c^A u_0^{(\alpha,\theta)} f(\alpha/\theta) d\alpha \right) f(\cdot,\theta) d\theta \end{cases}.$$

But the reader can observe that r and π work in a rather similar way. So if x stands for either r or π, we obtain that:

$$\partial_x S(r,\pi) = \partial_x S_C(r,\pi) + \partial_x S_{\bar{C}}(r,\pi) =$$

$$\left(\left(\int_{\alpha(r,\pi,\theta)}^{A} u_e^{(\alpha,\theta)}(r,\pi) \cdot f(\alpha/\theta) d\alpha \right) f(\cdot,\theta) \right)\Big|_{\theta=\theta(r,\pi)} \times \partial_x \theta(r,\pi) \tag{4}$$

$$+\int_0^{\theta(r,\pi)} \left(\int_{\alpha(r,\pi,\theta)}^{A} \partial_x u_e^{(\alpha,\theta)}(r,\pi) f(\alpha/\theta) d\alpha - \left(u_e^{(\alpha,\theta)}(r,\pi) f(\alpha/\theta) \right)\Big|_{\alpha=\alpha(r,\pi,\theta)} \partial_x \alpha(r,\pi,\theta) \right) f(\cdot,\theta) d\theta \tag{5}$$

$$-\left(\left(\int_c^A u_0^{(\alpha,\theta)} \cdot f(\alpha/\theta) d\alpha \right) f(\cdot,\theta) \right)\Big|_{\theta=\theta(r,\pi)} \times \partial_x \theta(r,\pi). \tag{6}$$

Since $\theta(r,\pi) = \bar{r}\left(c - \frac{\pi \cdot m(\pi)}{r} \right)$ and $\alpha(r,\pi,\theta) := \frac{\theta r}{\bar{r}} + c - \left(rc - m(\pi) \right)$, we also remark that:

$$\begin{cases} \text{(i)}\, u_e^{(\alpha,\theta)}(r,\pi)\Big|_{\theta=\theta(r,\pi)} = \alpha - (1-r)\cdot c - \pi \cdot m(\pi) - \bar{r}\left(c - \frac{\pi \cdot m(\pi)}{r} \right)\frac{r}{\bar{r}} = \alpha - c = u_0^{(\alpha,\theta)}\Big|_{\theta=\theta(r,\pi)} \\ \text{(ii)}\, \alpha(r,\pi,\theta)\Big|_{\theta=\theta(r,\pi)} = \bar{r}\left(c - \frac{\pi \cdot m(\pi)}{r} \right)\frac{r}{\bar{r}} + c - \left(rc - \pi m(\pi) \right) = c \\ \text{(iii)}\, u_e^{(\alpha,\theta)}(r,\pi)\Big|_{\alpha=\alpha(r,\pi,\theta)} = \theta\frac{r}{\bar{r}} + c - \left(rc - \pi m(\pi) \right) - (1-r)\cdot c - \pi \cdot c(\pi) - \theta\frac{r}{\bar{r}} = 0 \end{cases}.$$

By (i) and (ii) the first (4) and the third (6) term in the preceding sum simplify, and (iii) reduces the second (5). We can therefore say that:

$$\partial_x S(r,\pi) = \int_0^{\theta(r,\pi)} \left(\int_{\alpha(r,\pi,\theta)}^{A} \partial_x u_e^{(\alpha,\theta)}(r,\pi) \cdot f(\alpha/\theta) d\alpha \right) f(\cdot,\theta) d\theta$$

Now remember that $\partial_\pi u_e^{(\alpha,\theta)}(r,\pi) = -(\pi m'(\pi) + m(\pi)) < 0$ for all $\pi > 0$ and that the only feasible contract for which $\pi = 0$ is $(r,\pi) = (0,0)$, we can therefore say that:

$$\forall (r,\pi) \in \mathcal{F} \text{ and } (r,\pi) \neq (0,0), \quad \partial_\pi S(r,\pi) = -(\pi m'(\pi) + m(\pi)) \int_0^{\theta(r,\pi)} \int_{\alpha(r,\pi,\theta)}^{A} dF < 0,$$

where $\int_0^{\theta(r,\pi)} \int_{\alpha(r,\pi,\theta)}^{A} dF = P(r,\pi)$ the proportion of households who accept the waste management contract. This proves (i) of proposition 2.

Let us move to (ii) of proposition 2. Since $e(r) = \frac{r}{\bar{r}}$, we observe that $\partial_r u_e^{(\alpha,\theta)}(r,\pi) = c - \frac{\theta}{\bar{r}}$. Moreover, since $\forall \theta < \theta,\ (r,\pi) =: \bar{r}\left(c - \frac{\pi \cdot m(\pi)}{r} \right)$, we can say that:

$$c - \frac{\theta}{\bar{r}} > c - \bar{r}\left(c - \frac{\pi \cdot m(\pi)}{r} \right)\frac{1}{\bar{r}} = \frac{\pi \cdot c(\pi)}{r} \geq 0.$$

This implies that:

$$\forall (r,\pi) \in \mathcal{F} \text{ and } (r,\pi) \neq (0,0), \quad \partial_r S(r,\pi) = \int_0^{\theta(r,\pi)} \left(c - \frac{\theta}{\bar{r}} \right) \left(\int_{\alpha(r,\pi,\theta)}^{A} f(\alpha/\theta) d\alpha \right) f(\cdot,\theta) d\theta > 0.$$

Now let us denote by

$$\bar{\Theta}(r,\pi) := \int_0^{\theta(r,\pi)} \theta \frac{\left(\int_{\alpha(r,\pi,\theta)}^{A} f(\alpha/\theta) d\alpha \right)}{\int_0^{\theta(r,\pi)} \int_{\alpha(r,\pi,\theta)}^{A} dF} f(\cdot,\theta) d\theta$$

the average disutility of the effort for households who accept the waste management contract. This gives us:

$$\forall(r,\pi)\in\mathcal{F} \text{ and } (r,\pi)\neq(0,0), \quad \partial_r S(r,\pi)=\left(c-\frac{\overline{\Theta}(r,\pi)}{\overline{r}}\right)P(r,\pi)\cdot$$

Proof of lemma 2.

The proof is obvious since the surplus $S(r,\pi)$ is continuous and the set $\mathcal{F}$ of feasible contracts is compact.

Proof of proposition 3.

Remember that $\left(r^*,\pi^*\right)\in \underset{(r,\pi)\in\mathcal{F}'}{\arg\max}\, S(r,\pi)$ with $\mathcal{F}'=\left\{(r,\pi)\in\left[0,\overline{r}\right]\times\left[0,1\right]: r=\min\left\{\frac{\pi\cdot\left(m(\pi)+c\right)}{c},\overline{r}\right\}\right\}$

Point (i): $\exists\pi_{\sup}<1$ given by $\frac{\pi_{\sup}\cdot\left(m(\pi_{\sup})+c\right)}{c}=\overline{r}$ such that $\pi^*\le\pi_{\sup}$.

Let us first verify that $\pi_{\sup}$ exists and is smaller than 1. To see this, let us observe that $f(\pi)=\frac{\pi\cdot\left(m(\pi)+c\right)}{c}$ is increasing in π and let us remember that we have assumed that $m(0)=0$ and $m(1)>c$. The range of f is therefore given by $f\left(\left[0,1\right]\right)=\left[0,f(1)\right]$ with $f(1)>2$. Since $\overline{r}\in\left[0,1\right]$, there always exists a unique $\pi_{\sup}(\overline{r})<1$ solving $\frac{\pi_{\sup}\cdot\left(m(\pi_{\sup})+c\right)}{c}=\overline{r}$.

Now let us verify that $\pi^*\le\pi_{\sup}$. Assume the contrary. Since $f'(\pi)>0$, it is immediate by the definition of $\mathcal{F}'$ that $r^*=\overline{r}$. But the same holds for $\pi'=\frac{\pi^*+\pi_{\sup}}{2}<\pi^*$. Now remember by proposition 2 that $\partial_\pi S(r,\pi)<0$; it follows that $S(\overline{r},\pi')>S(\overline{r},\pi^*)$ which contradicts the fact that $\left(r^*,\pi^*\right)$ is an optimal solution.

Point (ii): $s^*=r^*\cdot c-\pi^*\cdot m(\pi^*)=\pi^*\cdot c$.

Since $f'(\pi)>0$, by step 1 we know that $f(\pi^*)\le f(\pi_{\sup})=\overline{r}$. It follows by the definition of $\mathcal{F}'$, that $r^*=\frac{\pi^*\cdot\left(m(\pi^*)+c\right)}{c}$ or in other words that $r^*\cdot c-\pi^*\cdot m(\pi^*)=\pi^*\cdot c$.

Point (iii): $\exists\pi:\left[0,\overline{r}\right]\to\left[0,\pi_{\sup}\right]$, with the property that $r=\frac{\pi(r)\cdot\left(m(\pi(r))+c\right)}{c}$.

The same arguments as in the first part of the proof of point (i) apply.

Proof of proposition 4.

Since the search for an optimal contract reduces to the computation of a waste reduction rate that satisfies

$r^*\in\underset{0\le r\le\overline{r}}{\arg\max}\, S(r,\pi(r))$.

Point (i): $r^*\neq0$.

Let us compute $\lim_{r\to0}\partial_r S(r,\pi(r))$. We observe by proposition 2 that:

$$\lim_{r\to0}\partial_r S(r,\pi(r))=\lim_{r\to0}\left(\partial_r S(r,\pi)\big|_{\pi=\pi(r)}\right)+\lim_{r\to0}\left(\partial_\pi S(r,\pi)\big|_{\pi=\pi(r)}\right)\cdot\lim_{r\to0}\frac{d\pi(r)}{dr}$$

with

$$\partial_r S(r,\pi)=\left(c-\frac{\overline{\Theta}(r,\pi)}{\overline{r}}\right)P(r,\pi) \quad \partial_\pi S(r,\pi)=-(\pi m'(\pi)+m(\pi))P(r,\pi), \ \frac{d\pi}{dr}=\frac{c}{\pi(r)\cdot m'(\pi(r))+m(\pi(r))+c}.$$

Now remember that $m(\pi)$ is increasing and convex; it follows that $\lim_{\pi\to0} m'(\pi)$ is bounded. Since $P(r,\pi(r))\in\left[0,1\right]$, $m(0)=0$ and $\pi(0)=0$, we can state that $\lim_{r\to0}\partial_\pi S(r,\pi(r))=0$. But we can also observe that $\frac{d\pi}{dr}=\frac{c}{\pi(r)\cdot m'(\pi(r))+m(\pi(r))+c}\in\left[0,1\right]$, so, by continuity, the same holds for $\lim_{r\to0}\frac{d\pi}{dr}$. We can therefore say (see proof of proposition 2) that:

$$\lim_{r\to 0}\partial_r S(r,\pi(r)) = \lim_{r\to 0}\left(\partial_r S(r,\pi)\big|_{\pi=\pi(r)}\right) = \int_0^{\lim_{r\to 0}\theta(r,\pi(r))}\left(c-\frac{\theta}{\overline{r}}\right)\left(\int_{\lim_{r\to 0}\alpha(r,\pi(r),\theta)}^{A} f(\alpha/\theta)d\alpha\right)f(\cdot,\theta)d\theta .$$

Now let us observe that:

(i) $\lim_{r\to 0}\alpha(r,\pi(r),\theta) = \lim_{r\to 0}\left(\theta\frac{r}{\overline{r}} + \pi(r)\cdot m(\pi(r)) + (1-r)\cdot c\right) = c$.

(ii) $\lim_{r\to 0}\theta(r,\pi(r)) = \lim_{r\to 0}\overline{r}\left(c-\frac{\pi(r)\cdot m(\pi(r))}{r}\right) = \overline{r}c$ since, by L'hôpital's rule,

$$\lim_{r\to 0}\frac{\pi(r)\cdot m(\pi(r))}{r} = \lim_{r\to 0}\left(m(\pi(r)) + \pi(r)m'(\pi(r))\right)\frac{d\pi}{dr} = 0 .$$

(Remember that $\lim_{\pi\to 0} m'(\pi)$ is bounded and $\frac{d\pi}{dr}\in[0,1]$). We can therefore say that:

$\lim_{r\to 0}\partial_r S(r,\pi(r)) = \int_0^{\overline{r}c}\left(c-\frac{\theta}{\overline{r}}\right)\left(\int_c^A f(\alpha/\theta)d\alpha\right)f(\cdot,\theta)d\theta$. But we have assumed that F is absolutely continuous with strictly positive density, hence $\left(\int_c^A f(\alpha/\theta)d\alpha\right) > 0$ and $f(\cdot,\theta) > 0$. We deduce that:

$$\lim_{r\to 0}\partial_r S(r,\pi(r)) > \int_0^{\overline{r}c}\left(c-\frac{\overline{r}c}{\overline{r}}\right)\left(\int_c^A f(\alpha/\theta)d\alpha\right)f(\cdot,\theta)d\theta = 0 .$$

From that point of view, it is impossible that $r^* = 0$.

Point (ii): $s^* = \pi(r^*)c > 0$.

This follows directly from point (ii) of proposition 3 and the fact that $r^* > 0$.

Point (iii): the marginal condition.

Since $r > 0$, we know from the Kuhn-Tucker first-order conditions that:

$$\begin{cases}\partial_r S(r,\pi(r)) + \partial_\pi S(r,\pi(r))\dfrac{d\pi}{dr} - \lambda = 0\\ \lambda\left(r-\overline{r}\right) = 0 \quad r-\overline{r}\le 0 \quad \lambda\ge 0\end{cases}$$

or equivalently that: $\partial_r S(r,\pi(r)) + \partial_\pi S(r,\pi(r))\frac{d\pi}{dr}\ge 0$ with equality if $r<\overline{r}$.

By replacing the different derivatives by their value we finally obtain:

$\left(c-\frac{\overline{\Theta}(r^*,\pi^*)}{\overline{r}} - (\pi^* m'(\pi^*) + m(\pi^*))\frac{d\pi}{dr}\right)P(r^*,\pi^*)\ge 0$ with equality if $r<\overline{r}$.

Proof of lemma 3.

Let us remember that each household solves:

$$\max_{(x,e,f)\in\{0,1\}\times[0,1]\times[0,1]} \alpha x - \theta e - \alpha x - \left((1-f)\cdot d + f\cdot c\cdot\pi\right)\cdot\min\left\{x-\overline{r}\cdot e, 0\right\} .$$

Point (i): $\pi = \frac{d}{c}$.

For $\pi = \frac{d}{c}$, the household is indifferent between all values of f. We can therefore say that $f = 1$ is an optimal strategy. Moreover by $\pi = \frac{d}{c}$ the agency minimizes its control costscompared with a TPT in which $\pi > \frac{d}{c}$.

Point (ii): $\forall(\alpha,\theta)\in[0,A]\times[0,\Theta]$, $V_{TPT}^{(\alpha,\theta)} = \max\left\{0, \alpha-(a+d) + \max\left\{d\cdot\overline{r}-\theta, 0\right\}\right\}$.

If $\pi = \frac{d}{c}$, the previous program becomes $\max_{(x,e,f)\in\{0,1\}\times[0,1]} \alpha x - \theta e - \alpha x - d\cdot\min\left\{x-\overline{r}\cdot e, 0\right\}$ soif $x = 0$, we can say that

$e = 0$ is the optimal effort and the indirect utility is givenby $V^{(\alpha,\theta)}(0) = 0$. If $x = 1$, the optimal effort is respectively $e = 1$ or $e = 0$ when $\theta \le \overline{r} \cdot d$ or $\theta > \overline{r} \cdot d$. It follows that $V^{(\alpha,\theta)}(1) = \alpha - a - d + \max\{\overline{r} \cdot d - \theta, 0\}$. Since each consumer chooses the best solution between both options, we can conclude that:

$V_{TPT}^{(\alpha,\theta)} = \max\{0, \alpha - a - d + \max\{\overline{r} \cdot d - \theta, 0\}\}$.

Point (iii): the budget constraint is given by $a + d \ge c + \frac{d}{c} m\left(\frac{d}{c}\right) - \overline{r}\left(c - d\right)\left(1 - P_{TPT}(a,d)\right)$.

Let us fist remember that for $\pi = \frac{d}{c}$, no fines are collected. It follows that the waste disposal agency collects an ADF for each unit of goods sold on the market, a complete disposal fee paid by the households performing no effort and a reduced fee for those who set their effort at $e = 1$. From that point of view, the agency obtains on average per effective buyer: $a + d \cdot \left(1 - P(a,d)\right) + d \cdot \left(1 - \overline{r}\right) \cdot P(a,d)$, where $P_{TPT}(a,d)$ denotes the proportion of effective buyers who provide no effort. The agency incurs a cost given by (i) the total waste management cost for the households who make no effort, (ii) a reduced cost for those that reduce their amount of waste, and (iii) the monitoring cost (bearing in mind that $\pi = \frac{d}{c}$) that applies to all the effective buyers. It therefore spends on average per effective buyer: $c \cdot P_{TPT}(a,d) + c \cdot \left(1 - \overline{r}\right) \cdot \left(1 - P_{TPT}(a,d)\right) + \frac{d}{c} \cdot m\left(\frac{d}{c}\right)$. After rearrangement, the budget constraint is therefore given by $a + d \ge c + \frac{d}{c} \cdot m\left(\frac{d}{c}\right) - \overline{r} \cdot \left(c - d\right) \cdot \left(1 - P_{TPT}(a,d)\right)$.

Proof of proposition 5.

Point (i): $\forall(\alpha,\theta)$, $V_{WMC}^{(\alpha,\theta)} \ge V_{TPT}^{(\alpha,\theta)}$.

If a WMC, given by $(\overline{r}, c - (a + d) + \overline{r}d, \overline{r}\frac{d}{c})$, is implemented, we know by section 1 that the utility of a household of type (α,θ) is given by

$$V_{WMC}^{(\alpha,\theta)} = \max\{0, \alpha - c, \alpha - (a + d) + \overline{r}d - \theta, \alpha - \left(a + d\right)\} \tag{7}$$

If $a + d \ge c$, the second term dominates the last one: it can therefore be forgotten. Moreover if we replace c in the second term by $(a + d)$, we can say that:

$V_{WMC}^{(\alpha,\theta)} \ge \max\{0, \alpha - (a + d), \alpha - (a + d) + \overline{r}d - \theta\} = \max\{0, \alpha - (a + d) + \max\{d \cdot \overline{r} - \theta, 0\}\} = V_{TPT}^{(\alpha,\theta)}$ (by lemma 3) .

Now, if $a + d < c$, the second term is dominated by the last one, so that

$V_{WMC}^{(\alpha,\theta)} = \max\{0, \alpha - (a + d) + \overline{r}d - \theta, \alpha - \left(a + d\right)\} = \max\{0, \alpha - (a + d) + \max\{d \cdot \overline{r} - \theta, 0\}\} = V_{TPT}^{(\alpha,\theta)}$.

This also means that every buyer accepts the WMC, since the strategy of buying the good and refusing the contract is dominated.

Point (ii):

if $a + d \ge c$, the optimal WMC strictly dominates the optimal TPT.

It is immediate that our WMC, given by $(\overline{r}, c - (a + d) + \overline{r}d, \overline{r}\frac{d}{c})$, satisfies (IC), since the strategy of buying the good and cheating (last term of (7)) is dominated by the strategy of buying the good and refusing the contract (second term of (7)). Moreover, (PC) is satisfied since all the households belonging to:

$\{(\alpha,\theta) \in [0, A] \times [0, \Theta] : \alpha \ge (a + d) - \overline{r}d + \theta \text{ and } \theta \le c - (a + d) + \overline{r}d\}$

participate in the WMC and since $c - (a + d) + \overline{r}d > 0$ (see remark 2) this set has a non-empty interior so that its measure is strictly positive. Let us now check that (BBC) is true as long as the TPT satisfies a budget constraint (see lemma 3). Under this WMC, the agency only collects c per effective buyer (since IC is true no fine is collected) and spends (1) the total waste treatment cost for buyers who refuse the contract, (2) the subsidy paid to households who accept the WMC, (3) the remaining waste treatment cost for the latter, and (4) the monitoring cost of the households who accept the contract. So if we denote by $P_{WMC}(a,d)$ the proportion of buyers that refuse the WMC, we can say that the budget constraint, per effective buyer, is given by:

$$c \geq c \cdot P_{WMC}(a,d) + \left(c - (a+d) + \overline{r}d\right) \cdot \left(1 - P_{WMC}(a,d)\right) + (1-\overline{r}) \cdot c \cdot \left(1 - P_{WMC}(a,d)\right) + \overline{r}\frac{d}{c} m\left(\overline{r}\frac{d}{c}\right) \cdot \left(1 - P_{WMC}(a,d)\right).$$

By rearranging this expression, we can verify that:

$$a + d \geq c + \overline{r}\frac{d}{c} m\left(\overline{r}\frac{d}{c}\right) - \overline{r}\left(c-d\right).$$

But we know that the related TPT satisfies the budget constraint. Hence by lemma 3, we can say that:

$$a + d \geq c + \frac{d}{c} m\left(\frac{d}{c}\right) - \overline{r}\left(c-d\right) \cdot \left(1 - P_{TPT}(a,d)\right)$$

$$\geq c + \frac{d}{c} m\left(\frac{d}{c}\right) - \overline{r}\left(c-d\right) \text{ since } P_{TPT}(a,d) \leq 1 \text{ and } \pi_{TPT} = \frac{d}{c} \leq 1$$

$$> c + \overline{r}\frac{d}{c} m\left(\overline{r}\frac{d}{c}\right) - \overline{r}\left(c-d\right) \text{ since } \overline{r} < 1 \text{ and } m(\pi), m'(\pi) > 0.$$

We can therefore say that the WMC that we have associated with a TPT is feasible and, by point (i), allocates more utility to each household. But this WMC is less efficient than our optimal WMC. We can therefore conclude that an optimal WMC dominates (at least weakly) a TPT. Moreover, the last equation tells us that the WMC that we have introduced saves money compared with the TPT. We can therefore distribute this benefit and conclude that the optimal WMC strictly dominates a TPT when $a + d \geq c$.

Point (iii): $a + d < c$.

By point (i), we know that $\forall(\alpha,\theta)$, $V_{WMC}^{(\alpha,\theta)} = V_{TPT}^{(\alpha,\theta)}$ and that every buyer accepts the WMC. This has several consequences. First, since the utility allocation is the same, the set of buyers who accept the contract without cheating is exactly the set of agents who buy the good and perform the effort under a TPT. This is given by:

$$A := \left\{(\alpha,\theta) \in [0,A] \times [0,\Theta] : \alpha \geq (a+d) - \overline{r}d + \theta \text{ and } \theta \leq \overline{r}d\right\}.$$

With a similar argument, we can also show that the set of buyers who accept the contract and cheat is the same as the set of agents who buy the good and perform no effort under a TPT, so that: $P_{WMC}(a,d) = P_{TPT}(a,d)$. Since we only consider the case in which $(a+d) < c + d \cdot \overline{r}$ (see remark 2), and $d > 0$ (otherwise we are back to a pure ADF), it is simple to show that A has a non-empty interior. This guarantees that PC is true.

It remains to verify that the budget constraint is verified and is not binding. Under a WMC, its computation is similar to point (ii); we simply have to take into account that (i) a fine is now collected and (ii) every buyer is controlled and receives a subsidy. We obtain the following constraint per unit of effective buyers.

$$c + P_{WMC}(a,d) \cdot \left(\overline{r}\frac{d}{c}\right) \cdot c \geq c \cdot P_{WMC}(a,d) + \left(c - (a+d) + \overline{r}d\right) + (1-\overline{r}) \cdot c \cdot \left(1 - P_{WMC}(a,d)\right) + \overline{r}\frac{d}{c} m\left(\overline{r}\frac{d}{c}\right).$$

By rearranging this expression, we now verify that:

$$a + d \geq c + \overline{r}\frac{d}{c} m\left(\overline{r}\frac{d}{c}\right) - \overline{r}\left(c-d\right) \cdot \left(1 - P_{WMC}(a,d)\right).$$

But now $P_{WMC}(a,d) = P_{TPT}(a,d)$. So we know, since the PT satisfy BBC, that:

$$a + d \geq c + \frac{d}{c} m\left(\frac{d}{c}\right) - \overline{r}\left(c-d\right) \cdot \left(1 - P_{WMC}(a,d)\right)$$

and since $\overline{r} < 1$ and $m(\pi), m'(\pi) > 0$ we conclude as in point (ii) that:

$$a + d > c + \overline{r}\frac{d}{c} m\left(\overline{r}\frac{d}{c}\right) - \overline{r}\left(c-d\right) \cdot \left(1 - P_{WMC}(a,d)\right).$$