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Day-of-the-week and jump effects in international investment sentiment indices

Abstract

This study investigates the jump characteristics and day-of-the-week effects in international investment sentiment indices using an autoregressive conditional jump intensity (ARJI) model. The empirical results find the day-of-the-week effect by means of an ANOVA test in international investment sentiment indices. Thus, this study further tests the Monday effect in the ARJI model and finds that the time-varying jump intensity model is feasible for the modeling of volatility indices. Consequently, the international investment sentiment indices are able to provide valuable information so that traders can adequately implement their investment strategies.

Keywords: the day-of-week effect, ARJI model, Jump-diffusion model.
JEL Classification: G14, C22, D82.

Introduction

The volatility index (VX) is derived from the market price form option market and measures the fear gauge sentiment so as to enable portfolio managers, options traders and investors to implement their investment strategies. Numerous studies have focused on the Chicago Board Options Exchange's market volatility index (VIX), which depicts the expected level of volatility in the S&P500 index over the next 30 days. Several countries recognize the VX as an essential financial instrument and have each gradually compiled volatility indices[1]. A few studies have examined non-U.S. volatility indices and Whaley (2009) has emphasized that examining the issue of volatility indices in different countries is a topic that cannot be neglected[2]. Therefore, the primary purpose of this paper is to examine the day-of-the-week and jump effects of the volatility indices for different countries.

With regard to the day-of-the week effect and asset volatility, the fact that the volatility is observed to be different on each day of the week is one of the most well-known market anomalies and has important implications for investors and management as they implement investment strategies and engage in portfolio selection in an effort to reach their goal. Several studies have found evidence of day-of-the-week effects in international stock, futures, bond, foreign exchange markets, etc. However, there is limited evidence with respect to a similar day-of-the-week effect for the volatility index[3]. Berument and Kiymaz (2001) pointed out that the predicted volatility patterns are used for hedging and speculative purposes. Moreover, the investor who seeks to estimate future market volatility and recognizes the day-of-the-week pattern faces a variety of speculative and hedging decisions. However, Fleming, Ostdiek and Whaley (1996) first only found evidence of a day-of-the-week effect for the volatility index in the U.S. Thus, examining the day-of-the-week effect for volatility indices in different countries has an important bearing on investment strategies.

In recent years, studies on the modeling of financial market volatility have combined diffusion and jump processes, and investors will make incorrect financial and economic decisions when they fail to comprehend the jump features. Wagner and Szimayer (2004) were the first to investigate the jumps in the implied volatility index based on the jump-diffusion process. Wagner and Szimayer (2004), Dotsis, Psychoyios and Skiadopoulos (2007), Becker, Clements and McClelland (2009), Bollerslev, Kretschmer, Pigorsch and Tauchen (2009), Duan and Yeh (2010) and Lin and Lee (2010) found evidence of significant jumps in the volatility index. Moreover, Chan and Maheu (2002), Maheu and McCurdy (2004), Chiu, Lee and Chen (2005) and Lee and Lee (2009) consistently demonstrated that the discontinuous autoregressive conditional jump intensity (hereafter, ARJI) model can more precisely measure the jump-diffusion process for a single asset. Consequently, it is relevant to investigate whether the volatility indices focus on the jump behavior using the ARJI and to examine the day-of-the-week effects in different countries' markets by incorporating time-varying jump behavior.

This empirical study contributes to this line of research by determining whether the day-of-the-week effect of volatility indices in fact exists and whether volatility indices exhibit jump behavior in different countries. The present study applies a powerful time-varying jump model by Chan and Maheu (2002) and Maheu and McCurdy (2004) to investigate the jump

[1] For example, the Belgium 20 Volatility Index, VSTOXX Volatility Index, VDAX-New Volatility Index, Nikkei Stock Average Volatility Index, Vkospi Volatility Index, FTSE 100 Volatility Index, etc.

[2] See, for example, Nikkinen and Sahlstrom (2004), Wagner and Szimayer (2004), Nikkinen, Sahlstrom and Vahamaa (2006) and Aijo (2008).

[3] See, for example, Cross (1973), Gibbons and Hess (1981), Rogalski and Tinic (1986), Bohl, Goodfellow and Bialkowski (2010), Lim, Ho and Dollery (2010), Muhammad and Rahman (2010) and Mutairi (2010).

behavior of volatility indices for different countries. To our knowledge, this study is the first to apply the ARJI model in volatility indices; thus, the current research hopes to fill the existing gap in the literature. It is found that the ARJI model reveals strong evidence of jump-diffusion behavior in the different countries, while the traditional GARCH model only finds evidence of diffusion behavior. Furthermore, evidence of a Monday effect is found in the volatility indices for different countries.

The remainder of this paper is organized as follows. Section 1 describes the data and methodology underlying the ARJI model. The empirical results are presented in section 2, and the final section concludes.

1. Data and methodology

Daily volatility indices are used for the sample period from January 2, 2006 to August 31, 2010. The indices are BVIX, EVIX, GVIX, JVIX, KVIX, UKVIX and VIX[1]. All data were obtained from Datastream. This study uses the daily VX log returns, R_t, which is calculated as the logarithmic difference in the daily volatility index as $R_t = (\ln P_t - \ln P_{t-1}) \times 100$, where P_t and P_{t-1} are the closing volatility index in time periods t and t-1, respectively.

The ARJI model is applied with the jump intensity obeying an ARMA process and incorporating the GARCH effect. The ARJI model can be expressed as follows:

$$R_t = \mu + \mu_1 M + \sum_{i=1}^{p} \varphi_i R_{t-i} + \varepsilon_{1,t} + \varepsilon_{2,t}, \quad (1)$$

where M is the Monday dummy variable which is equal to 1 when the day is Monday, otherwise 0. $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ denote a normal and jump innovation, respectively. $\varepsilon_{1,t}$ is contemporaneously independent of $\varepsilon_{2,t}$ $\varepsilon_{1,t}$ is assumed to be:

$$\varepsilon_{1,t} = h_t z_t, \; z_t \sim NID\left(0,1\right) \text{ and}$$
$$h_t = \omega + \sum_{i=1}^{p} \beta_i h_{t-i} + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2. \quad (2)$$

The jump stochastic process is assumed to follow a Poisson distribution with a time-varying conditional intensity parameter, λ_t. The Poisson distribution with parameter λ_t conditional on Ω_{t-1} is assumed to describe the arrival of a discrete number of jumps, where $n_t \in \{0, 1, 2,\ldots\}$ over the interval $[t - 1, t]$. The conditional density of n_t is as follows:

$$P(n_t = j \mid \Omega_{t-1}) = \frac{e^{-\lambda_t} \lambda_t^j}{j!}, \; j = 0,1,2,... \quad (3)$$

The conditional jump intensity λ_t is the expected number of jumps conditional on the information set Ω_{t-1}, which is parameterized as:

$$\lambda_t = \lambda_0 + \rho \lambda_{t-i} + \gamma \zeta_{t-j}. \quad (4)$$

λ_t is related to the conditional jump intensity and ζ_{t-1} which is defined as:

$$\zeta_{t-1} \equiv E\left[n_{t-1} \mid \Omega_{t-1}\right] - \lambda_{t-1} =$$
$$= \sum_{j=0}^{\infty} j \; P\left(n_{t-1} = j \mid \Omega_{t-1}\right) - \lambda_{t-1}, \quad (5)$$

where $P(n_{t-1} = j \mid \Omega_{t-1})$, called the filter, is the ex post inference on n_{t-1} given the information set Ω_{t-1}, and $E[n_{t-1} \mid \Omega_{t-1}]$ is the ex post judgment of the expected number of jumps from $t - 2$ to $t - 1$ and λ_{t-1} is the conditional expectation of $n - 1$ given the information set Ω_{t-2}. Therefore, ζ_{t-1} represents the change in the conditional forecast of n_{t-1} by the econometrician as the information set is updated.

The jump size, $\pi_{t,k}$, is assumed to be independently drawn from a normal distribution. The jump-size distribution is:

$$\pi_{t,k} \sim NID\left(\theta, \delta^2\right) \quad (7)$$

and the jump component influencing the VX log returns from $t - 1$ to t is:

$$J_t = \sum_{k=1}^{n_t} \pi_{t,k}. \quad (8)$$

Therefore, the jump innovation associated with period t is expressed as:

$$\varepsilon_{2,t} = J_t - E\left[J_t \mid \Omega_{t-1}\right] = \sum_{k=1}^{n_t} \pi_{t,k} - \theta \lambda_t. \quad (9)$$

The conditional variance of returns is decomposed into two components: a smoothly developing conditional variance component related to the diffusion of past news impacts and the conditional variance component associated with the heterogeneous information arrival process which generates jumps. The conditional variance of returns is:

$$Var\left(R_t \mid \Omega_{t-1}\right) = Var\left(\varepsilon_{1,t-1} \mid \Omega_{t-1}\right) +$$
$$+ Var\left(\varepsilon_{2,t-1} \mid \Omega_{t-1}\right) = h_t + \left(\theta^2 + \delta^2\right) \; \lambda_t. \quad (10)$$

The likelihood function is constructed as follows. The conditional density of returns follows a normal distribution and j jumps and is normally distributed as

[1] The Volatility Index for the Belgium 20 Volatility Index in Belgium, VSTOXX Volatility Index in Europe, VDAX-New Volatility Index in Germany, Nikkei Stock Average Volatility Index in Japan, Vkospi Volatility Index in South Korea, FTSE 100 Volatility Index in UK and Volatility Index in U.S (hereafter, BVIX, EVIX, GVIX, JVIX, KVIX, UKVIX and VIX).

$$f\left(R_t \mid n_t = j \;, \Omega_{t-1}\right) = \left(2\pi\left(h_t + j\delta^2\right)\right)^{-1/2} \exp\left(-\frac{\left(R_t - \mu - \mu_1 - \sum_{i=1}^{p} \varphi_i R_{t-i} + \theta\lambda_t - \theta j\right)^2}{2\left(h_t + j\delta^2\right)} \right). \quad (11)$$

Furthermore, Maheu and McCurdy (2004) propose and provide an ex post distribution for the number of jumps, n_t. The filter is contracted as:

$$P\left(n_t = j \mid \Omega_t\right) = \frac{f\left(R_t \mid n_t = j, \Omega_{t-1}\right) P\left(n_t = j \mid \Omega_{t-1}\right)}{f\left(R_t \mid \Omega_{t-1}\right)}. \quad (12)$$

Integrating out all jumps during one unit interval, the conditional probability density function can be expressed as:

$$f\left(R_t \mid \Omega_{t-1}\right) = \sum_{j=0}^{\infty} f\left(R_t \mid n_t = j, \Omega_{t-1}\right) P\left(n_t = j \mid \Omega_{t-1}\right). \quad (13)$$

Therefore, the likelihood function can be expressed as:

$$L(\Psi) = \sum_{t=1}^{T} \log f\left(R_t \mid \Omega_{t-1}; \Psi\right), \quad (14)$$

where $\Psi = (\mu, \mu_1, \phi_i, \omega, \alpha_j, \beta_i, \theta, \delta, \lambda_0, \rho, \gamma)$. This study does not take time-varying jump variables into account and the ARJI model reduces to a constant Jump (CJ) model when setting $\rho = \gamma = 0$. When it does not take jump variables into account, the ARJI model reduces to a GARCH model when setting $\theta = \delta = \lambda_0 = \rho = \gamma = 0$.

2. Results

The descriptive statistics are shown in Table 1. The means (standard deviations) of the volatility indices range from 22.186 to 29.158 (from 9.311 to 12.823), there being a large value for Japan (Japan) and a small value for Belgium (Germany), respectively. The means (standard deviations) of the VX log returns range from -0.00876 to 0.072419 (from 9.311 to 12.823), there being a large value for the US (US) and a small value for Korea (Germany), and only the VX log returns is negative in Korea. The volatility indices and the VX log returns exhibit positive skewness and also kurtosis. As for the Ljung-Box Q and Q^2 test for examining the serial correlation of the squared returns, both statistics with four lags are significant at the 1% level. This indicates that the volatility indices and the VX log returns exhibit autocorrelation, linear dependence and strong ARCH effects. The respective index and the VX log returns for BVIX, EVIX, GVIX, JVIX, KVIX, UKVIX and VIX are illustrated in Figures 1 to 7.

Table 1. Descriptive statistics

Panel A: Volatility indices							
	BVIX	EVIX	GVIX	JVIX	KVIX	UKVIX	VIX
Mean	22.186	26.209	23.143	29.158	26.941	23.4	23.674
SD	9.779	11.385	9.311	12.823	11.588	10.627	12.305
Skewness	1.491	1.717	1.868	2.062	2.238	1.871	1.681
Kurtosis	2.962	3.721	4.206	4.834	5.932	4.709	3.29
Minimum	8.56	12.708	11.91	13.64	14.15	10.74	9.89
Maximum	69.47	87.513	74	91.45	89.3	78.69	80.86
J-B	871.1***	1256.9***	1551.8***	1911.8***	2650.8***	1773.0***	1074.7***
Q(4)	4425.9***	4375.9***	4401.9***	4310.4***	4348***	4405.1***	4404.6***
Q^2(4)	4165.3***	4081.6***	4154.8***	4231.8***	4161.3***	4163.1***	4171.5***
Panel B: The VX log returns							
Mean	0.075	0.060	0.044	0.031	-0.009	0.061	0.072
SD	6.322	6.054	5.539	6.440	5.608	6.094	7.172
Skewness	0.251	0.939	0.709	0.937	1.156	0.528	0.633
Kurtosis	3.559	3.416	3.694	3.179	6.419	1.486	4.385
Minimum	-32.182	-24.913	-27.034	-22.432	-21.588	-21.946	-35.059
Maximum	32.238	32.768	29.983	33.627	41.572	25.542	49.601
J-B	633.919***	742.506***	764.681***	642.876***	2229.35***	162.131***	1007.632***
Q(4)	23.918***	12.614***	5.821***	22.833***	24.024***	6.0891***	33.246***
Q^2(4)	205.83***	197.71***	134.76***	98.11***	123.31***	90.435***	58.568***

Notes: Q and Q^2 are the Ljung-Box test for serial correlation in the standardized residuals and square standardized residuals. SD denotes standard deviations. *** Denote significance at the 1% level.

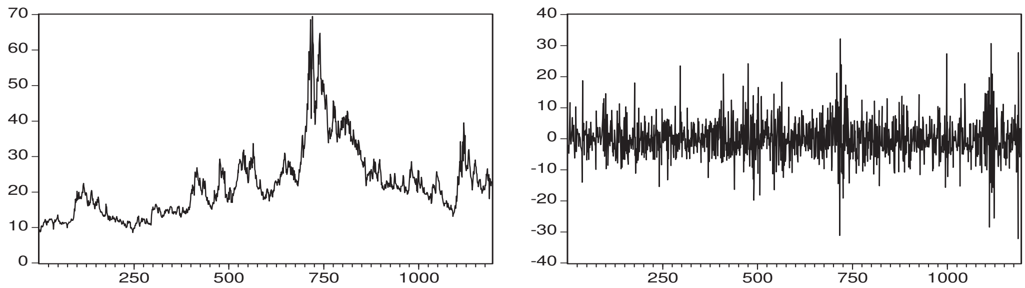

Fig. 1. Volatility index (right) and VX log returns (left) for Belgium

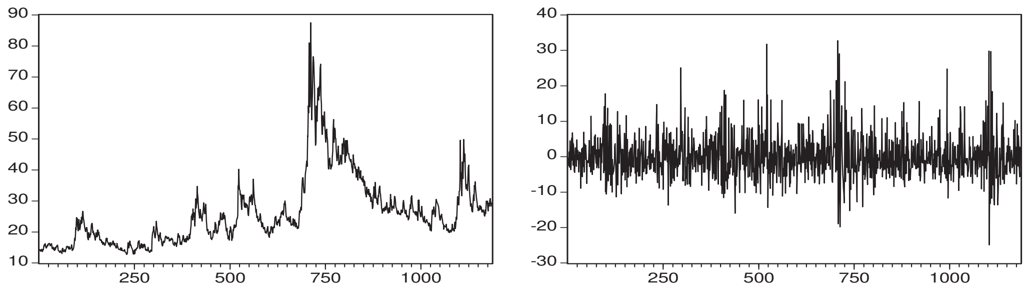

Fig. 2. Volatility index (right) and VX log returns (left) for Europe

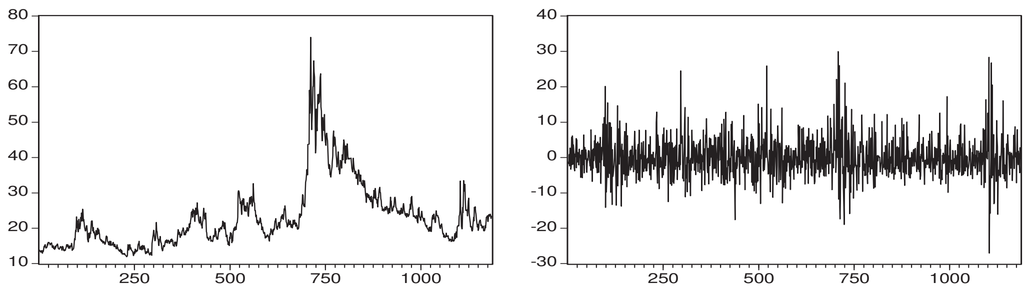

Fig. 3. Volatility index (right) and VX log returns (left) for Germany

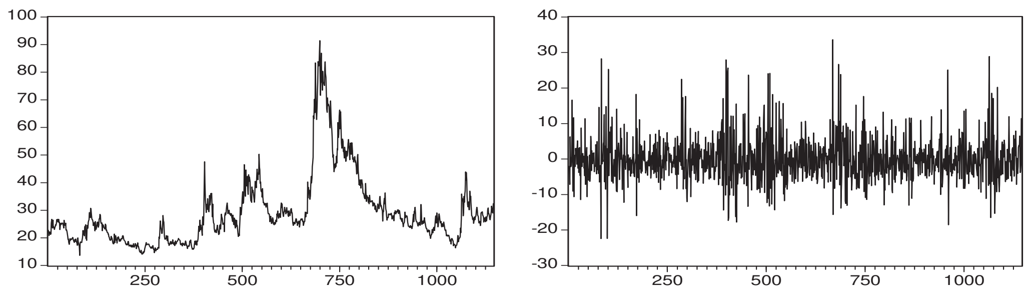

Fig. 4. Volatility index (right) and VX log returns (left) for Japan

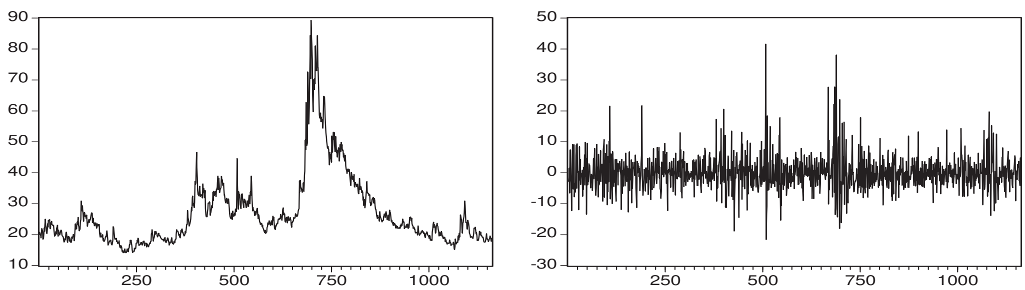

Fig. 5. Volatility index (right) and VX log returns (left) for Korea

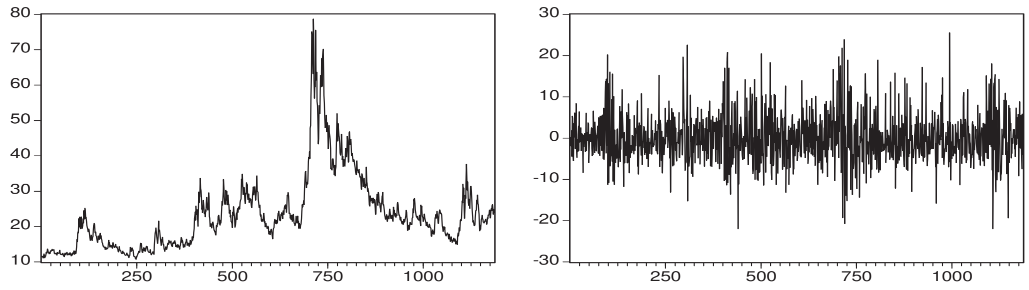

Fig. 6. Volatility index (right) and change in volatility index (left) for the UK

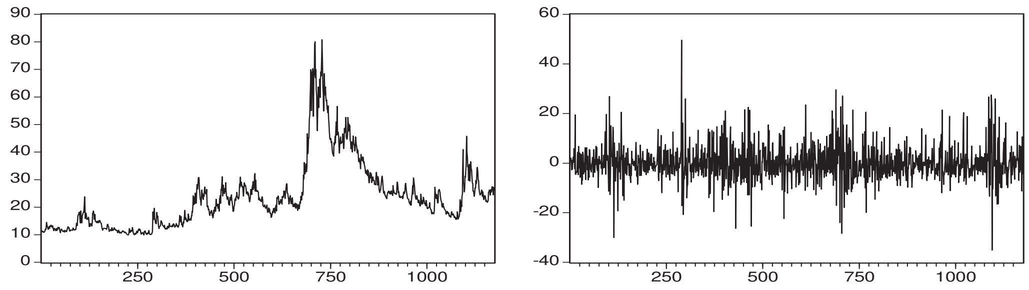

Fig. 7. Volatility index (right) and VX log returns (left) for the US

The results of the augmented Dickey-Fuller (ADF) unit root tests on the log volatility indices are presented in Table 2. Based on the use of the ADF tests, we provide the results for the non-stationarity of the levels of the log volatility indices, as well as the results for the stationarity of the first differences. The results presented in Table 2 indicate that all of the series are non-stationary in levels, but stationary in first-order differences, suggesting that all of the volatility indices are integrated, with an order of 1, $I(1)$. Therefore, this paper uses the VX log returns to analyze the day-of-the-week effect and the jump effect.

Table 2. Unit root test for the volatility indices

Items	Level			1st difference		
	C	C&T	Non	C	C&T	Non
BVIX	-2.662*	-2.799	-0.795	-23.984***	-23.981***	-23.992***
EVIX	-2.506	-2.771	-0.724	-18.404***	-18.399***	-18.409***
GVIX	-2.335	-2.406	-0.634	-19.904***	-19.901***	-19.901***
JVIX	-2.257	-2.337	-0.745	-23.504***	-23.494***	-23.514***
KVIX	-2.418	-2.388	-1.000	-7.599***	-7.606***	-7.602***
UKVIX	-3.251*	-3.453*	-1.172	-36.660***	-36.647***	-36.674***
VIX	-2.211	-2.351	-0.748	-21.326***	-21.321***	-21.332***

Notes: ** and *** denote significance at the 5% and 1% levels, respectively. C, C&T and Non indicate that the models have constant, constant and trend and non constant and no trend.

Table 3 contains mean statistics for all the VX log returns and the day-of-the-week effect test. First, the mean statistics of the VX log returns on Monday are consistently positive in these markets. Thus, this study uses the F1 tests to reject significantly the null hypothesis, indicating that the VX log returns for each day does not exactly equal 0. It is then found that the F2 tests do not significantly reject the null hypothesis, indicating that the VX log returns on Tuesday to Friday exactly equal each other[1]. These results are consistent with the increase in the VX log returns from Friday to Monday in different countries, implying that the investor will engage in various high volatility strategies when the VX log returns is high on Monday, meaning that the investor can obtain a higher option premium[2]. These results are consistent with the findings of Gonzalez-Perez and Guerrero (2013). In the present paper, it is found that Monday is more positive than other days; hence, the Monday effect is further incorporated into the ARJI model to analyze the jump effect on the VX log returns in different countries.

[1] The main difference is that the other types of securities are long-lived securities, whereas the volatility index levels are based on options that expire in 30 days. Thus, it is not surprising that Monday volatility levels (and returns) are higher than other days.

[2] Gonzalez-Perez and Guerrero (2013) indicated that an investor can design a profitable volatility-trading strategy based on Monday effect. For instance, an investor can sell a larger number of deep out-of-the-money calls and buy a just out-of-the-money call.

Table 3. Mean statistics and day-of-the-week effect test

Items	BVIX	EVIX	GVIX	JVIX	KVIX	UKVIX	VIX
Monday	1.245	1.732	1.390	1.676	2.259	1.993	0.956
Tuesday	-0.256	-0.216	-0.028	-0.109	-0.187	-0.107	-0.669
Wednesday	-0.375	-0.473	-0.587	-0.150	-0.415	-0.543	-0.476
Thursday	0.129	-0.219	0.135	-0.657	-0.707	-0.291	0.284
Friday	-0.354	-0.482	-0.664	-0.435	-0.973	-0.636	0.346
F1	2.757**	5.698***	5.154***	4.485***	12.782***	7.338***	1.954**
F2	0.343	0.151	1.286	0.370	0.867	0.381	1.229

Notes: ** and *** denote significance at the 5% and 1% level, respectively. The F1 (F2) test is the hypothesis for the mean equal to 0 from Monday to Friday (from Tuesday to Friday).

Panels A to C in Table 4 list the model diagnosis for the GARCH, CJ and ARJI models for the VX log returns. The GARCH, constant jump and ARJI model reveal that the Q, Q^2 and ARCH tests of residuals are all insignificant, implying that no serial correlation exists. The GARCH, CJ and ARJI models are adequate. Thus, this paper employs a likelihood ratio test to compare the ARJI model with the GARCH and CJ models in Panel D in Table 4[1]. The empirical results reveal that the LR1 and LR2 tests are significant at the 1% level, implying that the ARJI model provides a superior fit to the GARCH and CJ models. Consequently, the ARJI model is feasible for modeling financial market volatility indices.

Table 4. The model diagnosis and LR test

Panel A: The model diagnosis for the GARCH model							
Country	BVIX	EVIX	GVIX	JVIX	KVIX	UKVIX	VIX
Q (20)	14.427	20.116	18.82	14.021	17.912	24.616	21.006
Q^2 (20)	19.478	9.194	8.881	6.91	6.258	12.07	9.178
ARCH(10) Test	7.868	3.962	5.221	2.967	3.782	6.465	7.549
Panel B: The model diagnosis for the constant jump model							
Q (20)	16.351	23.035	23.032	13.418	22.842	26.451	22.140
Q^2 (20)	21.170	22.551	24.910	25.985	9.368	17.460	9.288
ARCH(10) Test	7.923	12.007	10.115	5.962	2.883	7.179	7.931
Panel C: The model diagnosis for the ARJI model							
Q (20)	14.831	19.580	22.146	9.892	12.919	23.619	16.214
Q^2 (20)	20.241	11.015	10.845	11.690	7.285	18.497	4.033
ARCH(10) Test	5.992	7.148	6.203	6.116	4.950	11.670	2.323
Panel D: The functional value and LR test							
FV for GARCH	-3767.951	-3686.116	-3586.014	-3620.599	-3512.688	-3715.001	-3858.174
FV for constant jump	-3732.070	-3623.908	-3535.674	-3553.803	-3425.599	-3667.169	-3754.756
FV for ARJI	-3724.667	-3600.041	-3522.368	-3531.900	-3407.858	-3630.804	-3732.709
LR1 test	86.567***	172.151***	127.293***	177.398***	209.660***	168.393***	250.930***
LR2 test	14.805***	47.735***	26.612***	43.806***	35.481***	72.730***	44.094***

Notes: ** and *** denote significance at the 5% and 1% levels, respectively. FV denotes functional value. The LR1 (LR2) test indicates that L_R and L_U are the GARCH model (CJ model) and ARJI model, respectively.

Table 5 presents the empirical results which indicate that the coefficient of the Monday effect is significantly positive with the VX log returns. The estimated short-run and long-run (α and β) persistence of the shocks are positive and significant, except the short-run persistence that is not significant for BVIX and JVIX. All the VX log returns reveal significant β sensitivity to its own past volatility. The volatility sensitivity places KVIX, VIX, GVIX, UKVIX and EVIX (0.899, 0.903, 0.942, 0.934 and 0.942) in the high volatility sensitivity group, while BVIX and JVIX (0.554 and 0.268) are placed in the relatively low volatility group. This study notes the strong GARCH effect and the persistence of the conditional variance, with the parameters $\alpha + \beta$ ranging from 0.907 to 0.958 and being quite high, which is an indication of a covariance stationary model with a high persistence and long memory in the conditional variance, although it is considerably lower for BVIX and JVIX.

[1] The test statistic $LR = -2(L_R - L_U) \sim \chi^2(v)$, where L_R (L_U) and v denote the functional value of the restricted (unrestricted) model and the number of restricted conditions, respectively.

Table 5. The estimated results of the ARJI model

	BVIX	EVIX	GVIX	JVIX	KVIX	UKVIX	VIX
Panel A: Parameter estimates							
μ	-0.262	-0.303**	-0.183 **	-0.357 **	-0.445 **	-0.202	-0.078
μ_1	2.055 ***	2.589 ***	1.706 ***	2.182 ***	2.850 ***	2.504 ***	1.821***
ϕ_1	-0.052*	-0.023	-0.042	-0.096 **	-0.037	-0.035	-0.109 ***
ϕ_2	-0.015	-0.066**	-0.062 **	-0.061 **	-0.014	-0.033	-0.100***
ω	7.308 **	0.169	0.267 **	9.285 ***	1.107 ***	0.576 **	0.564 ***
α	0.031	0.016 **	0.025 ***	0.044 **	0.008	0.011 **	0.022 ***
β	0.554 **	0.942 ***	0.924 ***	0.268 *	0.899 ***	0.934 ***	0.903 ***
θ	2.577 ***	5.744***	6.389***	5.065 ***	5.668 ***	6.595 ***	5.123 ***
δ^2	53.764 ***	6.855 **	0.223	36.952***	26.724 ***	6.74	24.323***
λ_0	0.017 **	0.047 **	0.025 ***	0.022 **	0.021 **	0.017 **	0.047 ***
ρ	0.554 ***	0.652 ***	0.740 ***	0.563 ***	0.454***	0.741 ***	0.516 ***
γ	0.397 ***	0.269***	0.203 ***	0.386***	0.493 ***	0.235 ***	0.423***
Panel B: The diffusion-jump variance							
Total variance	38.592	33.914	29.143	40.528	28.904	37.509	49.154
The variance in jumps %	43%	62%	53%	52%	43%	54%	62%
The variance in diffusion %	57%	38%	47%	48%	57%	46%	38%
Panel C: Unconditional variance							
Jump innovations	2.302	5.382	3.946	3.152	2.263	3.297	4.911
Diffusion	17.610	4.024	5.235	13.496	11.903	10.473	7.520

Note: ** and *** denote significance at the 5% and 1% level, respectively. Total Variance of VX log returns, the variance in jumps % and the variance in diffusion % are $V_t = h_t + \lambda_t(\theta^2+\delta^2)$, $\frac{\lambda_t(\theta^2+\delta^2)}{V_t}$ and $\frac{h_t}{V_t}$. Unconditional variance of jump innovations and unconditional variance of diffusion are $\frac{\omega}{(1-\alpha-\beta)}$ and $(\theta^2+\delta^2)\frac{\lambda_0}{(1-\rho)}$.

The jump-size means (θ) for all VX log returns are positive and range from 2.577 to 6.389, implying the jumps' positive impact on the conditional means of the returns. The jump-size variances (δ^2) for BVIX, EVIX, JVIX, KVIX and VIX are significant at the 1% level and larger than GVIX and UKVIX. As regards jump intensity, the parameters (λ_0, ρ and γ) for the VX log returns are all statistically significant, providing evidence of time-variation in the arrival of jump events. The jump frequency (λ_0) displays statistical significance at the 5% level, indicating significant jump behaviors on the VX log returns. The ρ parameters for the arrival of jump event range from 0.454 to 0.740, implying that a high probability of many (few) jumps today tends to be followed by a high probability of many (few) jumps tomorrow (Chan and Maheu, 2002). The γ parameters for the effect of the most recent intensity residual range from 0.185 to 0.892. The lagged intensity residual and jump clustering of the VX log returns are all statistically significant at the 1% level, indicating that the jump frequency within the sample period is not a constant and that the arrival process can systematically deviate from its unconditional mean as demonstrated by Bates (1996), Chan and Maheu (2002), Maheu and McCurdy (2004), Chiu, Lee and Chen (2005) and Lee and Lee (2009).

The unconditional jump intensity ranges from 0.038 to 0.135 (0.038 for BVIX and KVIX, 0.05 for JVIX and 0.066 for UKVIX, 0.096 for GVIX, 0.097 for VIX and 0.135 for EVIX), indicating that these findings imply about 10, 34, 24, 13, 10, 16 and 24 jumps on average per year in the long run in BVIX, EVIX, GVIX, JVIX, KVIX, UKVIX and VIX[1]. Moreover, the estimated results of the ARJI model report that the jumps have the phenomenon of jump clustering; therefore, VIX may have four jumps within a month but no jumps for the next month.

Panel B in Table 5 shows the results of the diffusion-jump variance based on the VX log returns[2]. The variance caused by the jump process of the VX log returns contributes 42.7%-61.8% of the total variance. We find that the importance of jump risks can be determined based on the ratio of the jump variance to the total variance; therefore, the jump variance plays a crucial role in the market volatility indices. These findings highlight that investors cannot be overlooked.

[1] The unconditional jump intensity is $\lambda_0/(1-\rho)$, according to Maheu and McCurdy (2004).

[2] Accordingly, this study adopts the model developed by Chan and Maheu (2002), in which the total variance is estimated via $V_t = h_t + \lambda_t(\theta^2+\delta^2)$, namely, the total variance of VX log returns is divided into the variance caused in the jump process ($\lambda_t(\theta^2+\delta^2)$) and that caused in the diffusion process (h_t), or the ratio of the jump variance to the total variance, that is $[\lambda_t(\theta^2+\delta^2)]/V_t$.

Panel C in Table 5 shows that unconditional variance of jump innovations and unconditional variance of diffusion correspond to a long-term average value of the conditional variance in jump and diffusion. Thus, the effects of jumps on the VX log returns are the average variances due to jumps which range from 2.263 to 5.382 (2.302 for KVIX, 2.302 for BVIX, 3.152 for JVIX, 3.297 for UKVIX, 3.946 for GVIX, 4.911 for VIX and 5.382 for EVIX). Then, the effects of diffusion on the VX log returns are the average variances due to diffusion which range from 4.024 to 17.610 (4.024 for EVIX, 5.235 for GVIX, 7.520 for VIX, 10.473 for UKVIX, 11.903 for KVIX, 13.496 for JVIX and 17.610 for BVIX). The jumps (diffusion) in the EVIX are more (less) frequent and have a larger (smaller) effect on the VX log returns than the other volatility indices, implying these results may be understood as speculators engaging in trade activities.

Conclusion

This study applies a powerful time-varying jump model based on the ARJI model of Chan and Maheu (2002) and Maheu and McCurdy (2004) to investigate the fundamental and jump characteristics and the day-of-the-week in international investment sentiment indices.

The empirical results demonstrate that the VX log returns on Mondays are larger than on other days, implying that the investor engages in various high volatility strategies. The ARJI model is feasible based on the modelling of financial market volatility indices and the results indicate that the VX log returns have a strong GARCH effect as well as time-variation in the arrival of jump events. Finally, this study finds that the jump variance plays a crucial role in the market volatility indices. The empirical results provide valuable information to understand the jump-diffusion process for the VX log returns so that the traders can adequately implement their investment strategies.

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