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The stock index futures hedge ratio with structural changes

Abstract

This paper estimates the optimal stock index futures hedge ratio for S&P 500, FTSE 100, and Nikkei 225 stock indexes using bivariate GARCH model with structural changes (bivariate ICSS-GARCH). The article uses ICSS (Iterated Cumulative Sums of Squares) algorithm proposed by Inclan and Tiao (1994) to identify time points of structural changes in the financial time series. The results show that except for FTSE 100, the bivariate ICSS-GARCH model does not outperform the OLS and OLS-CI models. However, the bivariate ICSS-GARCH model has better performance than the bivariate GARCH model for all three markets. The finding suggests the necessity to incorporate structural changes in GARCH models and shows the importance to consider structural changes when estimating the hedge ratios.

Keywords: hedge ratio, structural changes, ICSS algorithm.
JEL Classification: G11, C32.

Introduction

Stock is one of the major investment instruments for investors. Since system risk cannot be diversified away by portfolio formation, stock investors would bear the system risk. With the existence of stock index futures markets, investors can hedge their stock positions with stock index futures to prevent the value of their stock portfolios from being affected by the system risk. Thus, the next important issue is how to calculate the optimal hedge ratio.

One of the ways to hedge risk is using a naïve hedging strategy. Investors establish stock index futures positions equal in magnitude but opposite sign to the stock portfolio. The hedge ratio is equal to one. However, only in the absence of basis risk on the spot commitment day, the naïve hedge can fully reduce risk.

In order to get the optimal hedge ratio under the existence of basis risk, Ederington (1979) uses ordinary least squares (OLS) to estimate the hedge ratio and notices that the hedge ratio is less than one in most cases. Even though the OLS technique accounts for basis risk, it ignores the fact that spot and futures prices often have a unit root and are cointegrated (Wahab and Lashgari, 1993). Hence, integrating an error correction term into OLS can improve the hedging performance (Ghosh, 1993).

The hedge ratios estimated by OLS without and with an error correction term are both constant over time. However, the hedge ratio should be time-varying. Chang, Chou and Nelling (2000) point out that when stock market volatility increases, the demand for hedging will increase. Their result implies that the hedge ratio should vary with spot volatility. Therefore, many studies such as Park and Switzer (1995), and Yeh and Gannon (2000) estimate time-varying hedge ratios using bivariate ARCH and GARCH models.

The hedge ratio may also depend on the level of price volatility. When there are structural changes in volatility, which are caused by political, social or economic events and unknown in advance, the optimal hedge ratio can vary. Lamoureux and Lastrapes (1990) consider that because of a failure to take account of structural changes in the financial time series, an ARCH/GARCH model may overestimate the persistence in variance, which means that the impact of shocks on volatility does not die out quickly under the model. Wilson, Aggarwal, and Inclan (1996) also suggest that if hedgers account for structural changes in variance, the portfolio will be correctly hedged. Furthermore, ignoring structural changes may significantly overestimate the degree of volatility transmission (Ewing and Malik, 2005; Arago-Manzana and Fernandez-Izquierdo, 2007; Marcelo, Quiros, and Quiros, 2008). Therefore, the hedge ratio estimated by either a bivariate ARCH or GARCH model without considering structural changes may not be optimal.

Mansur, Cochran and Shaffer (2007) use a GARCH model with structural changes to compute the optimal hedge ratio of exchange rates and show that the ratio can improve hedging performance. This is because those structural changes have influence on the volatility structure of the joint spot and futures distribution. Stock markets often experience structural changes (Aggarwal, Inclan and Leal, 1999). However, few studies have explored that whether structural changes affect the hedge ratio of stock markets. Our paper fills the gap in the literature by estimating the optimal stock index futures hedge ratio based on a bivariate GARCH model with structural changes.

Our results show that except for FTSE 100, the bivariate ICSS-GARCH model does not outperform the OLS and OLS-CI models. However, hedged portfolios constructed by the bivariate ICSS-GARCH model have better performance than those by the bivariate GARCH model across all the three markets.

Our finding provides a justification for incorporating structural changes in GARCH models and shows that it is important to consider structural changes when estimating the hedge ratio. Our results are consistent with the Mansur, Cochran and Shaffer (2007), who show that ICSS-GARCH model outperforms the basic GARCH model for exchange rates markets.

The remainder of this paper is organized as follows. Section 1 describes the theory of futures hedging. Section 2 provides literature review of hedge ratio estimation, section 3 lays out the methodology to estimate hedge ratio. Section 4 describes the data. Section 5 provides the results of the hedge ratio estimation. The final section concludes.

1. Theory of futures hedging

The concept of futures hedging has been proposed in the early stages. Keynes (1930) asserts that hedgers, owning spot endowment and executing short hedge, are willing to pay risk premium to speculators to be exempted from price risk. The next question for hedgers is how to get the optimal hedge ratio. There have been many different theoretical approaches to deriving the optimal hedge ratio, depending on objective functions.

The objective function of the naïve hedging strategy is risk avoidance. Working (1953), however, argues that the objective function of hedgers is expected profit maximization. In particular, because of basis risk, short hedgers would hedge if the basis is expected to fall and would not hedge if the basis is expected to rise.

Johnson (1960) and Stein (1961) propose portfolio theory to incorporate risk avoidance of the naïve hedging strategy with Working's expected profit maximization. The objective function is to minimize the variance of a hedged portfolio. Although the minimum variance (MV) hedge ratio ignores the expected return of the hedged portfolio, it is often used in many studies because it is simple to understand and to estimate.

In this paper, we adopt the MV approach to deriving the optimal hedge ratio. Nevertheless, following Kroner and Sultan (1993), we start with the objective function within the mean-variance framework to show that the optimal mean-variance hedge ratio is the same as the MV hedge ratio, under the assumption that the futures price follows a martingale process[1].

Assume that an investor holds a portfolio, including one unit in the spot market and a short position of $-b$ units in the futures market. The payoff of this portfolio, r, is:

$$r = s - bf, \quad (1)$$

where s and f are the price changes of spot and futures, respectively.

Assume further that the expected utility function of the investor can be expressed in the mean-variance framework,

$$E[U(r)] = E(r) - \gamma Var(r), \quad (2)$$

where γ is the degree of risk aversion, $\gamma > 0$. The objective function of the investor is as follows:

$$\underset{b}{Max}\, E[U(r)] = \underset{b}{Max}\left\{E(s) - bE(f) - \gamma\left[\sigma_s^2 + b^2\sigma_f^2 - 2b\sigma_{sf}\right]\right\} \quad (3)$$

By solving the first-order condition, the optimal hedge ratio, b^*, is:

$$b^* = \frac{\sigma_{sf}}{\sigma_f^2} + \frac{-E(f)}{2\gamma\sigma_f^2}. \quad (4)$$

The first term of the right hand side is the optimal hedge ratio under the objective function of variance minimization, and the second term is the hedge ratio under the objective function of payoff maximization. Short hedgers would increase short futures position if futures prices are expected to fall and decrease otherwise. We assume that futures prices follow a martingale process, so equation (4) can be rewritten as:

$$b^* = \frac{\sigma_{sf}}{\sigma_f^2}. \quad (5)$$

b^* is a constant hedge ratio, and is often estimated by the ordinary least squares estimator from a time-series regression of changes in spot prices on changes in futures prices.

However, the joint distribution of spot and futures price changes is time varying, so the constant hedge ratio may not be suitable for practice use. Consider the dynamic process below.

$$r_t = s_t - b_{t-1} f_t, \quad (6)$$

where s_t and f_t are spot and futures price changes from time t-1 to t respectively. b_{t-1} is the short futures position at time t-1.

The expected utility function of the investor is as follows:

$$E_t[U(r_{t+1})] = E_t(r_{t+1}) - \gamma\sigma_t^2(r_{t+1}). \quad (7)$$

The subscript t under the expectation operator and the variance symbol are used to emphasize that they

[1] Pok, Poshakwale and Ford (2009) investigate hedging effectiveness of dynamic and constant models in the emerging market of Malaysia. Particularly, they use both minimum variance and expected utility maximum method to measure in-sample and out-of-sample hedging performance.

are calculated conditional on available information at time t. By solving the first-order condition, the optimal hedge ratio, b_t^*, is:

$$b_t^* = \frac{\sigma_{s_{t+1}f_{t+1}}}{\sigma_{f_{t+1}}^2} + \frac{-E_t(f_{t+1})}{2\gamma\sigma_{f_{t+1}}^2}. \quad (8)$$

We further assume that futures prices follow a martingale process, so equation (8) can be rewritten in the following manner:

$$b_t^* = \frac{\sigma_{s_{t+1}f_{t+1}}}{\sigma_{f_{t+1}}^2}. \quad (9)$$

The difference between b_t^* and b^* comes from the replacement of the constant unconditional variance by the time varying conditional variance. b_t^* varies with time to capture the price change caused by new information.

2. A review of hedge ratio estimation

2.1. Constant hedge ratio. It is convenient for hedgers to estimate the hedge ratio by ordinary least squares (OLS) technique. In addition, the OLS technique accounts for basis risk. Ederington (1979) uses OLS to estimate the hedge ratio of GNMA, T-Bill, wheat, and corn futures markets. They notice that hedge ratios are less than one in most cases, contrary to the naïve hedging strategy.

However, the OLS technique ignores the fact that spot and futures often share a unit root and are cointegrated. Using Standard and Poor 500 (S&P 500) index and the Financial Times 100 index, Wahab and Lashgari (1993) show that cash and futures markets are cointegrated and it is appropriate to represent each series as an error correction process. Therefore, integrating an error correction term into OLS (OLS-CI) can improve the hedging performance. For example, Ghosh (1993) documents that hedge ratios estimated by an error correction model have smaller forecast errors than those by an OLS model.

2.2. Time-varying hedge ratio. The hedge ratios estimated by OLS and OLS with an error correction term are both constant over time. However, the hedge ratio should be time-varying. Baillie and Myers (1991) argue that since optimal hedge ratios depend on the conditional distribution of price movements, they will almost certainly vary over time as this conditional distribution changes. In other words, they argue that the time-invariant optimal hedge ratio is inappropriate. In addition, Chang, Chou and Nelling (2000) point out that when stock market volatility increases, the demand for hedging will increase. Their results also imply that hedge ratios should vary with spot volatility.

Many studies have used ARCH/GARCH models to estimate time-varying hedge ratios for commodity, interest rate, and stock index futures. For example, Cecchetti, Cumby and Figlewski (1988) note that as expectations about risk and return changed, hedge ratios of Treasury bonds futures estimated by the univariate ARCH framework vary from 0.52 to over 0.91. Myers (1991) notes that bivariate GARCH models have theoretical advantages over OLS models. Using wheat futures contracts traded at the Chicago Board of Trade, he shows that the bivariate GARCH model provides superior hedging performance to the OLS model. Baillie and Myers (1991) use a bivariate GARCH model to estimate hedge ratios for six commodities futures contracts such as beef, coffee, corn, cotton, gold, and soybeans, and find that a constant hedge ratio is quite costly for some commodities. Choudhry (2004) compares the hedging effectiveness of an OLS model with that of a bivariate GARCH model. Using Australian, Hong Kong, and Japanese stock futures markets, his results show that the time-varying GARCH hedge ratios outperform the constant ratios in most of the cases. Pok, Poshakwale and Ford (2009) investigate hedging effectiveness of dynamic and constant models in the emerging market of Malaysia. The results show that out of sample hedging performance of dynamic GARCH models in the Malaysian emerging market is as good as the one reported for the highly developed markets in the previous literature.

Kroner and Sultan (1993) and Park and Switzer (1995) further apply a bivariate GARCH error correction model to estimate time-varying hedge ratios. The error correction term of the model is used to capture the long-run relationship between spot and futures prices. These two studies examine the issue using foreign currency futures and stock index futures respectively and both find that the bivariate GARCH error correction model improves the hedging performance over several other models such as the naïve, OLS, and OLS-CI models.

2.3. Time-varying hedge ratio and structural changes. Due to structural changes, which are caused by factors unknown a priori such as political, social and economic events, the hedge ratios estimated by a bivariate ARCH or GARCH model may not be optimal. Lamoureux and Lastrapes (1990) consider that failing to take account of structural changes in the financial time series may cause an ARCH/GARCH model to overestimate the persistence in variance. That is, the model assumes that impacts of shocks on volatility do not die out quickly. This model misspecification in persistence may has influence on the hedging performance.

However, previous few studies have explored whether a GARCH model with structural changes can improve hedging performance. The study of Mansur, Cochran and Shaffer (2007) is an exception. They argue that structural changes have influence on the volatility structure of the joint distribution of spot and futures prices. Therefore, they use a GARCH model considering structural changes to compute the hedge ratios in currency futures and show that the model can improve hedging performance. Stock markets often experience structural changes (Aggarwal, Inclan and Leal, 1999). However, few studies have explored that whether structural changes affect the hedge ratio of stock markets. Our paper fills the gap in the literature by estimating the stock index futures hedge ratios using a bivariate GARCH model with structural changes.

Recently, some studies have emphasized that GARCH model with structural changes can improve hedging performance. Lien and Yang (2010) suggest that daily currency risk can be better hedged with currency futures when controlling for unconditional variance breaks in the bivariate GARCH model. Arago and Salvador (2011) employ several multivariate GARCH models to estimate the optimal hedge ratios for the Spanish stock market. They show that more complex models including sudden changes in volatility outperform the simpler models in hedging effectiveness both with in-sample and out-of-sample analysis.

Many approaches have been proposed to find structural changes. Lamoureux and Lastrapes (1990) arbitrarily assume that structural changes in the unconditional variance occur at every 302 observations over a range of 4,228 observations. Similar to Lamoureux and Lastrapes (1990), Kearns and Pagan (1993) trim the data by separately omitting observations on returns whose absolute value exceeded x per cent, where x equals to 20, 15, 10, 7.5, 3, and 1. Diebold (1986) argue that persistent movements in variance may be due to a failure to include policy regime dummies for the conditional variance intercept. Lastrapes (1989) confirms the argument of Diebold (1986), improves the ARCH model fitness, and reduces volatility persistence by accounting for monetary policy regime shifts in the model. Ackert and Racine (1997) directly consider October 1989 crash event as a structural change. However, these approaches are either arbitrary or biased. Therefore, a statistical procedure that can effectively detect the actual structural changes would be required.

The ICSS (Iterated Cumulative Sums of Squares) algorithm proposed by Inclan and Tiao (1994) is often used to identify time points of structural changes in the financial time series. For example, the approach has been used in studies such as Wilson, Aggarwal and Inclan (1996), Aggarwal, Inclan and Leal (1999), Malik (2003), Malik and Hassan (2004), and Malik, Ewing and Payne (2005). They all conclude that when structural changes detected by the ICSS algorithm are incorporated into ARCH/GARCH models, the persistence of financial asset volatility overestimated by ARCH/GARCH models decreases dramatically.

3. Methodology

3.1. A bivariate GARCH model. Kroner and Sultan (1993) propose a bivariate GARCH (1,1) error correction model to estimate b_t^*:

$$s_t = \alpha_{0s} + \alpha_{1s}(S_{t-1} - \delta F_{t-1}) + \varepsilon_{st}, \tag{10}$$

$$f_t = \alpha_{0f} + \alpha_{1f}(S_{t-1} - \delta F_{t-1}) + \varepsilon_{ft}, \tag{11}$$

$$\begin{bmatrix} \varepsilon_{st} \\ \varepsilon_{ft} \end{bmatrix} \Bigg| \Psi_{t-1} \sim N(0, H_t), \tag{12}$$

$$\begin{gathered} H_t = \begin{bmatrix} h_{ss,t} & h_{sf,t} \\ h_{sf,t} & h_{ff,t} \end{bmatrix} = \begin{bmatrix} h_{s,t} & 0 \\ 0 & h_{f,t} \end{bmatrix} \times \\ \times \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \times \begin{bmatrix} h_{s,t} & 0 \\ 0 & h_{f,t} \end{bmatrix}, \end{gathered} \tag{13}$$

$$h_{s,t}^2 = v_{0s} + v_{1s}\varepsilon_{s,t-1}^2 + v_{2s}h_{s,t-1}^2, \tag{14}$$

$$h_{f,t}^2 = v_{0f} + v_{1f}\varepsilon_{f,t-1}^2 + v_{2f}h_{f,t-1}^2, \tag{15}$$

$$b_t^* = \frac{\hat{h}_{sf,t}}{\hat{h}_{ff,t}}, \tag{16}$$

where S_{t-1} and F_{t-1} are the natural logarithm of spot and futures prices at time t-1 respectively. Ψ_{t-1} is the information set at time t-1. The term $(S_{t-1} - \delta F_{t-1})$ is the error correction term, which captures the co-movement and long-run stable equilibrium between stock and futures prices. The error correction term should be added into the bivariate time series model if two variables are cointegrated (Engle and Granger, 1987). b_t^* is the time-varying hedge ratio.

3.2. The ICSS algorithm. The methodology we will use to detect structural changes in the variance of an observed time series is based on the ICSS (Iterated Cumulative Sums of Squares) algorithm proposed by Inclan and Tiao (1994). The analysis assumes that variance of a time series is stationary over an initial period until the occurrence of a structural change, caused by an exogenous shock. The variance is then stationary again until the next shock occurs. The process is repeated through time and yields an unknown number of structural changes in the variance.

Let $\{a_t\}$ be a series with zero mean and with unconditional variance σ_t^2, and t = 1, …, T. The variance within each interval is denoted by τ_j^2, j = 0, 1, …, N_T, and given as follows:

$$\begin{aligned} &\sigma_t^2 = \tau_0^2, \quad 1 < t < k_1 \\ &\sigma_t^2 = \tau_1^2, \quad k_1 < t < k_2 \\ &\quad \dots \\ &\sigma_t^2 = \tau_{N_T}^2, \; k_{N_T} < t < T, \end{aligned} \tag{17}$$

where k_1, k_2, …, k_{N_T} are the set of time points when structural changes occur.

Inclan and Tiao (1994) use a cumulative sum of squares approach to estimate the number of changes in variance and time points of structural changes. Let

$$C_k = \sum_{t=1}^{k} a_t^2, \; k = 1,...,T, \tag{18}$$

be the (mean-centered) cumulative sum of the squares from the first observation of the series to the kth point in time. Define the statistic D_k as follows:

$$D_k = \left(\frac{C_k}{C_T}\right) - \frac{k}{T}, \; k = 1,...,T, \tag{19}$$

with $D_0 = D_T = 0$.

If a time series has no structural changes in variance, the D_k statistics will oscillate around zero. On the contrary, if the series contains one or more structural changes, the D_k statistics will drift either upward or downward away from zero. Critical values used to detect a significant structural change in variance are obtained from the distribution of D_k statistics under the null hypothesis of homogeneous variance. The null hypothesis is rejected if the maximum absolute value of D_k is greater than the critical value. Define k^* to be the value of k at which $\max_k|D_k|$ is attained. k^* is taken as an estimate of the structure-change point if $\max_k \sqrt{T/2}\,|D_k|$ exceeds a predetermined boundary. The term $\sqrt{T/2}$ is required for standardizing the distribution.

Under the null hypothesis of homogeneous variance, $\sqrt{T/2}\,D_k$ behaves like a Brownian bridge asymptotically. The critical value of 95th percentile of the asymptotic distribution of $\max_k \sqrt{T/2}\,|D_k|$ is 1.358. Following Aggarwal, Inclan and Leal (1999), we also set the critical value to be 1.358. However, if the analyzed series contains multiple structure-change points, D_k statistics is insufficient to find these due to masking effects. To solve the problem, Inclan and Tiao (1994) propose an iterative scheme that uses D_k statistics to systematically identify any possible structure-change points at different pieces of the series.

3.3. A bivariate GARCH model with structural changes. In the univariate ARCH/GARCH model, when structural changes detected by the ICSS algorithm are incorporated directly into conditional variance in form of dummy variables, the overestimated persistence of variance decreases dramatically (Wilson, Aggarwal and Inclan, 1996; Aggarwal, Inclan and Leal, 1999; Malik, 2003; Malik and Hassan, 2004; and Malik, Ewing and Payne, 2005). Structural changes also affect the volatility structure of the joint spot and futures distribution. Therefore, following Mansur, Cochran and Shaffer (2007), we use a bivariate GARCH model with structural changes to estimate the hedge ratios for stock index futures. The model is as follows:

$$s_t = \alpha_{0s} + \alpha_{1s}(S_{t-1} - \delta F_{t-1}) + \varepsilon_{st}, \tag{20}$$

$$f_t = \alpha_{0f} + \alpha_{1f}(S_{t-1} - \delta F_{t-1}) + \varepsilon_{ft}, \tag{21}$$

$$\left.\begin{bmatrix} \varepsilon_{st} \\ \varepsilon_{ft} \end{bmatrix}\right| \Psi_{t-1} \sim N(0, H_t), \tag{22}$$

$$\begin{aligned} H_t = \begin{bmatrix} h_{ss,t} & h_{sf,t} \\ h_{sf,t} & h_{ff,t} \end{bmatrix} = \begin{bmatrix} h_{s,t} & 0 \\ 0 & h_{f,t} \end{bmatrix} \times \\ \times \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \times \begin{bmatrix} h_{s,t} & 0 \\ 0 & h_{f,t} \end{bmatrix}, \end{aligned} \tag{23}$$

$$h_{s,t}^2 = v_{0s} + v_{1s}\varepsilon_{s,t-1}^2 + v_{2s}h_{s,t-1}^2 + \sum_{i=2}^{n} d_{s,i}D_{s,i}, \tag{24}$$

$$h_{f,t}^2 = v_{0f} + v_{1f}\varepsilon_{f,t-1}^2 + v_{2f}h_{f,t-1}^2 + \sum_{j=2}^{m} d_{f,j}D_{f,j}, \tag{25}$$

$$b_t^* = \frac{\hat{h}_{sf,t}}{\hat{h}_{ff,t}}, \tag{26}$$

where i and j are structural changes detected by the ICSS algorithm, and n and m are the number of structural changes for spot and futures returns respectively. $D_{s,i}$ ($D_{f,j}$) are dummy variables. If the ICSS algorithm detects a shift in the volatility of spot or futures returns, $D_{s,i}$ ($D_{f,j}$) take a value of one from each point of structural change onwards, zero elsewhere. As a result of intercept specification in the conditional variance, we add only m-1 and n-1 structural changes in the conditional variance to avoid the problem of multicollinearity. b_t^* is the time-varying hedge ratio.

4. Data description

Data for the study are obtained from Datastream database. The daily prices used in this study are S&P 500 index and futures, FTSE 100 index and futures, and Nikkei 225 index and futures. The three futures contracts are traded in Chicago Mercantile Exchange (CME), London International Financial Futures Exchange (LIFFE), and Osaka Securities Exchange (OSE) respectively. At any time, S&P 500 index futures has eight contracts outstanding with delivery in the March quarterly cycle. For FTSE 100 index futures, the three nearest quarterly months (March, June, September and December) will be listed. The contract months of Nikkei 225 Futures are 5 near contracts in the March quarterly cycle. The prices of contracts with the nearest expiration date are used for the study.

The daily prices are transformed into weekly rates of returns based on Wednesday prices. When there is no trading on a given Wednesday, the last trading day before Wednesday is used to compute returns. The sample period is from January 1, 1989 to December 31, 2006, including 939 weekly observations[1]. Returns are defined as changes in the logarithmic prices, $R_t = \ln(p_t) - \ln(p_{t-1})$. The reason for using weekly rather than daily data is twofold. The first one is that weekly returns contain less noise than daily measures. The other one is weekly hedging adjustments would incur lower hedging cost than the daily adjustment strategy.

Summary statistics of price and return series for the three financial markets are presented in Table 1 (see Appendix). Panel B reports the result of the return series. The sample mean for the S&P 500 and FTSE 100 stock index spot and futures returns are significantly different from zero, but those for Nikkei 225 are close to zero. The sample kurtosis and Jarque-Bera statistics show that the return series are not normally distributed.

There is evidence of serial correlation in S&P 500 and FTSE 100 stock index and futures return series. Thus, a conditional-mean equation to take account of the serial correlation in the returns is required[2]. In addition, the $Q_2(8)$ and $Q_2(16)$ statistics indicate significant serial correlation in the squared returns across all the markets, which suggests the need to model the conditional heteroscedasticity.

[1] I have no access rights to Datastream due to contract expiration, so the sample period stops in December 2006.

[2] We identify the best-fitting specification of conditional-mean equation by Box-Jenkins techniques for S&P 500 and FTSE 100 stock index and futures. The partial autocorrelation function suggests that the ARMA (‖1, 7‖, 0) model would be appropriate for S&P 500 spot and futures return series, and ARMA(‖1‖, 0) model would be appropriate for FTSE 100 spot and futures return series. The statistics are shown in Table 5, Table 6, and Table 7.

5. Empirical results

5.1. Unit root test. To test for the stationarity of prices and returns, the augmented Dickey-Fuller test (ADF) is used. The null hypothesis is that a time series has a unit root. We consider three different specifications as follows.

$$\Delta Y_t = \gamma Y_{t-1} + \sum_{i=2}^{p} \beta_i \Delta Y_{t-i+1} + \varepsilon_t, \tag{27}$$

$$\Delta Y_t = \alpha_0 + \gamma Y_{t-1} + \sum_{i=2}^{p} \beta_i \Delta Y_{t-i+1} + \varepsilon_t, \tag{28}$$

$$\Delta Y_t = \alpha_0 + \gamma Y_{t-1} + \alpha_2 t + \sum_{i=2}^{p} \beta_i \Delta Y_{t-i+1} + \varepsilon_t, \tag{29}$$

where Y can denote either price or return for spot and futures. The appropriate number of lagged differences, p, is determined by BIC criterion. Specifically, we choose a number as p from zero to 20 to minimize the BIC criterion.

The difference between the three equations is the presence of drift term or time trend. Equation (27) is like a pure random walk model, equation (28) adds a drift term (α_0), and equation (29) extends both a drift and a time trend (t). In all cases, if the parameter γ is significantly different from zero, series Y does not contain a unit root, indicating that series Y is stationary.

Table 2 (see Appendix) presents the results of unit root tests conducted on prices and returns of spot and futures. The results confirm the presence of a unit root in the logarithmic price indices, but there is no evidence of a unit root in their first differences, i.e. returns. Unit root tests indicate that spot and futures series for each index are both nonstationary in prices, and returns are stationary. Because that spot and futures series are integrated of order one, $I(1)$, tests for cointegration can be undertaken. Therefore, the next step is to test whether spot and futures prices are cointegrated.

5.2. Cointegration test. Two time series are said to be cointegrated if they share a common trend. That is, there is a long-term equilibrium relationship between the two variables. Engle and Granger (1987) initiate the cointegration test technique. They propose the Engle-Granger two-step procedure to test whether two nonstationary time series are cointegrated. The main advantage of their method is its simplicity.

Based on the Engle-Granger two-step procedure, the first step is regressing spot prices on futures prices. The equation is as follows:

$$S_t = \eta + \delta F_t + e_t, \tag{30}$$

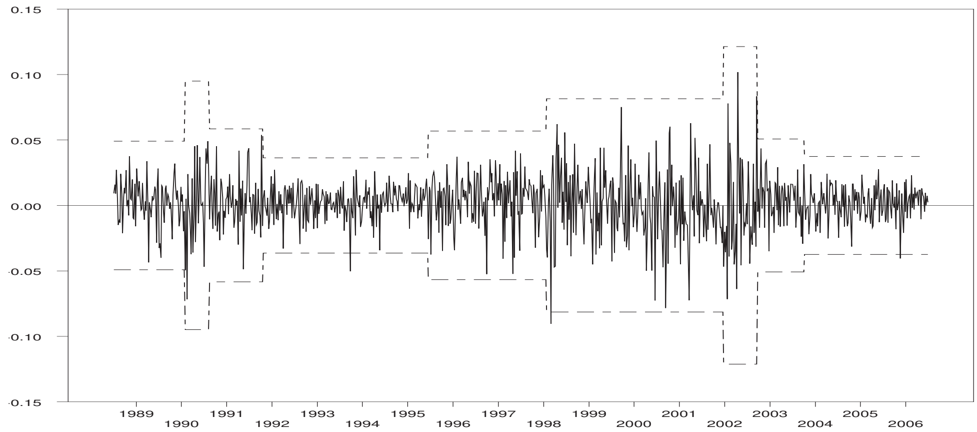
where S_t and F_t are spot and futures prices in the logarithmic form respectively. Residual, e_t, can be regarded as temporary deviations from the long-run equilibrium. The second step is testing residuals, e_t, for unit roots. The null hypothesis is that spot and futures are non-cointegrated. If γ (as shown in equations (27), (28), and (29)) is significantly different from zero, e_t residual is stationary, indicating that spot and futures prices are cointegrated.

Table 3 (see Appendix) presents the Engle-Granger cointegration results for ADF unit root tests on residuals from the cointegrating regression. The cointegrating parameters, δ, are highly significant and are approximately unity. γ are all significantly different from zero, providing evidence of cointegration in spot and futures markets. The results are also consistent with Wahab and Lashgari (1993), who show that the cash and futures markets are cointegrated. Therefore, the error correction term ($S_{t-1} - \delta F_{t-1}$), which captures the co-movement and long-run stable equilibrium between stock and futures prices, can be included in the model to estimate the optimal hedge ratio.

5.3. Structural changes detected by the ICSS algorithm. Table 4 (see Appendix) reports the number and dates of structural changes in variance identified by the ICSS algorithm for weekly stock index spot and futures returns. For each period, Table 4 also provides the level of annualized standard deviation for weekly returns and numbers of observations per period.

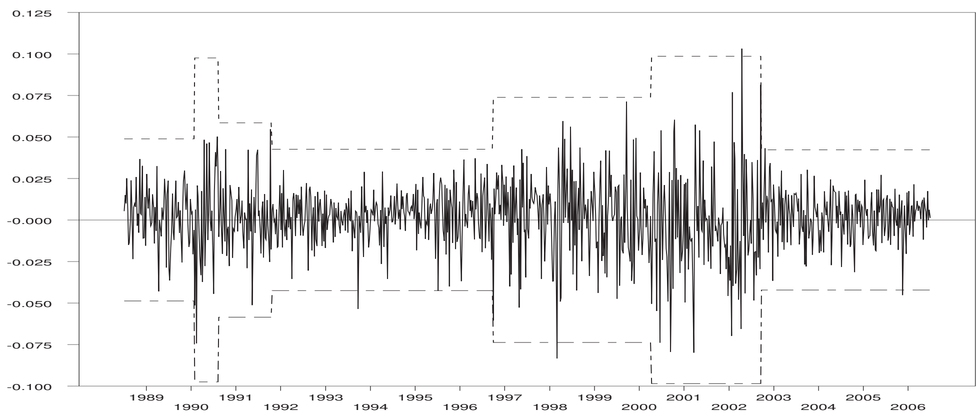
The numbers of structural changes are not the same in spot and futures for each market, with the exception of Japan markets. The S&P 500 and FTSE 100 stock index have eight break points in variances, and their futures have six break points; Nikkei 225 stock index spot and futures both have nine break points. Except the seventh structural change in Japan, change points of futures markets either lead or synchronize those of spot markets.

The regimes of different variance structures are shown in Figures 1 and 2 for S&P 500, Figures 3 and 4 for FTSE 100, and Figures 5 and 6 for Nikkei 225. Boundaries are set to be ± 3 standard deviations, where the standard deviation is the unconditional volatility calculated within each regime period.



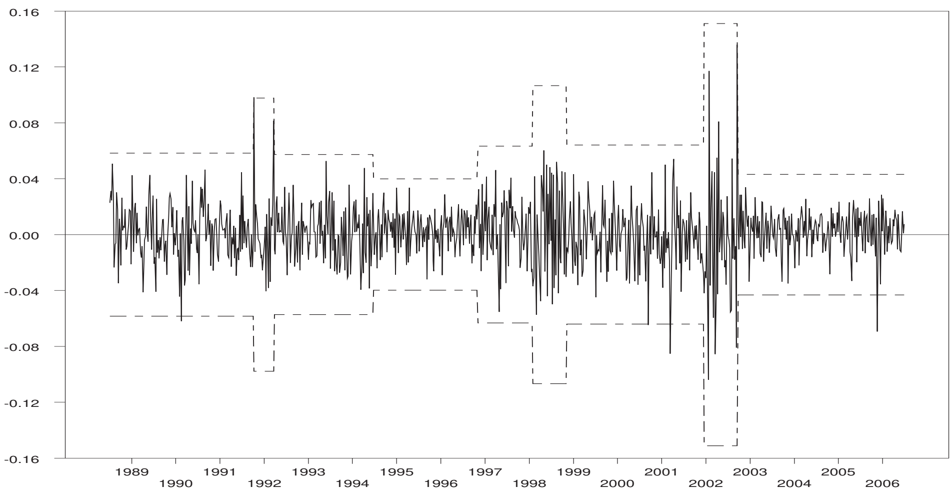
Notes: Doted lines specify boundaries of ± 3 standard deviations. Structural changes are detected using the ICSS algorithm. Sample period is from January 1, 1989 to December 31, 2006.

Fig. 1. S&P 500 stock index weekly returns



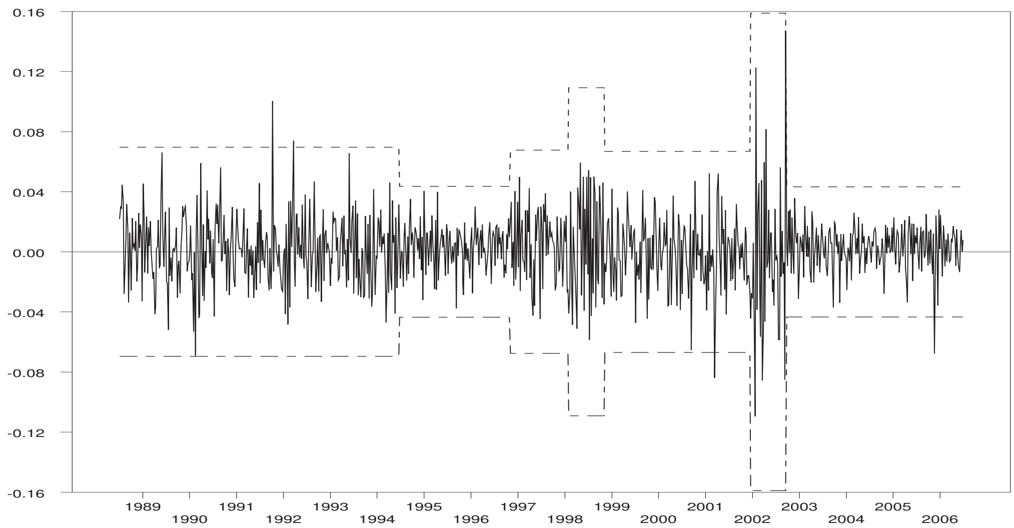
Notes: Doted lines specify boundaries of ± 3 standard deviations. Structural changes are detected using the ICSS algorithm. Sample period is from January 1, 1989 to December 31, 2006.

Fig. 2. S&P 500 stock index futures weekly returns



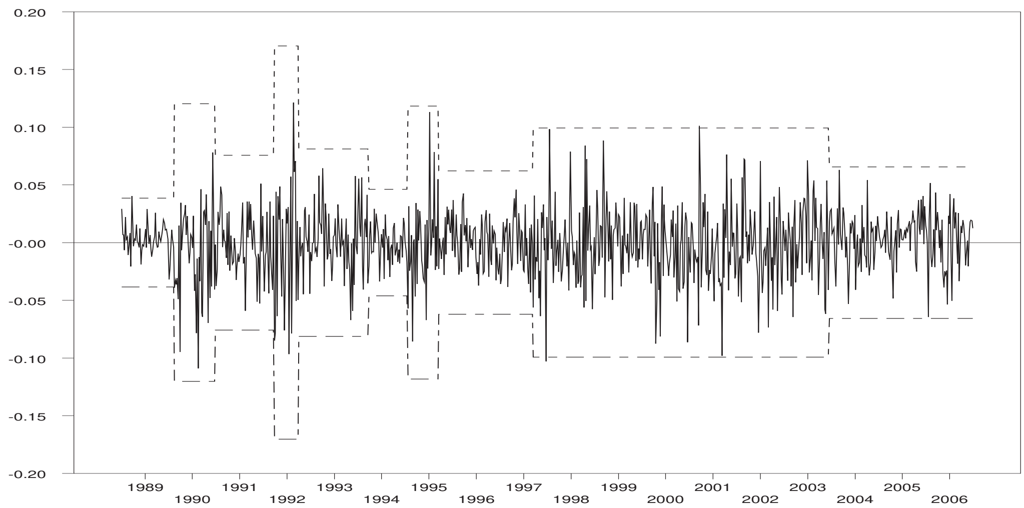
Notes: Doted lines specify boundaries of ± 3 standard deviations. Structural changes are detected using the ICSS algorithm. Sample period is from January 1, 1989 to December 31, 2006.

Fig. 3. FTSE 100 stock index weekly returns



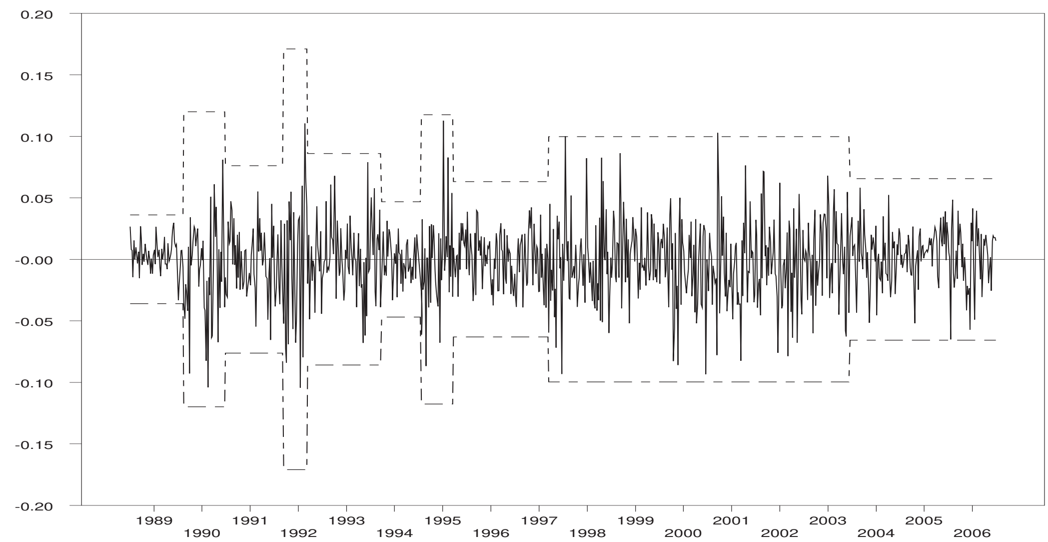
Notes: Doted lines specify boundaries of ± 3 standard deviations. Structural changes are detected using the ICSS algorithm. Sample period is from January 1, 1989 to December 31, 2006.

Fig. 4. FTSE 100 stock index futures weekly returns



Notes: Doted lines specify boundaries of ± 3 standard deviations. Structural changes are detected using the ICSS algorithm. Sample period is from January 1, 1989 to December 31, 2006.

Fig. 5. Nikkei 225 stock index weekly returns



Notes: Doted lines specify boundaries of ± 3 standard deviations. Structural changes are detected using the ICSS algorithm. Sample period is from January 1, 1989 to December 31, 2006.

Fig. 6. Nikkei 225 stock index futures weekly returns

5.4. Constant hedge ratio – OLS and OLS-CI. Table 5 (see Appendix) reports the results from constant hedge ratio models, OLS and OLS-CI. The OLS and OLS-CI models are obtained by imposing $v_{1s} = v_{2s} = v_{1f} = v_{2f} = \alpha_{1s} = \alpha_{1f} = 0$ and $v_{1s} = v_{2s} = v_{1f} = v_{2f} = 0$ in equations (10) to (16). The hedge ratios estimated by these two models are constant, because conditional variance is assumed to be time-invariant.

As expected, the coefficient of the error correction term is significant for each time series. In addition, the likelihood ratio (LR) test statistics under the null hypothesis H_0: $\alpha_{1s} = \alpha_{1f} = 0$, presented at the bottom of Table 5 are all significant at the 1% level. These results show that the OLS-CI model fits the data better than the OLS model. Consistent with the empirical results of Kroner and Sultan (1993), and Mansur, Cochran and Shaffer (2007), hedge ratios estimated by the OLS model are smaller than those estimated by the OLS-CI model across the financial markets except for S&P 500.

5.5. Time-varying hedge ratio – bivariate GARCH. Table 6 (see Appendix) reports the results from the bivariate GARCH model. The bivariate GARCH model is obtained by imposing the restriction $d_{s,i} = d_{f,j} = 0$ in equations (20) to (26). The GARCH coefficients (v_{1s}, v_{2s}, v_{1f}, and v_{2f}) are all significant at the 1% level across all the stock indices, implying that it is appropriate to include the GARCH specification in the hedge ratio estimation model.

The likelihood ratio test statistics, LR_1, under the null hypothesis H_{01}: $v_{1s} = v_{2s} = v_{1f} = v_{2f} = \alpha_{1s} = \alpha_{1f} = 0$, is significant at the 1% level. These results show that the bivariate GARCH model fits the data better than the OLS model. In addition, the likelihood ratio test statistics, LR_2, under the null hypothesis H_{02}: $v_{1s} = v_{2s} = v_{1f} = v_{2f} = 0$, is also significant at the 1% level. This shows that the bivariate GARCH model fits the data better than the OLS-CI model.

5.6. Time-varying hedge ratio – ICSS-GARCH. Table 7 (see Appendix) reports the results from the bivariate ICSS-GARCH model. Many coefficients of structural change dummy variables are at least significant at the 10% level for each stock index. After including structural changes in the model, some GARCH coefficients (v_{1s}, v_{2s}, v_{1f}, and v_{2f}) are insignificant for each markets. It implies that it is also appropriate to include the GARCH specification in the hedge ratio estimation model. However, the GARCH effect is only partially captured by the structural changes detected by the ICSS algorithm. Therefore, a more complete analysis should allow for both the GARCH effect and the structural change effect. In addition, the likelihood ratio test statistics LR_1, LR_2, and LR_3, which compare the OLS, OLS-CI, and bivariate GARCH models with the bivariate ICSS-GARCH model, show that the bivariate ICSS-GARCH model fits the data well for each stock index.

5.7. Hedging performance comparison. Considering that an investor holds a portfolio, including one unit in the spot market and a short position of $-b_t$ units in the futures market, the following basic profit model is used.

$$\pi_t = s_t - b_t f_t, \tag{31}$$

where π is the rate of return. The objective function of minimum variance (MV) hedge ratio is to minimize the variance of the hedged portfolio, in other words, to minimize the variance of π.

In order to compare the hedging performance of the four estimation models, we use equation (31) to construct a portfolio and calculate the variance of π.

Table 8 (see Appendix) presents the results. Surprisingly, the bivariate ICSS-GARCH model does not outperform the OLS and OLS-CI models, except for FTSE 100. The hedge ratio estimated by the bivariate ICSS-GARCH model yields the lowest variance for FTSE 100 markets. However, for S&P 500 and Nikkei 225 markets, their variance of 0.1171 and 0.2976 are higher than those of the OLS and OLS-CI models. The best model for S&P 500 market is either OLS or OLS-CI. The best model for Nikkei 225 market is OLS.

In particular, the bivariate ICSS-GARCH model has better performance than the bivariate GARCH model for all the three markets. It may imply that structural changes identified by the ICSS algorithm indeed have influence on the volatility structure of the joint spot and futures distribution. In order to increase the hedging performance, it is necessary to include the structural changes in the bivariate GARCH model. Our results are consistent with the Mansur, Cochran and Shaffer (2007), who show that ICSS-GARCH outperforms the basic GARCH model for exchange rates markets.

Conclusion

Calculating hedge ratios for stock index futures is an important issue to prevent the value of a stock portfolio from being affected by systematic risk. Many studies use bivariate ARCH and GARCH models to estimated time-varying hedge ratios. However, Lamoureux and Lastrapes (1990) suggest that failing to take account of structural changes in the financial time series can cause ARCH/GARCH models to overestimate the persistence in variance. A market with structural changes in variance experiences important information shocks, and these shocks can be incorporated into the volatility generating process of the other market. Therefore, ignoring structural changes may significantly overestimate the degree of volatility transmission (Ewing and Malik, 2005; Arago-Manzana and Fernandez-Izquierdo, 2007; Marcelo, Quiros and Quiros, 2008). Furthermore, these structure changes can have influence on the volatility structure of the joint spot and futures distribution (Mansur, Cochran and Shaffer, 2007).

In this study, we use the bivariate GARCH model with structural changes (bivariate ICSS-GARCH) to estimate the stock index futures hedge ratios for S&P 500, FTSE 100, and Nikkei 225 stock indexes. We use the ICSS (Iterated Cumulative Sums of Squares) algorithm proposed by Inclan and Tiao (1994) to identify time points of structural changes in the financial time series. Our results show that except for FTSE 100, the bivariate ICSS-GARCH model does not outperform the OLS and OLS-CI models. However, the bivariate ICSS-GARCH model has better performance than the bivariate GARCH model for all the three markets. Our findings suggest the necessity to incorporate structural changes in GARCH models and show the importance to consider structural changes when estimating the hedge ratios. Our results are consistent with the Mansur, Cochran and Shaffer (2007), who show that the ICSS-GARCH model outperforms the basic GARCH model for exchange rates markets.

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Appendix

Table 1. Descriptive statistics

Panel A shows the natural logarithm of stock index spot and futures prices. Panel B shows returns of spot and futures. Returns are defined as the change in logarithmic prices, i.e. $r_t = \ln(p_t) - \ln(p_{t-1})$. J-B is the Jarque-Bera test for normality. $\rho(j)$ denote the jth order autocorrelation of index and returns. $Q_1(j)$ represent the Ljung and Box (1978) test statistic for no serial correlation in the series up to lag j. $Q_2(j)$ represent the Ljung and Box (1978) test statistics for no serial correlation in the squares of the series up to lag j.

		Mean	Std dev	Skewness	Kurtosis	J-B	$\rho(8)$	$\rho(16)$	$Q_1(8)$	$Q_1(16)$	$Q_2(8)$	$Q_2(16)$
Panel A: The natural logarithm of stock index spot and futures prices												
S&P 500	Spot	6.6083***	0.5178	-0.3168***	-1.4646***	99.63***	0.9943	0.9881	7514.88***	15007.54***	7512.25***	14999.03***
	Futures	6.6126***	0.5176	-0.3104***	-1.4687***	99.48***	0.9942	0.9880	7514.07***	15005.01***	7511.42***	14996.33***
FTSE 100	Spot	8.2799***	0.3589	-0.2912***	-1.2075***	70.32***	0.9869	0.9751	7451.75***	14781.16***	7451.43***	14780.47***
	Futures	8.2857***	0.3555	-0.2686***	-1.2216***	69.68***	0.9860	0.9743	7442.64***	14759.55***	7442.89***	14760.17***
Nikkei 225	Spot	9.7316***	0.3493	0.1066	-0.2466	4.16	0.9705	0.9415	7317.06***	14260.59***	7317.73***	14261.83***
	Futures	9.7342***	0.3530	0.1228	-0.2576	4.96*	0.9707	0.9421	7319.20***	14268.09***	7319.77***	14268.80***
Panel B: Spot and futures stock index returns												
S&P 500	Spot	0.0017**	0.0207	-0.1704**	2.1242***	180.90***	-0.0413	-0.0511	22.39***	34.53***	153.25***	278.81***
	Futures	0.0017**	0.0211	-0.2086***	1.9678***	158.15***	-0.0491	-0.0411	24.18***	35.92***	159.89***	290.11***
FTSE 100	Spot	0.0013*	0.0219	0.1461*	3.7584***	555.40***	-0.0294	0.0326	22.87***	27.84**	185.69***	211.57***
	Futures	0.0013*	0.0234	0.1823**	3.3954***	455.77***	-0.0415	0.0297	23.82***	29.40**	179.28***	200.21***
Nikkei 225	Spot	-0.0006	0.0293	-0.1102	1.3212***	70.12***	0.0283	-0.0411	5.47	11.19	89.29***	107.54***
	Futures	-0.0006	0.0295	-0.1057	1.1914***	57.22***	0.0251	-0.0338	1.66	8.46	90.83***	111.50***

Notes: ***, **, and * indicate significance at the 1%, 5%, and 10% level, respectively.

Table 2. ADF unit root test

Panel A shows stock index spot and futures prices in the natural logarithmic form. Panel B shows returns of spot and futures. Returns are defined as the change in the logarithmic prices, i.e. $r_t = \ln(p_t) - \ln(p_{t-1})$. ADF models are as follows:

Model 1: $\Delta Y_t = \gamma Y_{t-1} + \sum_{i=2}^{p} \beta_i \Delta Y_{t-i+1} + \varepsilon_t.$

Model 2: $\Delta Y_t = \alpha_0 + \gamma Y_{t-1} + \sum_{i=2}^{p} \beta_i \Delta Y_{t-i+1} + \varepsilon_t.$

Model 3: $\Delta Y_t = \alpha_0 + \gamma Y_{t-1} + \alpha_2 t + \sum_{i=2}^{p} \beta_i \Delta Y_{t-i+1} + \varepsilon_t.$

where Y denotes the series of interest. The appropriate number of lagged differences, p, is determined by the BIC criterion. Specifically, we choose a number as p from zero to 20 to minimize the BIC.

		Model 1		Model 2		Model 3	
		p	γ	p	γ	p	γ
Panel A: Spot and futures stock index							
S&P 500	Spot	1	2.6842	1	-1.4138	1	-1.3116
	Futures	1	2.6470	1	-1.3900	1	-1.3205
FTSE 100	Spot	1	1.9457	1	-1.7252	1	-1.7558
	Futures	1	1.8458	1	-1.7221	1	-1.8080
Nikkei 225	Spot	0	-0.6917	0	-1.8330	0	-1.5173
	Futures	0	-0.7049	0	-1.8426	0	-1.5112
Panel B: Spot and futures stock index returns							
S&P 500	Spot	0	-33.3957*	0	-33.6351*	0	-33.6464*
	Futures	0	-33.5844*	0	-33.8166*	0	-33.8258*
FTSE 100	Spot	0	-33.7228*	0	-33.8389*	0	-33.8467*
	Futures	0	-34.5868*	0	-34.6897*	0	-34.6944*
Nikkei 225	Spot	0	-30.6591*	0	-30.6569*	0	-30.6903*
	Futures	0	-30.2271*	0	-30.2252*	0	-30.2586*

Note: * indicate significance at the 1% level.

Table 3. Engle and Granger's two-step co-integration test

The first step is regressing spot prices on futures prices. The equation is as follow. $S_t = \eta + \delta F_t + e_t$, where S_t and F_t are spot and futures prices in the logarithmic form. The second step is testing residuals, e_t, for unit roots. The ADF models are the same as those shown in Table 2.

	η	δ	Model 1		Model 2		Model 3	
			p	γ	p	γ	p	γ
S&P 500	-0.0066*** (-3.56)	1.0003*** (3599.47)	11	-31.41***	11	-31.41***	11	-31.47***
FTSE 100	-0.0835*** (-17.05)	1.0094*** (1709.07)	13	-10.47***	13	-10.47***	13	-10.48***
Nikkei 225	0.0998*** (18.82)	0.9895*** (1816.31)	11	-17.75***	11	-17.74***	11	-17.73***

Notes: ***, **, and * indicate significance at the 1%, 5%, and 10% level, respectively.

Table 4. Structural changes in volatility

Panel A: S&P 500				
	No. of structural changes	Time period	Std	Observations per period
Spot	8	Jan. 11, 1989-Aug. 1, 1990	0.0164	82
		Aug. 8, 1990-Feb. 13, 1991	0.0317	28
		Feb. 20, 1991-Apr. 22, 1992	0.0195	62
		Apr. 29, 1992-Dec. 13, 1995	0.0121	190
		Dec. 20, 1995-Jul. 22, 1998	0.0189	136
		Jul. 29, 1998-Jun. 19, 2002	0.0271	204
		Jun. 26, 2002-Mar. 19, 2003	0.0404	39
		Mar. 26, 2003-Mar. 31, 2004	0.0169	54
		Apr. 7, 2004-Dec. 27, 2006	0.0125	143

Table 4 (cont.). Structural changes in volatility

Panel A: S&P 500				
	No. of structural changes	Time period	Std	Observations per period
Futures	6	Jan. 11, 1989-Aug. 1, 1990	0.0163	82
		Aug. 8, 1990-Feb. 13, 1991	0.0325	28
		Feb. 20, 1991-Apr. 22, 1992	0.0195	62
		Apr. 29, 1992-Mar. 26, 1997	0.0142	257
		Apr. 2, 1997-Oct. 4, 2000	0.0246	184
		Oct. 11, 2000-Mar. 19, 2003	0.0329	128
		Mar. 26, 2003-Dec. 27, 2006	0.0141	197
Panel B: FTSE 100				
	No. of structural changes	Time period	Std	Observations per period
Spot	8	Jan. 11, 1989-Apr. 8, 1992	0.0194	170
		Apr. 15, 1992-Sep. 23, 1992	0.0326	24
		Sep. 30, 1992-Dec. 21, 1994	0.0191	117
		Dec. 28, 1994-Apr. 30, 1997	0.0133	123
		May 7, 1997-Jul. 29, 1998	0.0211	65
		Aug. 5, 1998-May 5, 1999	0.0356	40
		May 12, 1999-Jun. 12, 2002	0.0214	162
		Jun. 19, 2002-Mar. 19, 2003	0.0504	40
		Mar. 26, 2003-Dec. 27, 2006	0.0144	197
Futures	6	Jan. 11, 1989-Dec. 21, 1994	0.0232	311
		Dec. 28, 1994-Apr. 30, 1997	0.0145	123
		May 7, 1997-Jul. 29, 1998	0.0225	65
		Aug. 5, 1998-May 5, 1999	0.0364	40
		May 12, 1999-Jun. 12, 2002	0.0223	162
		Jun. 19, 2002-Mar. 19, 2003	0.0530	40
		Mar. 26, 2003-Dec. 27, 2006	0.0144	197
Panel C: Nikkei 225				
	No. of structural changes	Time period	Std	Observations per period
Spot	9	Jan. 11, 1989-Feb. 14, 1990	0.0128	58
		Feb. 21, 1990-Dec. 26, 1990	0.0401	45
		Jan. 2, 1991-Mar. 25, 1992	0.0252	65
		Apr. 1, 1992-Sep. 30, 1992	0.0568	27
		Oct. 7, 1992-Mar. 23, 1994	0.0271	77
		Mar. 30, 1994-Jan. 18, 1995	0.0154	43
		Jan. 25, 1995-Sep. 13, 1995	0.0394	34
		Sep. 20, 1995-Sep. 10, 1997	0.0207	104
		Sep. 17, 1997-Dec. 10, 2003	0.0331	326
		Dec. 17, 2003-Dec. 27, 2006	0.0219	159
Futures	9	Jan. 11, 1989-Feb. 14, 1990	0.0120	58
		Feb. 21, 1990-Dec. 26, 1990	0.0400	45
		Jan. 2, 1991-Mar. 11, 1992	0.0254	63
		Mar. 18, 1992-Sep. 9, 1992	0.0570	26
		Sep. 16, 1992-Mar. 23, 1994	0.0287	80
		Mar. 30, 1994-Jan. 18, 1995	0.0156	43
		Jan. 25, 1995-Sep. 20, 1995	0.0392	35
		Sep. 27, 1995-Sep. 10, 1997	0.0210	103
		Sep. 17, 1997-Dec. 10, 2003	0.0332	326
		Dec. 17, 2003-Dec. 27, 2006	0.0219	159

Table 5. Constant hedge ratios – OLS and OLS-CI models

The OLS-CI model is as follows:

$$s_t = \alpha_{0s} + \alpha_{1s}(S_{t-1} - \delta F_{t-1}) + \varepsilon_{st},$$

$$f_t = \alpha_{0f} + \alpha_{1f}(S_{t-1} - \delta F_{t-1}) + \varepsilon_{ft},$$

$$H_t = \begin{bmatrix} h_{ss,t} & h_{sf,t} \\ h_{sf,t} & h_{ff,t} \end{bmatrix},$$

$$b^* = \frac{\hat{h}_{sf,t}}{\hat{h}_{ff,t}}.$$

The OLS model is obtained by imposing the restriction $\alpha_{1s} = \alpha_{1f} = 0$. L represents the log-likelihood values. LR is the likelihood ratio test for model superiority between the OLS and OLS-CI models and is χ^2 distributed with 2 degrees of freedom. The numbers in the parentheses are t-statistics computed using White (1980) heteroscedastic consistent standard errors under the null hypothesis that the coefficient is zero.

	S&P 500		FTSE 100		Nikkei 225	
	OLS	CI	OLS	CI	OLS	CI
$\alpha_{0s} \times (10^3)$	2.2043*** (3.34)	1.9755*** (19.10)	1.4921** (2.27)	1.3979*** (11.14)	-0.6062*** (-3.41)	-0.6308*** (-3.85)
s_{t-1}	-0.2333*** (-7.76)	-0.0955*** (-20.16)	-0.1665*** (-6.35)	-0.0880*** (-15.17)	-	-
s_{t-7}	-0.0589** (-2.29)	-0.0625*** (-13.68)	-	-	-	-
α_{1s}	-	-0.1458*** (-7.72)	-	0.2205*** (8.38)	-	-0.2044*** (-11.59)
$\alpha_{0f} \times (10^3)$	2.2147*** (3.33)	1.9767*** (18.75)	1.4971** (2.12)	1.3908*** (10.40)	-0.6178*** (-3.46)	-0.6470*** (-3.92)
f_{t-1}	-0.2388*** (-8.14)	-0.0959*** (-19.84)	-0.1810*** (-6.97)	-0.0920*** (-15.98)	-	-
f_{t-7}	-0.0576** (-2.25)	-0.0606*** (-13.30)	-	-	-	-
α_{1f}	-	0.1185*** (6.11)	-	0.3804*** (13.67)	-	0.1829*** (10.45)
$h_{ss} \times (10^3)$	0.4300	0.4210	0.4741	0.4710	0.8600	0.8596
$h_{sf} \times (10^3)$	0.4330	0.4240	0.4973	0.4937	0.8495	0.8514
$h_{ff} \times (10^3)$	0.4465	0.4373	0.5384	0.5343	0.8688	0.8687
ρ_{sf}	0.9881*** (751.57)	0.9881*** (3759.83)	0.9843*** (685.19)	0.9842*** (2927.61)	0.9827*** (2371.35)	0.9852*** (2634.81)
b^*	0.9698	0.9696	0.9237	0.9240	0.9778	0.9801
L	8012.80	8030.28	7791.27	7796.03	7243.35	7318.63
LR	34.96***		9.52***		150.56***	

Notes: ***, **, and * indicate significance at the 1%, 5%, and 10% level respectively.

Table 6. Hedge ratio – bivariate GARCH model

The bivariate GARCH model is as follows:

$$s_t = \alpha_{0s} + \alpha_{1s}(S_{t-1} - \delta F_{t-1}) + \varepsilon_{st},$$

$$f_t = \alpha_{0f} + \alpha_{1f}(S_{t-1} - \delta F_{t-1}) + \varepsilon_{ft},$$

$$H_t = \begin{bmatrix} h_{ss,t} & h_{sf,t} \\ h_{sf,t} & h_{ff,t} \end{bmatrix} = \begin{bmatrix} h_{s,t} & 0 \\ 0 & h_{f,t} \end{bmatrix} \times \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \times \begin{bmatrix} h_{s,t} & 0 \\ 0 & h_{f,t} \end{bmatrix},$$

$$h_{s,t}^2 = v_{0s} + v_{1s}\varepsilon_{s,t-1}^2 + v_{2s}h_{s,t-1}^2,$$

$$h_{f,t}^2 = v_{0f} + v_{1f}\varepsilon_{f,t-1}^2 + v_{2f}h_{f,t-1}^2,$$

$$b_t^* = \frac{\hat{h}_{sf,t}}{\hat{h}_{ff,t}}.$$

L represents the log-likelihood values. LR_1 and LR_2 denote the likelihood ratio test for the bivariate GARCH model fitting the data better than the OLS model and OLS-CI model respectively, and are χ^2 distributed with 6 and 4 degrees of freedom respectively. $Q_1(j)$ and $Q_2(j)$ are the Ljung-Box (1978) test statistics for no serial correlation in the standardized and squared standardized residuals up to lag j. $Q_1(j)$ and $Q_2(j)$ are asymptotically χ^2 distributed. The critical values at the 5% level are 15.51 for j = 8 and 26.30 for j = 16. The numbers in the parentheses are t-statistics computed using White (1980) heteroscedastic consistent standard errors under the null hypothesis that the coefficient is zero.

	S&P 500	FTSE 100	Nikkei 225
$\alpha_{0s} \times (10^3)$	2.2705*** (4.21)	2.2544*** (3.40)	1.0919 (1.32)
s_{t-1}	-0.1428*** (-6.97)	-0.0641* (-1.72)	-
s_{t-7}	-0.0367* (-1.71)	-	-
α_{1s}	-0.1200 (-0.55)	0.5861** (2.27)	-0.1267 (-0.65)
$\alpha_{0f} \times (10^3)$	2.2714*** (4.13)	2.3277*** (3.38)	1.1152 (1.35)
f_{t-1}	-0.1419*** (-6.77)	-0.0705* (-1.78)	
f_{t-7}	-0.0415* (-1.90)		
α_{1f}	0.0970 (0.68)	0.8296*** (2.77)	0.2432 (1.25)
$v_{0s} \times (10^3)$	0.0118** (2.33)	0.0086** (2.18)	0.0456*** (3.42)
v_{1s}	0.0708*** (3.87)	0.0637*** (4.29)	0.0695*** (4.19)
v_{2s}	0.8972*** (33.84)	0.9159*** (57.53)	0.8738*** (32.78)
$v_{0f} \times (10^3)$	0.0127** (2.51)	0.0074** (2.05)	0.0480*** (3.49)
v_{1f}	0.0664*** (4.36)	0.0613*** (4.32)	0.0710*** (4.34)
v_{2f}	0.9002*** (39.09)	0.9233*** (65.97)	0.8702*** (32.81)
ρ_{sf}	0.9868*** (835.44)	0.9831*** (565.36)	0.9852*** (791.72)
Average b_t^*	0.9665	0.9226	0.9796
Max b_t^*	1.0677	1.0126	1.0673
Min b_t^*	0.8998	0.7712	0.8857
L	8125.90	7897.06	7391.35
LR_1	226.20***	211.58***	296.00***
LR_2	191.24***	202.06***	145.44***
$Q_1(8)$ [spot], [futures]	[5.58], [7.08]	[5.20], [5.71]	[5.97], [4.42]
$Q_1(16)$ [spot], [futures]	[18.23], [18.37]	[7.71], [8.90]	[13.93], [11.75]
$Q_2(8)$ [spot], [futures]	[11.25], [9.50]	[7.18], [7.34]	[18.65**], [23.55***]
$Q_2(16)$ [spot], [futures]	[14.94], [13.23]	[13.54], [13.13]	[25.72*], [28.81**]

Notes: ***, **, and * indicate significance at the 1%, 5%, and 10% level, respectively.

Table 7. Hedge ratios – the bivariate ICSS-GARCH model

The bivariate ICSS-GARCH model is as follows:

$$s_t = \alpha_{0s} + \alpha_{1s}(S_{t-1} - \delta F_{t-1}) + \varepsilon_{st},$$

$$f_t = \alpha_{0f} + \alpha_{1f}(S_{t-1} - \delta F_{t-1}) + \varepsilon_{ft},$$

$$H_t = \begin{bmatrix} h_{ss,t} & h_{sf,t} \\ h_{sf,t} & h_{ff,t} \end{bmatrix} = \begin{bmatrix} h_{s,t} & 0 \\ 0 & h_{f,t} \end{bmatrix} \times \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \times \begin{bmatrix} h_{s,t} & 0 \\ 0 & h_{f,t} \end{bmatrix},$$

$$h_{s,t}^2 = v_{0s} + v_{1s}\varepsilon_{s,t-1}^2 + v_{2s}h_{s,t-1}^2 + \sum_{i=1}^{n} d_{s,i}D_{s,i},$$

$$h_{f,t}^2 = v_{0f} + v_{1f}\varepsilon_{f,t-1}^2 + v_{2f}h_{f,t-1}^2 + \sum_{i=1}^{m} d_{f,i}D_{f,i},$$

$$b_t^* = \frac{\hat{h}_{sf,t}}{\hat{h}_{ff,t}}.$$

L represents the log-likelihood value. LR_1 and LR_2 denote the likelihood ratio test for model superiority between the bivariate ICSS-GARCH model and the OLS and OLS-CI models respectively, and are χ^2 distributed with 6 and 4 degrees of freedom respectively. LR_3 test statistics test for model superiority between the bivariate ICSS-GARCH model and the bivariate GARCH model, and are χ^2 distributed with 6, 4, and 12 degrees of freedom for S&P 500, FTSE 100, and Nikkei 225 respectively. $Q_1(j)$ and $Q_2(j)$ are the Ljung-Box (1978) test statistics for no serial correlation in the standardized and squared standardized residuals up to lag j. $Q_1(j)$ and $Q_2(j)$ are asymptotically χ^2 distributed. The critical values at the 5% level are 15.51 for j = 8 and 26.30 for j = 16. The numbers in the parentheses are t-statistics computed using White (1980) heteroscedastic consistent standard errors under the null hypothesis that the coefficient is zero.

	S&P 500	FTSE 100	Nikkei 225
$\alpha_{0s} \times (10^3)$	2.3066*** (3.72)	1.9156*** (3.28)	0.3611 (0.53)
s_{t-1}	-0.1332*** (-3.41)	-0.0421 (-1.29)	-
s_{t-7}	-0.0436* (-1.68)	-	-
α_{1s}	-0.1116 (-0.46)	0.2523* (1.71)	-0.1684 (-0.90)
$\alpha_{0f} \times (10^3)$	2.3071*** (3.60)	1.9127*** (3.20)	0.3326 (0.48)
f_{t-1}	-0.1317*** (-3.28)	-0.0420 (-1.21)	-
f_{t-7}	-0.0452* (-1.73)	-	-
α_{1f}	0.1121 (0.45)	0.4691*** (2.70)	0.2426 (1.26)
$v_{0s} \times (10^3)$	0.0729*** (10.77)	0.2848*** (9.29)	0.5099*** (8.87)
v_{1s}	0.0519*** (2.89)	0.0260 (1.06)	0.0339 (1.60)
v_{2s}	0.8514*** (23.62)	0.4036*** (11.78)	0.6593*** (13.60)
$v_{0f} \times (10^3)$	0.0699*** (8.70)	0.3828*** (8.97)	0.4296*** (7.65)
v_{1f}	0.0505*** (2.77)	0.0377 (1.30)	0.0373* (1.75)
v_{2f}	0.8608*** (27.36)	0.3429*** (7.30)	0.7084*** (16.42)
$d_{s,2} \times (10^3)$	-0.0460*** (-2.94)	-0.0150 (-1.12)	0.0340 (0.25)
$d_{s,3} \times (10^3)$	0.0039 (0.31)	-0.0015 (-0.12)	0.5630* (1.81)
$d_{s,4} \times (10^3)$	0.0073* (1.89)	0.1007 (1.25)	-0.9021*** (-3.13)
$d_{s,5} \times (10^3)$	0.0099 (1.48)	0.0661 (0.86)	-0.1072** (-2.23)

Table 7 (cont.). Hedge ratios – the bivariate ICSS-GARCH model

	S&P 500	FTSE 100	Nikkei 225
$d_{s,6} \times (10^3)$	0.0050 (0.87)	-0.2060*** (-4.02)	0.1930** (2.09)
$d_{s,7} \times (10^3)$	-0.0357** (-2.15)	0.4780*** (3.22)	-0.1837** (-2.10)
$d_{s,8} \times (10^3)$	-0.0014 (-0.68)	-0.6188*** (-4.06)	0.1849*** (5.46)
$d_{s,9} \times (10^3)$	-	-	-0.1521*** (-4.51)
$d_{f,2} \times (10^3)$	-0.0460*** (-2.86)	0.0663 (0.63)	0.0360 (0.27)
$d_{f,3} \times (10^3)$	0.0080 (0.62)	0.0301 (0.34)	0.4471** (2.19)
$d_{f,4} \times (10^3)$	0.0103* (1.86)	-0.2124*** (-4.05)	-0.7244*** (-4.49)
$d_{f,5} \times (10^3)$	0.0043 (0.83)	0.5503*** (3.04)	-0.1072** (-1.99)
$d_{f,6} \times (10^3)$	-0.0307** (-2.41)	-0.7208*** (-3.84)	0.1631* (1.75)
$d_{f,7} \times (10^3)$	-	-	-0.1532* (-1.76)
$d_{f,8} \times (10^3)$	-	-	0.1508*** (4.80)
$d_{f,9} \times (10^3)$	-	-	-0.1262*** (-4.15)
ρ_{sf}	0.9877*** (814.98)	0.9849*** (765.54)	0.9882*** (1354.89)
Average b_t^*	0.9642	0.9210	0.9820
Max b_t^*	1.0389	1.0083	1.3670
Min b_t^*	0.8978	0.8189	0.8118
L	8164.02	7962.10	7483.30
LR_1	302.44***	341.66***	479.90***
LR_2	267.48***	332.14***	329.34***
LR_3	76.24***	130.08***	183.90***
$Q_1(8)$ [spot], [futures]	[4.27], [6.10]	[4.75], [5.22]	[4.70], [3.22]
$Q_1(16)$ [spot], [futures]	[16.75], [16.29]	[8.09], [8.37]	[17.24], [14.99]
$Q_2(8)$ [spot], [futures]	[14.38*], [11.05]	[23.77***], [27.33***]	[21.55***], [24.16***]
$Q_2(16)$ [spot], [futures]	[22.71], [17.78]	[30.27**], [35.33***]	[28.37**], [31.25**]

Notes: ***, **, and * indicate significance at the 1%, 5%, and 10% level respectively.

Table 8. Hedging performance comparison of different models

Hedging performance is measured by the variance of π, where $\pi_t = s_t - b_t f_t$. The hedge ratio, b_t, is estimated by OLS, OLS-CI, bivariate GARCH, and bivariate ICSS-GARCH models. The numbers in the table are multiplied by 10^4.

	S&P 500	FTSE 100	Nikkei 225
OLS	0.1141	0.1565	0.2956
OLS-CI	0.1141	0.1565	0.2957
GARCH	0.1172	0.1480	0.3020
ICSS-GARCH	0.1171	0.1460	0.2976