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On optimal integration of financial markets

Abstract

This paper focuses on the interactions in a credit network and insists on the differences between the attitude that the agents take while passing from a micro to a macroeconomic context.

The following research has been mainly inspired by the literature of Gallegati (2009) and Stiglitz (2010). The authors stresses the transformation of the interaction process between agents as single units and market agents representing nations, whether they make part of a financial union or not, and in the event of external financial crisis taking place and affecting the network participants.

As a result of the analysis on such a context, the author has tried to detect an "optimal degree of financial opening" that will help to guarantee the improvement of social welfare even in cases of financial crisis.

Keywords: convesse technologies and expected utility, credit networks, diversification, risk transferability, individual robustness, avalanches of bankruptcies, autarchy and liberalization, theory of game with focal point, optimal partial integration.
JEL Classification: G18, G02, G15, F36, D89, E60.

Introduction

Current economics, strongly characterized by indissoluble connecting integrations between the local and the global sphere, is weak in developing models conveying an often extremely simplified image of reality.

This research tries to understand whether from the view of financial interactions it will be possible to realize convenient mutual trades among the competitors in a market, for instance between fund givers and receivers, both on a micro and macro level.

As economic science usually states and demonstrates, we will not be able to refer to a "general theory" to explain the *paradigm shift* between micro and macroeconomic aspects. Thus, we can't but refer to experiential facts and model future behaviors on their results, referring to the latest and the most specific related branches of research, from which, thanks to the contributions of several authors, we have been inspired in this research. For instance, we have explained the frailty of a financial system as deriving in part, from the lack of control of the latest financial tools, referring to Bester's model (1985), passing to the conditions a credit cycle present, explained by Fratianni (2008).

Nevertheless it is mainly from consulting the latest interpretations of Gallegati et al. (2008), Battiston, Delligati et al. (2009) and particularly of Stiglitz (2010) that this work has been produced. This literature has been analyzed in details and, in the meanwhile, re-edited in a personal, original and coherent contribution to the literature from which it takes inspiration. Dealing with researches on decisions taken by Economic Politics inside the Economics of Financial Trades, it has been relevant integrating with different other topics of financial and economic theories in uncertain conditions. That's how notions of theories on "Portfolio management" (Markowitz, 1952), on agents' behaviors in case of information asymmetry (Spence, 1973; Rothschild-Stiglitz, 1986), on the decisions taken by the agents according to others behaviors, (Nash, 1951; Schelling, 1960), and on the course of actions of the agents to evaluate an uncertain event (von Neumann-Morgenstern, 1947), have been crucial to give a multidimensional focus to the research.

A new kind of "systemic holistic balance" will be defined inside the field of globalized finance, emerging from the interaction of new financial tools combined from modern economies and deregulation policies implemented all over the world. This process will take place unless some element of "instability" will emerge and trigger recursive effects as resounding as those of the expansion cycle.

Then, we will look upon the interconnections realized at a superior level, taking into account the contemporary high standard of interconnections together with the elements we have previously considered. Agents (nowadays nations) will benefit from joining together into systemic entities coherent with the "exchange of goods" and with the "mobility factors". Moreover, in cases in which it is possible or convenient to realize a "system of fixed exchange rates", among the joining countries the theory of "Optimum Currency Area" (Mundell, 1961) will take place. In this case both the decrease of asymmetric shocks and the direct decrease of divergences into the same system, would allow an improvement to the joining countries with respect to the moment they entered the system.

What is undeniable is, no matter which level of financial integration it is, we are able to define the direction some certain economic trends may take in

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the event of distorted behaviors due to a natural interaction of free markets. Such a sovereignty will soften the differences within the global and the local markets and will allow the economical agents to act according to concerted, functional, effective and target-oriented schedules or plans.

1. Incompatibility between non-convex technologies and concave utility functions in imperfect markets

In an article published for the series of AER (2010) working papers, Stiglitz explains: "*The intuition behind why integration should be desirable was based on "convexity": with convex technologies and concave utility functions, risk sharing was always beneficial. The more globally the world economy is integrated, the better risks are "dispersed." But if technologies are not convex, then risk sharing can lower expected utility*" (Stiglitz, 2010, p. 5).

Here follows a detailed analysis of the meaning of such intuition.

Stiglitz refers to *concave utility functions* to represent "adverse risk investors" who "do not invest" if "actuarially fair bets" are proposed, that it to say those with null expected value.

This is the necessary condition, although not sufficient, to define the existence of a Markovian market, where the decision-maker chooses "options for economic efficiency" in investments. According to Markowitz, the investor "maximizes the expected utility of yields" and "is adverse to risks" following that he will choose according to a multi-criteria perspective: with fixed expected yield maximizes the risk, $\left\{\overline{E(r)} \rightarrow \min \sigma_P\right\}$; with fixed risk maximizes the expected yield $\left\{\overline{\sigma_P} \rightarrow maxE(r)\right\}$.

"A sufficient condition" is that "the investor acts in a perfect market", for financial integration to guarantee a fairer distribution of risks.

This is generally valid for theoretical models that agree on the *hypothesis of convexity of technologies* (when referring to the technical rate of substitution of productive factors, long as much as an isoquant of production) for concave functions of production (meaning the Cobb-Douglas functions), in relation with decreasing results of factors.

Nevertheless, the non-convexity is spread in the world, meaning with this concept the hypothesis of convex functions of production, which isoquants result, as a consequence, to be concave.

In this case, the economic theory shows that such a *problem in a constrained minimization of production costs* is solved by matching the technical rate of substitution and the ratio of production input prices. What is peculiar is that the point of minimal cost is defined in the intersection between the isoquant curve and the isocost line on the y axis (*corner solution*). The representation is in Figure 1.

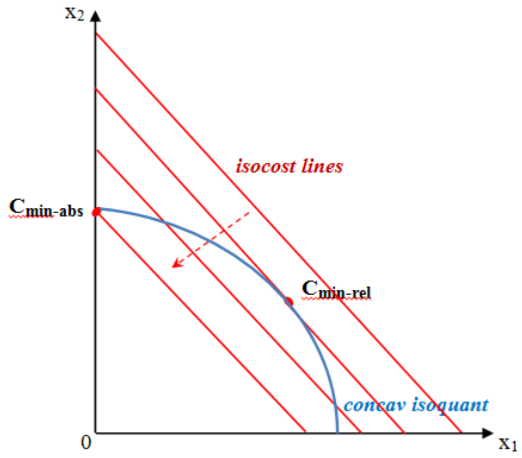


Fig. 1. Solution of a problem of constrained minimization with non-concave technologies

The achieved result doesn't differ if it is reinterpreted according to financial analysis. The same logic is applied. Markowitz in fact, suggests to fix an objective (the expected production level, represented by the isoquant in the Figure 1) and afterwards to minimize the other objective (costs combination, represented by the isocost lines). As a consequence in the financial area, if the *fix objective* is the "level of yields expected by the investors" (represented in Figure 1 by the isoquant curve) and the *factor to be minimized* is the "variancy of portfolio yields" (represented in Figure 1 by the isocost lines, as well as all the possibilities in which the effective yield differs from the expected yield) everything results again in a corner solution; hence, the comparison is done.

According to this logic, we consider appropriate to "model" a portfolio constituted by two types of bonds (*two assets portfolios*) as in the case of a production level (Y) that depends, in theory, on the existance of two factors, i.e. the capital and the labor.

Our question, at this point, is "how will the entrepreneur or the rational investor behave when choosing between these two factors?"

The answer is already given in Figure 3. *He will focus on just one of two assets*, if the optimal solution is the corner solution.

What is important to stress is the *"misunderstanding" of the considered solution when referring to a portfolio analysis.* In fact the benefit of diversification consists in the possibility to add to a portfolio, bonds with different performances to sustain the risk of the inverter. For this reason to reject in advance x_1 and to accept $C_{min-abs}$ as a unique solution to the problem, seems to be risky if the solution is not found in the problem context. Only in case of market “efficiency” the investors will choose for inverting all their endowment in x_2 factor/bond, prevailing on x_1 which, as a consequence, will be discarded.

How is it possible to manage “efficient portfolios” if market conditions do not allow to distinguish among different market assets? That is to say, the operators might not know or be able to apply a distinction in choosing their investments, having to settle for a *second-best* balance situation (i.e. in $C_{min\text{-}rel}$ point, where an inefficient factor has been applied as well).

Institutional, regulatory and contextual elements – in relation with the current phase of the economical cycle – bear heavily on the achievement of a *first-best* economical balance ($C_{min\text{-}rel}$ point in Figure 1 representing the ideal goal) being not recommended in terms of welfare. It is clear, this way, that if the aggregation of more convenient assets causes higher “deadweight losses” than the expected individual benefits, it would be convenient to reconsider diversification. If following concentration approaches cause a loss in welfare deriving from not taking advantage of the economical trade, in this case it would have been possible to obtain a higher marginal benefit. *Pareto improvement* while adopting diversification, notwithstanding the possibilities for some certain subjects to reach a personal advantage. Again and again we face the contraposition between efficiency and equity effects originating from market power (profits deriving from polar system as those of “perfect competition” and “monopoly” which are to each other as the derivative profits expected from a risk-free well diversified portfolio, is to a portfolio coinciding with only one “risky” asset that allows better market results).

Actually, the constraints to reach a *first-best* condition, in some certain markets, exist to guarantee equity and efficiency; in other markets, for instance in the financial ones, the weakening of regulation has caused heavy redistribution and speculation effects.

As a consequence of this situation and as a consequence of the modalities for structuring investments portfolios in financial markets under uncertain conditions, we derive the following assumptions: “the agents that derive extra-profits from growing results of factors, in relation with convexity and with the increasing trends in their production factors (i.e. with non-convexity of technologies), might shift the inefficiency of various risks form on the subjects that will grant for them”. This is in fact, what Stiglitz (2010) foresees in his work.

2. From diversifiability to risk trasferability

A direct consequence of the “aggregation mechanisms in investments decisions” – when accepting as a solution to the problem of bound minimization, the cornet solution in Figure 1, even if markets are not efficient – is the decrease of the (total) expected utility in relation to the risk sharing; that is to say in case only few agents would be able to manage dominant bonds “dumping” the inefficient ones on the market, the result would be a collapse of the market.

Hence, it seems necessary to establish a direct connection to the securitization mechanism.

As an intuition the possibility to obtain favorable results in a leveraging phase – that is to say during the credit cycle expansion (Fratianni, 2008) – can reduce the bonus for the (medium) risk among the community of inverters.

The result will be the “flattening” of the function of utility of the decisors – i.e. the compensation of inverted capitals or as in our case, the increasing of prices for assets collected through sub-prime loans – conditioned by the probability of real occurrence of the event.

This is analytically due to the Certainty-Equivalent (*C.E.*) approaching the Expected Value ($E[X]$) in the “bet” in a first step, and in the conveyance of the endowment to the asset considered “dominant”, afterwards. Therefore, given $[C.E. = \{E[X]\} \pm \rho\}]$, where “$\rho$” stands for bonus for the risk, if $\rho \to 0, = \rangle$ $\rangle\ (C.E. \equiv E[X])$: condition of neutral-risk.

The representation is given in Figure 2. $U[X]$ is the function of utility of an individual i and $X = \{x_1, x_2\}$ is instead a random vector because its outcomes are random variables.

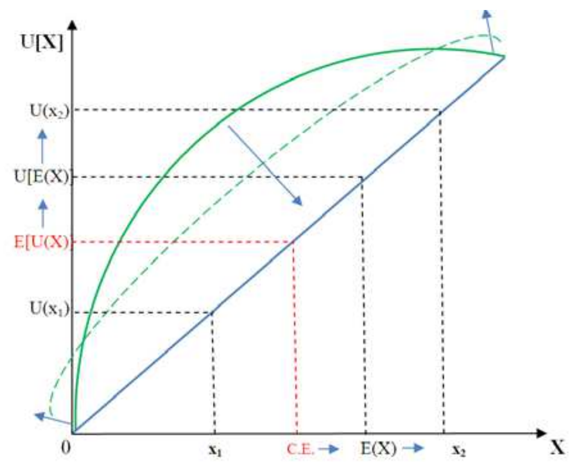


Fig. 2. The cancellation of risk appetite for investors

This confirms how the attitude towards the risk can be distorted, not considering anymore this attitude as a subjective presumption ex-ante to the individual investment, but as it will rather be a direct consequence of the Expected Value of the bet in the moment when it tends to one. This will be likely to happen in peculiar economic junctures as in those instable and ephemeral ones we are referring to.

In such a case, as in a "flock effect" the random vector could be acquired as the function of a unique random variable, bond x_2, considered inefficient – because the rush to achieve high levels of profitability, besides the immediate search for liquidity, brings to discard x_1 bond; The latter, as long as it is mastered in a individual confrontation, brings beneficial effects if considered from the diversification perspective of Markowitz. Considering x_1 as a corrupted random variable, which value is null ($k = 0$), and discarding it beforehand, leads us to the weakness existing at the beginning of the current financial crisis: the confusion between "transferability" and "diversification".

As it is not possible anymore, in a bet, to refer to the Expected Utility $E[U(X)]$, as a suitable and simplified operator to evaluate random events, that is to say considering the generic investor as "neutral to the risk" it will be for him $E[U(X)] = U[E(X)]$ and $C.E. = E(X)$ – where the distortions will further a speculative attitude. This would not happen in efficiency market conditions, where the operators (lending institutions, or from a macro perspective even countries) would attribute the right value to a mastered investment, by adopting a cautious attitude. That is to say, in case the bond x_1 is not discarded, certainly the operators will present $C.E. < E[X]$ as they will be risk-adverse going backward, the flattening of the function of utility, as a replay to speculative "flock effects" derives from the exclusion from investors' portfolios of bonds considered inefficient.

The achieved concentration depends on the assumption that, with non-convex technologies connected with functions of production expected to originate increasing yields, the diversification might be sidestepped and even it would be more convenient to seize on corner solution. This represents the misunderstanding on transferability and the demonstration of the failures originated by confusing it with diversifiability, due to the intention to cancel the part of non-diversifiable risk, deriving from a Markowitz approach. Indeed, such a dynamic cannot function if applied to incomplete markets.

3. Financial strength and frailty in credit networks

A research study on the existing relation among the *risk diversification*, the financial integration rate (defined as density), and the level of "*individual robustness*" deriving from financial shocks, has been proposed by Battiston, Delli Gatti, Gallegati, Greenwald and Stiglitz in a working paper of *NBER* in 2009.

Referring to the main results of such research it will be possible to shift paradigm among micro and macroeconomics evaluations. What we will demonstrate will be used for the analysis of Stiglitz's model (2010), as well.

In the following analysis we will refer to the *risk*, – both on an individual and collective level – to be affected by a *negative shock* – meaning an inversion of the yields of a risky assets – generated somewhere in globalized economy.

There are mainly two ways through which "shocks" can be spread in a financial network. Shocks can be "entirely" or "partially" transferred from one node to the closest ones, or can be transferred and spread on other nodes. The first process implies holding the original risk level through the write off of the nodes while the latter implies the multiplication of the original risk level. We will refer to the first as "*risk sharing*", and to the second as "*contagion*" (Gallegati et al., 2008).

In the related literature the *contagion*, instead, is defined as a synonym for the spread of financial stresses, through the connections that facilitate the diffusion of a shock.

Indeed, such a definition is very close to the concept of *risk sharing*, while the real *contagion* occurs when the correlation among output yields of different agents connected in the network (referring to financially integrated countries) increases during crisis periods, being a countertrend to the hypothesis in which it stabilizes during stability periods.

In Battiston-Delligatti et al. (2009) work, a network density is defined as the average degree of nodes to which a *k-th* agent (k_i) is connected. When a node is connected to every other node of the system, the network is *complete* (maximum density).

In financial integrated systems (with high density) diversification reduces the idiosyncratic risk, but leads to a "propagation of financial stresses", which, exceeding certain levels of density, will originate a contagion. Therefore, we wonder whether *higher network density* might solve or lead to systematic risk. The answer to our question is given by shaping a model originating from the definition of a certain parameter "ρ_i", assumed as "*measurement of financial robustness*" of an agent *i-th* connected to the others in a credit-network. Examples of such measurement could be the *equity ratio* (meant as a reciprocal of financial leverage as described above) or the *credit rating*. If we choose for the first interpretation: ρ_i[1] $\in[0,1]$, when ρ_i lowers below a certain level "*bankruptcy threshold*", the agent undergoes a crisis: ($\rho_i = 0$).

[1] Stiglitz in his demonstration of the model (2010) assumes by implication that $\rho_i = \rho_j$. This way it is possible to focus on cases of potential convenience of financial integration, regardless of endogenous factors as the "financial robustness" of the agents, which would immediately originate the perception of financial credibility.

On the other side, when ($\rho_i = 1$) the agent is *self-financed* and, therefore, he does not need to establish external relations or to resort to the credit market. With these assumptions the authors define a "*law of movement of agent robustness"* exemplified as follows:

$$\rho_{i(t+1)} = \underbrace{\sum_{j=1}^{\kappa_i} W_{ij}[\rho_J(t) + \sigma_i \xi_j(t)]}_{\textit{Risk Sharing}} + \underbrace{h(\rho_i(t))}_{\textit{Trend Reinforcement}}, \quad (1)$$

where W_{ij} is the weight of the closest agent j in the network of the agent i relations; κ_i is the number of i nodes, "the dimension of neighbourhood"; $\sum_{j=1}^{\kappa_i} W_{ij}\rho_j$ is the assessed average of the "neighbourhood robustness"; ξj is an idiosyncratic "normally spread shock" that hits the robustness of the closest agent j; $\sum_{j=1}^{\kappa_i} W_{ij}\xi_j$ is the assessed average of shocks hitting those agents who are making part of the network to which belongs i; σ_i is the specific risk (variancy) of agent i; term h is an *increasing function* of the "agent's robustness history" and does not depend on the robustness of the neighbourhoods (while holding feedback for itself). Therefore, it is a measurement of the *reinforcement trend*.

The equivalent formulation of the model in equation 1, *in continuous time*, is easy to obtain by deriving ρ_i respect to (t):

$$\frac{d\rho_i}{dt} = \rho_i' = (\sum_{j=1}^{k_i} W_{ij}\rho_J - \rho_i)dt + \\ + \sum_{j=1}^{k_i} W_{ij} \bullet d\xi_j + h(\rho_i)dt, \quad (2)$$

where ρ_i' is a "(instantaneous) measurement" of financial robustness of i agent, technical expression of the robustness concept grounded on such model.

The message in the model is that the connections among the agents (connectivity) allow "the diffusion of idiosyncratic risk in the network".

Therefore, a shock that hits the agent i damages the robustness of agent j as well, and vice versa. If the agent's frailty i increases, thanks to the network and its *density*, the agent j might "disperse" the transmitted stress, spreading it – through a fraction – among the participants to the network.

This is the "benefit of risk sharing" we have discussed so far and of which we have given an analytical formulation.

The other side of the coin is represented by the fact that connectivity, might instead, lead to a dynamic of unfettered *trend reinforcement*.

Let's suppose that for some certain reasons (i.e. considering deregulation, financial innovation, undemanding monetary policies, the tendency to concentration in the "most profitable" markets, and so on) the agents are boosted to spread the risk because they believe that by spreading it through the globalized market they might reabsorb, at least, an infinitesimal fraction of it.

Such beliefs have sustained the expansion of a credit cycle that has caused a speculative bubble, which exponentially inflated by a never existing before connectivity model, has blown up beforehand with resounding effects. The reason of the lack of control on it, it is due to the functioning, according to the authors, of a "*financial accelerator*".

Supposing that the agent i suffers a negative shock that reduces his robustness in such a measure to lead his partners to worsen the credit conditions offered in a stability condition. If the variation of the robustness of the agent i, described as $\left\{\Delta_i = [\rho_i(t) - \rho_i(t-dt)] > \left[-\xi\frac{\sigma_i}{\sqrt{k}}\right]\right\}$, where the latter represents the maximum *threshold* endurable by the shared network – implies a *trend reinforcement* of $h = (-\alpha)$ entity, the reaction of agent i might be individual bankruptcy, with costs to redistribute among the participants to the network. In this transformation the value of "α" represents the "*sensitivity*" of the reaction of the neighbours j ($j \neq i$) to the worsening of robustness of i. It is, therefore, the cost that i will have to pay to compromise his position and to weaken the other agent's position.

Such a symbology (the ‹-α›) will be drawn on with the development of Stiglitz's model (2010), when the output expected by agent i will depend on the composition of his capital, and in case the probability of disaster will be realized, the effective loss endured by i will correspond to (-α).

In such situations, if it would not be possible to interrupt, as a last option, the relations with the agent i, because for instance the interconnectivity level (density) is too high, when the agent will go bankrupt there will be a further specific transfer of stress to connected partners. *This kind of shock* is different from the idiosyncratic one, because *it could lead to partners bankruptcy* as well. In such a condition, "*cascades or avalanches of bankruptcies*" would be the direct consequence of the worsening of "financial frailty" on a systemic level. Such a dynamic of propagation of financial stresses inside a credit network, caused by the bankruptcy of one agent taking part to the network, represents what we define a "financial accelerator".

On the other hand, in absence of *trend reinforcement* – that is to say if as an hypothesis we exclude "the non diversifiable/systematic risk" from the discussed model – the benefit acquirable from an integration/system would be a total resetting of the agents (idiosyncratic) risk; hence, we can demonstrate that the resetting of systemic risk nullifies the specific risk, through the diversification of the latter.

From a Markowitz point of view, $((\xi_i \equiv \sigma_{ij}) = 0$ and $h = 0$, the equation 2 becomes:

$$\frac{d\rho_i}{dt} = \rho_i' = \left(\sum_{j=1}^{k_i} W_{ij}\rho_J - \rho_i\right)dt, \quad (3)$$

$$\sum_{j=1}^{k_i} W_{ij} = \left(\frac{1}{k}\right).$$

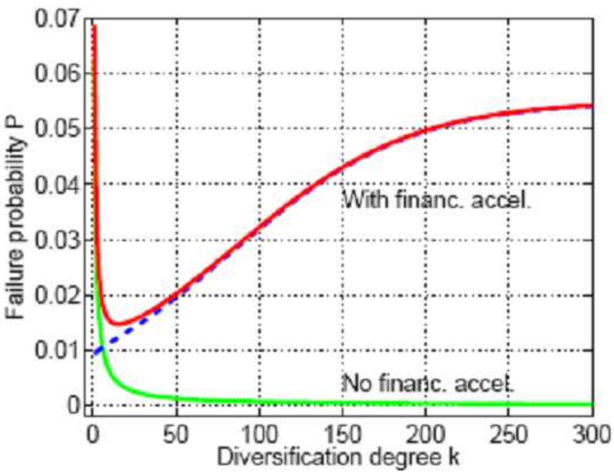
Therefore, we verify, with such hypothesis, how the "*combined probability to go bankrupt*", specified as the opposite of its *robustness*, which is also imagined as the reciprocal of the financial leverage, is:

$$P_f^{rs} = \left(\frac{1}{\rho_i'} \equiv L\right) = \frac{\sigma^2}{k} \quad (4)$$

with $\sigma^2 = \sum_{k=1}^{n} \sigma_i^2$(total variancy), and with P_f^{rs}, *probability of bankruptcy through risk sharing* (without systemic risk).

If $k \to \infty$, that is to say with an indefinite increase of the number of participants, the possibility of a systemic bankruptcy (excluding systematic risk) is cancelled.

The relation between "systemic risk" and "specific risk", that is to say the possibility to assume the existence or nonexistence of a financial accelerator (event in which $h = 0$) is shown in the following Figure 3.



Source: Battiston et al. (2009).

Fig. 3. Convenience on diversification

The search for a "mechanism" to control the *financial accelerator*, meaning the indicator that records the systemic level of financial robustness and, therefore, the probability of bankruptcy of the network as a consequence of systemic shocks during unstable periods, seen that it is not possible to exclude in reality the systematic risk, has been the subject of further analysis by Stiglitz (2010).

We will discuss such a mechanism together with the reasons that have caused its formulation and its significance in policy terms in the following sections.

4. Adjustments on the degree of financial integration: Stiglitz's proposal (2010)

Stiglitz (2010), in financial upheavals situations, suggests imposing "*restrictions of the relations in financial markets*". This is because the architecture of financial integration might affect the probability of risk of a cascade bankruptcy, as a consequence of "contagion" phenomena, hence to put a strain on systemic risk. Therefore, having to choose between two polar systems (full integration or autarchy) in unstable economic conditions, through the use of a simplified mathematical model, autarchy might result to be superior; on the other side, we will demonstrate that by loosening the closure degree, there exists an interval in which liberalization can increase social welfare.

By limiting cash outflow (e.g. through systems of taxation on exchange markets, or on short movement of capitals) by protecting own markets, by adopting positions of protectionism – if some certain hypothesis subsists – it would be likely to limit the possibilities of contagion that cause crisis of global reach and might function exactly as a circuit break works, to avoid the spreading a failure to the whole circuit, when there is a failure in a power line. The author supports his thesis by elaborating an analytical model that depending on proper variances, according to the degree of closure-opening of financial markets, demonstrates how the existence of a "*circuit break*" might even lead to the increase of social welfare.

4.1. Interpretation and explanation of the Stiglitz's model (2010). If we consider the output of a country i as a function of a random variable S_i, adopted as the available capital of the country (*Own capital* + *Return on debit*), that is to say:

$$Y_i^{\exp} = f(S_i). \quad (5)$$

In the world there exist n countries, each one with a different structuring of venture capital S.

To maximize the availability of capital own by each of them, we assume that each S_i is a function of investments of portfolio subjected to an expected yield X_i such as:

$$E(X_i) = \mu, Var(X_i) = \sigma_i^2 e Cov(X_i, X_j) = \rho_{ij} \bullet \sigma_i \sigma_j,$$

with $\rho \geq 0$, $\forall_i \neq j$; where obviously $\sigma_{ij} = 0$ corresponding with the hypothesis of risk of null contagion.

The following elaboration comes from Gallegati et al. (2008), to which Stiglitz's work is directly connected. By considering such theory we are able to match micro and macroeconomics effect together.

Every agent, as previously said, can operate in autarchy and obtain a yield $R_i = X_i$ (*v.c.*) or can share the risk with the others in an express way by acquiring a share of other's agents portfolio investments or by lending them money. The singularity of the risk sharing derives from the non-repayment of the loan when the agent goes bankrupt.

According to this structuring, if the achieved yield is lower than the *threshold* θ, the agent goes bankrupt. We consider this way $D_i = 1_{[Xi < \theta]}$ the *default indicator* for agent *i* "without integration", and $D_i = 1_{[\bar{x}_i < \theta]}$ the *default indicator* for agent *i* "with integration".

The "bankruptcy estimated quotas", respectively "in absence" and "in presence" of financial integration are:

$$S_{no-link} = E\left[\frac{\sum iD_i}{n}\right] \text{ and } S_{link} = E\left[D\right] = P(\bar{X} < \theta).$$

The ratio of these two indicators is given by

$\varphi = S_{link}/S_{no-link}$, with $0 \le \varphi \le 1$.

These indicators are used to realize a comparison among the agents "*individual incentives*" to participate to a financial integration; obviously the integration of φ value as close as possible to zero, is required.

By assuming:

$$U_i = U(R_i) + u(R_i - R) - CD_i \qquad (6)$$

the utility of agent *i*, where $R_i = X_i$, $R = \bar{X}$ (*market yield*), D_i is the default-indicator and $C \ge 0$ is the individual cost of bankruptcy. By assuming the agent-country is "*risk adverse*", we have:

$U', u' > 0$ and $U'', u'' < 0$.

Therefore, the utility of agent *i* positively depends on his expected yield, insofar as his yields exceeds markets yields, and negatively depend on the risks and costs of bankruptcy.

Therefore, *without financial integration*, each country obtains an expected utility of :

$$E(U)_{no-link} = E(U) + E(u) - CS_{no-link}. \qquad (7)$$

With financial integration, *i* obtains, instead:

$$E(U)_{link} = E(U) - CS_{link}. \qquad (8)$$

Obviously integration will happen each and every time $E(U)_{link} > E(U)_{no-link}$, in function of φ.

In Stiglitz's model, the production is linear in *S*, at the condition that *S* is higher than the critic level S^*; when $[S \le S^*: f(\Delta_i)]$ there happen "*avalanches of bankruptcies*", and a loss of *C* is given.

The value of capital *S* – as both Gallegati (2008) and Battiston (2009) have delved into, through labor integration – includes already all the information on the possibility to have individual defaults; for this reason the decision to integrate or not is a direct consequence of the possibilities to neutralize the individual bankruptcy expectancies.

We assume that $S_i = -\alpha_1$[1] with (p) probability and that $S_i = -\alpha_2$ with probability $(1 - p)$, such as, to simplify, we consider that the expected output without bankruptcy is equal to zero ($Y_i^{exp} = 0$), then we have:

$$\rho\alpha_1 = (1 - \rho)\alpha_2. \qquad (9)$$

Introducing a further simplification, we assume $S^* = 0$, that is to say $[S \le 0]$ and supposing $|C| < |\alpha_1|$ and $\alpha_2 < |\alpha_1|$, and in addition $\rho < 0,5$[2].

The assumption of $|C| < |\alpha_1|$ tallies with the basic hypothesis on the diversification convenience: the model of an integrated financial system certainly allows to reduce idiosyncratic risk effects; hence, the maximum loss, being part of a system will be inferior to the individual loss of a country, even if in such unusual cases, it brings more serious effects[3].

With these preliminary remarks we have found that "*there exists a light probability of* "*non correlated*" *disaster among the different countries*". We will demonstrate such assumption considering different hypothesis.

Option A: Autarchy. "Considering the importance of international capital flows for world economy, "financial protectionism" might be as much significant as commercial protectionism. It is usually measured by regulation restraints on flows of international capital (i.e regulations on inflows and outflows, on quantity and costs, on external partnership obligations)"[4].

In the event a decision of financial closure might be taken, the expected output will be:

[1] Let's note how, not by coincidence, $-\alpha$ coincides with Battiston's logic (2009) in describing the mechanism of *trend-reinforcement*, when "pain threshold" is ridden out by *i*, and S_i corresponds to the expectations on individual bankruptcy expectances of Gallegati (2008).

[2] We mean that the disaster will happen every time the expected output is lower than zero, and every time that the loss of *C* is lower than the minimum assumable value of *S* in case it is $p = 1$, and every time that the same value α_1 is higher than α_2 if $p = (1 - p) = 0.5$, and every time that the probability to have a capital deficit is assumed as lower than 0.5 or as an equivalent, that the probability to have an available capital surplus is higher than 0.5.

[3] Let's think about the default of Argentina in the first years of 2000 and let's compare it with the effects of current financial crisis on financially integrated countries.

[4] CBE (February 2009), *Monthly Report*.

$$Y_i^{\exp\,(before\ liberalization)} = -\rho C + (1-\rho)\alpha_2{}^{1},$$

for equation (9) and properly summing up the terms, it becomes:

$$Y_i^{\exp\,(b.l.)} = -\rho(\alpha_1 - C) :> 0. \quad (10)$$

Being $\alpha_1 > C$ for hypothesis, *any (p) we consider, the probability of "non correlated" disaster is actually negligible*.

Option B: Full integration. Now we consider integration in N countries.

This possibility implies all the considerations we have done so far in relation with the *risk sharing* convenience and those we have done on risk sharing becoming contagion, according to the exceeding of the threshold.

In this representation we assume, to simplify, that there are only two countries i and j, and particularly that $\rho_i = \rho_j$, considered an implicit function of the following formulation, is regardless of the degree of financial robustness.

Therefore, if a "*full liberalization*" is agreed, it will be:

$$\rho(\Sigma_i S_i \le 0)^2 = [1-(1-\rho)^{N=2}]. \quad (11)$$

This shows that both the countries will go bankrupt if only one of them gets a bad yield.

It will follow that the two countries expected output after liberalization will be:

$$Y_i^{\exp\,(after\ liberalization)} = \left[-C\left(1-(1-\rho)^2\right) + (1-\rho)^2\alpha_2\right]. \quad (12)$$

At the end it is possible to decide which strategy to carry out regardless of the other country strategy through a comparison between the two achieved yields $Y^{\exp}$. Since: $[(1-(1-p)^2) > p]$, we infer that $C^{a.l.} > C^{b.l}$.

The interpretation of such result is that, "*Having to choose between two polar regimes*" – full integration or autarchy – *liberalization leads to an inevitable welfare decrease*.

Option C: Partial integration. Let's consider what happens in the event that countries have a different strategy: gradually opening inside a certain range, that is to say to realize a "*partial liberalization*". In such event we assume that bankruptcy happens if $[\Sigma_i S_i/N \le K < 0]$[3].

Stiglitz demonstrates that, for $N = 2$, there is a critic value of p, such as $p \le p^*$ (equal to $\alpha_2 \ge \alpha_2^{*}$[4] liberalization is a welfare reducer: if disasters are rare but significant, liberalization is undesirable.

The critic variable p^* is defined as:

$$p^* = (1-\varsigma)(1-2\varsigma), \quad (13)$$

$$\text{where } \{\varsigma\} \equiv 2K/\alpha_1. \quad (14)$$

Option C$_1$: An interpretation of "optimal" partial integration. Solving the quadratic equation (13) in ς – after having assumed $p^* = 0.5$ (maximum acceptable result to be able to affirm that such combined probability of disaster is higher than the probability of success) it derives that, for values of $0.19 < \varsigma \le 0.49$ *liberalization increases social welfare*. This is *"optimal partial" liberalization;* the "*circuit break" to the financial accelerator* according to Stiglit is therefore "ς".

Every time that a partial opening of capital flows implies that the available capital S_i is structured to have as a return on debit a quote which is on average $K = \left\lfloor \frac{\varsigma}{2}\bullet\alpha_1 \right\rfloor \equiv 0{,}175\bullet\alpha_1$, if $\bar{\varsigma} = \frac{0{,}19+0{,}49}{2} \equiv 0{,}34\bar{9}$ – expressed K from equation 14 – *there will be an increase of social welfare*.

Option C$_2$: "Undesiderable" partial integration. If $\varsigma = 0$ $(K = 0)$ *liberalization is never desiderable*, because:

$[\Sigma_i S_i/N \le 0]$ and $p^* = 1$.

As a general rule, *if the model is extended to N countries*, which critic value of p^* still satisfies the condition $p \le p^*(N, K)$, *liberalization will never be desiderable.* This derives from the fact that if N goes to infinity (that is to say with the indefinite increase of participants to partial integration) in the hypothesis of $\varsigma = 0$ because $K = 0$, for the law of large numbers, we will have:

$$\lim_{n\to\infty}\left(\frac{\sum S_i}{N} = 0^-\right) \text{ with } p^* = (1-0)(1-2\times 0) = 1.$$

The result of such limit is an infinitesimal of order lower than zero, such that the condition to realize systemic bankruptcy will be valid.

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[1] Stiglitz substitutes value C with α_1 because the aim for developing the model is the evaluation of the possible bankruptcy effects.

[2] It is the "systemic probability", of i from 1 to N, hence two countries of our example, go bankrupt together. This is equal to the opposite event of probability of the two countries to have a positive yield, which is the result of compatible events, according to the law of compound probabilities.

[3] That is to say bankruptcy happens if the average available capital in N countries is lower than a certain "K" (the equivalent of S^* when we considered the effects for *i-th* country), and if *such average capital* is itself lower than zero.

[4] This will be the probability of partial combined disaster, higher than the correlated probability and, therefore, the expected value of positive yield with partial integration will be lower than the one obtained before liberalization.

5. A personal interpretation on Stiglitz's model according to a theory of game scheme

From the model we have analyzed so far we infer that the liberalization of financial capital flows, is never desiderable. There is, instead, a certain dimension of financial capital flows ($\varsigma \equiv 2K/\alpha_1$) to allow the increase of social welfare; *such result appears to be convenient only if liberalization besides being partial, concerns a finite number of countries.*

The achieved results can be schematized referring to what emerges from a non cooperative variable-sum game between two players, countries *i* and *j*; we have adopted the simplification for which two *countries are symmetrical in everything* and we have assumed that partial liberalization, to simplify, implies an inflows/outflows of capital equal to the half of the total.

The demonstration of the basic formulation of adopted strategies to define the payoff and its explications have been given in the Appendix.

Table 1. Matrix1 (referring to option *A*)

i/j	Autarchy	Liberalization
Autarchy	0,0*	0, -11.8
Liberalization	-11.8, 0	-6.7, -6.7

Table 2. Matrix 2 (referring to option C_1)

i/j	Autarchy	Partial	Liberalization
Autarchy	0, 0*	0, -2	0, -11.8
Partial	-2, 0	6, 6**	-4.45, -9.35
Liberalization	-11.8, 0	-9.35, -4.45	-6.7, -6.7

Notes: * initial Nash equilibrium; ** final Nash equilibrium with *focal-point*.

For an intuitive demonstration of the game results we propose the example of a result obtained by combining *liberalization and partial integration*:

$$L-P:\left\{\begin{array}{l}\left(\frac{1}{2}+\frac{\overleftarrow{1}}{4}\right)\left[\left(1-(1-\rho)^2\right)(-\alpha_1)_i+(1-\rho)^2(\alpha_2)_i\right]+\\+\left(\frac{\overleftarrow{1}}{4}\right)\left[\rho*\left(-\alpha_1^*\right)_j+\left(1-\rho^*\right)\left(\alpha_2^*\right)_j\right]\end{array}\right\},$$

$$\left\{\begin{array}{l}\left(\frac{3}{4}\right)\left[\rho*\left(-\alpha_1^*\right)_j+\left(1-\rho^*\right)\left(\alpha_2^*\right)_j\right]+\\+\left(\frac{1}{4}\right)\left[\left(1-(1-\rho)^2\right)(-\alpha_1)_i+(1-\rho)^2(\alpha_2)_j\right]\end{array}\right\}=-9.35,-4.45.$$

The use of numerical coefficients, i.e. [1/2 + 1/4] = 3/4 in the strategy L-P, corresponding to P-L strategy, on the opposite, means that country *i*, while choosing for a total liberalization, will accept to exchange half of its capital with country *j*, the latter instead, while adopting a partial liberalization, will grant to *i* only a fourth of its own capital (half of the half).

The expected payoffs related to the adopted strategy are instead inserted within square brackets; therefore we infer $[(1-(1-\rho)^2)(-\alpha_1)_i + (1-\rho)^2)(\alpha_2)_i]$ referring to option B "of full integration", related to equation 12, in which *C* has been substituted with $(-\alpha_1)_i$, which represents the level of return on debit that leads to individual bankruptcy or the *financial accelerator* "*h*", that triggers "bankruptcy cascades" in the moment the value ($-\alpha$) is reached. By inserting such value in the function that defines the expected output we would like to test, at this stage, rather than the effects of an individual bankruptcy, the potential output referring to the decision of integrating or not, where ($-\alpha$) would represent only a limit value, which is possible to avoid thanks to the occurred integration; on the other side, $\left[\rho*\left(-\alpha_1^*\right)_j+\left(1-\rho^*\right)\left(\alpha_2^*\right)_j\right]$ refers to option C, where ρ^* has been assumed as the combined probability to go bankrupt, not anymore individual in the moment a network is set up. The same standard, with proper adjustments, has been adopted for all the strategies.

The message given by Stiglitz, as it is reported in Matrix 1, is that – in financial crisis circumstances – the mechanism of free market might automatically produce an allocation of efficient equilibrium (A-A), according to what is supposed in the first general theorem of welfare economics.

The *problem of stagnation*, in such event countries might be closed as a consequence of a financial crisis that has hit on them, as the second general theorem of welfare economics states, that they might be solved provided that it is possible to reallocate the resources in a planned way. In Matrix 2 the problem of *Nash mixed equilibrium* is solved by the same players, who, while being rational agents, will choose for the expected result that ensure them the best yield (as demonstrated by Stiglitz, the one that increases social welfare). The considered result is (P-P) (*the "focal point"*), as suggested by Schelling (1960).

6. Suggestion for further prosecution of the research

Through this framework the author wanted to suggest, how it would be desiderable that policy-makers adopt a strategy of partial financial integration, during crisis periods, with the aim to bypass the impasse of a financial crisis.

In this research a simplified model has been presented. Such model aims to represent the circumstances for the desirable realization of financial integration processes, starting with the analysis of the conditions that might foster integration on a macroeconomic level, therefore, demonstrating how potential bankruptcies might instead derive from aggregation processes on a microeconomic level. For some certain countries, in fact, the integration will be appropriate (because the probability of collective bankruptcy is lower than the individual one, and the economic expected yield inside the network is higher than the achievable one on a individual level), hence integration will always represent a benefit.

Moreover, the increase of the participants number and their connecting degree (density) will guarantee a fair distribution of idiosyncratic risk which will be spread up to such a level to vanish according to the indefinite increase of the agents. The peculiarity of the interconnection has been proved by the existence of a strengthening factor of financial risk (*financial accelerator*), that increases with the increase of the participants and of the individual bankruptcies. We have observed how in the moment the value of interconnection exceeds a certain threshold, *cascade bankruptcies* take place and even involve those agents unrelated with the original default.

We have noticed how the evolution and the distortion of modern finance, when *diversifiability* is confused with transferability on a microeconomic level, might engender individual bankruptcies. If these bankruptcies damage the fundamentals of macroeconomic agents, because of interconnections deriving from financial globalization or because of existing agreements on a supranational level, or because of the pervading worldly interbank relations, then the bankruptcy of a country will degenerate into a financial global crisis. The current crisis represents, in fact, a very appropriated example. Moreover, we have seen how in the event we exclude the (non diversifiable) "systematic risk" from the theoretical representation it will be possible to nullify the negative effect of the *financial accelerator* on the "level of systemic robustness"; this is the only case in which diversification will always be beneficial because of bringing the cancellation of specific risks to the limit, as the financial and the Markowitz portfolio theories has already suggested.

When instead, this is not possible and the individual bankruptcies are even the natural consequence of flaws in partnerships, due to the weakening of monitoring level on a systemic level, then speculative and predatory behaviors will arise, especially during the expansion of a credit cycle and generally anticipating the blowout of a speculative bubble.

With the aim to avert phenomena such as systemic financial crisis, this work has taken the direction of researching potential levels of optimal financial integration, convenient to override stagnation on a global level and to re-activate the economic recovery, starting from the originating source of the crisis.

A simplified solution has been adopted in this work in the form of a theoretical representation that challenges the architecture of global finance as it appears today, while being inspired by different authors we have considered going over the subject through a representation in terms of game theory.

It would be worth to verify if the equilibrium conditions defined in our research, would be repeatable in the moment in which simplifying hypothesis would fail, and would break the contagion channels, both in favorable and in crisis periods. It would be required to put in practice an insurance mechanism among the participants, ruled by a supranational supervisor, acting as stabilizer during peak times, both in the ascendant and descendent phases; the reduction of procyclicality would derive from the introduction of the obligation to set aside resources, during expansion phases, to use them during crisis moments. Through such an insurance market, where it is possible to distinguish between high quality and low quality debtors, it would be possible to establish some "punishment mechanism" when agents might behave as high-risky ones (to avoid individual bankruptcy where systemic defaults derive from) and "incentives" to promote deserving behaviors of low risk among the agents (through the grant of premiums, originating from a taxation on those who attain losses).

A close examination on this subject is given through cues coming from Rothschild and Stiglitz (1976). The structuring of such theoretical model is functional to the framework defined in this research project, because it is possible, on a theoretical level, to model an insurance market functioning through "bilateral comparisons" among the insured parties. If from Stiglitz's (2010) simplifying hypothesis re-interpreted through a scheme of games theory, every agent *i-th* would consider appropriate to realize a partial financial integration during financial instability periods, through proper actions of economical policy.

From bilateral comparison among macroeconomics agents, according to Rothschild-Stiglitz approach (1976), it might be possible to extend far beyond some limits in modelling our framework, where, among other hypothesis, we have stated that financial robustness levels among agents are equivalent ($\rho_i = \rho_j$).

If this hypothesis fails, it might anyway occur the definition of an equilibrium of partial optimal financial integration, that would satisfy the incentives to participate in such a trade through contracts stipulated in function of the participants financial worthiness.

A "separating equilibrium" would be defined among the agents *i-th* taking part in the financial integration, where all the participants find convenient realizing a partial financial integration, as defined, so far.

This way, in the economics doctrine, the idea of realizing "insuring mechanism against asymmetrical shocks" inside a monetary union, has been advanced. These mechanisms might work according to "public" insurance plans (defined by a potential centralization of budget management on a systemic level) or according to "private" insurance plans (operating exclusively through financial markets). An insurance mechanism that directly plays on the optimal level of financial integration among countries, represents in my opinion, an incremental improvement of the private insurance mechanism, because revised through elements inclining it to the public sector. Though this has already reduced behaviors of moral hazards among the participants, the mechanism might be strengthen by defining punishment and incentive measures ruled by the policymakers, such as discouraging possible deviation strategies of a super-game turning into collusive. If the profit deriving from deviation, would be of lower entity than the costs of quitting the financial union then, none of the participants would find convenient to quit the game. In such a case the participants would accept a very burdensome loss in the short term but necessary in long terms, rather than loosing those benefits yielded as an aid by the financial union to re-establish the initial super game.

Conclusions

The reference to the events of the last months inside the European Monetary Union, concerning the crisis of Government Debt in Greece and the speculation attacks on Italy, is not accidental. The observation on aid strategies defined by the ECB and the IMF together, and the costs that these countries will have to bear to recover trustworthiness inside the Union are all conditions that appear concurring with our research.

Moreover, we have seen how in the event we exclude from the theoretical representation the (non diversifiable) "systematic risk" it would be possible to nullify the negative effect of the *financial accelerator* on the "level of systemic robustness". This is the only case in which diversification will always be beneficial because of bringing to the limit the cancellation of specific risks, as the financial and the Markowitz portfolio theories have already suggested.

When instead, this is not possible and the individual bankruptcies are even the natural consequence of flaws in partnerships, due to the weakening of monitoring level on a systemic level, then speculative and predatory behaviors will arise, especially during the expansion of a credit cycle and generally anticipating the blow-out of a speculative bubble.

With the aim to avert phenomena such as systemic financial crisis, this work has taken the direction of researching potential levels of optimal financial integration, convenient to override stagnation on a global level and to re-activate the economic recovery, starting from the originating source of the crisis.

A simplified solution has been adopted in this work in the form of a theoretical representation that challenges the architecture of global finance as it appears today, while being inspired by different authors we have considered the subject through a representation in terms of game theory.

If we redefine the level of financial integration when imported financial crisis occurs, while adopting the simplification of only two agents ($N = 2$) having to decide on the degree of financial integration, improvements in their social welfare will be achieved in the moment the result of the non-cooperative game is a Nash equilibrium with *focal point:* that is to say the result of a partial financial integration strategy applied by both two players. The condition for this event has been defined by supposing that every agent does not exceed a certain value of the return on debit, described as a certain fraction of the value originating the individual bankruptcy. The functioning of such result is framed in a simplified model where there is still room for further revisions. We have assumed, in fact, that financial robustness is equivalent for every participant in the network and that the involved countries do not differ for expected results. At the same time, we have supposed, to simplify the calculation, that the level of partial integration implies interchanges of value equal to the half of the full financial integration. Further critics might arise if we would observe the individual debt load measured by the Government debt load and its growth level, and we would also observe the different levels of tax burden and expected growth.

If then, though the applied simplifications, the main policy direction would still tend to realize an Optimum Currency Area, according to Mundell theory (1961), seen the causes and effects of derivative financial crisis, it would be absolutely damaging not considering the "correct level of financial integration" among elements such as the *high labor mobility*, the *high degree of opening to trading among regions*, and *the lack of asymmetrical shocks*, that might reduce the costs of a Monetary Union.

These are some reasons for which international public institutions might act through the re-definition of the same concept of globalism, and might review competitive positions by playing "on the optimal integration of financial markets".

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Appendix

1. Justification to the payoff of Matrix 1 and 2, as a revision of the results achieved by Stiglitz in terms of applied strategies in a non-cooperative variable sum between two players (countries *i* and *j*). We want to remind how a simplification for which two countries are completely symmetrical and for which the partial liberalization implies the inflows/outflows of capital equal to the half of the total liberalization, has been applied.

From these suppositions we have originated a mechanism to define the expected result in terms of strategy adopted by the game player and by the strategy that the other player will adopt, while considering all the possibilities in the case.

The result of the payoff of Matrix 1 has been explained as follows, given the hypothesis for which $\rho < 0.5$, $|C| < |\alpha_1|$ and $|\alpha_2| < |\alpha_1|$.
To achieve numerical results we have chosen to attribute the following values, in respect to *hp*: $\rho = 0{,}3$; $|\alpha_1| = 100$; $|\alpha_2| = 80$; $|C| = 90$, such as

A-A: $\{\rho(-\alpha_1)_i + \rho(\alpha_1)_i\}$; $\{\rho(-\alpha_1)_j + \rho(\alpha_1)_j\} = 0; 0,$

L-A: $$\left\{\left(\frac{1}{2}+\frac{1}{2}\right)\left[\left(1-(1-\rho)^2\right)(-\alpha_1)_i + (1-\rho)^2(\alpha_2)_i\right] + 0\right\};\{0\} = -11{,}8;\ 0,$$

A-L: $$\{0\};\left\{\left(\frac{1}{2}+\frac{1}{2}\right)\left[\left(1-(1-\rho)^2\right)(-\alpha_1)_i + (1-\rho)^2(\alpha_2)_i\right] + 0\right\}; = 0; -11{,}8,$$

L-L: $$\left\{\begin{array}{l}\frac{1}{2}\left[\left(1-(1-\rho)^2\right)(-C_i) + (1-\rho)^2(\alpha_2)_i\right] \\ +\frac{1}{2}\left[\left(1-(1-\rho)^2\right)(-C_j) + (1-\rho)^2(\alpha_2)_j\right]\end{array}\right\},$$

$$\left\{\begin{array}{l}\frac{1}{2}\left[\left(1-(1-\rho)^2\right)\left(-C_j\right)+(1-\rho)^2(\alpha_2)_j\right] \\ +\frac{1}{2}\left[\left(1-(1-\rho)^2\right)\left(-C_i\right)+(1-\rho)^2(\alpha_2)_i\right]\end{array}\right\}=-6,7;-6,7.$$

The result of the payoff of Matrix 2 has been explained as follows, given the hypothesis for which: $\rho < 0.5$, $|C| < |\alpha_1|$ and $|\alpha_2| < |\alpha_1|$ and moreover $\rho \le \rho*$, $\alpha_2 \ge \alpha_1$, hence $\alpha_2* \ge \alpha_2$.

To achieve numerical results we have chosen to attribute the following values, in respect to *hp*: $\rho = 0,3$; $|\alpha_1| = 100$, $|\alpha_2| = 80$; $|C| = 90$; $\rho^* = 0,4$; $\alpha_1^*=110$; $\alpha_2^* = 70$, such as:

A-A: $\{\rho(-\alpha_1)_i + \rho(\alpha_1)_i\}$; $\{\rho(-\alpha_1)_j + \rho(\alpha_1)_j\} = 0; 0,$

P-P: $$\left\{\frac{1}{2}\left[\rho^*(-C_i)+(1-\rho^*)(\alpha_2^*)_i\right]+\frac{1}{2}\left[\rho^*(-C_j)+(1-\rho^*)(\alpha_2^*)_j\right]\right\},$$

$$\left\{\frac{1}{2}\left[\rho^*(-C_j)+(1-\rho^*)(\alpha_2^*)_j\right]+\frac{1}{2}\left[\rho^*(-C_i)+(1-\rho^*)(\alpha_2^*)_i\right]\right\}=6;6,$$

P-A: $$\left\{\left(\frac{1}{2}+\frac{1}{2}\right)\left[\rho^*(-\alpha_1^*)_i+(1-\rho^*)(\alpha_2^*)_i\right]+0\right\};\{0\}=-2;0,$$

A-P: $$\{0\};\left\{\left(\frac{1}{2}+\frac{1}{2}\right)\left[\rho^*(-\alpha_1^*)_j+(1-\rho^*)(\alpha_2^*)_j\right]+0\right\};=0;-2,$$

L-L: $$\left\{\frac{1}{2}\left[\left(1-(1-\rho)^2\right)(-C_i)+(1-\rho)^2(\alpha_2)_i\right]+\frac{1}{2}\left[\left(1-(1-\rho)^2\right)(-C_j)+(1-\rho)^2(\alpha_2)_j\right]\right\},$$

$$\left\{\begin{array}{l}\frac{1}{2}\left[\left(1-(1-\rho)^2\right)\left(-C_j\right)+(1-\rho)^2(\alpha_2)_j\right] \\ +\frac{1}{2}\left[\left(1-(1-\rho)^2\right)\left(-C_i\right)+(1-\rho)^2(\alpha_2)_i\right]\end{array}\right\}=-6,7;-6,7,$$

L-A: $$\left\{\left(\frac{1}{2}+\frac{1}{2}\right)\left[\left(1-(1-\rho)^2\right)(-\alpha_1)_i+(1-\rho)^2(\alpha_2)_i\right]+0\right\};\{0\}=-11,8;0,$$

A-L: $$\{0\};\left\{\left(\frac{1}{2}+\frac{1}{2}\right)\left[\left(1-(1-\rho)^2\right)(-\alpha_1)_j+(1-\rho)^2(\alpha_2)_j\right]+0\right\};=0;-11,8,$$

L-P: $$\left\{\begin{array}{l}\left(\frac{1}{2}+\frac{\bar{1}}{4}\right)\left[\left(1-(1-\rho)^2\right)(-\alpha_1)_i+(1-\rho)^2(\alpha_2)_i\right] \\ +\left(\frac{\bar{1}}{4}\right)\left[\rho^*(-\alpha_1^*)_j+(1-\rho^*)(\alpha_2^*)_j\right]\end{array}\right\},$$

$$\left\{\begin{array}{l}\left(\frac{3}{4}\right)\left[\rho^*(-\alpha_1^*)_j+(1-\rho^*)(\alpha_2^*)_j\right] \\ +\left(\frac{1}{4}\right)\left[\left(1-(1-\rho)^2\right)(-\alpha_1)_i+(1-\rho)^2(\alpha_2)_i\right]\end{array}\right\}=-9,35;-4,45,$$

P-L: $$\left\{\left(\frac{3}{4}\right)\left[p^*(-\alpha_1^*)_i+(1-p^*)(\alpha_2^*)_i\right]+\left(\frac{1}{4}\right)\left[\left(1-(1-\rho)^2\right)(-\alpha_1)_i+(1-\rho)^2(\alpha_2)_i\right]\right\},$$

$$\left\{\begin{array}{l}\left(\frac{1}{2}+\frac{\bar{1}}{4}\right)\left[\left(1-(1-p)^2\right)(-\alpha_1)_j+(1-p)^2(\alpha_2)_j\right] \\ +\left(\frac{\bar{1}}{4}\right)\left[p^*(-\alpha_1^*)_i+(1-p^*)(\alpha_2^*)_i\right]\end{array}\right\}=-4,45;-9,35.$$