“Credit risk analysis and the KMV Black & Scholes model: a proposal of correction and an empirical analysis”

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Abstract

The first aim of this work is to propose a correction to the KMV Black and Scholes model to assess credit risk. The second aim is to perform a comparative empirical analysis, by applying the *modified* and the *non-modified* KMV Black and Scholes model and comparing the results.

The proposed corrections to the KMV Black and Scholes model regard: (1) the potential coupon detachment for the assessment of the current value of shares, and (2) the use of the t-Student probability distribution to replace the Normal distribution.

The proposed model was first tested on a sample of bankrupt firms (listed on the Dax), in order to validate the assumptions. After this preliminary phase, the same model has been applied on a sample of twenty five high-tech companies listed on the Italian Stock Exchange. Finally, the results of the previous studies have been compared with the ones obtained applying the non-modified model.

The paper maintains that the corrections proposed in this paper can be useful to support a more accurate estimate in assessing the credit risk and an improved perception of a firm's intrinsic default risk.

Keywords: credit risk, KMV Black and Scholes, distance to default, expected default frequency, Italian Stock Market.
JEL Classification: G12, G32, G33.

Introduction

The credit risk. Many economists consider the financial crisis still affecting several countries as the most severe since the 1929 crisis. Governments have launched bank rescue programs setting aside funds for thousands of billions Euros, through an unprecedented action in terms of allocated resources and of coordination among governments in the perspective of the global dimension of the crisis.

Over the past decade an increase in the number of insolvencies concerning bonds issued by private companies at an international level and, recently also in Italy, has been observed. In the same period, bonds issued by emerging countries have not been refunded. Given such defaults and the recent economic crisis, the development of an adequate analysis of credit risk and of its components has become crucial.

The definition and quantification of credit risk is extremely complex. Firstly, credit risk can be defined as the possibility for the borrower not to meet the financial obligation previously assumed in an agreement, thereby causing a loss for the creditor counterparty (Ammann, 2001).

Actually, such definition is not exhaustive, since it only takes into account the extreme situation of an insolvent debtor. A loss of value of the credit condition may also originate from a deterioration of the debtor's financial and economic conditions affecting the debtor's will and possibility to meet financial obligations incurred, despite the debtor is not totally insolvent.

Therefore, two different risk credit paradigms have been outlined:

- The default-model paradigm, according to which the loss of credit *exclusively* occurs as a consequence of the debtor's insolvency.
- The mark-to-market paradigm, according to which the variation of credit value occurs as a consequence of a downgrade of the debtor's rating, thereby classifying the default as an extreme event.

According to Altman's theory (Altman, 1968), it is necessary to differentiate static insolvency from dynamic insolvency. In particular, the former occurs when a company has a negative net capital, while the latter occurs when the company's cash-flow does not cover all payments owed. Subsequently, some authors in the literature have linked the concept of default risk to the concept of financial distress.

The paper is organized as follows. Section 1 analyzes the literature regarding the models assessing credit risk, with a special reference to models based on real options. Section 2 describes the KMV Black and Scholes (KMV B&S) model highlighting its weaknesses. Section 3 proposes possible adjustments that may overcome the weaknesses of the model itself; the *modified* KMV B&S model will, therefore, be proposed. Section 4 illustrates the application of the *modified* KMV B&S model for the risk credit analysis applied to a sample of Italian high-tech companies.

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1. Literature review: the structural models, based on the options theory, for the evaluation of credit risk

We can identify three different groups of models for credit risk evaluation: (1) structural credit risk models; (2) reduced form models; and (3) methodologies taken from the field of artificial intelligence and operational research (hybrid models). In this section the models of groups (1) and (2) are analyzed, which are those more strictly linked to the research described in this paper.

Structural credit risk models rely on the notion of claim priority and limited liability, which allows a firm's equity and debt be viewed as contingent claims that partition the asset value of the firm. Black and Scholes (1973) were the first to formally consider equity as a call option on the firm's asset value. However, it was the corporate bond pricing model by Merton (1974) that popularized the structural approach to model risky corporate debts.

Black and Cox (1976) extended Merton's model to a first passage model, whereby bondholders can force the reorganization or the bankruptcy of the firm if its value falls to a specific value (trigger value). Leland and Toft (1996), and Longstaff and Schwartz (1995) proposed methods for estimating the default probability of risky corporate debt. Leland and Hayne (1994) and Leland and Toft (1996) derived closed-form solutions relating on the value of long-term corporate debt and optimal capital structure to firm risk, taxes, bankruptcy costs and bond covenants. Longstaff and Schwartz (1995), with a semi-closed form solution, developed a two factor model to value risky debt, assuming that interest rates follow a mean reverting stochastic process and that there are deviations from strict absolute priority.

Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997) studied the importance of strategic debt service on risky debt spreads. Ericsson and Reneby (1998) represented company's debt as a discrete coupon risky bond. Collin-Dufresne and Goldstein (2001) extended Longstaff and Schwartz (1995) model to consider a stationary leverage ratio of the firm. Huang and Huang (2003) calibrated several structural models including models incorporating time-varying asset risk premium and jump-diffusion process.

Perli and Nayda (2004) proposed a structural model for revolving retail credit that employing exactly the same approach of the corporate models, considering that a consumer is in default if its assets are lower than a certain threshold.

Recently, several authors adopted a different approach to valuate the risk debt (for example, Ericsson et al., 2005) and used credit default swaps as a proxy to companies default risk. These authors argue that structural models provide prices to those derivatives coherent with those observed on the markets.

Although the structural approach is based on a powerful and compelling interpretation of a firm's credit risk, implementation is complicated by the fact that the firm's asset values cannot be directly observed, as assumed, for example, by Jarrow and Turnbull (2000). It seems that the pertinent parameters of a structural credit risk model cannot be estimated. In fact, the implementation difficulty motivates an alternative approach known as reduced-form, which considers corporate default as an event governed by an exogenous shock that is not based on the firm's asset value failing to cover its debt obligation.

There are several studies within this approach: in Madan and Unal (2000) a model for the value of assets and liabilities is presented and default occurs when a single and random loss, occurring at a random time, is larger than the value of equity. Duffie and Lando (2001) were the first to show that structural models can be transferred into reduced form models by limiting the information set of the market participants. Giesecke (2006) extends this idea to the case where the default barrier is unobservable; Zhou (2001) adds an unpredictable jump time to the asset value process, and Collin-Dufresne et al. (2002) assume that investors receive lagged information about the asset value process. Further models that assume incomplete information about the asset value process are the ones by Giesecke and Goldberg (2004), and by Cetin et al. (2004), and Guo et al. (2005a; 2005b). Bakshi et al.'s (2006) model is a reduced-form model based on Vasicek-type state variables.

The model by Kealhofer, McQuown and Vasicek (McQuown, 1993; Kealhofer, 1993; Vasicek, 1984), who founded the KMV Corporation in the late 80's, is a model much used in practical applications.

In the following section the model, known as the KMV Black and Scholes model, is described. The main weak points of the model will be analyzed and the *modified* KMV B&S model – overcoming those limitations – will be proposed.

2. The KMV B&S model for the evaluation of credit risk and its weaknesses

This section illustrates the structure of the KMV B&S model, with special reference to its criticalities, and it aims to suggest some improvements through a series of new approaches.

2.1. Description of the model. The model assessing a firm's default probability developed by the KMV Corporation is based on the so-called expected default frequency (EDF). Such procedure employed in the KMV model is based on 4 steps (Crosbie, 2003):

1. Estimation of asset value and volatility.
2. Calculation of the Distance to Default;
3. Calculation of the Expected Default Frequency (EDF).
4. Calculation of the default rate for a given level of Distance to Default.

Interesting results have been obtained by applying the Black & Scholes formula to the Credit Monitor model by KMV. The final phase of such model may rely on mathematical and financial features rather than being based on an empirical model. The former features if the default probability is calculated by applying the Black and Scholes approach – and, therefore, starting from the hypothesis of a normal distribution of the values of the firm's asset – instead of calculating it by making reference to a sample of companies. The default probability has been defined by KMV as the probability for the value of the assets to fall below the default point. Summing up the aforesaid in analytical terms, we obtain:

$$P_t = \Pr\left[A_t \le D_{pt} \middle| A_0 = A_t\right] = \\ = \Pr\left[\ln(A_t) \le \ln(D_{pt}) \middle| A_0 = A_t\right], \tag{1}$$

where P_t is the default probability at a given time "t" (between 1 and 7 years), while A_t and D_{pt} are, respectively, the market value of the company's assets and its default point, both referred to the t time in space. Moreover, it is given that the evolution of the asset value over time follows the law:

$$dA = \mu A dt + \sigma_A A dz.$$

As a consequence of the Black & Scholes model, we can conclude that the value of the asset in "t" is (hypothesizing that its value in zero is equal to A):

$$\ln(A_t) = \ln(A) + \left(\mu - \frac{\sigma_A^2}{2}\right)t + \sigma_A\sqrt{t}\varepsilon. \tag{2}$$

where μ and ε represent the expected profit deriving from the firm's activities, and an aleatory component on the profit itself.

If we combine (1) and (2), the default probability may be expressed as follows:

$$P_t = \Pr\left[\ln(A_t) = \ln(A) + \left(\mu - \frac{\sigma_A^2}{2}\right)t + \\ + \sigma_A\sqrt{t}\varepsilon \le \ln(D_{pt})\right] \tag{3}$$

That is, after having carried out some simple steps:

$$P_t = \Pr\left[-\frac{\ln\frac{A}{D_{pt}} + \left(\mu - \frac{\sigma_A^2}{2}\right)t}{\sigma_A\sqrt{t}} \le \varepsilon\right]. \tag{4}$$

Assuming now that (ε) is a random draw of a standard normal distribution, we can then define the default probability in terms of cumulative standard normal distribution, i.e.:

$$P_t = N\left[-\frac{\ln\frac{A}{D_{pt}} + \left(\mu - \frac{\sigma_A^2}{2}\right)t}{\sigma_A\sqrt{t}} \le \varepsilon\right]. \tag{5}$$

Considering that the distance to default indicates how far a company is from bankruptcy, we can then state that in the Black & Scholes procedure *DD* is equal to:

$$DD = \frac{\ln\frac{A}{D_{pt}} + \left(\mu - \frac{\sigma_A^2}{2}\right)t}{\sigma_A\sqrt{t}}. \tag{6}$$

It has been observed, in fact, that if the value of the activities is defined by (2), the difference between this value and the Default Point (i.e. the Market Net Worth) is exactly equal to the numerator of the above-mentioned formula. If we consider that the denominator of the formula above is made up by the volatility of the profits, we can conclude that (6) actually reproduces the classical formula of the Distance to Default. From the combination of the last two formulas we can infer the following:

$$EDF = N(-DD). \tag{7}$$

Under the hypothesis of the neutrality towards risk, the expected profit of the assets (μ) equals to an free risk rate (r), at any time, because the return of an asset without risk must be the risk-free rate (see Black and Sholes models). Therefore, (5) becomes as follows:

$$P_t = N\left[-\frac{\ln\frac{A}{D_{pt}} + \left(\mu - \sigma_A^2\right)t}{\sigma_A\sqrt{t}}\right]. \tag{8}$$

We have, thus, determined, the Distance to Default through the KMV B&S model.

2.2. Main weaknesses of the KMV B&S model. The application of the Black & Scholes model provides the considerable advantage of allowing the calculation of the default probability through a purely mathematical-financial approach (i.e. without any reference to empirical sets of data), despite some intrinsic limits. According to the authors, this model presents more criticalities, but the aim of this paper is try to overcome this inaccuracies:

1. Equations (5) and (6) show that the Distance to Default (DD) and, subsequently, the Expected Default Frequency (EDF), take into consideration only the assets increase generated by the rate $\left(\mu - \frac{\sigma_A^2}{2}\right)$, thereby neglecting cash outflows linked to dividends and interests.
2. The empirical distribution taken as a reference by KMV attributes higher probability levels to extreme events than the ones of the normal probability distribution. In other words, the normal shows more limited effects than the distribution taken as a reference by KMV and resulting from empirical data.

The following section proposes a simplified model that attempts to overcome the above mentioned criticalities. Moreover, the empirical research carried out is also described and it refers to the application of the simplified model on a sample of Italian high-tech firms.

3. The modified KMV B&S model and its application on a sample of Italian high-tech firms

This section illustrates the *modified* KMV B&S Model and its application on a sample of Italian high-tech firms. The last section compares the results obtained with those of the original KMV B&S (non-modified) model. In particular we will illustrate:

1. The proposed modified KMV B&S model.
2. The characteristics of the selected sample.
3. The methodology.
4. The comparison and analysis of the results obtained.
5. Discussion and comments.

3.1. Proposal of correction: the modified KMV B&S model. This section illustrates two proposals of correction for the two above-mentioned criticalities. We maintain that if they are applied to the KMV B&S model, such corrections could improve the results obtained by the application of the model itself. The proposals of amendments are:

1. To consider also any coupon detachment.
2. To consider the t-Student probability distribution as a replacement of the normal distribution.

As for correction (1), the insight allowing us to consider any potential dividends is easily understandable: in the calculation of the future value of assets and of its standard deviation, through the Black & Scholes formula, one of the sets of input data is the current value of market shares (E_0), which is calculated as the current price of the share multiplied by the number of shares on the market ($P_0 \times N^o Shares = E_0$). When a coupon detachment occurs it is assumed that the price of the share decreases as a result of the detachment itself. Such decrease accounts for 80% of the amount of the dividend at the date of detachment, also taking into account the fiscal effect.

The idea is the following: to discount the dividend, to subtract it from the current price ($P' = P_0 - 0,8 \times \times (D_{iv}e^{-rt})$) and then to calculate the current value of shares on the market by using the new price ($E_1 = P' \times \times N^o Shares$). The current value of the shares calculated as such will also consider any dividends, thus affecting the estimate of the asset and its standard deviation.

As for correction (2), one of the main differences between the estimates carried out through the empirical distribution of KMV Corporation and the ones carried out through a normal distribution is, as stated above, the remarkable difference of the value of probability that the two distribution functions associate to extreme events: more technically, an empirical distribution has characterized of fat tails than a normal distribution. In order to overcome this limitation, and given the numerousness of firms in the sample assessed, the Student's t-distribution with 5 degrees of freedom has been proposed for the calculation of the default probability. The Student's t-distribution, as shown in the figure below, looks like the standard Normal distribution; however, its repercussions are slightly more severe, in other word this distribution is characterized of a kurtosis greater than normal distribution, and this peculiarity according to the authors, allow us to replicate the empiric distribution of KMV (Figure 1).

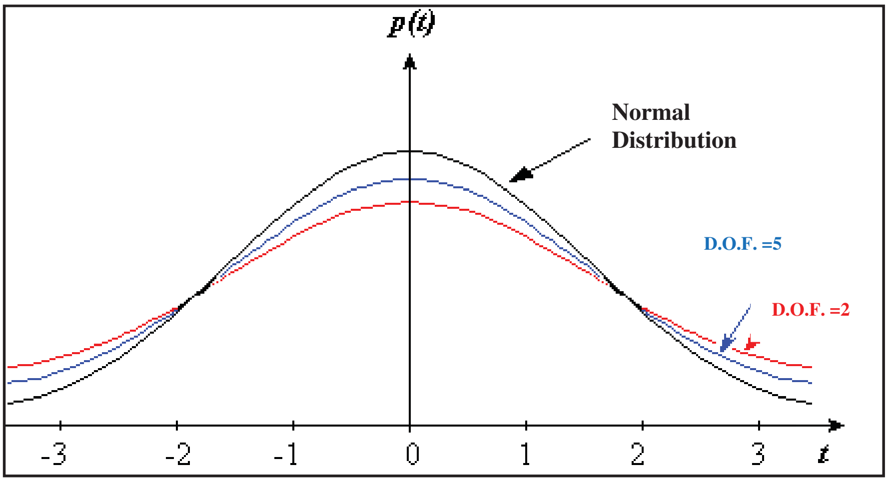

Fig. 1. Normal and Student's t-distribution (2 and 5 degrees of freedom)

3.2. The sample of firms and data source. The analysis comes from tests done on a set of firms including both bankrupt and high-tech companies. The latter are quoted on the Stock Exchange and belong to the following categories: mobile, software and computer services, technology hardware and fixed line telecommunication. Table 1 below lists all the firms taken into consi-deration.

Table 1. The sample of high-tech and bankrupt firms considered

Sample of analysis				Bankrupt firms
Acotel	Cad It	Cdc	Eutelia	Ackermans & Van (XET)
Buongiorno	Dada	Dmt	Fastweb	Agr group (XET)
	Engineering II	Eems Italia	Retelit	Amt Amstd. Moleculr (XET)
	Exprivia	Esprinet	Telecom Italia	Arco Vara (XET)
	Full Six	Eurotech		Atlas Copco 'B'(XET)
	KR Energy	IT Way		Austrian Airlains (XET)
	Noemalife	Olidata		Banco Guipuzcoano (XET)
	Reply	Screen Sevice		
	Tas			
	Tiscali			
	Txt E-Solution			

The high-tech sector has been chosen because competitiveness in this field is more based on intangible elements than in other sectors: the assets for these firms are largely immaterial. Therefore, it is easier for a firm in this sector to face difficulties as compared to other sectors, such as, for example, basic sectors, where investments in tangible resources may confer greater stability to the firms and, thus, lower exposure to credit risk.

Data has been collected from Thomson DataStream Advance, one of the most comprehensive databases available, and taken from the firms' financial reports drawn in 9 consecutive years (from 2001 to 2009).

3.3. Methodology for credit risk evaluation. The first step of the application of the KMV B&S model is the definition of the following data, for each firm in the sample: (1) total debt; (2) shares volatility; (3) risk-free interest rate; (4) expectation horizon; (5) net amount of the last dividend paid per share; (6) time to coupon detachment (in months); (7) current price of a share; (8) number of shares issued; (9) default point.

As concerns the risk-free interest rate, the rate usually employed as a benchmark is the *Libor/swap rate*. The credit default swap rates are usually employed to assess traders' interest rates in order to evaluate derivatives and, thus default probabilities; credit default swaps are agreements offering insurance against the risk of a borrower firm not repaying a loan. The implicit interest rate of these agreements is equal to the *Libor/swap rates* less 10 *basis points* (b.p.).

In order to assess default probability we employed the risk-free *Euro swap rates* less 10 *basis points*. Once the balance-sheet items have been collected, and the risk-free interest rate has been defined (Euro swap rates – 10 b.p.), the procedure for the calculation of default risk requires the use of the Microsoft Excel Solver tool.

In order to define the default point we must calculate the amount of the debt considered. The default point is defined as the sum of short-term debt + half of the medium/long-term debt.

After having acquired such data, we can calculate the current value of shares (E), through the formula illustrated below:

$(E = P' \times N^{o} Shares)$, where

$(P' = P_0 - 0{,}8 \times (D_{iv} e^{-rt}))$.

Once the current value of shares is available, we have all the items to assess the asset (A), and its volatility (σ_A) by means of the Black & Scholes formula. Subsequently, Distance to Default (DD), Market Net Worth (MNW) and Expected Default Frequency (EDF) are defined. At this stage, we look for the values of the asset and its volatility able to satisfy all the conditions required, by means of the Excel Solver tool, thereby finally defining DD, MNW and EDF.

From a computational point of view, we ended up with the following hypotheses:

1. The dividend resulting from the application of the modified model is the same of the last dividend that we have at disposal. In general, it is possible to estimate the dividend using the Gordon-Shapiro method or similar one.
2. We assume the borderline EDF basis point number equal to 2500.

4. Results

The results obtained through the application of the modified KMV B&S model are reported below. They are divided in two groups: (1) the first is a set of firms that went bankrupt during the year subsequent to the last balance available; (2) the second concerns the lasting ones, that are the subject of our study. Furthermore, these results have been compared to the ones obtained applying the non-modified model.

The figures in Appendix report the expected temporal horizon on the abscissa axis (from 1 to 7 years), whereas the levels of default probability are expressed in basis points on the ordinate axis.

4.1. Bankrupt firms. The application of modified model points out that the EDF (Expected Default Frequency) of the bankrupt firms is greater than 2500 basis point during the first year of previsions (Figure 1c). The firm that has the highest EDF is Banco Guipuzcoano, with 3884.73 basis points. The others are characterized by EDFs included in an interval between the 2500 and the 3000 basis points.

4.2. Sample of analysis. *4.2.1. Mobile manufacturing companies.* The application of the modified model points out that the EDF (Expected Default Frequency) of the Acotel group has an average of 28.954 basis points in the expected temporal horizon (7 years), and the highest default probability occurs in the second year (Figure 4a).

The Buongiorno S.p.A. has far higher EDF values than the Acotel group: the default probability hits a peak (801.96 b.p.) in the first year, and it decreases in the following years reaching a level of 14.85 b.p. in the seventh year. The average value of the EDF is 3.27% (327.214 b.p.).

If we apply the non-modified KMV B&S model (Figure 4b) lower levels of default probability are registered: the levels of default probability for the Acotel group are almost null, whereas Buongiorno S.p.A. hits a peak in the first year (1.423 b.p.). The levels of EDF decrease gradually in the remaining years.

When taking into account the Distance to Default (Figure 4c) the lowest DD values are detected in the second year, both for Acotel and for Buongiorno (4.77 and 2.53, respectively), while the highest values are registered in both cases in the last year of the forecasts.

4.2.2. Software and computer service firms. Figure 5a highlights the EDF of the Tas firm, which has a probability of almost 20.53% (2053.65 b.p.) for a default to occur in the first year, while in the second year the EDF reaches 14.71% (1471 b.p.). The EDF then gradually tends to zero in the remaining years.

Tiscali also displays a high level of EDF with 7% of probability (702.08 b.p.) for the default to occur in the first year, 5.12% (512.45 b.p.) for the default to occur in the second year, and, finally, 1.26% (126.42 b.p.) in the third year. Other companies, such as Exprivia (525.9 b.p.), Dada (504.63 b.p.) and TXT-E solution (433.23 b.p.) have the highest values of EDF across the first year.

The application of the non-modified model registers lower EDF values for all the firms belonging to this category: Tas has a default probability of 8.58 b.p. in the first year; this value halves in the second year and is close to zero in the third year. Worth mentioning is the value of Tiscali for the first year (1.08 b.p.). The remaining firms have almost null probability levels (Figure 5b).

The corresponding DD figure underlines that Cad It has the highest DD value in the first year, followed by Noemalife, Reply and Engineering. During the second and the third year hierarchies remain unaltered, while from the fifth to the seventh year the highest DD value is displayed by Noemalife. It is evident that the companies with higher default probabilities are those characterized by a lower Distance to Default (Figure 5c).

4.2.3. Technology hardware firms. Through the application of the modified model it has been observed that Olidata has 17.25% (1725.21 b.p.) default probability in the first year; in the second year the EDF value decreases to 9.75% (974.97 b.p.); and in the remaining years the probability values flatten (Figure 6a). The Cdc company hits a peak of 10.78% (1078 b.p.) in the first year, and its EDF values decrease gradually in the remaining years in keeping with the trend of the majority of the firms belonging to this category. Dmt (8.70%, 870.61 b.p.) and Eurotech (6.86%, 686.45 b.p.) have relevant default probabilities during the first year, while the remaining firms display lower EDF values.

The application of the original model highlights probability levels close to zero, a feature shared by all sampled firms. Olidata shows the highest EDF values (5.58 b.p.), a probability which is clearly lower than the percentage point. Cdc, Dmt and Eurotech respectively register the following levels: 2.52 b.p; 1.68 b.p.; 1.035 b.p. The remaining EDF levels are close to zero (Figure 6b).

Like the DD figures above, the Distance to Default figure (Figure 6c) displays increasing curves: during the first year the average is 2.975, which increases over the years up to 8.05 in the last year.

4.2.4. Fixed line telecommunication firms. The application of the modified model shows that Eutelia is the firm displaying the highest default probability during the first year, 18.51% (1851.7 b.p.), see Figure 7a. During the second year, the EDF remains at considerable levels, i.e., 17.14% (1714.77 b.p.). The probability levels curve, tends to decrease over time and – at the seventh year – the EDF reaches 0.22% (21.85 b.p.). The remaining firms, including Telecom Italia, are rather stable, since their EDF levels are slightly higher than 1% (Fastweb − 0.98%, Retelit − 1.06% and Telecom Italia − 0.65%). During the second year peaks are reached by Fastweb, Retelit and Telecom, while Eutelia registers a slight decrease in its EDF levels. From the third year onwards, probability levels progressively decrease up to the seventh year when EDF levels are well below the percentage point.

The application of the non-modified model indicates that Eutelia is the firm with the highest EDF levels: 6.23 b.p. in the first year and 5.18 in the second year, then plummeting in the third year (1.1 b.p.) and becoming close to zero in the remaining years. The other firms belonging to this category display EDF levels close to zero (Figure 7b).

As for the Distance to Default parameter in this category, the DD values have an average of 3.50 b.p. across the first year, and then slightly decrease in the second year (3.20) to increase again up to an average of 8.32 (Figure 7c) in the seventh year.

The overall analysis of the sample shows that the firm with the highest default probability is Tas, followed by Eutelia and Olidata. They also share a common characteristic, i.e. their debt structure feature short-term debts much higher than the medium/long-term debts. As the description of the methodology calculating the Expected Default Frequency points out, this feature affects the definition of the default point and, consequently, also the EDF.

Discussion and conclusions

When calculating a firm's default probability there are three relevant levels of information: the financial data, the market prices of shares and debts, and, finally, the subjective estimates of the firm's future risks and perspectives. By considering that financial data mainly indicates what happened in the past while market prices are the result of the forecasts on the future of a firm, we can observe that the default probability calculation procedure is forward and backward looking. This is quite evident since this procedure should be able to combine historical data and market data.

The inclusion of market shares and debt prices among the essential elements for the calculation of the default probability is not based on the assumption that markets are fully efficient; the aim is only to underline that the outcomes obtained by making reference to market data represent a satisfactory result which could not be easily achieved through alternative methods.

The model described in this paper allows to assess the default probability through both market and budget data, so to provide an adequate and real-like estimate. Similarly to other mathematical and financial models – among which the structural models based on the theory of options – the modified model may serve as a "guideline" to assess the EDF of a given firm. However, some variables often neglected by these models (i.e. the firm's immaterial and human components and the sector where the firm operates) must be taken into account. The combination of such data allows a clearer vision of a firm's credit risk (Iazzolino, Pietrantonio, 2005; Iazzolino et al., 2012). For example, the firm Eutelia, belonging to the sample employed in this paper, has experienced a very gloomy period and it has been declared insolvent in June 2010. The reasons for this situation are also strictly related to the human resources capital working in the firm.

Our assumption to consider the EDF of default as 2.500 basis points has been validated thanks to the tests done on the first sample of firms that went bankrupt.

As compared with the non-modified KMV B&S model, the model proposed in this paper allows us to have:

- better perception of the intrinsic default risk of a company. For example, the same firm registers EDF levels of about 23% (2298.94 b.p.) if the modified model is applied, while its EDF levels would be 0.086% (8.58 b.p.) with estimates carried out through the non-modified model. The first value is surely more relevant for analysts;
- better evidence of the differences between two firms or among a set of companies, since the differences in relative terms are more clearly outlined. The comparison between the results of the data from two firms is carried out through numerical values in tens, rather than through values almost close to zero.

If we combine two factors below, more accurate estimates can be achieved. On the one hand, the innovation components characterizing the modified model are the theoretical prerequisites to obtain resuls which are more consistent with reality; on the otherhand, such results, highlighting the greater difference in default probability levels obtained through the comparison with the non-modified model, allow us to achieve a better definition of credit risk.

However, an analogy between the (non-modified and the modified) models is evident: the firms displaying relatively low (or vice versa high) EDF levels are the same, despite the enormous difference among their absolute values. The evaluation scale changes. This implies that the changes applied to the original model do not alter the results achieved; instead they try to improve estimates and to attribute more accurate and appropriate values to such estimates, thereby providing a clearer picture of the firm's default risk.

As previously shown, the application of the non-modified model to the sample of firms provides EDF values extremely low in absolute terms: the highest EDF levels across the first year are registered by the TAS firm (0.085%), followed by Eutelia (0.062%) and Olidata (0.055%). Such levels, however, stabilize below the percentage point (Figure 1). The application of the modified model shows that TAS, Eutelia and Olidata have EDF levels for the first year equal to 20.53%, 18.5% and 17.25%, which are far above the percentage point (Figure 2) and define the firm's credit risk more clearly.

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Appendix

This section shows the figures for the application of the models used for analysis.

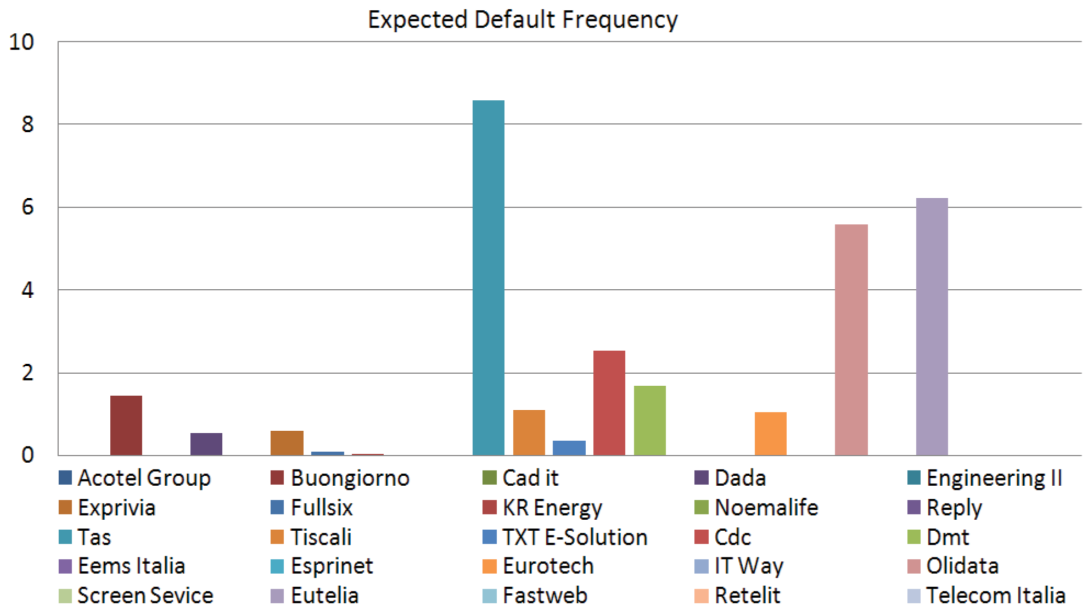


Fig. 1. EDF values of firms in the sample during the first year of forecast (unmodified model)

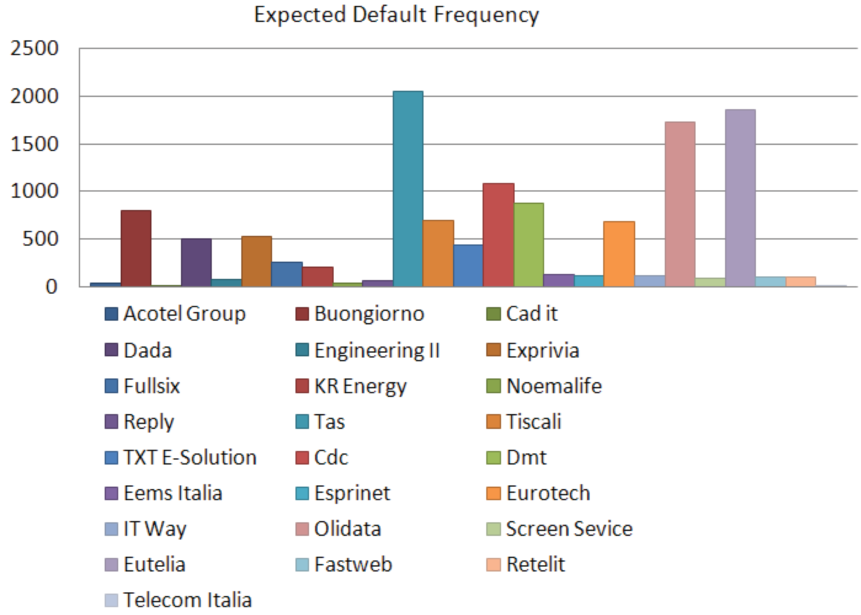


Fig. 2. EDF values of firms in the sample during the first year of forecast (modified model)

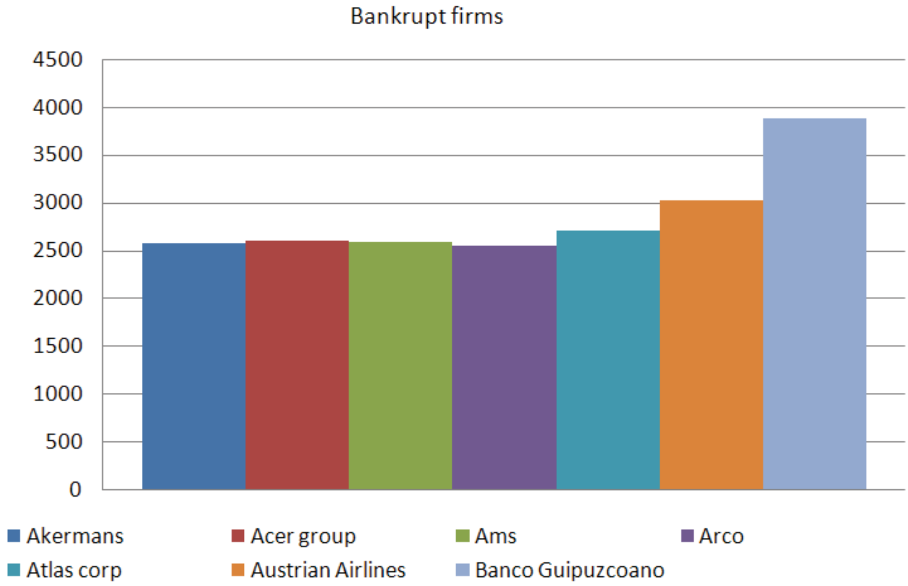
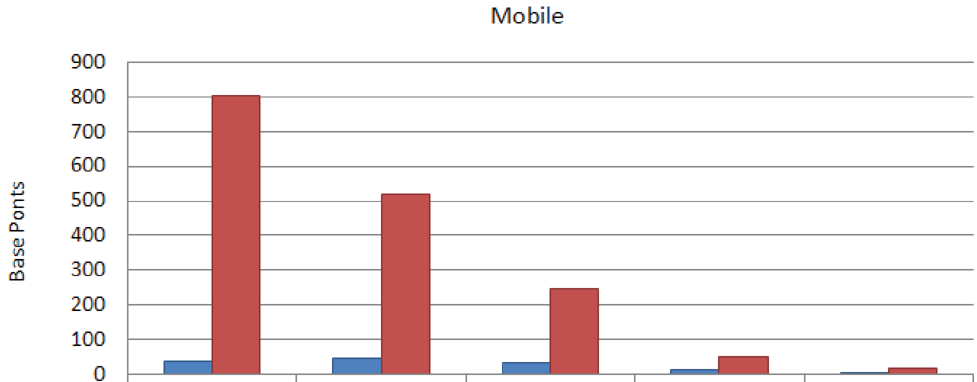
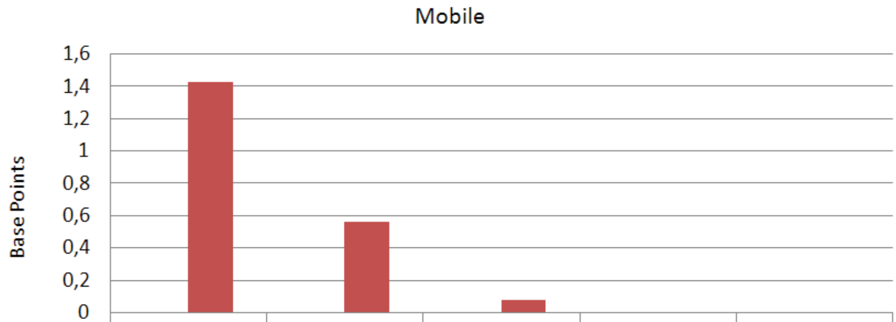


Fig. 3. EDF of bankrupt firms during the first year of forecast



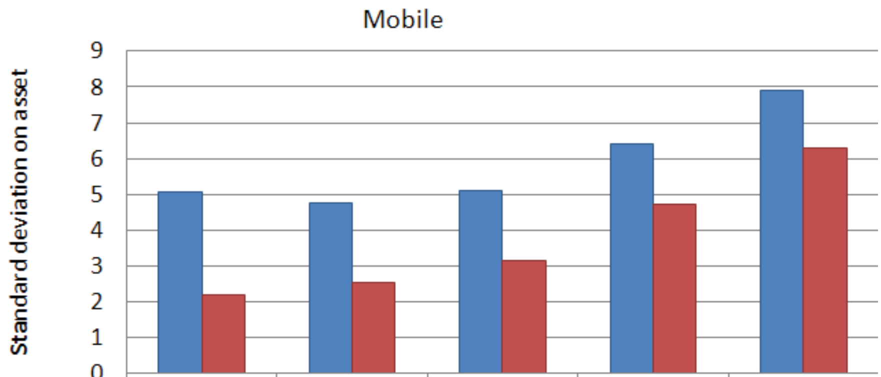
	1	2	3	5	7
Acotel Group	38,84	49,88	37,14	13,69	5,22
Buongiorno	801,96	520,83	246,6	51,83	14,85

Fig. 4a. EDF of firms belonging to the mobile category (modified model)



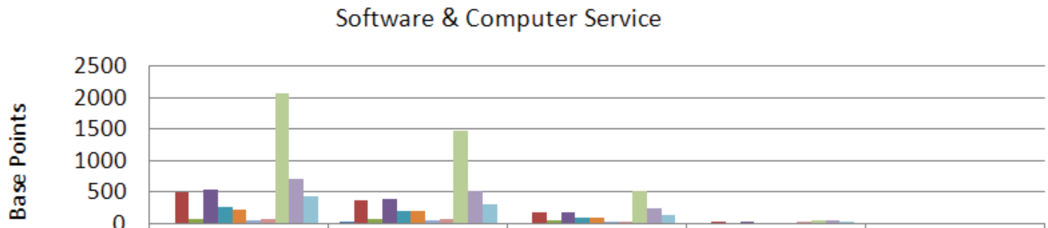
	1	2	3	5	7
Acotel Group	2E-05	8E-05	1E-05	7E-05	0
Buongiorno	1,423	0,56	0,074	0,00011	0

Fig. 4b. EDF of firms belonging to the mobile category (unmodified model)



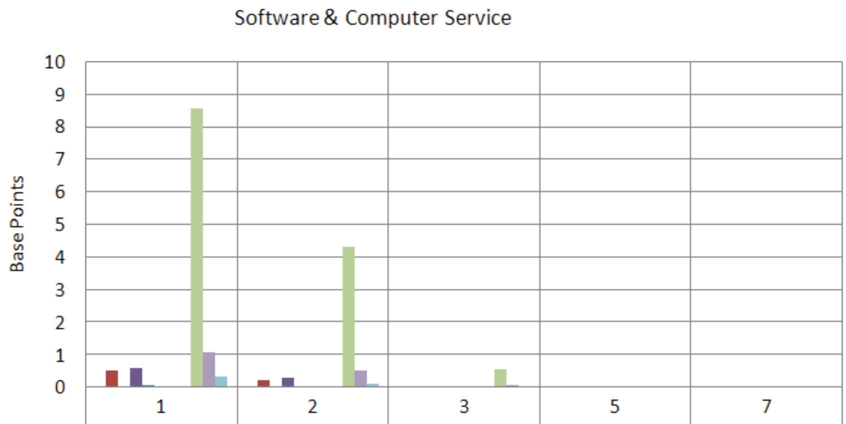
	1	2	3	5	7
Acotel Group	5,06	4,77	5,11	6,41	7,9
Buongiorno	2,19	2,53	3,17	4,73	6,29

Fig. 4c. DD of firms belonging to the mobile category



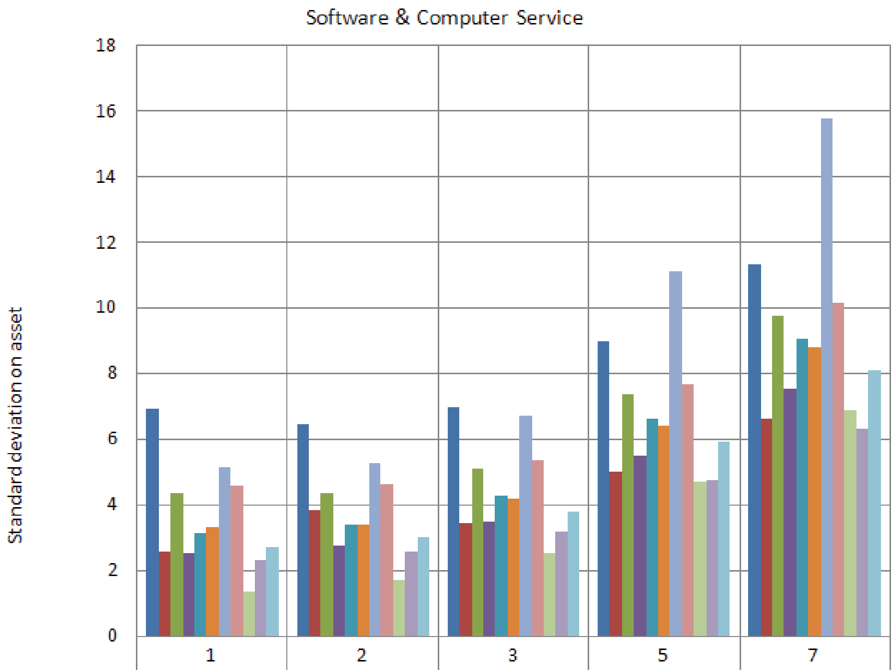
	1	2	3	5	7
Cad it	9,69	13,41	9,5	2,89	0,95
Dada	504,63	372,62	186,68	41,15	12
Engineering II	74,72	73,03	37,6	7,35	1,93
Exprivia	525,9	396,43	172,4	27,18	6,48
Fullsix	263,87	192,01	80,1	11,86	2,72
KR Energy	211,53	194,82	88	13,62	3,18
Noemalife	36,77	33,26	11,25	1,03	0,18
Reply	59,31	58,75	30,53	15,41	1,59
Tas	2053,65	1471,5	525,87	53,59	10
Tiscali	702,08	512,45	247,92	51,69	14,68
TXT E-Solution	433,23	298,35	126,42	19,79	4,7

Fig. 5a. EDF of firms belonging to the software & computer services category (modified model)



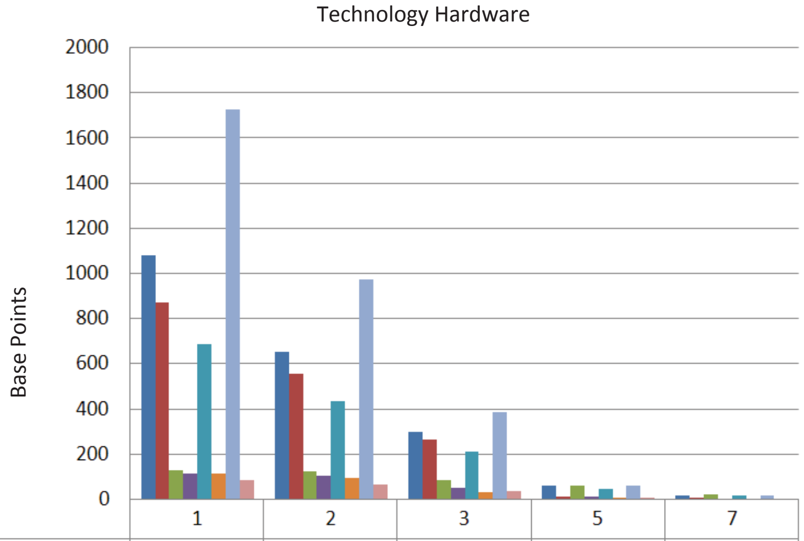
	1	2	3	5	7
Cad it	0	0	0	0	0
Dada	0,5189	0,24	0,03	2,91E-05	0
Engineering II	0,000732	0,000656	1,67E-05	0	0
Exprivia	0,572	0,285	0,023	0	0
Fullsix	0,092	0,033	0,001	0	0
KR Energy	0,046	0,035	0,0015	0	0
Noemalife	1,45E-05	7,6E-06	0	0	0
Reply	0,00023	0,00021	4,3E-06	0	0

Fig. 5b. EDF of firms belonging to the software & computer services category (unmodified model)



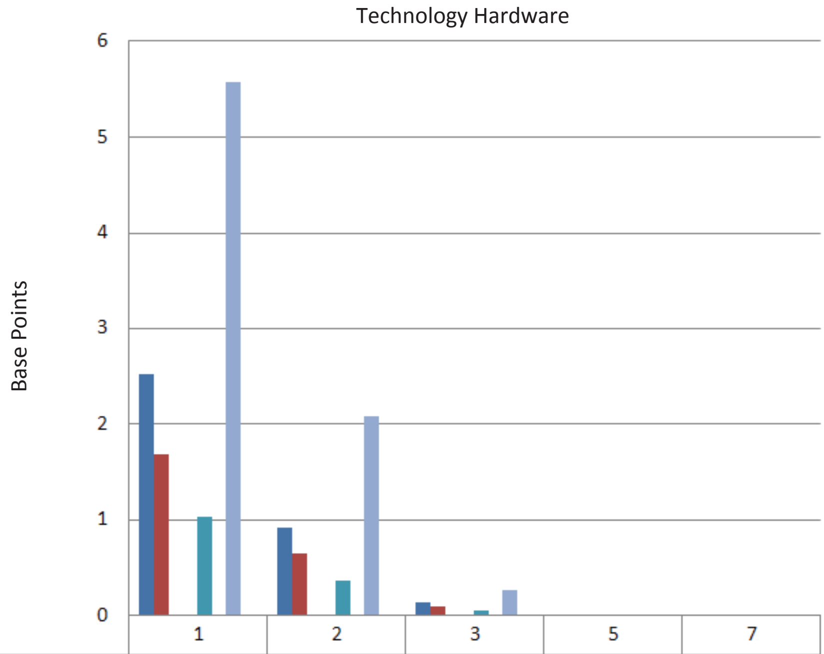
	1	2	3	5	7
Cad it	6,91	6,44	6,95	8,95	11,3
Dada	2,56	3,81	3,42	5	6,6
Engineering II	4,33	4,36	5,1	7,34	9,74
Exprivia	2,53	2,76	3,5	5,5	7,54
Fullsix	3,11	3,4	4,26	6,61	9,06
KR Energy	3,31	3,39	4,16	6,41	8,77
Noemalife	5,13	5,25	6,69	11,1	15,73
Reply	4,58	4,59	5,35	7,66	10,15
Tas	1,36	1,71	2,53	4,69	6,86
Tiscali	2,29	2,55	3,17	4,73	6,31
TXT E-Solution	2,69	3	3,8	5,9	8,07

Fig. 5c. DD of firms belonging to the software & computer services category



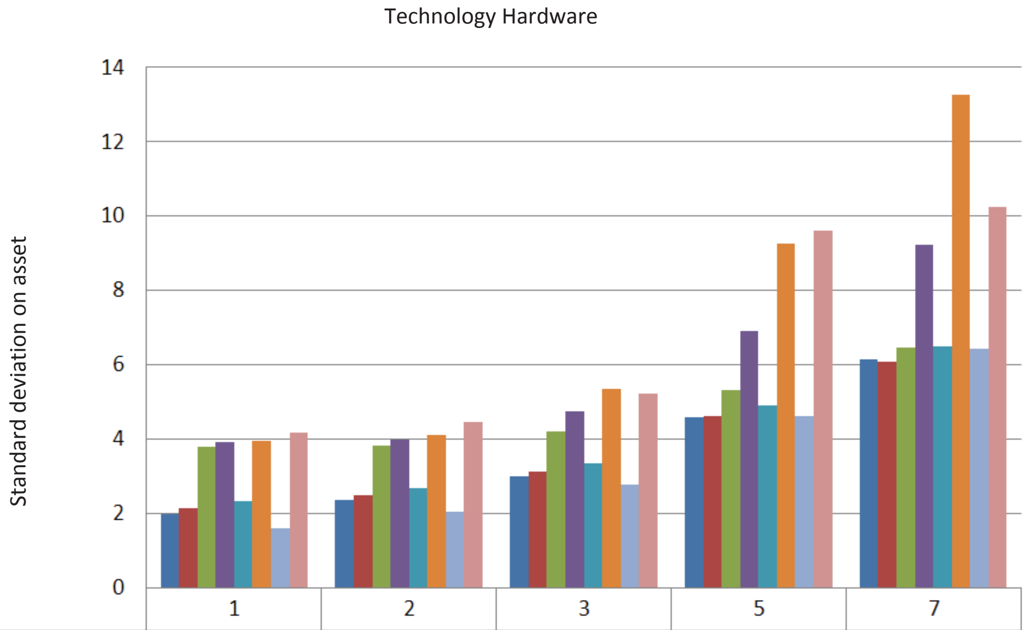
	1	2	3	5	7
Cdc	1078	651	295	59	16,5
Dmt	870,61	554	264,67	10,63	5,13
Eems Italia	127,78	123,14	84	58,05	17,18
Esprinet	112,1	104,7	52	9,7	2,5
Eurotech	686,45	435,3	208,08	44,73	12,95
IT Way	110,08	93,93	30,2	2,48	0,43
Olidata	1725,21	974,97	385,7	57,5	13,46
Screen Sevice	85,9	66,8	34,27	7,4	2,06

Fig. 6a. EDF of firms belonging to the technology hardware category (modified model)



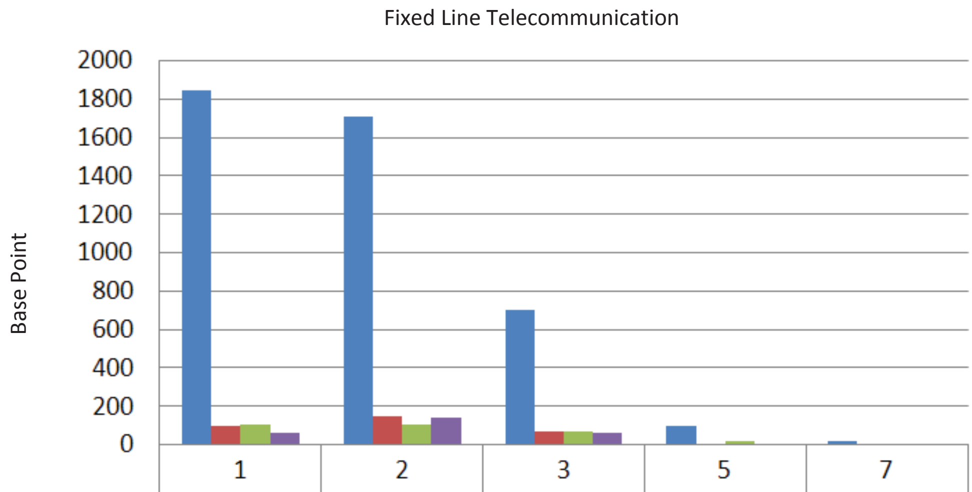
	1	2	3	5	7
Cdc	2,52	0,92	0,127	0,00022	0
Dmt	1,68	0,645	0,092	0,0002	0
Eems Italia	0,0075	0,0065	0,0012	5E-06	0
Esprinet	0,0044	0,0033	0,00011	0	0
Eurotech	1,035	0,362	0,043	4,8E-05	0
IT Way	0,00414	0,0021	3E-06	0	0
Olidata	5,58	2,09	0,265	0,0002	0
Screen Sevice	0,0014	0,001	4E-05	0	0

Fig. 6b. EDF of firms belonging to the technology hardware category (unmodified model)



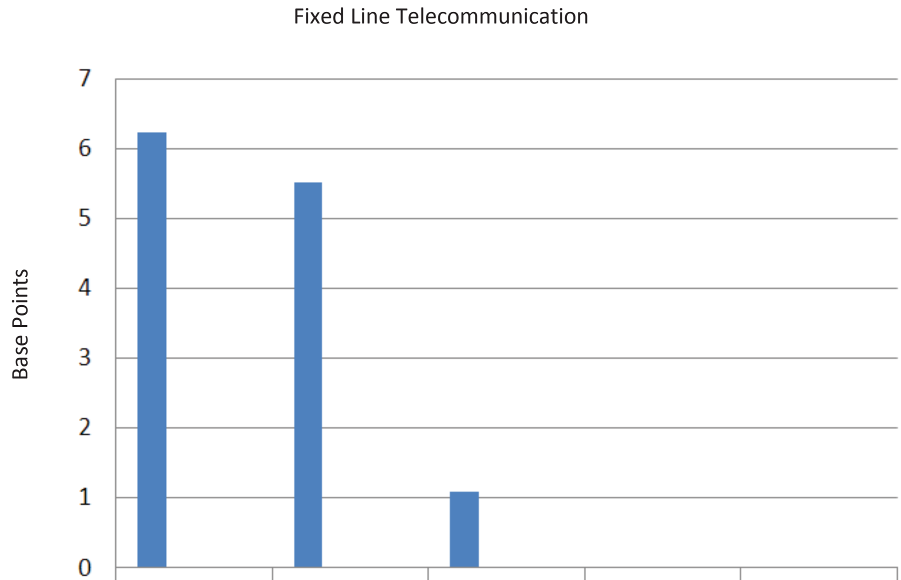
	1	2	3	5	7
Cdc	1,96	2,36	3	4,59	6,15
Dmt	2,12	2,49	3,11	4,6	6,09
Eems Italia	3,78	3,82	4,21	5,3	6,47
Esprinet	3,91	3,98	4,73	6,91	9,24
Eurotech	2,31	2,68	3,33	4,9	6,5
IT Way	3,93	4,1	5,36	9,25	13,28
Olidata	1,59	2,03	2,78	4,62	6,43
Screen Sevice	4,18	4,45	5,21	9,62	10,24

Fig. 6c. DD of firms belonging to the technology hardware category



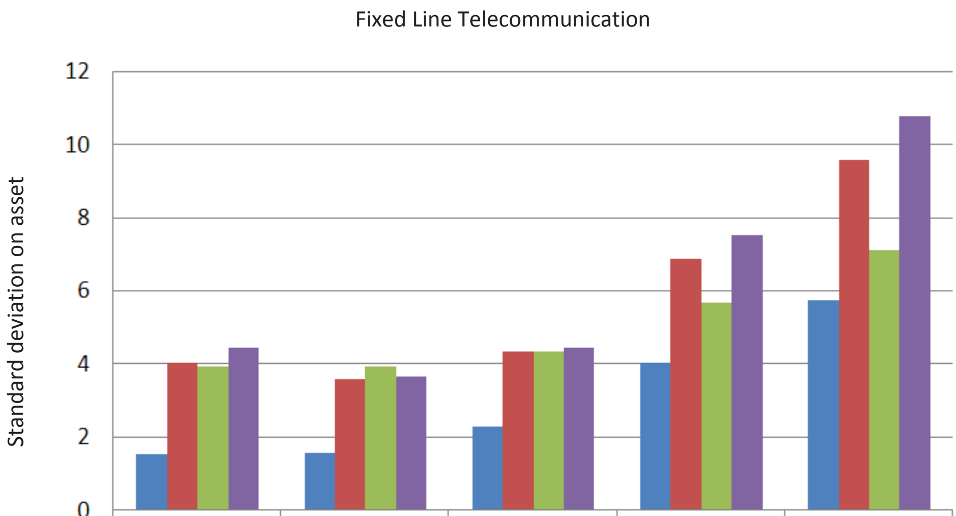
	1	2	3	5	7
Eutelia	1851,7	1714,77	707,23	98,04	21,85
Fastweb	98,23	152,65	73,49	9,78	2,08
Retelit	106,5	110,07	72,21	23,32	8,39
Telecom Italia	65,85	142,79	65,97	6,46	1,17

Fig. 7a. EDF of firms belonging to the fixed line telecommunication category (modified model)



	1	2	3	5	7
Eutelia	6,23	5,52	1,1	0,0025	0
Fastweb	0,0025	0,015	0,00067	0	0
Retelit	0,0036	0,0041	0,00062	0	0
Telecom Italia	0,0004	0,011	0,00039	0	0

Fig. 7b. EDF of firms belonging to the fixed line telecommunication category (unmodified model)



	1	2	3	5	7
Eutelia	1,53	1,59	2,29	4,05	5,77
Fastweb	4,05	3,61	4,35	6,9	9,59
Retelit	3,96	3,93	4,37	5,69	7,13
Telecom Italia	4,47	3,68	4,46	7,54	10,8

Fig. 7c. DD of firms belonging to the fixed line telecommunication category