“Obesity overtaken by leanness as a repeated game: social networks and indirect reciprocity”

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Obesity overtaken by leanness as a repeated game: social networks and indirect reciprocity

Abstract

Recent research shows that social networks appear to explain obesity and leanness. A conceptual model of how these networks cause such an effect can be derived from economic and biological studies of the Iterated Prisoner’s Dilemma (IPD). This analysis has potential implications for the evolution of norms and policies designed to combat obesity. The paper first briefly considers some results in game theory, then turns to ecological and economic models based on the IPD. Alternative results are considered in light of social networking theory. Finally, it considers possible policy responses to obesity.

Keywords: obesity, social networks, game theory.

JEL Classification: I1, I10, I19, I18.

Introduction

Obesity in the United States, and increasingly internationally, is linked to socioeconomic status (SES) (Sobal and Stunkard, 1989). In the U.S. a poor person is more likely to be obese than a person who is not poor, all else equal (Ball and Crawford, 2005). Poverty creates numerous constraints that inhibit physical exercise and encourage the consumption of calorie-dense food (Drewnowski, 2003; Drewnowski and Specter, 2004). The linkage is especially pronounced among certain subpopulations of the poor: African-American females, Hispanics, and Native Americans (Zhang and Wang, 2004; Runge, 2009). It would seem that reducing disparities between rich and poor would lead to reductions in obesity. But there is an alternative interpretation to observed obesity patterns, which while acknowledging the importance of reducing poverty, does not trace obesity to differences in SES as such, but to the reference groups and social networks in which different strata of society live and communicate. These differences in social networks, while correlated with SES, may be more powerful explanations than SES alone in explaining obesity. In this paper, we propose a game-theoretic interpretation of social interaction and norm formation and simulate various interactions within subgroups to test the hypothesis that peer group effects can be powerful in explaining an “obseogenic niche”.

This perspective was explored empirically by Christakis and Fowler, who published an expanded version of their New England Journal of Medicine article (Christakis and Fowler, 2007) in Connected (2009). They showed that social networks could help explain patterns of obesity (among other behaviors). Using data from the Framingham Offspring Study, they found a person’s chances of becoming obese increased by 57 percent if they had a friend who became obese. The percentages were 40 percent for siblings and 37 percent for spouses. Moreover, the connection was not found among geographical neighbors or among those who had stopped smoking. Christakis and Fowler emphasized that the increase in U.S. obesity from 23 percent to 31 percent from 1971-2002, with 66 percent of U.S. adults overweight, ‘cannot be explained by genetics’, concluding that the spread of obesity in social networks ‘suggests that it may be possible to harness the same force to slow the spread of obesity’, and that networks ‘might be exploited to spread positive health behaviors’ (see Wing and Jeffrey, 1999; Christakis and Fowler, 2007). As they observed, norms may be particularly relevant (emphasis added) (Christakis and Fowler, 2007, p. 377-378).

Other recent work supports the social norms approach to obesity (see Nolan et al., 2008). These norms, upheld by a principle of reciprocity, can link individuals through their shared approach to social interactions even in relatively large and seemingly disconnected groups (Sugden, 1984, 2004). Furthermore, they may cut across lines of SES, confounding efforts to link obesity or leanness to SES alone. Manski (2000) considers social norms and the role of social groupings in eliciting expectations of others’ behavior. Kapinos and Yakusheva (2010) explore whether students assigned to the same dormitory influence each others’ weight gain in college. Hoekstra et al. (2010) consider similar peer effects on physical fitness in squadrons of Air

1 Cohen-Cole and Fletcher (2008) (CCF) offered detailed criticism of the statistical inferences drawn by Christakis and Fowler, but could not perform an exact replication due to data limitations. One of the main issues concerns the direction of causation: do social networks lead individuals to become obese together, or do the already obese form social networks? Our analysis takes their criticism into account by examining the coevolution of endogenous and environmental factors explaining obesity. In a detailed reply to CCF, Fowler and Christakis (2008, p. 1387) argue that for all specifications of their model, there is broad consistency with the CCF estimates, and that the Christakis and Fowler estimates fall within the confidence intervals of CCF.
Force Academy cadets assigned at random. Since dorm or squadron assignments are not based on previous weight or fitness, they argue against the claim that social network formation is endogenous to these characteristics.

In this paper, we examine the difficulty of getting a norm of leanness to take root and spread within and across networks. A variety of game-theoretic tools can be used to model strategic interactions within and outside of social networks. Here we use the Iterated Prisoner’s Dilemma (IPD) to demonstrate how social networks can affect body type choice over time. We use the IPD game framework for several reasons, apart from its general acceptance in the literature. First, it captures the essential payoff structure associated with body type choice: returns to a body type choice have private costs and benefits which are affected by the choices of others in their network and those outside it. Second, the repeated framework allows for the evolution of choices and networks over time and for a particular body choice to become dominant within and across social networks (for a discussion of norm evolution, see Jackson, 2008).

Our analysis has potential implications for the evolution of norms and policies designed to combat obesity. The paper first briefly considers how an IPD game can model strategic interactions among individuals within and across networks. We then consider the circumstances under which an individual would find it in their best interest to retain their original body type or to adopt another body type. This binary choice provides a simple framework to consider the private and public benefits and costs of such behavior (see Schelling, 1973). Next, we consider the circumstances under which a sub-population of obese, when joined by a sub-population of lean people, would find it in their best interest to choose leanness as a body type. Finally, we consider possible policy responses to obesity given our findings.

1. Body weight: a game theoretic approach

To introduce the notion that the payoff to an individual choosing healthy weight (be lean or BL) or obesity (be obese or BO) depends on the behavior of others we use a one-period normal form game where the payoff a player receives from his body type choice will be a function of the other’s choice. When we distinguish between leanness and obesity in this game we are not primarily focused on their physical manifestations; there is a continuum of body types between lean and obese that we cannot hope to capture in our reduced games of strategic interaction between people. Instead we consider body type choice a collection of decisions that are either consistent with becoming leaner and the associated benefits and costs or over time (BL) or activities that tend to lead to weight gain and its associated benefits and costs over time (BO).

The two options can be viewed as a choice between cooperation and defection where choosing leanness (BL) is the choice of cooperation and choosing obesity (BO) represents defection. Obesity is a defection from the social perspective because society will be worse off due to the well-documented social costs of obesity, including lost productivity, absences, underperformance, higher insurance premia, and large government expenditures on health care. Conversely, by maintaining a lean body type, a person helps keep societal health care costs to a minimum. Let the social benefit that a player receives from the other player choosing BL be given by $b_0 > 0$.

Body type choice also creates private benefits and costs. Overweight individuals suffer from increased morbidity and mortality (Runge, 2007) and worsening mental health (Oswald and Powdthavee, 2007). Obesity can also limit other social and economic opportunities, only some of which can be quantified. The benefits of obesity are less time spent exercising and more time spent working or pursuing other interests and the satisfaction from eating without restrictions. Let the net private benefit of choosing BO be normalized at 1. The private benefits to BL are primarily the inverse of obesity’s private costs, including avoided medical costs and greater social and economic opportunities than those faced by the obese. Let the private benefits associated with BL be given by $b_1 > 0$ and its cost, including time spent exercising and the distress of resisting excess caloric intake, is given by $c$. Therefore, the net private benefits to BL are $b_1 - c$. The payoff structure to this game is presented in Table 1.

Table 1. The payoff structure

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Player 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Be lean (BL)</td>
<td>Be lean (BL)</td>
</tr>
<tr>
<td>Be obese (BO)</td>
<td>$b_1 + 1$, $b_1 - c$</td>
</tr>
</tbody>
</table>

1 Buskens and Weesie (2000) proposed a general model of cooperation in social networks, using a Heterogeneous Trust Game. More recently, Oswald and Powdthavee (2009) have analyzed the impacts of obesity on the psychological distress of populations in Great Britain and Germany.  

2 The continuum in physical body types is expressed by the Body Mass Index (BMI). Interestingly there is some question as to whether BMI accurately captures obesity and leanness (Burkhauser and Cawley, 2008).

3 In other work applying this analysis to commons dilemmas, cooperative strategies in the commons are termed “stinting”, or holding back, which fits this case, at least figuratively (Runge, 1981).
As long as the personal net benefit associated with BL is one or less \((b_s - c \leq 1)\), BO is the dominant individual strategy. However, if \(b_s + b_o - c > 1\) then the optimal strategy set from society’s perspective is \((BL_1, BL_2)\) (see Note 1 in the paper’s Appendix). In this case the divergence in optimal private and public strategies makes this a PD\(^1\).

The sub-optimal outcome of this game can be overcome by some form of outside enforcement (Sen, 1967) or social expectations. With respect to the latter influence, the more assured Player 1 is that Player 2 will play BL, the more reason Player 2 has to play BL. In other words, social relationships are key because they can overcome the selfish incentive to choose BO and instead lead to socially optimal reciprocal strategies (see Sen, 1967; Runge, 1981, 1984). And expectations, in turn, can be influenced by social norms (Runge, 1984; Fehr and Fischbacher, 2002; Blanchflower et al., 2009).

These norms may vary significantly across subpopulations and these subpopulations can ‘lock in’ to a BL equilibrium.

Thomas Schelling (1973) demonstrated graphically how assurance can overcome selfish behavior in a population of \(N\) players who either play BL or BO, or “binary externalities.” In panel (A) of Figure 1 (See Appendix) the payoffs to BO always dominate BL (a multiperson PD). Panel (B) shows the opposite: the payoffs to BL always dominate the payoffs to BO. Panel (C) shows a situation where playing BO is an individual’s best strategy when most other people play BO but at some ‘critical mass’ of others playing BL \((n^* \text{ in the graph})\) the payoff to BL begins to dominate that of BO (Schelling, 1960, p. 91; Runge, 1986). In his illustration Schelling accredited this switch in best response to the ‘pain of conspicuousness’. In our case the cost of this pain overrides any benefits associated with free-riding on the growing social benefits created by an increasingly lean population. We will return to these situations below, showing how subpopulations can lock in to a (BO, BO) or a (BL, BL) equilibrium, and how a critical mass can tip a subpopulation from one equilibrium to the other \(^2\).

\(^1\) The normal form above will not be a PD game if the net private benefits of BL are always greater than the private benefits associated with BO (here \(b_s - c > 1\)). In that case leanness is a strategy that both individuals will choose.

\(^2\) Myerson (2009, p. 5), in a review of the contributions of Thomas Schelling to the role of multiple equilibria in games of strategy, noted that in such games ‘anything in a game’s environment or history that focuses the players’ attention on one equilibrium may lead them to expect it, and so rationally to play it. This focal point effect opens the door for cultural and environmental factors to influence rational behavior.’ In a famous example, Schelling showed that if dimes and pennies were distributed at random on a checkerboard in equal number, and each chose only to move closer to coins like itself, within two moves the board was completely segregated into two dissimilar subpopulations.

2. Evolution of norms: a repeated game framework

A payoff system that supports defection in a one-round PD game may support other outcomes if the game is iterated repeatedly (IPD), allowing for a form of contingent reciprocity (Taylor, 1976; Axelrod and Hamilton, 1981; Axelrod, 1984; Axelrod and Dion, 1988; Fehr and Fischbacher, 2002). First, we consider under what circumstances an individual in a network nested within a larger population will choose to be lean or obese (we will consider the returns to leanness and obesity across a population of networks and the circumstances under which a switch to leanness is beneficial to all members of the population, no matter their network, in the next section). Whether or not an individual will adopt leanness in the IPD game is a function of, among other things, the probability of continued interaction between players (i.e., the compactness or strength of the networks) and the extent to which people discount the future. We employ a specific form of the multi-person IPD developed by Nowak and Sigmund, and explicated by McElreath and Boyd (2007) (see also Stephens et al., 2002; Nowak and Sigmund, 2004; Fowler and Christakis, 2010)\(^3\).

Assume that two players from some population are matched for perpetuity in a strategic game over body type decisions. This match could represent the relationship between two friends or family members. Alternatively, player 2 could be viewed as a composite of the multiple individuals in player 1’s network (assuming that player 1’s network members are homogenous in their body choice strategy). One equilibrium strategy in IPD’s is “tit-for-tat” (see Axelrod, 1984; Axelrod and Dion, 1988). If a player employs the “tit-for-tat” (TFT) strategy then he/she will play BL in the first round. He/she finds that his/her match is another TFTer, where such matches are represented by TFT|TFT, he/she will continue to play this strategy indefinitely. Otherwise if he/she is matched with a person who always chooses BO (ABO), where such matches are represented by TFT|ABO or ABO|TFT, he/she will choose BO in the second round of interaction and continue to choose BO indefinitely. An ABOer will always play BO no matter their partner. In this game structure continued rounds of play are probabilistic; at some point a player may stop interacting with their partner (network). In the strategic relationships modeled in this section an individual never changes networks.

\(^3\) In this sense, the evolving norms reflect both social networks and the context or environment, which co-evolve. These are the endogenous and contextual interactions that are so difficult to separate, especially in empirical work (Cohen-Cole and Fletcher, 2008, p. 1384).
Once a person no longer interacts with their network they are no longer relevant to this game.

The probability of further interaction in any round beyond the first between paired TFTers is given by $w_{\text{TFT}}$ while the probability of subsequent interaction between two ABOers is given by $w_{\text{ABO}}$. A TFTer will interact with an ABO partner in future rounds with probability $w_{\text{TFT}}$ (and vice-versa). Here we assume $w_{\text{TFT}} \leq w_{\text{ABO}}$; the probability of further interactions between two similar types is more likely than interactions between two opposite types (Cavalli-Sforza and Feldman, 1983, p. 2017). As the probability of social interaction increases, the 'compactness' or density of the interaction increases\(^1\).

Therefore, by assuming $w \leq w_{\text{TFT}}$ we assume that TFTers who switch to playing ABO after the first round of interaction with their network will find themselves in a less compact relationship than before due to divergent norms. TFTers and ABOers have rates of time preference given by $\delta_{\text{TFT}} \in [0, 1]$ and $\delta_{\text{ABO}} \in [0, 1]$, respectively, where TFTers who adopt an obese body take on rate $\delta_{\text{ABO}}$. We generally assume that $\delta_{\text{TFT}} < \delta_{\text{ABO}}$. In other words, choosing obesity is associated with a higher rate of time preference (time rate preferences are state-dependent) (see Komlos et al., 2003). However, the assumption of a disparity in time preference can be dropped and $\delta_{\text{TFT}} = \delta_{\text{ABO}}$ (or we could even set $\delta_{\text{TFT}} > \delta_{\text{ABO}}$).

The expected (discounted) payoff to the TFT player assuming their match is a TFTer is\(^2\):

$$V(TFT|TFT) = b_s + b_0(n_{\text{TFT}}) - c + \left( \frac{w_{\text{TFT}}}{1 + \delta_{\text{TFT}}} \right) (b_s + b_0(n_{\text{TFT}}) - c) + \left( \frac{w_{\text{TFT}}}{1 + \delta_{\text{TFT}}} \right)^2 (b_s + b_0(n_{\text{TFT}}) - c) \ldots$$

(1)

Once the two players stop interacting with their opposites their payoffs are 0. The social benefit from leanness is proportional to the number of TFT individuals in the population. In round 1 this population is given by $n_{\text{TFT}}$. In round 2 and beyond the population of TFTers is equal to the number of TFTers in relationships with other TFTers, given by $n_{\text{TFT}\mid\text{TFT}}$ (i.e., if there are 5 TFT|TFT relationships in round 2 then $n_{\text{TFT}\mid\text{TFT}} = 10$). Let $b_0$ be increasing in $n_{\text{TFT}}$ and $n_{\text{TFT}\mid\text{TFT}}$ at a decreasing rate. By definition $n_{\text{TFT}\mid\text{TFT}} \leq n_{\text{TFT}}$. We assume the same relationships among $b_0$, $b_s$, and $c$ as in the one-round PD game above; namely, $b_0 + b_s - c > 1$ as long as $n_{\text{TFT}} \geq 2$ in round 1 and $n_{\text{TFT}\mid\text{TFT}} \geq 2$ in all subsequent rounds.

Equation (1) simplifies to:

$$V(TFT|TFT) = \frac{(b_s + b_0(n_{\text{TFT}}) - c)(1 + \delta_{\text{TFT}}) + (b_0(n_{\text{TFT}\mid\text{TFT}}) - b_0(n_{\text{TFT}}))w_{\text{TFT}}}{1 + \delta_{\text{TFT}} - w_{\text{TFT}}}.$$  

(2)

As shown in Note 2 of the Appendix, the overall payoff to a TFTer when the other player is a TFTer increases as the probability of subsequent interactions $w_{\text{TFT}}$ increases and as the rate $\delta_{\text{TFT}}$ decreases. Hence, TFTers are selfishly interested in increasing the odds of social interactions in the future with their matched TFTer and will engage in activities that increase the probability of such interactions. Moreover, their payoff to such interaction increases the more they value the future relative to the present, as reflected by a low rate of time preference.

Now suppose a TFTer is paired with an ABOer. In this case the expected payoff to the TFT player is,

$$V(TFT|ABO) = b_s + b_0(n_{\text{TFT}}) - c + \frac{w_{\text{TFT}}}{1 + \delta_{\text{ABO}}}(b_0(n_{\text{TFT}\mid\text{ABO}}) + 1)\left( \frac{w_{\text{TFT}}}{1 + \delta_{\text{ABO}}} \right)^2 \ldots$$

(3)

Equation (3) simplifies to:

$$V(TFT|ABO) = b_s + b_0(n_{\text{TFT}}) - c + \frac{b_0(n_{\text{TFT}\mid\text{ABO}}) + 1)w_{\text{TFT}}}{1 + \delta_{\text{ABO}}}.$$  

(4)

\(^1\) Note that this interaction does not necessarily imply geographic proximity.

\(^2\) The payoff is the same if we assume each subsequent interaction involves a different lean TFT individual (McElreath and Boyd, 2007). All payoffs in this section can be interpreted in such an alternative way. We will use this structure later.
According to equation (4), the payoff to a TFTer networked with an ABOer decreases in $\delta_{ABO}$ and increases in $w$ (see Note 3 in the Appendix).

Now suppose player 1 is an ABOer and interacts with a TFTer. Player 1’s expected payoff includes ‘free-riding’ on her partner’s and the larger public’s leanness. In this case player 1 is partly responsible for the reduction in the public benefit of leanness in the second round and beyond given his/her influence on his/her network partner’s body type choice.

$$V(ABO|TFT) = b_0(n_{TFT}) + 1 + (b_0(n_{TFT|TFT}) + 1) \frac{w}{1 + \delta_{ABO}} + (b_0(n_{TFT|TFT}) + 1) \left( \frac{w}{1 + \delta_{ABO}} \right)^2 + \ldots$$

(5)

Equation (5) reduces to:

$$V(ABO|TFT) = \frac{(b_0(n_{TFT}) + 1)(1 + \delta_{ABO}) + w(b_0(n_{TFT|TFT}) - b_0(n_{TFT}))}{1 + \delta_{ABO} - w}.$$  

(6)

The payoff to an ABO individual in an ABO|TFT network decreases in $\delta_{ABO}$ and increases in $w$ (see Note 4 in the Appendix). Therefore, an ABO individual benefits from increased interaction with the newly obese in their network and a lower time preference rate. Finally, if both players in a network are ABOers then they remain so over time and the (discounted) expected body type choice payoff to both is:

$$V(ABO|ABO) = b_0(n_{TFT}) + 1 + (b_0(n_{TFT|TFT}) + 1) \frac{w_{ABO}}{1 + \delta_{ABO}} + (b_0(n_{TFT|TFT}) + 1) \left( \frac{w_{ABO}}{1 + \delta_{ABO}} \right)^2 + \ldots$$

(7)

Equation (7) reduces to:

$$V(ABO|ABO) = \frac{(b_0(n_{TFT}) + 1)(1 + \delta_{ABO}) + w_{ABO}(b_0(n_{TFT|TFT}) - b_0(n_{TFT}))}{1 + \delta_{ABO} - w_{ABO}}.$$  

(8)

As before, the (discounted) expected payoff increases as the expected frequency of subsequent interactions in the network increases and $\delta_{ABO}$ decreases (see Note 5 in the Appendix).

Therefore, in all network combinations a player’s value function will increase in the expected frequency of subsequent interactions with their partners, whether their partner is their type at first or not. While these results are conventional, we now turn to the more important question of under what conditions it pays for an individual to abandon their a priori norm and adopt the competing norm.

$$\frac{(b_0(n_{TFT}) + 1)(1 + \delta_{TFT}) + b_0(n_{TFT|TFT})w_{TFT}}{1 + \delta_{TFT} - w_{TFT}} > \frac{1 + \delta_{ABO} + w_{ABO}(n_{TFT|TFT})}{1 + \delta_{ABO} - w}.$$  

(9)

Of particular interest is the parameter $b_0(n_{TFT|TFT})$. How large does the positive social externality of leanness have to be in the second round and beyond for leanness to dominate in a TFT|TFT relationship?

$$b_0(n_{TFT|TFT}) > \frac{(1 + \delta_{TFT} - w_{TFT})(1 + \delta_{ABO}) + (1 + \delta_{TFT})(c - b_0)(1 + \delta_{ABO} - w)}{w_{TFT}(1 + \delta_{ABO}) - w(1 + \delta_{TFT})}.$$  

(10)

3. Conditions for switching strategies

First, consider the conditions that need to prevail in order for leanness in TFTers to remain the optimal body type in a strategic relationship between two TFTers. This will occur as long as $V(TFT|TFT) > V(ABO|TFT)$. Otherwise a TFT individual will have incentive to switch to an obese body type in round 1 (i.e., act as an ABOer). For now assume that player 1 is aware of the body type in their network before they choose their body type (but not vice-versa). $V(TFT|TFT) > V(ABO|TFT)$ when,

...
Inequality (10) highlights several points. First, as $n_{TFT|TFT}$ increases, all else equal, inequality (10) is more likely to hold. Second, the greater the likelihood of future interactions between TFTers (given by an increasing $w_{TFT}$), the lower the net cost of leanness ($c - b_s$), and the lower the lean TFTer’s rate of time preference the more likely leanness is to continue even if the public benefits of leanness are relatively low. Otherwise TFT individuals may be tempted to act like ABOers.

The effect of $\delta_{ABO}$ on defection likelihood requires a bit more analysis. Let $b_s(n_{TFT|TFT})$ be a threshold value for which any $b_s(n_{TFT|TFT})$ greater than $b_s(n_{TFT|TFT})$ causes inequality (10) to hold. When we calculate the change in $b_s(n_{TFT|TFT})$ with respect to a change in $\delta_{ABO}$, we get

$$
\frac{\partial b_s(n_{TFT|TFT})}{\partial \delta_{ABO}} = \frac{1 + \delta_{TFT} - w_{TFT}(1 + \delta_{TFT})(1 + c - b_s)w}{((1 + \delta_{ABO})w_{TFT} - (1 + \delta_{TFT})w)^2}.
$$

This fraction is always equal to or less than 0 ($c - b_s \geq -1$). In words, the greater the rate of time preference associated with BO the smaller player 2 can credibly threaten a priori that a defection to obesity will result in reduced interaction in the future (a sort of shunning punishment that promotes the ‘pain of conspicuousness’) then the temptation to defect is curtailed (see Boyd et al., 2010).

In fact, a divergence in social compactness across networks or time preference across types is necessary to support leanness as an equilibrium strategy decreases as well. Therefore, if player 2 can credibly threaten a priori that a defection to obesity will result in reduced interaction in the future (a sort of shunning punishment that promotes the ‘pain of conspicuousness’) then the temptation to defect is curtailed (see Boyd et al., 2010).

Now let us consider the case of two matched ABO individuals and the likelihood of defection to leanness on one of the player’s part. An ABO individual would find it profitable to switch to leanness for the first round (but revert back to ABO after the first round as stipulated by the rules of the IPD) if they know they are playing a fellow ABOer and $V(TET|ABO) > V(ABO|ABO)$.

If rates of time preference and relationship compactness are the same across subpopulations, i.e., $\delta_{TFT} = \delta_{ABO}$ and $w = w_{TFT} = w_{ABO}$, then inequality (9) reduces to:

$$
(1 + \delta)(b_s - c - 1) > 0.
$$

This inequality cannot hold given the payoff structure of $b_s - c \leq 1$. This is just a reiteration of the normal form PD game: leanness is never a winning strategy if the compactness of relationships and discount parameters on the incentive to defect are conventional. The most interesting and policy-relevant result is associated with the effect of $w$ on possible defection. The change in $b_s(n_{TFT|TFT})$ with respect to a change in $w$ is:

$$
\frac{\partial b_s(n_{TFT|TFT})}{\partial w} = \frac{1 + \delta_{TFT} + 1 + \delta_{TFT} - w_{TFT}}{(1 + \delta_{ABO})w_{TFT} - (1 + \delta_{TFT})w)^2}.
$$

This fraction is always equal to or greater than 0. As the likelihood of future interaction between the defecting TFTer and their TFT partner decreases ($w$ falls), the value of $b_s$ sufficient to support leanness as an equilibrium strategy decreases as well. Therefore, if player 2 can credibly threaten a priori that a defection to obesity will result in reduced interaction in the future (a sort of shunning punishment that promotes the ‘pain of conspicuousness’) then the temptation to defect is curtailed (see Boyd et al., 2010).

In fact, a divergence in social compactness across networks or time preference across types is necessary to support leanness as an equilibrium body type choice in the IPD game. If the social compactness parameter is equal across all network combinations, i.e., $w = w_{TFT} = w_{ABO}$, but $\delta_{ABO}$ and $\delta_{TFT}$ are different, then inequality (9) will hold for certain values of $b_s$, $c, b_s(n_{TFT|TFT}), \delta_{ABO}$ and $\delta_{TFT}$. Conversely, if rates of time preference are equal across types but social compactness differs across network combinations, inequality (9) will hold for certain values of $b_s$, $c$, $b_s(n_{TFT|TFT})$, and $\delta$ (see Note 6 in the Appendix). If rates of time preference and relationship compactness are the same across subpopulations, this can be viewed as player 1’s punishment for defection from obesity. In this case let the private benefits of temporary leanness be the relevant threshold value. If,
then an individual in an ABO subpopulation will gain more by adopting a new norm of leanness all

\[
\frac{1 + \delta_{ABO} + w_{ABO} h_0(n_{TFT})}{1 + \delta_{ABO} - w_{ABO}} \leq 1 + \frac{(h_0(n_{TFT}) + 1)w}{1 + \delta_{ABO} - w}
\]

or \( h_0(n_{TFT}) \leq \frac{w - w_{ABO}}{w_{ABO} - w} \).

No matter the relationship between \( w \) and \( w_{ABO} \), the right-hand side of this last inequality is either negative or 0. The value of \( b_0(n_{TFT}) \) can never be negative and 0 implies there are no TFTers in the population. Therefore, inequality (17) can never hold in a mixed population. In other words, it will never be in an ABO’s interest to defect to leanness temporarily if they know they are to be matched up with another obese individual.

This situation is consistent with empirical evidence that obese subpopulations tend to remain so and breaking out of this suboptimal equilibrium, even temporarily, is extremely difficult (Popkin and Udry, 1998; Hedley et al., 2004; Lee et al., 2009). This circumstance represents an ‘obesogenic niche’ (Wells, 2009). It also suggests why it is very hard for an ABOer in a network of other ABOers to adopt a low rate of time preference when all of his/her network peers have higher rates of time preference. We turn next to the question of whether, and how, one type of network can overtake or ‘invade’ another.

4. Overtaking obese social networks with leanness strategies

The IPD game described above is limiting for several reasons. It does not allow individuals to change networks over time, to change behavior based on these new networks, and does not explicitly consider the effect that the number of lean people versus obese in the population affects people’s body type decisions. Here we amend our IPD game to take these dynamics into account.

In the field of evolutionary biology scientists study the invasion or overtaking of one subpopulation by another. Here we postulate that the same can happen to a subpopulation of obese via the social network mechanism. In particular, a population threshold may be reached where networks will begin to choose leanness because the network begins to pay more than obesity (see panel (C) of Figure 1 in the Appendix).

There is empirical support in favor of a social network switching its norms as the norm becomes more prevalent in surrounding networks. In a 2009 study, Salvy et al. (2009b) found that social interactions, especially with friends, can actually substitute for excess caloric consumption, noting:

‘Individuals are influenced by the eating and activity norms set by those around them, and the results of the present study suggest that friendship can provide an alternative to eating. Drawing on these findings and from the work of others, we contend that decreasing sedentary behavior and increasing leisure activity may require the social structure of meaningful relationships with friends, as friendship may help to promote or ‘socialize’ active lifestyles’ (p. 211).

This view was reinforced in a 2009 report by Koehly and Loscalzo, who argued that ‘by capitalizing on the structure of the network system, a targeted intervention that uses social relationships in families, schools, neighborhoods and communities may be successful in encouraging healthful behaviors among children and their families’ (p. 1).

In a recent unpublished paper, O’Malley and Christakis (2010) find that individuals with similar BMIs are likely to ‘stick together’ in social networks, while those with different BMIs are likely to dissolve such ties, consistent with our results. Similar results occur in simulation studies (Bahr et al., 2009).

As briefly noted in the introduction, in a study of Air Force Academy undergraduates randomly assigned to different squadrons of 30 students that function as social networks, Hoekstra et al. (2010) were able to overcome many of the identification problems which led to criticism of social network studies (eg., Cohen-Cole and Fletcher, 2008). The results were broadly confirmed by Christakis and Fowler (2007; 2009). Specifically, when measures

\[ b_s - c > \frac{1 + \delta_{ABO} + w_{ABO} h_0(n_{TFT})}{1 + \delta_{ABO} - w_{ABO}} - \frac{(h_0(n_{TFT}) + 1)w}{1 + \delta_{ABO} - w} \]

given that \( b_s - c \leq 1 \), a necessary condition for inequality (15) to ever hold is,
of physical fitness within 30 person subpopulations of Air Force cadets are compared, using indicators over and above BMI, if half of the ‘alters’ in a subpopulation (squadron) is unfit, the probability of an ego fails the basic fitness test triples. As the authors’ state: ‘In equilibrium, our estimates imply that each out-of-shape individual creates two additional out-of-shape individuals through their social actions, thus supporting the provocative notion that obesity spreads on a person-to-person basis’ (p. 15).

Another empirical example supportive of the shifting norms hypothesis is a study based on the weight of 30-60-year-old American women from 1976 to 2000 and data from the CDC’s Behavioral Risk Factor Surveillance System (BRFSS) from 1994 and 2002 (Burke and Heiland, 2007). This econometric analysis considered how norms of ‘desired weights’ by women reflected endogenous upward shifts: in 1994, the average American woman weighed 147 pounds, while desired weight was 132 pounds. By 2002, the average had increased to 153 pounds, and the desired weight to 135 pounds. The authors conclude that these data support this empirical evidence. Suppose there is an out-of-shape individual creates two additional out-of-shape individuals through their social actions, thus supporting the provocative notion that obesity spreads on a person-to-person basis’ (p. 15).

In reality, the transition from one body type to another is costless in both directions.

In this game we use several simplifications. First, the evolution of a player’s body type could maintain some state between obesity and leanness; we just use the two polar extremes. Further, we do not define the length of time needed to complete a round; changes in body types, especially the extreme transitions, can take years. However, recall that our definition of obesity and leanness behavior is not the same as the eventual body type that emerges from this behavior. Instead individuals are choosing behaviors. While adding behavior choices that lie between lean and obese behavior and strictly defining the length of time periods would add more nuance to our treatment, we believe that the simpler exposition presented here still provides important insights into how networks and subgroups within a larger population affect body choice dynamics.

A lean TFTer will meet a fellow lean TFTer in any given round with probability \( r = (n_{TFT} - 1) / (n_{TFT} - 1 + n_{ABO}) \) and an ABOer with probability \( 1 - r = n_{ABO} / (n_{TFT} - 1 + n_{ABO}) \). Further, let \( z = n_{TFT} / (n_{TFT} - 1 + n_{ABO}) \) indicate the probability with which an obese ABOer will meet a lean TFTer in any given round. If we assume that the population \( N = n_{TFT} + n_{ABO} \) is larger then \( r \) and \( z \) converge to \( n_{TFT} / N \), the fraction of the population in a TFT|TFT relationship in any given round is \( r^2 \), and \( n_{TFT} \approx r^2 N^2 \). Finally, in this illustration we assume that the social benefit functional form is \( a X^{0.5} \). Therefore, \( B_0 = b_0(n_{TFT} + n_{ABO}) = a (r^2 N)_{0.5} \) and \( b_0 = b_0(n_{TFT}) = a (r^2 N)^{0.5} \).

In our game-theoretic structure the probability that a lean individual will create a new network with an obese counterpart (and then potentially revert back to a lean counterpart) is proportional to the sizes of the respective subpopulations. One could argue that the probability of a lean individual forming a new network with the obese is unlikely given peoples’ propensity to associate exclusively with their own types. Therefore, the formation and dissolution of networks and oscillations from leanness to obesity among individuals in the population is more dramatic in our setup than it might be in more “compact” subpopulations. In this case the expected payoff from body type decisions in the repeated game for an initially lean TFTer is:

1 In reality, the transition from one body type to another can cause health complications as well and these complications will vary according to the direction of transition and frequency of transitions (see Waring et al., 2010).

2 The fraction of the population in a ABO|TFT or ABO|TFT relationship in any given round is \( 2(1-r) \) and the fraction in a ABO|ABO relationship in any given round is \( (1-r)^2 \).
\[ V(TFT) = r \frac{(b_s + b_0 - c)(1 + \delta_{TFT}) + (B_0 - b_0)w_{TFT}}{1 + \delta_{TFT} - w_{TFT}} + (1 - r) \frac{(B_0 + 1)w}{1 + \delta_{ABO} - w}. \]  

Further, the expected repeated game payoff for an ABOer in this population is:

\[ V(ABO) = (1 - z) \frac{(b_s + 1)(1 + \delta_{ABO}) + (B_0 - b_0)w_{ABO}}{1 + \delta_{ABO} - w_{ABO}} + z \frac{(B_0 + 1)(1 + \delta_{ABO}) + (B_0 - b_0)w}{1 + \delta_{ABO} - w}. \]

We can use the payoffs from the previous section here because subsequent interactions with new but known-type individuals produce the same expected payoffs as playing the same known TFTer or ABOer from the first round on (see McElreath and Boyd, 2007).

A subpopulation of TFTers can outperform the ABOers, and may eventually shift the subpopulation norm in the direction of leanness, if \( V(TFT) > V(ABO) \). Analytically solving for the ratio \( w_{TFT}/N \) that just satisfies \( V(TFT) > V(ABO) \) is difficult. Instead we illustrate the norm competition by estimating the probability that \( V(TFT) > V(ABO) \) for a given value of \( r \) and a set of parameter values.

We consider 5 scenarios. For each scenario we evaluate the relationship between \( V(TFT) \) and \( V(ABO) \) at \( r = 0.100 \) one-thousand (1000) times; in each iteration parameter values are randomly drawn from statistical distributions unique to the scenario (i.e., the range of \( w_{TFT}, w_{ABO} \), etc. change across scenarios). We repeat this same process for the set of \( r \) values 0.105, 0.110,..., 0.900. Therefore, for each scenario we evaluate the relationship between \( V(TFT) \) and \( V(ABO) \) 161,000 times (1000 iterations x 161 values of \( r \)). In every simulation \( N = 1000 \), the distributions of \( b_s, b_o, \) and \( c \) never change, and \( B_0 \) is always given by \( a(r^2 \cdot N)^{0.5} \).

In the scenarios we do not experiment with changes in private and social returns because increases in these returns unequivocally increase the likelihood of \( V(TFT) > V(ABO) \) for all values of \( r \).

### Table 2. Parameter values for scenario analysis

<table>
<thead>
<tr>
<th>Scenario</th>
<th>( w_{TFT} )</th>
<th>( w_{ABO} )</th>
<th>( w )</th>
<th>( \delta_{TFT} )</th>
<th>( \delta_{ABO} )</th>
<th>Others</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[0.80, 0.95]</td>
<td>[0.80, 0.95]</td>
<td>[0.40, \text{min}(w_{TFT}, w_{ABO})]</td>
<td>[0.01, 0.15]</td>
<td>[0.05, 0.25]</td>
<td>( b_o - c \in [3.5, 1]; a = [0.025, 0.350] )</td>
</tr>
</tbody>
</table>

In Table 2 we define each parameter’s statistical distribution under each scenario. In scenario 1 the expected value of \( w_{TFT} \) and \( w_{ABO} \) are equal, the expected value of \( w \) is less than the expected value of \( w_{TFT} \) and \( w_{ABO} \) and the expected value of \( \delta_{TFT} \) is less than \( \delta_{ABO} \). In other words same-type networks have, on average, the same level of compactness, which is larger on average than the compactness of networks of cross-types. Further, lean TFTers value the future more. In scenario 2 the only difference from scenario 1 is that the expected value of \( w_{ABO} \) is larger than the expected value of \( w_{TFT} \) or, in words, networks of obese ABOers have, on average, a higher level of compactness than a network of TFTers. Scenario 3 is the same as scenario 2 except networks of lean TFTers have, on average, a higher level of compactness than networks of obese ABOers. Scenario 4 is exactly the same as scenario 1 except in this case obese ABOers value the future more. Finally, under scenario 5 all compactness measures and discount rates are, on average, indistinguishable across all network types and player types. In Figure 2 the number of times out of 1000 that \( V(TFT) > V(ABO) \) for each unique scenario \( r \) value combination is presented. Each curve represents a scenario.

If we momentarily ignore scenario 5, scenario 3 produces the greatest number of iterations where \( V(TFT) > V(ABO) \) at all levels of \( r \). This is not surprising given the number of advantages that TFTers have in this scenario; compactness among TFT networks is greater on average than it is among ABO networks and the rate of time preference is, on average, lower for TFTers. That scenarios 2 and 4 are the least likely to generate \( V(TFT) > V(ABO) \) is also not surprising given the advantages that ABOers have in these scenarios. In scenario 2 ABOers only have a network compactness advantage. In scenario 4, ABOers have a lower expected rate of time preference and the same expected compactness as among networks of TFTers. That these 2 scenarios produce very similar results (the scenario’s curves in Figure 2 are almost on top of each other) suggests that social network compactness is a larger determinant of iteration results than rates of time preference.

The biggest surprise from Figure 2 is scenario 5. At lower levels of \( r \) this produces more iterations where \( V(TFT) > V(ABO) \) than scenario 3 even though scenario 3 gives several more advantages to TFTer vis-à-vis ABOers. The dynamic that explains this surprising result is the compactness of networks formed by TFT-ABO pairs. At low levels of TFT invasion, high compactness among cross-type pairs provides lean TFTers with a much higher likelihood of continued future payoffs. For illustrative purposes, consider two populations. In the first scenario there is a distinct advantage to being lean: social network compactness among the lean is greater than the compactness of the other
two possible combinations of body types and rates of
time preference are lower for the lean. In the second
scenario there is no distinct advantage to being lean. At
low levels of $r$, however, the ability for TFTers in the
second population to form more compact social net-
works with ABOers (which will occur more frequently
at low $r$) leads to greater returns on average for second
population TFTers than their advantaged counterparts.

To summarize, if TFTers can form social networks
with ABOers that are just as compact as their net-
works with other lean TFTers then $V(TFT) > V(ABO)$
is more likely to occur at low levels of invasion even
if TFTers and ABOers are statistically indistinguish-
able in their characteristics and relationships. How-
ever, as the size of the TFT invasion becomes larger
$V(TFT) > V(ABO)$ is more likely to occur if TFTers
have social network compactness advantages and
lower rates of time preference. Given our payoff val-
ues (the private and public returns to leanness) the
point where this switch occurs is at $r$ value 0.35.

Discussion: implications for SES and obesity
This analysis generates a number of testable proposi-
tions and potential policy implications. First, it sug-
gests that subpopulations that have high rates of obe-
sity are in effect playing a repeated game in which ABO
has locked into an obesogenic niche. Unless they are
forced to interact with lean TFTers obese ABOers will
never have incentive to select leanness. Therefore,
poor black or Hispanic women or Native Americans
who are socially stratified or removed from regular
interactions with subpopulations that choose leanness
can be locked into obesity. Conversely, subpopulations
of TFTers playing leanness will tend to resist invasion
from ABOers if (1) their private costs of being lean are
low (health club membership and recreational exercise
are less costly at the margin as disposable income
rises); (2) the public benefits are high; (3) they have
lower rates of time preference; and (4) lean TFTers
that defect will pay for this anti-social behavior with
subsequently less compact social networks. This last
point is particularly important. Unlike obesity, lean-
ess is not a resilient equilibrium. The norm of lean-
ess will only remain viable in a subpopulation if de-
flection to obesity is accompanied by the threat of sub-
sequent social shunning within the TFT subpopulation
(i.e., the gap between $w_{TFT}$ and $w$ is large).

At policy level, governments can take several ap-
proaches to breaking up compact networks of obesity
(see Cawley, 2010). Algazy et al. (2010) comprehen-
sively analyzed a wide range of interventions to com-
batt obesity, concluding that “the best results are
achieved when entire communities join together to
address multiple causes of obesity simultaneously. The
Communities create social movements that make
healthy eating and exercise the norm” (p. 2). First,
explicit attempts to introduce lean role models
(TFTers) in obese subpopulations – preferably with
preexisting social links to the obese – in order to
change norms can be made (Wing and Jeffery, 1999).
As we saw in our simulation above, in general this
strategy has the best chance of success (changing the
population norm to leanness) when lean TFTers’ rates
of time preference are lower than those of the obese
and $w_{TFT} > w_{ABO} > w$. We say in general here because
the relation $w_{TFT} > w_{ABO} > w$ is not preferable when the
number of introduced lean role models is small relative
to the size of the obese subpopulation; in such cases
$w_{TFT} = w_{ABO} = w$ is more likely to lead to a norm shift.
A second approach that policy can take to reduce obe-
sity is to raise the personal benefits of leanness, reduc-
ing the costs of leanness via incentives and subsidies
(see Thaler and Benartzi, 2004), and increasing the
positive externalities associated with leanness.

We would argue that the policy maker should choose
the approach that generates the greatest reduction in
obesity per policy dollar spent. In other words, policy
can increase the probability that $V(TFT) > V(ABO)$ per
dollar spent by introducing lean role models to obese
subpopulations or by increasing the private and public
benefits ($b_1$ and $b_2$) associated with leanness and
decreasing the costs of maintaining it. The former
approach appears the most problematic given the lack of
policy approaches to increasing social network com-
 pactness and lowering people’s rates of time prefe-
 rence. In addition, the policy maker would need to
consider the number of introduced lean role models
appropriate for the expected relationships among $w_{TFT}$,
$w_{ABO}$, and $w$. Further, we have shown that increasing
social network compactness (especially in TFT and
ABO interactions) and lowering rates of time prefe-
 rence, especially among the obese, can re-enforce
an ABO norm when IPD game parameters differ by type.
Because we may never know exactly how compact
social networks are, or the level of discount rates, or if
they differ across subpopulations, by focusing on net
return policies, policy makers will avoid the potential
perverse result of re-enforcing obesity in a subpopu-
lation. In other words, by focusing on improving private
and public net returns to leanness, policy makers make
the safer policy choice. However, this policy calculus
could change as the regulator becomes more familiar
with social network dynamics and the latest advances
in network theory (Guimerá and Sales-Pardo, 2009). In
the future it may be the case that appropriately har-
nessing the power of social networks to affect norms,
including body choice norms, presents at least part of a
cost-effective approach to solving the global obesity
epidemic.

Acknowledgements
Our thanks to Martha Rogers for research assistance,
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lier version.
References


Appendix

Note 1
Assume $b_s - c \leq 1$. The societal payoff to both players playing obesity is 2. The societal payoff to both players playing thinness is $2(b_s - c + b_o)$. The societal payoff to one player playing thinness and the other playing obesity is $2(b_s - c) + b_o + 1$. Therefore, as long $b_s - c + b_o > 1$ then the greatest gain to society is from both players playing thinness. Given that $b_s - c \leq 1$, it follows that $b_o > 0$.

Note 2: Equation 2
The first derivative of equation (2) with respect to $w_{TFT}$ is,
$$\left[ (b_s + b_o (n_{TFT}) - c) (1 + \delta_{TFT}) \right] / \left( 1 + \delta_{TFT} - w_{TFT} \right)^2,$$
which is always greater than 0 if $n_{TFT} \geq 2$ and undetermined if $n_{TFT} < 2$.

The first derivative of equation (2) with respect to $\delta_{TFT}$ is,
$$\left[ w_{TFT} (c - b_s - b_o (n_{TFT})) \right] / \left( 1 + \delta_{TFT} - w_{TFT} \right)^2,$$
which is always less than 0 if $n_{TFT} \geq 2$ and undetermined if $n_{TFT} < 2$.

Note 3: Equation 4
The first derivative of equation (4) with respect to $w$ is,
$$\left( (b_o (n_{TFT}) + 1) (1 + \delta_{ABO}) \right) / \left( 1 + \delta_{ABO} - w \right)^2.$$
This derivative is always greater than 0.

Note 4: Equation 6
The first derivative of equation (6) with respect to $w$ is,
$$\left( b_o (n_{TFT}) + 1 \right) (1 + \delta_{ABO}) / \left( 1 + \delta_{ABO} - w \right)^2,$$
which is always greater than 0.

The first derivative of equation (6) with respect to $\delta_{ABO}$ is,
$$-w \left( b_o (n_{TFT}) + 1 \right) / \left( 1 + \delta_{ABO} - w \right)^2,$$
which is always less than 0.

Note 5: Equation 8
The first derivative of equation (8) with respect to $w$ is,
$$\left( b_o (n_{TFT}) + 1 \right) (1 + \delta_{ABO}) / \left( 1 + \delta_{ABO} - w \right)^2,$$
which is always greater than 0.

The first derivative of equation (8) with respect to $\delta_{ABO}$ is,
$$-w \left( b_o (n_{TFT}) + 1 \right) / \left( 1 + \delta_{ABO} - w \right)^2,$$
which is always less than 0.

Note 5A: Inequality 10
The derivative of the right hand side of inequality (10) with respect to $w_{TFT}$,
$$\frac{\left( 1 + \delta_{ABO} \right) (1 + \delta_{ABO} - w) (1 + c - b_o)}{\left[ w_{TFT} (1 + \delta_{ABO}) + w (1 + \delta_{TFT}) \right]^2},$$
is less than 0. In other words, the right-hand side of inequality (10) decreases in $w_{TFT}$.

The derivative of the right-hand side of inequality (10) with respect to $\delta_{ABO}$,

$$\frac{w_{TFT} \left[ (1 + \delta_{ABO})^2 - w \right]}{\left[ w_{TFT} (1 + \delta_{ABO}) + w (1 + \delta_{TFT}) \right]^2},$$

is greater than 0. In other words, the right-hand side of inequality (10) increases in $\delta_{ABO}$.

**Note 6**

Let $w = w_{TFT} = w_{ABO}$. Therefore, inequality (9) becomes,

$$\frac{(b_s - c)(1 + \delta_{TFT}) + wb_o \left( n_{TFT/FFT} \right)}{1 + \delta_{TFT} - w} > \frac{1 + \delta_{ABO} + wb_o \left( n_{TFT/FFT} \right)}{1 + \delta_{ABO} - w}.$$

Solving for $b_s - c$,

$$(b_s - c) > \frac{wb_o \left( n_{TFT/FFT} \right)((\delta_{TFT} - \delta_{ABO}) + (1 + \delta_{ABO})(1 + \delta_{TFT} - w)}}{(1 + \delta_{TFT})(1 + \delta_{ABO} - w)}.$$

Assume $\delta_{TFT} = 0.07$, $\delta_{TFT} = 0.10$, and $w = 0.8$ then the above becomes,

$$(b_s - c) > \frac{0.8b_o \left( n_{TFT/FFT} \right)(-0.03) + (1.1)(0.27)}{(1.1)(0.3)}.$$

$$\begin{align*}
(b_s - c) & > \frac{-0.024b_o \left( n_{TFT/FFT} \right) + 0.297}{0.33}.
\end{align*}$$

If $b_o \left( n_{TFT/FFT} \right) = 4$ then the above becomes,

$$(b_s - c) > \frac{-0.096 + 0.297}{0.33} = 0.61.$$

Therefore, if $\delta_{TFT} = 0.07$, $\delta_{TFT} = 0.10$, $w = 0.8$, $b_o \left( n_{TFT/FFT} \right) = 4$, and $(b_s - c) > 0.61$ (which is possible because $b_s - c \leq 1$) then $V(TFT|TFT) > V(ABO|TFT)$.

Let $\delta = \delta_{TFT} = \delta_{ABO}$. Therefore, inequality (9) becomes,

$$\frac{(b_s - c)(1 + \delta) + w_{TFT}b_o \left( n_{TFT/FFT} \right)}{1 + \delta - w_{TFT}} > \frac{1 + \delta + wb_o \left( n_{TFT/FFT} \right)}{1 + \delta - w}.$$

Solving for $b_s - c$,

$$(b_s - c) > \frac{1 + \delta + wb_o \left( n_{TFT/FFT} \right) - w_{TFT} \left( 1 + b_o \left( n_{TFT/FFT} \right) \right)}{1 + \delta - w}.$$

Assume $\delta = 0.07$, $w_{TFT} = 0.8$, and $w = 0.6$, then the above becomes,

$$(b_s - c) > \frac{0.27 - 0.2b_o \left( n_{TFT/FFT} \right)}{0.47}.$$

If $b_o \left( n_{TFT/FFT} \right) = 4$, then the above becomes,

$$(b_s - c) > \frac{0.27 - 0.8}{0.47} = -1.13.$$
Therefore, if $\delta = 0.07$, $w_{TFT} = 0.8$, and $w = 0.6$, $b_o \left( n_{TFT|TFT} \right) = 4$, and $(b - c) > -1.13$ (which is possible because $b \leq 1$) then $V(TFT|TFT) > V(ABO|TFT)$.

Table 2. Parameter values for scenario analysis

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$w_{TFT}$</th>
<th>$w_{ABO}$</th>
<th>$w$</th>
<th>$\delta_{TFT}$</th>
<th>$\delta_{ABO}$</th>
<th>Others</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[0.80, 0.95]</td>
<td>[0.80, 0.95]</td>
<td>[0.40, min{$w_{TFT}$, $w_{ABO}$}]</td>
<td>[0.01, 0.15]</td>
<td>[0.05, 0.25]</td>
<td>$b_s - c = [-5, 1]$; $a = [0.025, 0.350]$</td>
</tr>
<tr>
<td>2</td>
<td>[0.70, 0.85]</td>
<td>[0.70, 0.85]</td>
<td>[0.80, 0.95]</td>
<td>[0.05, 0.25]</td>
<td>[0.01, 0.15]</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>[0.80, 0.95]</td>
<td>[0.80, 0.95]</td>
<td>[0.80, 0.95]</td>
<td>[0.01, 0.15]</td>
<td></td>
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</tr>
<tr>
<td>4</td>
<td>[0.80, 0.95]</td>
<td>[0.80, 0.95]</td>
<td>[0.80, 0.95]</td>
<td>[0.01, 0.15]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: All distributions are uniform.