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Is the gain from a groundwater management policy insignificant?

Abstract

The point of departure of this work is Gisser and Sanchez's (1980) findings according to which an optimal groundwater management policy would not generate significant gain with respect to a situation with no optimal control. Their theoretical result is evidently checked for a fixed number of agents when the storage capacity of the aquifer is relatively large. The article proposes to add an open-access component into Rubio and Casino's (2001) adaptation from Gisser and Sanchez's (1980) seminal model, in order to make the number of farmers exploiting the resource endogenous. The author then shows that, at the stationary equilibrium, the Gisser and Sanchez's theoretical result does not persist anymore since the rent is zero at this state, although it is positive under optimal control of a water agency.

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Introduction

Serious depletion of aquifers is a major threat to many freshwater ecosystems all over the world. From a public economics angle, this phenomenon is due to the inefficiencies of aquifer exploitation. A frequently asked question is how we go about correcting these inefficiencies. In this context, we wish to address the following question: what is the size of the gains from a groundwater management policy?

In the seminal work of Gisser and Sanchez (1980) the "competitive" solution is analytically compared with the one of optimal control. The so-called competitive solution is defined as a "common property" situation with a fixed number of groundwater exploiters. Gisser and Sanchez's theoretical prediction is that if the storage capacity of the aquifer was relatively large, the two systems would be very close. The economic intuition for this is quite obvious because the storage capacity, at the limit, is so large that each agent becomes atomist, no longer having any impact on the groundwater stock when exploiting it. Nevertheless, their result has produced a wide literature about the economics of groundwater management (see Koundouri (2004) for a complete literature review) because of the policy implications.

Rubio and Casino (2001) confirmed Gisser and Sanchez's prediction when a strategic setting is added into the seminal model. In their framework, agents are able either to pursue path (open-loop equilibrium) or decision rule (closed-loop or feedback equilibrium) strategies. In order to model this, they adapted the Gisser and Sanchez's model by explicitly introducing a fixed number of homogeneous agents who are playing extraction strategies.

Koundouri and Christou (2006) tested the persistence of the Gisser and Sanchez's result with and without the presence of a backstop substitute (rain fed agriculture or desalinized water for instance) in a groundwater problem. Their finding is that it persists in the presence of a backstop substitute and not in its absence. Thus, they considered the possibility of exit from what we will call an irrigation race.

The main motivation of our work is to also consider entry, like it is the case in the literature on fisheries, through making the number of farmers endogenous. Gisser and Sanchez (1980) explained that, contrary to fish harvesting, when dealing with groundwater, the entry is restricted by land ownership. However, Gisser and Sanchez's theoretical result is that there is no gain from groundwater management when the storage capacity of the aquifer is relatively large. The explanation according to which the entry is restricted by land ownership becomes false within such a theoretical framework. Indeed, in his model, the main way through which the storage capacity of the aquifer can become higher lies in the increase of the covered area. An entry phenomenon can hence be at work in their groundwater exploitation model. From the best of our knowledge no paper on the economics of groundwater is addressing this question. However, the literature on fisheries has long been concerned with such problems of unregulated entry since the seminal paper of Gordon (1954). However the framework is quite different: in the fishery story, fish is both the resource and the product sold. This is not the case in the groundwater story where the product is an agricultural one. We will show that the implications of such specificity are weak.

More precisely, we will take Rubio and Casino's (2001) adaptation from Gisser and Sanchez's (1980) seminal model as a point of departure. They postulate a disaggregate water demand function built on the assumptions that all farmers are identical and that their number is fixed. We propose to make this number endogenous. For this purpose, we assume that farmers face climatic conditions such that they can either cultivate rain fed crops or irrigated one. If they choose irrigated crops, they begin to extract groundwater. We incorporate an opportunity cost into the basic formulation of the profit function of the farmers in order to take into account the outside possibility of rain fed crops.

Our main claim is that the consideration of an entry phenomenon in the groundwater exploitation problem can considerably increase the gain from a groundwater policy. Indeed, in the long run, the rents linked to the resource extraction are completely dissipated although it is not the case anymore under a benevolent water agency intervention. We thus show that the Gisser and Sanchez's theoretical result does not persist if we introduce the possibility of entry into an irrigation race.

In the next section, we will quickly recall the assumptions made by Rubio and Casino (2001) in order to adapt Gisser and Sanchez's (1980) model to a differential game framework. The reader is referred to Rubio and Casino's paper for more details. Section 2 will be devoted to the entry setting that we add into their basic model. We will then conduct, in section 3, a stationary analysis on the basis of this new setting. Finally, we will propose some concluding remarks and possible extensions. All technical proofs are relegated to the appendices.

1. The basic model

The basic model proposed by Gisser and Sanchez (1980) is a simplified representation of the economic, hydrologic and agronomic facts that must be considered relative to the irrigator's choice of water pumping. We are going to describe how Rubio and Casino (2001) adapted it. When the number of farmers N equals 1, both of the models coincide.

1.1. The farmer's revenues. The global demand for irrigation water is assumed to be negatively sloped linear function as follows:

$$W_t = g + kP_t\,, k < 0, g > 0\,,$$

where W_t is the amount of groundwater pumped at each time t, P_t the price of water at time t, k is the price coefficient and g is the intercept of the water demand function. In order to simplify notations, parameters denoting time are subsequently omitted unless otherwise stated.

Rubio and Casino (2001) furthermore postulate that all farmers are identical. The idea behind this symmetry assumption is to be able to solve their differential game analytically and to evaluate the effects of strategic behavior on private groundwater pumping. In order to be in phase with their basic model, we propose to make the same assumption. The authors then propose to write the aggregate rate of groundwater extraction as $W = Nw_i$, where N is the number of farmers and w_i is the pumping rate of the farmer i, and the individual demand functions as:

$$w_i = \frac{1}{N}\left(g + kP\right), i = 1, \ldots, N.$$

It is important to mention that with this specification, when the number of farmers increases, the individual demand for water is reduced. This specification is slowing down the over-exploitation of the resource caused by congestion effects, i.e., pumping cost externalities, and will prevent us to speak about a "tragedy of the commons" situation. Finally, Rubio and Casino proposed to write the farmer's i willingness-to-pay for groundwater use as:

$$\int_0^{w_i} P\left(w\right)dw = \frac{N}{2k}w_i^2 - \frac{g}{k}w_i,\ i = 1, \ldots, N.$$

Some basic static comparative allows us to see that when the number of farmers increases, the willingness-to-pay for groundwater is reduced ($k < 0$). If we assume that the agricultural production can either be made with irrigated or rain fed crops, this means that when the number of farmers irrigating increases, their willingness-to-pay for groundwater is decreasing because there are more products on the market and the selling price decreases[1]. This means that there are some intra-marginal losses.

In Gisser and Sanchez's model, the cost of extraction depends on the quantity of water extracted and on the depth of the water table. Like most groundwater models, costs vary directly with the pumping rate and inversely with the level of the water table (or, equivalently, the stock of water):

$$C\left(h, W\right) = \left(c_0 + c_1 h\right)W, c_1 < 0, c_0 > 0,$$

where h is the height of the aquifer, i.e., the water table elevation above some arbitrary level that is considered as being the bottom of the aquifer by Rubio and Casino, c_0 is the fixed (with respect to the aquifer height) cost linked with the hydrologic cone and c_1 is the marginal pumping cost per acre foot of water pumped per foot of lift.

As the unit groundwater pumping costs do not depend on the rate of extraction, Rubio and Casino propose to postulate that the individual farmer's withdrawal costs are as follows:

[1] We could have included in the model the supply and demand functions of the product but this does not change our main message. As a consequence, for clarity sake, we will not include them.

$$C_i\left(h,w_1\right)=\frac{1}{N}\left(c_0+c_1h\right)W=\left(c_0+c_1h\right)w_1.$$

It is here implicitly assumed that the well pump capacity constraint is nonbinding and that energy costs are constant along time. Moreover, sunk costs, replacement costs, and capital costs in general are ignored in this seminal formulation.

Finally, the farmer's i is net revenues per unit of time are equal to the willingness-to-pay for groundwater minus the extraction costs of this resource:

$$\frac{N}{2k}w_i^2-\frac{g}{k}w_i-C_i\left(h,w_i\right).$$

1.2. The simplified hydraulic model. The hydraulic model from Gisser and Sanchez is based on classical assumptions such as the "bathtub" one, which consists in postulating that the aquifer has parallel sides and a flat bottom. The differential equation that describes the water table as a function of time is obtained by equating inflows minus outlets with the impact on the water table:

$$AS\,\dot{h}=R+\left(\gamma-1\right)W,0<\gamma<1,$$

where AS denotes the storage capacity of the aquifer: area, A, time storage coefficient, S, which measures the average saturation of water in the aquifer; γ is the constant return flow coefficient of irrigation water; R denotes the deterministic and constant recharge.

2. The myopic solution with unregulated entry

In order to add unregulated entry into the seminal model, we assume that each farmer who wants to irrigate his lands located above the aquifer compares the benefits of this choice to outside opportunities (rain fed agriculture), which are represented by an opportunity cost (of the irrigation capital), s, assumed sunk and identical for each farmer.

The net farm rent per unit of time is total revenue minus total cost:

$$\frac{N}{2k}w_i^2-\frac{g}{k}w_i-C_i\left(h,w_i\right)-s,$$

where the number of farmers N can now possibly evolve along time.

2.1. The unregulated entry setting. In a dynamic perspective, when the revenues from irrigated agriculture exceed the opportunity cost, farmers enter into the irrigated agriculture race. If we now add the fact that this adjustment is taking time, we have the following law of motion that is governing the entry temporal phenomenon:

$$\dot{N}=\eta\left(\frac{N}{2k}w_i^2-\frac{g}{k}w_i-C_i\left(h,w_i\right)-s\right),N\left(0\right)=N_0>1,$$

where $\eta>0$ is an adjustment parameter and i denotes the last farmer who is entering into the irrigated agriculture race. This equation correspond to the seminal model of dynamic entry to a fishery, developed by Smith (1968), which is still used nowadays in the literature on fisheries: see for instance Sanchirico and Wilen (1999) who proposed to add a spatial component in the basic model. However, this specification suffers from vagueness. Indeed, the literature on fisheries measures the fishing effort as the number of boats that must be an integer variable. Some authors proposed to partially solve this issue by reasoning on a continuous variable like the number of fishing days. The same problem holds in our framework with the number of farmers. We could also have preferred to reason on a number of irrigation days but such a specification would have created another problem: the rationality of agents would thus have been ignored. Indeed, with such an interpretation of N, its dynamics would have suffered from not being based on an optimization process but on an evolutionary one. It is in order to stay in phase with Rubio and Casino framework that we chose to keep the number interpretation. We hope that the reader will not suffer from this imprecision.

This entry equation is defined in such a way that, in the long run, the number of agents exploiting the resource is characterized by the complete dissipation of the rents. Even if this setting is currently referred as a "tragedy of commons" one, it does not lead inevitably to the disappearance of the natural resource. As a consequence and in line with Ciriacy-Wantrup and Bishop (1975), we will rather refer to a "tragedy of unregulated entry". This is especially true in our model because the number of farmers is entering into their individual water demand function in such a way that the global demand is always the same one, hence having the same impact on the stock whatever the number of agents is.

2.2. The stationary equilibrium. As a first step, we chose to concentrate on the solution of myopic agents. In such a setting, each agent is a too small part of the whole to give serious considerations to how his pumping decision affects future water supplies. Such agents are not able to consider the inter-temporal effect of their current choices. So they maximize their current rents without taking into account the impact of their pumping decision on the groundwater stock:

$$\max_{w_i}\frac{N}{2k}w_i^2-\frac{g}{k}w_i-\left(c_0+c_1h\right)w_1-s.$$

The aquifer height are then determined thanks to their respective dynamic equations.

Proposition 1. The unique myopic (denoted *m*) stationary equilibrium (denoted *e*) with unregulated entry is stable and is characterized as:

$$N_e^m = \left[\frac{-R^2}{2sk(\gamma-1)^2} \right], w_e^m = \frac{2sk(\gamma-1)}{R}$$

$$h_e^m = \frac{-R}{kc_1(\gamma-1)} - \frac{g}{kc_1} - \frac{c_0}{c_1},$$

where $[x]$ denotes the integer part of x.

Tables 1 and 2 (see Appendix A) summarizes the analytical expressions at the stationary equilibrium when $N = 1$ (Gisser and Sanchez, 1980), when N is fixed (Rubio and Casino, 2001), and when N is endogenous (our modified framework with unregulated entry). As it can easily be checked, the aquifer height at the stationary equilibrium with unregulated entry is the same one as in the Gisser and Sanchez's so-called competitive case. Indeed, Rubio and Casino's introduction of the number of farmers does not affect the aggregate groundwater demand: when the number increases, the individual demand for the resource is reduced but the aggregate one remains the same. So an increase in the number of farmers (with respect to one) does not affect the aquifer height at the stationary equilibrium.

Proposition 2. The number of farmers at the myopic stationary equilibrium is:

1. Increasing with the natural recharge, R, with the percolation coefficient, γ, and with the price coefficient, k, because more groundwater is then available.
2. Decreasing with the opportunity cost, s, because it measures the cost of entry into the irrigated agriculture race.

3. The gain from a groundwater management policy

In order to measure the gain from a groundwater management policy when the number of farmers is endogenous, we are now going to compare the unregulated entry solution to the optimal one. Since our entry model is mainly a stationary one, we will focus on comparisons at the stationary equilibrium. We will derive results in the general case and then run some numerical illustrations.

3.1. The general case. In the optimal solution, the unregulated entry hypothesis does not hold: a water agency with perfect foresight is assumed to control both the volume of groundwater pumped (with a fiscal scheme on each unit of groundwater used for instance) and the number of farmers having an access to the aquifer (with a fiscal scheme on the irrigation capital for instance) that is assumed to be adjusted instantaneously. Thus, the dynamic equation of N does not hold anymore here. The symmetry of this solution is obvious: a water agency maximizing the rents accruing to homogenous farmers has never an incentive to discriminate between them because they are all identical. This benevolent agency's objective is to maximize the future value of the rent stream that is given by the sum of the individual ones, taking into account the impact of these decisions on the hydraulic system[1]:

$$\max_{w,N} \int_0^\infty N \left[\frac{N}{2k} w^2 - \frac{g}{k} w - (c_0 + c_1 h) w - s \right] e^{-rt} dt$$

$$s.t. \dot{h} = \frac{1}{AS} \left[R + (\gamma - 1) N w \right], \; h(0) = h_0 > 0$$

$$N \geq 1.$$

We need to constrain the number of farmers because of the sunk cost. Indeed, without this constraint, a number of agents lower than one (but different from zero) could be optimal from a water agency point of view.

Proposition 3. The unique optimal stationary solution (denoted *) is stable and can be analytically derived as:

$$N_e^* = 1, w_e^* = \frac{R}{(\gamma-1)},$$

$$h_e^* = \frac{-R}{kc_1(\gamma-1)} - \frac{g}{kc_1} - \frac{c_0}{c_1} + \frac{R}{rAS},$$

$$\lambda_e^* = \frac{c_1 R}{r(\gamma-1)}, \mu_e^* = s,$$

where γ denotes the shadow price of the aquifer height and μ denotes the Lagrange multiplier associated to the constraint on the number of farmers.

The optimum corresponds to Gisser and Sanchez's one (see Appendix A). Such a monopoly solution can be efficient because the opportunity cost, s, is sunk. This is the solution of a sole owner who may be imagined as either a private farmer or a government agency that owns complete rights to the exploitation of the groundwater. The aquifer height at the stationary equilibrium is also the same one as the Rubio and Casino's efficient one that, as

[1] The non-negativity constraints on the control variables are implicitly assumed and $h \geq 0$ is not imposed as a state constraint but as a terminal condition for simplicity: $\lim_{t\to\infty} h_t \geq 0$.

stressed by the authors, does not depend on the number of agents.

We now turn to the comparison of these values at the stationary equilibrium with the one obtained in the myopic case in order to check if the result of Gisser and Sanchez still holds. Since N_e^m cannot be lower than one and the aggregate demand does not change with the number of farmers (remind that $W = nw_i$), the individual volume of groundwater pumped at the myopic stationary equilibrium with unregulated entry is always lower or equal to the one obtained by a benevolent water agency. Concerning the aquifer heights at the two stationary equilibriums, we obtain the same difference as in the literature because these values do not depend on the number of agents:

$$h_e^* - h_e^m = \frac{R}{rAS} > 0.$$

So, our results differ from Rubio and Casino's one simply by the fact that the number of farmers and the individual amounts of groundwater pumped are different. However, this is of major interest: it has an impact on the individual rents and we even know from the setting with unregulated entry that the rents are completely dissipated at the stationary equilibrium. And this is true without any additional social damage linked to groundwater extraction. This leads us to the main point of this work.

Proposition 4. Making the number of agents endogenous in Rubio and Casino's model adapted from Gisser and Sanchez's one leads to a related result: when the storage capacity of the aquifer studied is relatively large, the aquifer height at the stationary equilibrium tends to be the same one in the regime of private extraction and in the one of a benevolent water agency. However, in such a setting, private extraction leads to the complete dissipation of the farmers' rent in the long run that destroys the theoretical result of Gisser and Sanchez because the rents of the myopic with unregulated entry case at the stationary equilibrium then equal zero and not the one characterizing the water agency solution. As a consequence, a "tragedy of the unregulated entry" can also be at work when dealing with groundwater extraction.

3.2. Numerical illustrations. We ran some simulations in order to illustrate this result to the Pecos Basin studied by Gisser and Sanchez (1980). For this purpose, we used the corresponding parameters presented in Table 3 (see Appendix C).

We first use the Gisser and Mercado's (1972) set of parameters. Concerning the sunk cost, s, we calibrated it in such a way that the rent of a private farmer at the myopic stationary equilibrium is equal to zero: s^m=17 233$. Notice that, by doing so, we implicitly assume that the number of farmers in the Basin at the dates of the authors' study is the equilibrium one. Note also that this value is of major importance: it fully determines the number of farmers at the stationary equilibrium.

We obtained the following results[1]:

$$N_e^m = 500 \qquad N_e^* = 1$$
$$w_e^m = 473\,\text{ac ft} \qquad w_e^* = 236\,986\,\text{ac ft}$$
$$h_e^m = 1525\,\text{ft} \qquad h_e^* = 1538\,\text{ft}$$
$$\pi_e^m = \Pi_e^m = 0\,\$ \qquad \pi_e^* = \Pi_e^* = 8.7\times 10^6\,\$\,,$$

where π denotes the rent of an individual farmer and Π is the aggregate value for all the N agents. We hence easily verify that the benefits from managing the system can be very high, even if the aquifer height difference is very low.

Furthermore, a numerical example allows to look in an easier way at what happens when the agents are foresight. The problem to solve then corresponds to the ith farmer's dynamic optimization problem defined as:

$$\max_{w_i} \int_0^\infty \left[\frac{N}{2k} w_i^2 - \frac{g}{k} w_i - \left(c_0 + c_1 h\right) w_i - s \right] e^{-rt} dt,$$

$$\text{s.t.} \dot{h} = \frac{1}{AS}\left[R + \left(\lambda - 1\right)\sum_{i-1}^{N} w_i \right], h(0) = h_0 > 0.$$

It can be solved thanks to two equilibriums concepts according to the behavioral assumptions made: open-loop or feedback. The resolution methods used will be the same as in Rubio and Casino (2001) and the number of farmers will be determined thanks to the dynamic equation of N.

We now propose to turn to the case of the high plains of Texas studied by Nieswiadomy (1985). The new sunk cost is calibrated as before and s^m=106 907$. With this new set of parameters, the myopic and optimal stationary equilibriums become the following one:

$$N_e^m = 7 \qquad N_e^* = 1$$
$$w_e^m = 64\,057\,\text{ac ft} \qquad w_e^* = 448\,400\,\text{ac ft}$$
$$h_e^m = 3156{,}106\,\text{ft} \qquad h_e^* = 3161\,\text{ft}$$
$$\pi_e^m = \Pi_e^m = 0\,\$ \qquad \pi_e^* = \Pi_e^* = 728\,632\,\$\,.$$

Here the number of groundwater users at the stationary equilibrium of the myopic case with unregulated entry

[1] Note that we did not need any estimate of the adjustment parameter, η, since we are at the steady state.

is very small. It is why we turned to more foresight behavioral assumptions of our agents: we are going to assume that they are either able to pursue path (open-loop equilibrium) or decision rule (feedback equilibrium) strategies. The open-loop with unregulated entry symmetrical[1] stationary equilibrium is characterized by the following system that is derived from the problem previously defined:

$$\begin{cases} \dfrac{N_e^{ol}}{k} w_e^{ol} - \dfrac{g}{k} - \left(c_0 + c_1 h_e^{ol}\right) + \dfrac{\lambda_e^{ol}(\gamma-1)}{AS} = 0 \\ \dfrac{1}{AS}\left[R + (\gamma-1) N_e^{ol} w_e^{ol}\right] = 0 \\ \eta\left(\dfrac{N_e^{ol}}{2k} w_e^{ol2} - \dfrac{g}{k} w_e^{ol} - \left(c_0 + c_1 h_e^{ol}\right) w_e^{ol} - s\right) = 0 \\ r\lambda_e^{ol} + c_1 w_e^{ol} = 0 \end{cases}$$

Because of the second degree equation, this system of equations admits two sets of possible solutions between which we can choose thanks to the positivity constraints on the control. Concerning the feedback equilibrium with unregulated entry, the resolution is much more complicated. Even if the reader can refer to Rubio and Casino's paper for more details, we recall that the resolution method is based on the assumptions of linear strategies such that: $\lambda^{fb} = \alpha h + \beta$. It hence begins with the computation of α. Once it has been expressed as a function of the other parameters (including N at this stage), the symmetrical[2] stationary equilibrium is given by the system that follows:

$$\begin{cases} \dfrac{N_e^{fb}}{k} w_e^{fb} - \dfrac{g}{k} - \left(c_0 + c_1 h_e^{fb}\right) + \dfrac{\lambda_e^{fb}(\gamma-1)}{AS} = 0 \\ \dfrac{1}{AS}\left[R + (\gamma-1) N_e^{fb} w_e^{fb}\right] = 0 \\ \eta\left(\dfrac{N_e^{fb}}{2k} w_e^{fb2} - \dfrac{g}{k} w_e^{fb} - \left(c_0 + c_1 h_e^{fb}\right) w_e^{fb} - s\right) = 0 \\ r\lambda_e^{fb} + c_1 w_e^{fb} - \dfrac{\lambda_e^{fb}(\gamma-1)\alpha}{AS} = 0 \end{cases}$$

As before, we have to choose the number of farmers inducing positive controls. Our computations finally lead us to the following numerical solutions:

$$\begin{array}{ll} N_e^{ol} = 7 & N_e^{fb} = 7 \\ w_e^{ol} = 64\,057 \text{ ac ft} & w_e^{fb} = 64\,057 \text{ ac ft} \\ h_e^{ol} = 3156{,}899 \text{ ft} & h_e^{fb} = 3156{,}862 \text{ ft} \\ \pi_e^{ol} = 1779\,\$ & \pi_e^{fb} = 1695\,\$ \\ \prod_e^{ol} = 12\,455\,\$ & \prod_e^{fb} = 11867\,\$. \end{array}$$

These simulations confirm our main result according to which, in a setting with unregulated entry, the gain from groundwater management is significant in the long run. However, the simulations show that this gain is reduced when agents are considered as being foresight. This is due to the fact that we had to round off the number of agents at the equilibrium.

Conclusion

The Gisser and Sanchez's theoretical result states that the numerical magnitude of benefits of optimally managing groundwater is insignificant. Our main claim is that it cannot persist when considering that entry is possible because, in such a setting, at the stationary equilibrium, the rents from irrigated agriculture equal zero, which is not the case within the framework of a benevolent water agency intervention. This point considerably differs from the literature on the economics of groundwater that omitting entry. The main explanation given by authors is that the access to an aquifer is limited by land ownership. However, each farmer can become very small in a "competitive" setting. The economics concept of entry can then be at work even if the area giving an access to the resource is limited.

In order to show this, we chose to follow the framework proposed by Rubio and Casino (2001) that is a priori internalizing some groundwater extraction externalities by constraining the aggregate water demand to be the same one whatever the number of agents is. However, even if some of these congestion external effects then disappear, some others are still remaining. And, in a setting in which the number of agents is exogenously fixed, this specificity helps to confirm the Gisser and Sanchez's theoretical result: the externalities do not hold anymore when the aquifer size becomes infinite (agents become a too small part of the whole). It is because of this a priori internalization that our setting with unregulated entry (characterized by a number of agents endogenous) leads to the complete dissipation of the rent without leading to the disappearance of the natural resource exploited. The force of the result relies on the fact that the gain of the management policy is significant even without adding any social cost (and hence externalities) of groundwater extraction in the basic model.

[1] For a proof of the symmetry of this equilibrium, refer to Rubio and Casino (2001).
[2] For a proof of the symmetry of this equilibrium, refer to Rubio and Casino (2001).

The main implication in terms of policy intervention is that when there is an unregulated entry to irrigated agriculture with groundwater, there is strong needs to manage this resource. Even if our model does not tell anything about which instruments to use, it leads to the conclusion that it's necessary to regulate both the amount of groundwater extracted and the number of farmers irrigating.

However, our model contains limits. First, a complete dynamic analysis of entry would consist in comparing the net present value of future rent stream in the different cases studied. In order to compute this one, it is necessary to fully characterize the paths leading to the stationary equilibriums. However, the equation of entry introduced in our work, which is largely based from the fishing literature, is a stationary one. Indeed, the economic rationality (profit maximization) would be affected if we used this equation in order to characterize dynamics. Nevertheless, this work path the way for future analysis of groundwater management that include the dynamics of the number of pumpers.

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Appendix A. Synopsis of results

Table 1. Aquifer height at the stationary equilibrium in the different settings

		Aquifer height
Gisser and Sanchez (1980)	Competitive	$-\frac{R}{kc_1(\gamma-1)}-\frac{1}{c_1}\left(\frac{g}{k}+c_0\right)$
	Efficient	$-\frac{R}{kc_1(\gamma-1)}+\frac{R}{rAS}-\frac{1}{c_1}\left(\frac{g}{k}+c_0\right)$
Rubio and Casino (2001)	Myopic	$-\frac{R}{kc_1(\gamma-1)}-\frac{1}{c_1}\left(\frac{g}{k}+c_0\right)$
	Open-loop	$-\frac{R}{kc_1(\gamma-1)}+\frac{R}{rASN}-\frac{1}{c_1}\left(\frac{g}{k}+c_0\right)$
	Feedback	$-\frac{R}{kc_1(\gamma-1)}-\frac{1}{c_1}\left(\frac{g}{k}+c_0\right)+\frac{R}{NAS\left\{r-\left[k(\gamma-1)(N-1)/NAS\right]\left[c_1-\alpha(\gamma-1)/AS\right]\right\}}$
	Efficient	$-\frac{R}{kc_1(\gamma-1)}+\frac{R}{rAS}-\frac{1}{c_1}\left(\frac{g}{k}+c_0\right)$
With entry	Myopic	$-\frac{R}{kc_1(\gamma-1)}-\frac{1}{c_1}\left(\frac{g}{k}+c_0\right)$
	Efficient	$-\frac{R}{kc_1(\gamma-1)}+\frac{R}{rAS}-\frac{1}{c_1}\left(\frac{g}{k}+c_0\right)$

Table 2. Other stationary equilibria

		Number of farmers, N	Private groundwater withdrawals, w	Shadow price of the stock
Gisser and Sanchez (1980)	Competitive	1	$-\frac{R}{(\gamma-1)}$	0
	Efficient	1	$-\frac{R}{(\gamma-1)}$	$\frac{c_1 R}{r(\gamma-1)}$
Rubio and Casino (2001)	Myopic	N	$-\frac{R}{(\gamma-1)N}$	0
	Open-loop	N	$-\frac{R}{(\gamma-1)N}$	$\frac{c_1 R}{r(\gamma-1)N}$
	Feedback	N	$-\frac{R}{(\gamma-1)N}$	$\frac{R}{N(\gamma-1)\left\{r-\left[k(\gamma-1)(N-1)/NAS\right]\left[c_1-\alpha(\gamma-1)/AS\right]\right\}}$
	Efficient	N	$-\frac{R}{(\gamma-1)N}$	$\frac{c_1 R}{r(\gamma-1)}$
With entry	Myopic	$\left[\frac{-R^2}{2sk(\gamma-1)^2}\right]$	$\frac{2sk(\gamma-1)}{R}$	0
	Efficient	1	$-\frac{R}{(\gamma-1)}$	$\frac{c_1 R}{r(\gamma-1)}$

Appendix B

Proof of proposition 1

In a myopic setting, the first order necessary condition for an interior solution is:

$$\frac{N}{k}w-\frac{g}{k}-\left(c_0+c_1 h\right)=0.$$

Remark 1: The second order condition is checked because $N/k < 0$.

Furthermore, because at the stationary equilibrium we have that:

$$\dot{h}=\dot{N}=0$$

and the following system yields:

$$\left\{\begin{array}{l}\frac{1}{AS}\left[R+(\gamma-1)N_e^m w_e^m\right]=0 \\ \frac{N_e^m}{2k}w_e^{m2}-\frac{g}{k}w_e^m-\left(c_0+c_1 h_e^m\right)w_e^m-s=0\end{array}\right\}.$$

Thus, if we express w_e^m in the first order equation, we obtain:

$$w_e^m=\frac{g}{N_e^m}+\frac{k\left(c_0+c_1 h_e^m\right)}{N_e^m}.$$

We then directly deduce the aquifer height at the stationary equilibrium from:

$$\dot{h}=0$$

that:

$$\frac{(\gamma-1)kc_1 h_e^m}{AS}=-\frac{R}{AS}-\frac{(\gamma-1)g}{AS}-\frac{(\gamma-1)kc_0}{AS}$$

$$\Leftrightarrow h_e^m=-\frac{R}{(\gamma-1)kc_1}-\frac{g}{kc_1}-\frac{c_0}{c_1}$$

and the number of farmers from:

$\dot{N}=0,$ i.e.:

$$\left[\frac{g}{N_e^m}+\frac{k}{N_e^m}\left(-\frac{R}{(\gamma-1)k}-\frac{g}{k}\right)\right]\left[\frac{g}{2k}+\left(\frac{1}{2-1}\right)\left(-\frac{R}{(\gamma-1)k}-\frac{g}{k}\right)-\frac{g}{k}\right]-s=0$$

$$\Leftrightarrow \frac{-R^2}{2(\gamma-1)^2 kN_e^m}-s=0$$

$$\Leftrightarrow N_e^m=\frac{-R^2}{2(\gamma-1)^2 ks}.$$

Finally,

$$w_e^m=\frac{-R/(\gamma-1)}{-R^2/2(\gamma-1)^2 ks}=\frac{2(\gamma-1)ks}{R}.$$

Remark 2: The local stability of this equilibrium is checked. Indeed, if we denote J the Jacobian matrix of:

$$F:\{h,N\}\rightarrow\left\{\frac{(\gamma-1)kc_1h+R+(\gamma-1)kc_0}{AS},\frac{g^2}{2kN}+\frac{g(c_0+c_1h)}{N}+\frac{g(c_0+c_1h)^2}{2N}-\left(\frac{g}{k}+c_0+c_1h\right)\frac{g+k(c_0+c_1h)}{N}-s\right\}$$

we have:

$$J\left(h_{e,}^m N_e^m\right)=\begin{bmatrix}\frac{(\gamma-1)kc_1}{AS} & 0\\ \frac{2c_1(1-\gamma)ks}{R} & \frac{2(\gamma-1)^2 ks^2}{R^2}\end{bmatrix}.$$

The local stability directly comes from the negativity of both eigenvalues:

$\frac{(\gamma-1)kc_1}{AS}$ and $\frac{2(\gamma-1)^2 ks^2}{R^2}$ of the Jacobian matrix, J, evaluated at the equilibrium.

Proof of proposition 2

These results directly come from some basic static comparative on the number of agents at the myopic stationary equilibrium with unregulated entry:

$$\frac{\partial N_e^m}{\partial R}=\frac{-R}{sk(\gamma-1)^2}>0,$$

$$\frac{\partial N_e^m}{\partial \gamma}=\frac{R^2}{sk(\gamma-1)^3}>0,$$

$$\frac{\partial N_e^m}{\partial k}=\frac{R^2}{2sk^2(\gamma-1)^2}>0, \tag{1}$$

$$\frac{\partial N_e^m}{\partial s}=\frac{R^2}{2s^2k(\gamma-1)^2}<0. \tag{2}$$

Proof of proposition 3

In order to solve the water agency problem, we propose to use the maximum principle. We define the current value Hamiltonian as:

$$H\left(h,w,N,\lambda\right)=N\left[\frac{N}{2k}w^2-\frac{g}{k}w-\left(c_0+c_1h\right)w-s\right]+\frac{\lambda}{AS}\left[R+\left(\gamma-1\right)Nw\right],$$

and the Lagrangian as:

$$L\left(h,w,N,\lambda,\mu\right)=H\left(h,w,N,\lambda\right)+\mu\left(N-1\right).$$

The necessary conditions yield:

$$N^*\left[\frac{N^*}{k}w^*-\frac{g}{k}-\left(c_0+c_1h^*\right)+\frac{\lambda^*\left(\gamma-1\right)}{AS}\right]=0, \tag{3}$$

$$\frac{N^*}{k}w^{*2}-\frac{g}{k}w^*-\left(c_0+c_1h^*\right)w^*-s+\mu^*+\frac{\lambda^*\left(\gamma-1\right)w^*}{AS}=0, \tag{4}$$

$$\dot{h}=\frac{1}{AS}\left[R+\left(\gamma-1\right)N^*w^*\right], \tag{5}$$

$$\dot{\lambda}=r\lambda^*+c_1w^*N^*, \tag{6}$$

along with the transversality conditions:

$$\lim_{t\to+\infty}e^{-rt}\lambda^*\geq 0,\ \lim_{t\to+\infty}e^{-rt}\lambda^*=0,\ i=0,...,N$$

and the complementary slackness one:

$$\mu^*\left(N-1\right)=0,\mu^*\geq 0,i=1,...,N.$$

At the stationary equilibrium, because of:

$$\dot{h}=\dot{\lambda}=0 \text{ and } N_e^*-1\geq 0,$$

we have that:

$$(1)\Leftrightarrow\frac{N_e^*}{k}w_e^*-\frac{g}{k}-\left(c_0+c_1h_e^*\right)+\frac{\lambda_e^*(\gamma-1)}{AS}=0$$

$$\Leftrightarrow \mu^*=s>0 \text{ in } (2)$$

$$\Leftrightarrow N_e^*=1$$

$$\Leftrightarrow\left\{\begin{array}{l} w_e^*=\dfrac{-R}{\left(\gamma-1\right)}\text{ in }\left(3\right)\\ \lambda_e^*=\dfrac{c_1R}{r\left(r-1\right)}\text{ in }(4)\\ h_e^*=\dfrac{-R}{c_1k\left(\gamma-1\right)}-\dfrac{g}{c_1k}-\dfrac{c_0}{c_1}+\dfrac{R}{rAS}\end{array}\right\} \tag{7}$$

Remark 3: The second order conditions are checked. Indeed, if we denote:

$$Hess\left(w,N,h\right)=\begin{bmatrix}\dfrac{N^2}{k} & \dfrac{2Nw-g}{k}-c_0-c_1h+\dfrac{\lambda}{AS}\left(\gamma-1\right) & -Nc_1\\ \dfrac{2Nw-g}{k}-c_0-c_1h+\dfrac{\lambda}{AS}\left(\gamma-1\right) & \dfrac{w^2}{k} & -c_1w\\ -Nc_1 & -c_1w & 0\end{bmatrix}.$$

The Hessian matrix of our Hamiltonian *H*, and if we compute this matrix at the equilibrium, we obtain:

$$Hess\left(w_e^*, N_e^*, h_e^*\right) = \begin{bmatrix} \frac{1}{k} & -\frac{R}{k(\gamma-1)} & -c_1 \\ -\frac{R}{k(\gamma-1)} & \frac{R^2}{(\gamma-1)^2 k} & \frac{c_1 R}{\gamma-1} \\ -c_1 & \frac{c_1 R}{\gamma-1} & 0 \end{bmatrix} \leq 0$$

that is concave.

Remark 4: The stability of this stationary equilibrium is checked. Indeed, we can write again the dynamic system that leads to our stationary equilibrium as:

$$\begin{pmatrix} \dot{h} \\ \dot{\lambda} \end{pmatrix} = \begin{pmatrix} \frac{R}{AS} + \frac{g(\gamma-1) + c_0 k(\gamma-1)}{AS} \\ \frac{c_1(g + c_0 k)}{1} \end{pmatrix} + \underbrace{\begin{pmatrix} \frac{c_1 k(\gamma-1)}{AS} & -\frac{k(\gamma-1)^2}{A^2 S^2} \\ \frac{kc_1^2}{1} & r + \frac{kc_1(\gamma-1)}{AS} \end{pmatrix}}_{A} \begin{pmatrix} h \\ \lambda \end{pmatrix},$$

where:

$$\det A = \frac{c_1 kr(\gamma-1)}{AS} < 0,$$

which means that the equilibrium has a saddle point property.

Proof of proposition 4

The result directly comes from the fact that the rents are dissipated at the stationary equilibrium.

Appendix C. Parameters values used for the simulations

The two sets of parameters used in our numerical examples are presented in Table 3:

- the first one comes from Gisser and Mercado (1972) empirical analysis of the Pecos Basin, in New Mexico[1],
- the second one from Nieswiadomy (1985) one applied to the High Plains of Texas[2].

Table 3. Parameters values

Symbol	Description	Gisser and Mercado's estimations	Nieswiadomy's estimations
k	Slope water demand function	-3 259 ac ft/yr	-134 337 ac ft/yr
g	Intercept water demand function	470 375 ac ft/yr	2 401 161 ac ft/yr
c_1	Slope pumping cost function	-0,035 $/ac ft/ft of lift	-0,035 $/ac ft/ft of lift
c_0	Intercept pumping cost function	125 $/ac ft	125 $/ac ft
γ	Return flow coefficient	0,27	0,20
AS	Storage capacity	135 000 ac/yr	645 696 ac/yr
R	Natural recharge	173 000 ac ft/yr	358 720 ac ft/yr
h_0	Initial water table elevation	3 400 ft above sea level	3 400 ft above sea level
N	Number of pumpers	500	7

Note: The number of pumpers item corresponds to a number of farmers in Gisser and Mercado (1972) and to a number of counties in Nieswiadomy (1985). 1 2

[1] The demand parameters estimate is based on 1968 dollars.
[2] The demand parameters estimate is based on 1967 dollars.