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A behavioral explanation of the asset allocation puzzle

Abstract

This paper combines a behavioral reward-risk model based on prospect theory with multiple investment accounts to explain the asset allocation puzzle, that is, the observation that investors violate the two-fund separation property of optimal mean-variance allocations. In a empirical analysis with U.S. data, the authors show that investors with preference according to the behavioral reward-risk model and multiple investment accounts, invest a higher proportion into bonds and large cap stocks as their risk tolerance diminishes, consistently with the empirical findings.

Keywords: portfolio selection, asset allocation puzzle, prospect theory, mental accounting.
JEL Classification: G11, D81.

Introduction

The modern portfolio theory of Markowitz (1952) is a rich source of intuition and also the basis for many practical decisions. Markowitz's seminal idea was to evaluate portfolios by using two opposing criteria: reward, measured by the portfolio's expected return, and risk, measured by the porfolio's variance. Under certain conditions, the mean-variance model of portfolio selection leads to two-fund separation (Tobin, 1958), that is, all investors hold a combination of the same portfolio of risky assets combined with the risk-free asset. Two-fund separation greatly simplifies the advice one should give to a heterogenous set of investors since the proportion of risky assets in the optimal portfolio is then independent of investor's risk aversion.

Even though practitioners adhere to the reward-risk methodology, their advices do not seem to follow the two-fund separation property[1]. This so-called asset allocation puzzle was first observed by Canner, Mankiw, and Weil (1997) who found that in practitioners' advice the more risk-averse is the client, the larger is the bonds-to-stocks ratio. Moreover, Wang (2003) identifies an asset allocation sub-puzzle, that is, in financial advisors' recommendations the proportion of large cap stocks relative to total holding of stocks increases with increasing risk aversion.

In this paper we combine the behavioral reward-risk model suggested by De Giorgi, Hens, and Mayer (2006) with the behavioral portfolio theory of Shefrin and Statman (2000) and provide a behavioral solution to the asset allocation puzzle and the sub-puzzle. The behavioral reward-risk model of De Giorgi, Hens, and Mayer (2006) is based on the trade-off between gains and losses that is incorporated in the prospect theory of Kahneman and Tversky (1979). The prospect theory is a descriptive model of preferences which assumes that alternatives are evaluated according to a reference-dependent, kinked (loss aversion) and convex-concave value function in addition to inverse S-shaped probability weighting functions. For given asset payoffs, De Giorgi, Hens, and Mayer (2006) define reward as the prospect theory value function applied over gains, while risk is the negative of the prospect theory value function applied over losses and normalized by the index of loss aversion. The normalization for losses implies that loss aversion describes the investor's tradeoff between gains and losses, while it doesn't impact how investors measure losses. Gains and losses are defined with respect to a subjective reference point, which describes investors' target returns or aspiration levels. Consequently, the risk measure describes the risk that assets' payoffs are below the reference point.

The reward and risk measures that De Giorgi, Hens, and Mayer (2006) defined based on prospect theory obviously depend on the parametrization of prospect theory by means of the utility index, as well as on the choice of the reference point which defines what is perceived as a gain and what is perceived as a loss. De Giorgi, Hens, and Mayer (2011) show that under some conditions for the utility indexes and the reference points, the behavioral reward-risk model of De Giorgi, Hens, and Mayer (2006) also satisfies the separation property for optimal portfolio allocations. Examples of reward and risk measures leading to this result are those defined by means of the piecewise-power value function suggested by Tversky and Kahneman (1992) and having the risk-free return as reference point, which are common specifications of prospect theory in behavioral finance.

In general, optimal solutions to the behavioral reward-risk model do not satisfy the two-fund separation property. For example, if reward and risk measures are defined according to a piecewise-power value function and the reference point is higher than the risk-free return and identical for all investors, then two-

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[1] The reward-risk methodology is the main paradigm used by financial advisors to communicate with clients.

fund separation is violated. Nevertheless, when the reference point is identical for all investors, deviations from two-fund separation in the behavioral reward-risk model are not systematic, e.g., the bonds-to-stocks ratio and the ratio between large cap stocks and the total holding of stocks are not monotonic as a function of investors' loss tolerance. However, this is due to the assumption that all investors possess the same reference point, which is not realistic, e.g., we expect investors with higher loss tolerance to also possess higher reference points. Consequently, we extend the behavioral reward-risk model in order to allow investors possessing different or even multiple (if multiple investment goals exist) reference points.

We combine the behavioral reward-risk model with the multiple-account version of the behavioral portfolio theory of Shefrin and Statman (2000). According to this theory investors possess different mental accounts which correspond to different aspiration levels, investments goals, or, in our framework, reference points[1]. Low aspiration accounts refer to need for security, while high aspiration accounts refers to hope for richness. In the behavioral reward-risk model, risk increases with the reference point, since the value and the probability of losses obviously increase when the reference point is higher. Therefore, investors with low degrees of loss tolerance mainly invest in accounts with low reference points, while investors with higher degrees of loss tolerance put a higher proportion of their wealth into accounts with high reference points. For each account, investors determine the minimum risk portfolio. Indeed, given their reference point for the corresponding mental account, investors' goal is to minimize the risk of being below the reference point, while higher reward refers to mental accounts with higher reference points or aspiration levels. Finally, investors allocate their wealth between the different accounts in order to maximize their total reward, given the loss constraint implied by their loss tolerance. This step is a simple linear program since investors treat mental accounts separately, e.g., aggregate the different accounts ignoring co-movements between the payoffs of accounts' specific portfolios.

We perform an empirical analysis on U.S. data. We specify the behavioral reward-risk model using the piecewise-exponential value function, as suggested by Kobberling and Wakker (2005) and De Giorgi and Hens (2006). We define investors' account using deterministic reference points, which corresponds to various target returns. We compute optimal portfolios as a function of investors' loss tolerance according to the behavioral reward-risk model with multiple accounts. We show that the bonds-to-stocks ratio is lower in high aspiration accounts relative to low aspiration accounts. Similarly, the ratio between large cap stocks and the total holding of stocks is higher for low-to-medium aspiration accounts relative to very high aspiration accounts. Since investors with lower loss tolerance mainly invest in low aspiration accounts, their portfolios also present a lower bonds-to-stocks ratio and a lower ratio between large cap stocks and total holding of stocks. These findings are consistent with the financial advisors' recommendations reported by Canner, Mankiw, and Weil (1997) and Wang (2003).

1. Related literature

In their seminal work, Canner, Mankiw, and Weil (1997) relax the key assumptions leading to the mean-variance two-fund separation theorem, which are: (1) existence of a riskless asset; (2) mean-variance objective functions; (3) investors use historical distributions; (4) assets can be freely traded (that is, there are no short-sale or borrowing constraints); (5) investors operate over a one-period planning horizon, and; (6) there is no background risk like human capital. However, they conclude that deviating from these assumptions does not provide satisfactory explanations of the recommended portfolio allocations, in particular of the relationship between the bonds-to-stocks ratio and risk aversion. Indeed, the authors state that it is hard to explain recommended portfolio allocations with a rational model.

Several authors have suggested solutions to the asset allocation puzzle. For this reason we make an effort here to precisely place our contribution in this literature. Some authors are skeptical about the conclusion of Canner, Mankiw, and Weil (1997) concerning the inconsistency of financial advisors with respect to the modern portfolio theory. Elton and Gruber (2000), for example, claim that the bonds-to-stocks ratio test is not sufficient to assert that financial advisors do not follow modern portfolio theory. In particular they point out that violations of two-fund separation might result from constraints that advisors are obliged to satisfy, like short-sale constraints, and the bonds-to-stocks ratio can be increasing or decreasing depending on the set of historical data or forecasted expected returns used to derive optimal strategies. They conclude that in order to test deviations of advisors' recommendations from the modern portfolio theory of Markowitz (1952) one should also take into account the input data used by financial advisors to derive the recommend allocations. Indeed, Siebenmorgen and Weber (2003) asked German advisors' to provide both recommended allocations for different investors and input data used to compute them. However, they found that recommended allocations were difficult to explain within the mean-variance model of Markowitz (1952) even when using the input data provided by the advisors.

[1] Mental accounting refers to the set of "cognitive operations used by individuals [...] to organize, evaluate, and keep track of financial activities" (Thaler, 1999). Here we consider the component of mental accounting which relates to the way investors assign different activities, or investment goals, to different (mental) accounts.

Shalit and Yitzhaki (2003) relax the assumption about mean-variance preferences and test the efficiency of financial advisors' portfolio allocations with respect to second-order stochastic dominance (SSD). They show that these allocations are not inefficient, that is, not dominated with respect to SSD by any alternative allocation. Therefore, even if the recommended allocations are not mean-variance efficient, they are optimal for at least one risk-averse expected utility maximizer, that is, there is no alternative allocation that is preferred by all risk-averse investors. Our concern about this approach is that the set of portfolio allocations that are not dominated by others with respect to second order stochastic dominance is large. Thus, several portfolios could be justified using second order stochastic dominance, while the fact that the bonds-to-stocks ratio of recommended allocations shows a specific shape as function of risk tolerance remains unexplained. Note that also in the mean-risk model introduced by De Giorgi (2005) investors select portfolios that are not dominated with respect to second order stochastic dominance, but two-fund separation is satisfied.

Other authors argue that a static portfolio model is not able to capture important aspects of the portfolio decision process and suggest relaxing assumption (5) above, advocating inter-temporal hedging activities as a solution of the asset allocation puzzle. Brennan and Xia (2000, 2002), Campbell and Viceira (2001, 2002), and Bajeux-Besnainou, Jordan, and Portait (2001) consider a dynamic model with stochastic interest rate where bonds can be used to hedge against the interest rate risk and show that the bonds-to-stocks ratio increases with risk aversion due to the hedging component of investors' optimal portfolios. Mougeot (2003) introduces a dynamic model with stochastic interest rate and estimation risk, that is, uncertainty about market's excess return, and shows that hedging components for estimation risk and interest rate risk can rationalize the asset allocation puzzle.

While dynamic models of portfolio selection represent a theoretical framework for solving the asset allocation puzzle that is very appealing to economists, Lioui (2007) shows that the results obtained are mainly driven by the assumption that bonds perfectly hedge the interest rate risk and the market price of risk is constant, and these assumptions lack empirical support. He shows that in a more realistic dynamic model for portfolio selection the asset allocation puzzle might even be more puzzling. Moreover, Wang (2003) points out that inter-temporal hedging might help explaining the bonds-to-stocks ratio, but it cannot explain why the proportion of large cap stocks relative to the total holding of stocks increases with risk aversion in financial advisors' recommendations.

Gomes and Michaelides (2004) provide a human capital explanation of the puzzle: investors face a stochastic uninsurable labor income and more risk adverse households invest a smaller percentage of their assets in stocks since they prefer labor income substitutes such as long-term bonds. Again, introducing human capital does not help understanding the sub-puzzle identified by Wang (2003).

This paper suggests a solution to the asset allocation puzzle by addressing assumption (2) above for mean-variance two-fund separation, that is, that investors possess mean-variance objective functions. Indeed, we provide a behavioral explanation of the asset allocation puzzle and the sub-puzzle assuming that investors possess reward-risk preferences founded in the prospect theory of Kahneman and Tversky (1979) and, additionally, have different mental accounts which correspond to different aspiration levels, or reference points. We are not the first suggesting a behavioral explanation to the asset allocation puzzle. Siebenmorgen and Weber (2003) use a static mean-variance portfolio selection model where investors are assumed to calculate portfolio's variance without taking correlations into account (pure risk), combined with a naive diversification criterion[1]. Wang (2003) also analyzes investors' choices in a static mean-variance portfolio model with pure risk (correlations are ignored), but instead of using naive diversification, investors are assumed to be averse to extreme losses. In the model of Wang (2003), aversion to extreme losses is given by a worst-case threshold which is assumed to depend on investors' tolerance to volatility. However, the author does not give any functional form to the relationship between volatility aversion and loss aversion.

Our solution to the asset allocation puzzle differs from these papers by the fact that we depart from the mean-variance setup of Markowitz (1952) and use a behavioral reward-risk model that is founded in the prospect theory, using the reference point which characterizes prospect theory preferences to define investors' mental accounts (Thaler, 1985, 1999). Extensive experimental evidence supports prospect theory as a descriptive model of decision-making under risk, showing that people have different attitudes to risk when facing gains or losses, and that people are loss averse, that is, dislike symmetric payoffs around their reference points. By contrast, mean-variance preferences treat gains and losses in the same way. Prospect theory also represents the natural framework to study portfolio selection with multiple mental accounts, which reflect different investment goals or reference points. Finally, aversion to

[1] Siebenmorgen and Weber (2003) assume that investors solve a tradeoff between pure risk and naive diversification, where naive diversification is measured as the distance from the $1/n$ portfolio strategy (see DeMiguel, Garlappi, and Uppal, 2007).

extreme losses can be easily incorporated into prospect theory; see Jalal, Jondeau, and Rockinger (2007) and Basili, Renó, and Zappia (2008). Our approach is also related to the recent behavioral literature applying prospect theory and loss aversion to explain low participation in equity markets, under-diversification, the disposition effect and the equity premium puzzle: see Benartzi and Thaler (1995), Barberis, Huang, and Santos (2001), Gomes (2005), Berkelaar, Kouwenberg, and Post (2004), Barberis, Huang, and Thaler (2006), Barberis and Huang (2008a, 2008b, 2009), Jin and Zhou (2008), Dimmock and Kouwenberg (2010), De Giorgi and Legg (2009), Bernard and Ghossoub (2010), He and Zhou (2011), De Giorgi, Hens, and Levy (2011), De Giorgi (2011). We add to this important literature by showing that prospect theory and mental accounting also helps explaining the asset allocation puzzle and the sub-puzzle.

The remainder of the paper is organized as follows. In section 2 we briefly describe the behavioral reward-risk model of De Giorgi, Hens, and Mayer (2006). Section 3 presents an empirical analysis of the asset allocation puzzle on U.S. data. The final section concludes the proposal.

2. The behavioral reward-risk model

In this section, we briefly describe the behavioral reward-risk model of De Giorgi, Hens, and Mayer (2006), which is based on the prospect theory of Kahneman and Tversky (1979). We assume a one-period portfolio model, where uncertainty is given by a finite state-space $\{1,..., S\}$, where each scenario $s = 1,..., S$ has probability $p_s \in (0,1)$. There are $K + 1$ assets with random gross returns R_k, $k = 1,..., K$, and $R = (R_1,..., R_{K+1})'$. We denote by $\lambda \in R^{K+1}$ the vector of assets weights and w_0 is the investor's initial wealth.

Prospect theory suggests modeling investors' preferences as follows:

Assumption 1 (Prospect Theory Preferences). Investors evaluate portfolio payoffs according to the value function:

$$V(X) = \sum_{s=1}^{S} v(X(s) - RP(X))\pi_s \tag{1}$$

for all portfolio payoffs $X = \lambda' R w_0$,

where v is a two-time differentiable function on $R\backslash\{0\}$, strictly increasing on R, strictly concave on $(0,\infty)$ and strictly convex on $(-\infty,0)$, with $v(0) = 0$; $\pi_s = w(p_s)$ and w is a differentiable, non-decreasing function from $[0,1]$ onto $[0,1]$ with $w(p) = p$ for $p = 0$ and $p = 1$ and with $w(p) > p$ ($w(p) < p$) for p small (large); $RP(X)$ is a subjective reference point, which might depend on the payoff X. If $RP(X) = RP$ for all X, then the reference point is fixed. If $RP(X) = (1 + \tau)\, q(X)$, where $q(X)$ is the price of portfolio X and $\tau > 0$, than the reference point corresponds to a fixed target return τ.

The prospect theory assumes that investors code payoffs in terms of gains and losses, that is, payoffs above and below a subjective reference point, respectively. Moreover, it also assumes different risk attitudes with respect to gains than with respect to losses (reflection principle). These assumptions suggest a natural way to define a reward-risk tradeoff that describes investors' preferences. De Giorgi, Hens, and Mayer (2006) rewrite the value function (1) by separating portfolio outcomes which are above the reference point and portfolio outcomes which are below the reference point and obtain

$$V(X) = \sum_{s=1}^{S} v(\max(0, X(s) - RP(X)))\pi_s - \\ - \beta \sum_{s=1}^{S} \left(\frac{-1}{\beta}\right) v(\min(0, X(s) - RP(X)))\pi_s, \tag{2}$$

where $\beta = \dfrac{\lim_x \to 0v'(x)}{\lim_x \to 0v'(x)} \geq 1$ is the index of loss aversion, as defined by Benartzi and Thaler (1995) and Kobberling and Wakker (2005).

Equation (2) can be seen as a tradeoff between risk and reward, where the reward and risk measures are

$$PT^{+}(X) = \sum_{s=1}^{S} v(\max(0, X(s) - RP(X)))\pi_s, \tag{3}$$

$$PT^{-}(X) = -\frac{1}{\beta} \sum_{s=1}^{S} v(\max(0, X(s) - RP(X)))\pi_s, \tag{4}$$

respectively. PT^+ measures positive deviations of assets' returns with respect to the reference point. By contrast, the reward measure ignores assets' payoffs which are below the reference point, that is, losses. Rather than affecting portfolio reward, we believe that losses should impact portfolio's risk. In fact, losses are determined by means of the risk measure PT^-. Finally, the index of loss aversion β measures the investor's tradeoff between gains and losses, since PT^- has been normalized by β and therefore does not account for loss aversion. The portfolio choice problem is:

$$\max PT^{+}(\lambda' R w_0) \text{ such that } PT^{-}(\lambda' R w_0) < pt^{-}, \tag{5}$$
$$\lambda' e = 1,\ \lambda \in R^{K+1}$$

where $e \in R^{K+1}$ is a vector of ones. The parameter pt^- represents investor's risk aversion and is closely related to it's index of loss aversion β. In the sequel, we will consider pt^- as the parameter reflecting investors' loss aversion. We call problem (5) the behavioral reward-risk model.

3. Empirical application

This section derives the optimal (PT^+, PT^-)-portfolio allocations between cash, bonds, small-mid caps and large caps using yearly historical returns from 1927 to 2007. We show that the prospect theory reward-risk model with mental accounting (Thaler, 1985, 1999) offers an explanation to the asset allocation puzzle and the sub-puzzle.

3.1. Data. We use yearly nominal returns of the CRSP market portfolio ("market"), of a bond index ("bonds"), and of the US one-month Treasury Bill ("cash"). In addition to this, we also include yearly nominal returns of two Fama and French US common stock portfolios formed on market capitalization of equity: small-mid caps ("small-mid") and large caps ("large")[1]. The equity data are obtained from Kenneth French's online data library; the bond index corresponds to the U.S. intermediate-term government bond index maintained by Ibbotson Associates; T-Bill data are also from Ibbotson Associates. The sample covers the period from January 1927 to December 2007, with a total of 81 yearly observations. Table 1 gives the summary statistics of our data. In our empirical analysis, similarly to Canner, Mankiw, and Weil (1997), we assume that cash is risk-free with a yearly gross return R0 = 1 + r = 1.0378 corresponding to the mean gross return of the T-Bill.

Table 1. The data

Panel A					
Assets	T-Bill	Bond index	Market	Small-mid	Large
Mean	3.78	5.34	12.01	15.63	11.74
Std. deviation	3.11	5.27	20.11	27.63	19.37
Skewness	0.95	1.16	-0.36	0.27	-0.37
Kurtosis	0.96	2.14	-0.02	0.96	0.00
Maximum	14.72	26.02	57.50	111.45	53.10
Minimum	-0.04	-5.17	-44.35	-46.82	-43.21
Panel B					
	T-Bill	Bond index	Market	Small-mid	Large
T-Bill	1.00	0.49	-0.03	-0.10	-0.02
Bond index	0.49	1.00	0.03	-0.03	0.06
Market	-0.03	0.03	1.00	0.93	0.99
Small-mid	-0.10	-0.03	0.93	1.00	0.89

Note: Panel A reports the summary statistics of yearly nominal returns (in %) for the Treasury Bill, the bond index, the CRSP market portfolio and the two Fama and French U.S. stock portfolios formed on market capitalization (small-mid and large). There are 81 observations ranging from 1927 to 2007. The bond index and the Treasury Bill data are from Ibbotson Associates, the stock portfolios and the CRSP market portfolio are from the Kenneth French data library (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/). Panel B reports the correlation between any pair of assets.

3.2. Prospect theory and two-fund separation. We compute optimal (PT^+, PT^-)-portfolios by solving problem (5) using historical distributions as proxies for future returns, that is, we assume that future returns are uniformly distributed on the set $\{r_t : t = 1927,..., 2007\}$, where r_t is the vector of observed returns for year t. We impose short-sale constraints. This is consistent with the observation that financial advisors' recommendations reported by Canner, Mankiw, and Weil (1997) and Wang (2003) only presents long positions on all asset classes. In general, the behavioral reward-risk model is non-convex and non-differentiable and thus we cannot apply Lagrange methods. We refer to De Giorgi, Hens, and Mayer (2007) for a detailed discussion of the numerical methodologies used to solve the (PT^+, PT^-)-optimization. We combine the behavioral reward-risk model (5) with the multiple-account version of the behavioral portfolio theory of Shefrin and Statman (2000). According to this theory investors possess different mental accounts, which correspond to different aspiration levels or investment goals. In the prospect theory reward-risk model this is captured by assuming that investors possess multiple reference points, which characterize their aspiration levels and the corresponding mental accounts. Low aspiration accounts reflect need for security, while high aspiration accounts reflects hope to achieve richness. Risk, as measured by PT^-, obviously increases with the reference point. Consequently, investors with low loss tolerance only invest into low aspiration accounts. For each account, optimal (PT^+, PT^-)-portfolios are derived, where reward and risk measures are defined using the account-specific reference point. Finally, wealth is allocated to the different accounts according to investors' loss tolerance. In performing this step, investors treat accounts' specific portfolios as separate entities ignoring dependencies between portfolios' payoffs.

We compute optimal (PT^+, PT^-)-portfolios for each account by solving problem (5). We specify reward and risk measures PT^+ and PT^-, respectively, using a piecewise-exponential value function

$$v(x) = \begin{cases} 1 - \exp(-\alpha x) & x \geq 0 \\ -\beta(1 - \exp(-\alpha x)) & x < 0 \end{cases} \qquad (6)$$

suggested by Kobberling and Wakker (2005) and De Giorgi and Hens (2006). The reference point is assumed to correspond to a fixed rate of return, that is, $RP(X) = (1 + \tau)\, q(X)$ for some $\tau \in R_+$, which corresponds to the account's aspiration level. We take

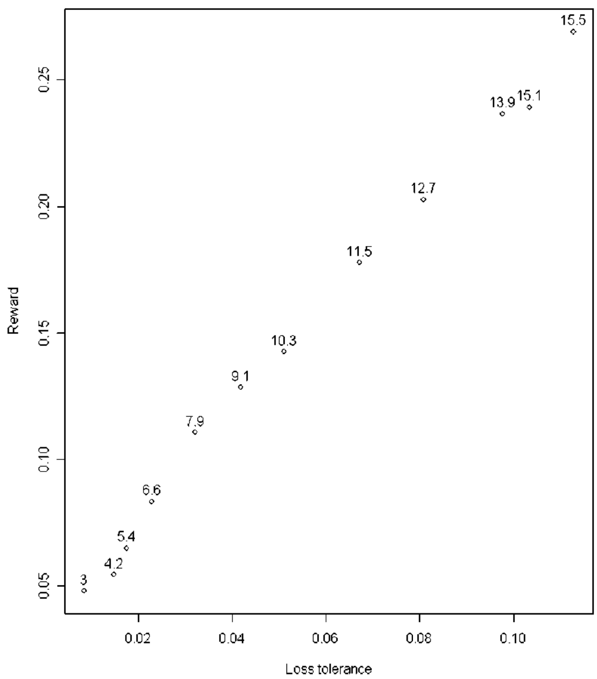
[1] The returns of the small-mid caps portfolio are obtained from the *Lo*30 and the *Med*40 portfolios of Fama and French. Based on the number of firms n_t^{Lo30} and n_t^{Med40} contained in these portfolios and the average market capitalization amc_t^{Lo30} and capitalization amc_t^{Lo30} and amc_t^{Med40}, we derive at each time t the weights $\lambda_t^{Lo30} = n_t^{Lo30}\ amc_t^{Lo30}/mc_t^{tot}$ and $\lambda_t^{Med40} = n_t^{Med40}\ amc_t^{Med40}/mc_t^{tot}$ where $mc_t^{tot} = n_t^{Lo30}\ mc_t^{Lo30} + n_t^{Med40}\ amc_t^{Med40}$. The returns of the small-mid caps portfolio are then obtained as $r_t^{sm} = \lambda_t^{Lo30}\ r_t^{Lo30} + \lambda_t^{Med40}\ r_t^{Med40}$. The large caps portfolio corresponds to the Hi30 portfolio of Fama and French.

reference points with values ranging from τ = 3% (mean inflation rate over the sample period) to τ = 15.5% (approximatively the mean return of small-mid cap stocks). Table 2 reports the minimum risk portfolios as function of the reference point, while Figure 1 displays their risk-reward tradeoff.

Table 2. Minimum risk portfolios

τ (%)	Cash	Bonds	Small-mid	Large
3.0	95	5	0	0
4.2	90	10	0	0
5.4	70	25	5	0
6.6	40	45	10	5
7.8	0	70	15	15
9.1	0	60	20	20
10.3	0	55	25	20
11.5	0	35	40	25
12.7	0	20	45	35
13.9	0	0	55	45
15.1	0	0	50	50
15.5	0	0	100	0

Note: The table reports minimum risk portfolios for the prospect theory reward-risk model for different reference points $RP(X)$ = = $(1 + \tau)\ q(X)$. The numbers are in percentage and rounded to nearest 5% for portfolio holdings. The reward and risk measures PT^+ and PT^-, respectively, are specified using a piecewise-exponential function $v(x) = 1 - \exp(-\alpha x)$ for $x \geq 0$ and $v(x) =$ $= -\beta(1 - \exp(\alpha x))$ for $x < 0$, $\alpha = 10$.



Note: The figure displays reward and risk for the minimum risk portfolios for the prospect theory reward-risk model with different reference points (which are indicated above the points). The prospect theory reward-risk model is specified using a piecewise-exponential value function $v(x) = 1 - \exp(-\alpha x)$ for $x \geq 0$ and $v(x) =$ $= -\beta(1 - \exp(\alpha x))$ for $x < 0$, $\alpha = 10$, and deterministic reference points $RP(X) = (1 + \tau)\ q(X)$. In order to compare reward and risk for portfolios computed assuming different reference points, reward is given as $\tau + PT^+$, while risk is given as PT^-.

Fig. 1. Reward-risk tradeoffs

The minimum risk portfolio minimizes PT^- for the corresponding reference point. We compute minimum risk portfolios since investors' loss tolerance only determine how wealth is allocated between the different accounts, while given a specific reference point all investors minimize the risk of missing the reference target return. As we expected, minimum risk portfolios strongly depend on the choice of the reference point. As the reference point increases, allocation to cash is reduced, while allocation to bonds and stocks increases and the ratio between bonds and stocks decreases. Finally, for high values of the reference point, minimum risk portfolios only contain stocks, which become small-mid cap stocks for extreme values of the reference point.

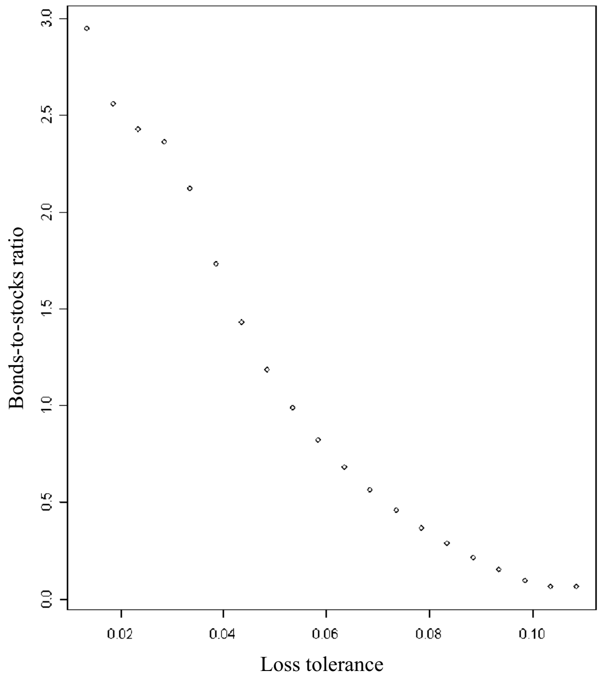
Investors' portfolios are obtained by allocating wealth between the different accounts. Let pt_k^- be the risk of the minimum risk portfolio for account k with corresponding reference point τ_k for $k = 1,..., K$, where $\tau_1 < \tau_2 < \ldots < \tau_K$. Obviously we have $pt_1 \leq pt_2 \leq \ldots pt_k^-$, since PT^- increases with the reference point. Let $\widetilde{pt}_k^+ = \tau_k + pt_k^+$ be the reward of the minimum risk portfolio for account k, plus the corresponding target return. Investors allocate wealth between the different accounts in order to maximize their total reward given the risk constraint implied by their loss tolerance. Moreover, the different accounts are considered separately, that is, the covariance between accounts' specific portfolios is ignored. Consequently, the investors' (global) portfolio choice problem can be written as a simple linear program:

$$\max \sum_{k=1}^{K} \zeta_k \widetilde{p}t_k^+, \text{s.t.} \sum_{k=1}^{K} \zeta_k \widetilde{p}t_k^- \leq pt^-, \ \sum_{k=1}^{K} \zeta_k = 1, \quad (7)$$

$$\zeta_k \geq 0 \text{ for } k = 1,..., K.$$

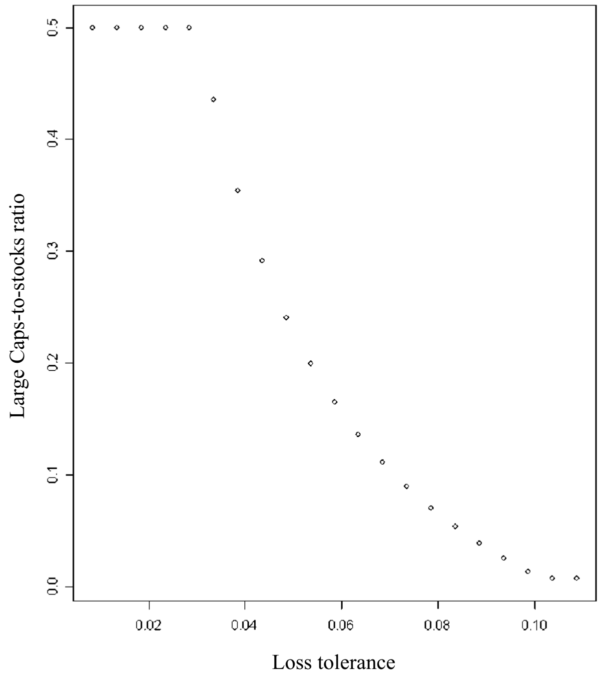
The mental accounts are defined for reference points $RP(X) = (1 + \tau)q(X)$, where τ takes values 3.0%, 4.2%,..., 15.5% as given in the first column of Table 2. We solve problem (7) for $pt^- \in [pt_1^-, pt_K^-]$, which represent investors' tolerance to losses. The range $[pt_1^-, pt_K^-]$ corresponds to the values of PT^- that can be reached when short-selling are not allowed, so it makes sense to restrict investors' tolerance to it. We obtain that investors with low tolerance to losses only invest into the accounts with reference points 3% and 7.8%, while investors with higher loss tolerance invests into the accounts with reference points 6.6%, 7.8% and 15.5%. Therefore, investors with higher loss tolerance also possess high aspiration accounts, while investors with lower loss tolerance mainly invest into low-aspiration accounts. Figure 2 shows the bonds-to-stocks ratio as function of investors' loss tolerance pt^{-}. The bonds-to-stocks ratio is decreasing due to the fact that investors with higher loss tolerance put a higher proportion of their wealth into high aspiration accounts.

Similarly, Figure 3 shows the ratio between the proportion invested in large cap stocks and the total holding of stocks. This ratio is constant for low degrees of loss tolerance, while it decreases as function of pt^{-} when the degree of loss tolerance is higher. These results are consistent with the portfolio recommendations reported by Canner, Mankiw, and Weil (1997) and Wang (2003).



Note: The figure shows the bonds-to-stocks ratio for optimal portfolios as function of the degree of loss tolerance pt^{-}. Investors' mental accounts correspond to the reference points presented in Table 2. The behavioral reward-risk model is specified using a piecewise-exponential value function $v(x) = 1 - \exp(-\alpha x)$ for $x > 0$ and $v(x) = -\beta v(-x)$ for $x < 0$, $\alpha = 10$.

Fig. 2. Bonds-to-stocks ratio with multiple accounts



Note: The figure shows the ratio between the proportion of large cap stocks and the total holding of stocks as function of the degree of loss tolerance pt^{-}. Investors' mental accounts correspond to the reference points presented in Table 2. The behavioral reward-risk model is specified using a piecewise-exponential value function $v(x) = 1 - \exp(-\alpha x)$ for $x \geq 0$ and $v(x) = -\beta(1 - \exp(\alpha x))$ for $x < 0$, $\alpha = 10$.

Fig. 3. Large cap stocks to total stocks ratios with multiple accounts

Conclusion

We have combined the behavioral reward-risk model of De Giorgi, Hens, and Mayer (2006) with the multiple accounts version of the behavioral portfolio theory of Shefrin and Statman (2000), assuming that investors might possess different and even multiple reference points. In an empirical analysis on U.S. data, we showed that the behavioral reward-risk model with multiple accounts explains both the asset allocation puzzle documented by Canner, Mankiw, and Weil (1997) and the sub-puzzle identified by Wang (2003).

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