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ARTICLE INFO	Khalid M. Dubas, Lewis Hershey, Inder P. Nijhawan and Rajiv Mehta (2011). Breakeven and profitability analyses in marketing management using R software. *Innovative Marketing* , *7*(3)
RELEASED ON	Thursday, 27 October 2011
JOURNAL	"Innovative Marketing "
FOUNDER	LLC “Consulting Publishing Company “Business Perspectives”

NUMBER OF REFERENCES **0**

NUMBER OF FIGURES **0**

NUMBER OF TABLES **0**

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Breakeven and profitability analyses in marketing management using R software

Abstract

An extensive literature in economics and business provides guidelines for profit maximization for firms in various market structures. However, these elaborate and sophisticated techniques and rules for profit maximization require appropriate estimation of a company's cost and revenue functions that are often difficult to obtain for many companies. So for many businesses, an important practical tool for profitability analysis and decision making is often the breakeven analysis that identifies the level of price and output where a firm's revenue equals its cost. So production and sales beyond that point generate profit. Although, breakeven analysis is easy to understand and use, its assumptions are often misunderstood or ignored resulting in its misuse.

The commonly used breakeven formula in business and marketing describes a special type of perfectly competitive firm that has no pricing power, faces a horizontal demand curve, a linear total revenue curve, a linear total cost curve, and a linear profit curve that increases indefinitely. A more typical perfectly competitive firm would instead have a U-shaped (or inverted S-shaped) total cost curve, and a quadratic profit function; thus, two breakeven points, and it would reach maximum profit at the point where its marginal revenue equals its marginal cost. Such firms would not require a marketing manager since they can sell all of their production at the given market price that is determined by the market demand and supply. However, most firms operate in an imperfectly competitive market and exercise some control over the price of their products; they also control their product quality, promotion, and distribution. Such firms require marketing managers for understanding and responding to the needs and wants of their customers and these managers can utilize appropriate breakeven and profitability analyses.

This article evaluates breakeven and profitability analyses for firms in perfectly competitive and imperfectly competitive markets. The conceptual and practical methods for profitability analysis are presented, and R, free mathematical and statistical software, is used to analyze various situations to guide marketing managers in their decision making. The use of this free and powerful software to facilitate analysis and decision making should empower the greatest number of decision makers all over the world. The role of product characteristics in determining product demand, pricing, and market shares is also presented here.

Keywords: breakeven analysis, profitability analysis, sales-costs-profit analysis, volume-costs-profit analysis, market structure, demand estimation, product characteristics, product features.

Introduction

Business organizations strive to be successful in achieving their missions and their goals. One essential business goal is to achieve desired profits. Marketing management works within the overall business framework to develop marketing plans involving market segmentation, targeted marketing mixes (product, price, promotion, and place) and appropriate positioning/repositioning of the product for each target market segment. Of the four marketing mix elements, only price generates revenue while the other three elements create costs (Kotler, 1999). So firms strive to increase their prices as high as their level of product differentiation, market structure, and their pricing power would allow. Successful firms accumulate a lot of information that helps them to make reasonable profitability analyses. Use of breakeven/profitability analyses, simulation modeling, and yield management for pricing is quite common in many companies and industries.

Perfectly competitive firm. Breakeven (cost-volume-profit) analysis is often utilized to understand the sales volume where profits begin to emerge. This tool is useful for evaluating new and existing products and projects. The typical discussion of breakeven analysis in the business literature assumes a given price (P) and linear total revenue (TR) curve; so the firm has a horizontal demand curve at the given market price. Its marginal revenue is equal to its average revenue, MR = AR. A linear total cost (TC) curve is also assumed that along with a linear TR curve, generates a linear profit curve. The breakeven point is reached at TR = TC or profit = 0. This describes a special type of firm within a perfectly competitive market; this firm has a linear profit function that increases indefinitely with increase in output that is always sold at the given market price. This is an unstable situation because the firm will continue to produce as much as possible since its output is always sold at the market price. Let us call this special firm an "unstable perfectly competitive firm". Changing the linear TC curve to a U-shaped TC curve, with upward sloping marginal cost (MC) curve, will stabilize this firm's situation and it will achieve an equilibrium between its supply and demand curves. A firm in a perfectly competitive market maximizes its profit at MR = MC which is its equilibrium point and this will lie between its two brea-

keven points that arise due to the fact that it will have a quadratic profit function. A firm in a perfectly competitive market does not require a marketing manager.

This special case of an unstable perfectly competitive firm tends to be applied across much of business and marketing literature to firms that operate in imperfectly competitive markets. It is important to understand the appropriate market structure (monopoly, oligopoly, monopolistic competition, perfect competition) of the firm under consideration and apply appropriate breakeven and profitability analyses for marketing management. Only firms in imperfectly competitive markets would require marketing managers to develop marketing plans, to study their customers' needs and wants, to achieve customer satisfaction, and build customer relationships that would generate profit for their firms.

Purpose and scope. This paper illustrates an overall conceptual framework for profitability analysis and breakeven analysis using examples and R software that is free and can be downloaded and installed from the Internet. Specifically, the following topics are presented in this article:

1. A generalized theoretical framework for profit maximization using marginal analysis and the implications of pricing power on the shape of demand curve, total revenue curve, and profit function.
2. The main ideas of breakeven (cost-volume-profit) analysis, its assumptions and uses. The methods for estimating revenue (sales forecasting) and various means of collecting and evaluating information for better decision making.
3. Yield management and price differentiation by organizations.
4. Market and competitor analysis using product characteristics and price to estimate market shares.
5. Use of R software for mathematical and statistical analysis.
6. Forecasting sales using price and marketing mix elements.

1. Literature review

1.1. Market potential, target sales potential, and sales forecasting. Market potential is the maximum possible total sales of a particular product or service, under ideal conditions, for the entire industry in a specific market for a specific time period. Sales potential is the share of market potential that an individual firm can ideally expect to achieve. Compared with sales potential, a sales forecast is what an individual firm realistically expects to achieve; typically the sales potential would be higher than the sales forecast since the former would be achieved only under ideal conditions and a firm's financial resources or changes in its external environment may not allow it to reach its sales potential. Sales and marketing literature (Palda, 1969, 1971; Spiro, Rich, and Stanton, 2008; Perreault, Cannon, and McCarthy, 2011) presents numerous methods for calculating market potential, sales potential, and sales forecasts and points out that the difficulty in developing an accurate sales forecast varies from situation to situation. Sales forecasts will be quite difficult to obtain for radically new products than for somewhat new or established products that enjoy stable sales. Other situations would require considering capacity limitations and product quality to forecast sales and manage demand.

Here are several sales forecasting methods and their data sources:

- Concept testing – respondents/potential customers.
- Survey of executive opinion, the Delphi technique – managers.
- Sales force composite – managers and salespeople.
- Survey of buyer intentions – customers.
- Moving average, exponential smoothing, and regression analysis – historical data.
- "Must do" approach and capacity-based approach – company operations.
- Test marketing – customers/potential users.

These techniques help a marketing manager to determine sales forecasts for a target market; however, the accuracy of these forecasts depends on the firm's ability to determine the correct marketing mix and understand the impact of the changing external environment on the firm. It should be noted that only a firm in an imperfectly competitive market would need to forecast sales and plan production accordingly since a firm in a perfectly competitive market can sell its entire production at the given market price even without any promotion at all since the customers have perfect information and all companies' products are homogeneous.

Products follow a product life cycle and the marketing mix must be adjusted as a product goes through the various stages of its life cycle. During the introduction stage of a product's life cycle, the pricing policy should be market skimming or market penetration. During the later stages of its life cycle, the price should match its competition; may use price dealing and price cutting (Perreault, Cannon, and McCarthy 2011). Sony, for example, used market penetration pricing to beat Toshiba in their competition between its Blu-ray DVD and Toshiba's HD DVD formats by cutting prices, establishing alliances with Samsung and Philips, and including Blu-ray players in its PS 3 consoles.

1.2. Types of innovations. A product is a bundle of features (or attributes or characteristics) and it can be modified by modifying its features. It is useful to describe different types of product innovations and indicate the degree of difficulties involved in forecasting

sales for each category. According to the Federal Trade Commission, a firm can call its product new only for up to six months after introduction to the market. To be called new, it must be entirely new or changed in substantial ways. Product innovations can be classified as follows:

1. New-to-the-world products or discontinuous innovations. These are radically new products that, if successful, create a new product category and require new skills and knowledge. These are least frequent innovations.
2. Me-too products. These products are new-to-the-company but not new-to-the-market so historical sales data are available for similar products that can be used to develop sales forecasts. These are more common innovations.
3. Product modifications involve modifying existing products.
4. Product line extension involves adding more products to a product line.
5. Product positioning/repositioning. Here the product is positioned/repositioned in the customer's mind by changing its advertising message and/or product features.

Sales forecasting for new-to-the-world products is most difficult while it is easier for other types of innovations since comparable data on similar or competing brands are often available for comparison.

1.3. Breakeven and profitability analyses. Several authors in business and marketing literature discuss the uses of breakeven analysis and all of them utilize the unstable perfectly competitive firm's breakeven analysis formula and/or diagram and apply them to all situations – including imperfectly competitive firms. Harris (1978) discusses the unstable perfectly competitive firm's breakeven analysis, a plot of TR, TC, FC, and a computer program in BASIC language for a mainframe computer and numerous applications of breakeven analysis. Since 1978, breakeven analysis has been more widely discussed and utilized in business literature and numerous breakeven analysis calculators are now available free online. Harris recommends use of breakeven analysis for the following situations:

- Profit planning or budgeting.
- Problem analysis at the level of a segment or whole business when financial results are unsatisfactory.
- Making quick assessment in advance of major changes like opening or closing plants, adding or eliminating sales territories or product lines.

Breakeven analysis is one of many techniques that have been developed to help management plan, coordinate, and control business operations for success. He notes that the basic breakeven analysis shows costs in two categories: *fixed* and *variable*. This classification is with respect to production. However, in a business the vast majority of costs are semi-variable; that is they vary with production but do not vary in direct proportion to volume. Thus, he suggests use of (1) *direct costs*, those costs that are incurred by producing a product like raw materials and fuel or making a sale like advertising and sales commissions, and (2) *period costs* that are incurred from the provision of capacity to make and sell and from keeping this capacity in readiness regardless of production or sales volume. Further, the period costs could be divided into two categories: (1) *capacity costs* that are required to provide operating capacity and organization, while (2) *discretionary costs* arise from specific management appropriations. Thus, like direct costs, the period costs can be influenced in the short run by management. Therefore, each organization should categorize its own costs and break them into their variable and fixed components for breakeven analysis. He recommends incorporating breakeven analysis concepts into the regular accounting and record keeping system by creating a chart of accounts and reporting financial results to management. Direct costing emphasizes the contribution approach, so the contribution of each segment is measured. This eliminates allocation of costs from outside a responsibility center using arbitrary methods. Direct cost system can transform managers from critical spectators to active participants in the management accounting process as they carry out planning, organization, and control activities.

He notes that revenue projection is a prerequisite for every modern management control technique. For example, sales forecasts are required for budgeting, production planning and inventory control; non-manufacturing organizations may express revenue as professional fees, rents, interest earned, merchandise sales, or royalties, etc. Individual segments of an organization plan their operations based on revenue potential. He notes the following difficulties in revenue projections:

- Revenue forecasting is not an exact science.
- The goals set are not reasonable.
- Company's personnel perform better or worse than expected.
- Unexpected changes in external environment.

He suggests the following steps for successful implementation of breakeven analysis:

- Proper understanding of breakeven formula and chart.
- The importance of proper terminology.
- The attainment of reasonable accuracy.
- The need to track results.
- The relation of breakeven techniques to direct costing.

- Start with past sales data for revenue projection.
- The estimate is usually for one year or less.
- Dollar or unit sales can be used as overall base to measure activity.
- A stable price is usually assumed.
- A constant sales mix is also assumed.
- Management should participate in revenue projection.
- Projection of sales should be within normal range of operations where volume-cost-profit relationships are predictable.

For sources of information for revenue projections, he recommends use of historical information and external sources of information from trade associations, government publications, trade journals, financial newspapers, professional marketing studies, chamber of commerce publications, business associations, and sales people.

Siegel, Shim, and Hartman (1992) describe the unstable perfectly competitive firm's breakeven analysis and related formulas as follows:

- Breakeven formulas are useful for all businesses. The three most common breakeven formulas are: (1) to determine breakeven point; (2) the margin of safety; and (3) the cash breakeven point.
- Breakeven analysis is used to organize thinking on important broad aspects of any business whether a hotel or an airline flight by determining the level of sales required to breakeven.
- The margin of safety is a measure of operating risk, the larger is the ratio the lesser is the risk in reaching the breakeven point. *Margin of safety* = [(*expected sales – breakeven sales*)/*expected sales*] x 100.
- The cash breakeven point is useful when a company has low cash on hand or the opportunity cost of holding excess cash is too high. The cash breakeven point is less than the usual breakeven point since noncash expenses are deducted from fixed costs. *Cash breakeven point = sales = variables costs + fixed cash cost*. It should be pointed out that elsewhere in the literature, the cash breakeven formula is also presented as (*Fixed costs – Depreciation)/Contribution margin per unit*.

They present these uses of breakeven formula and analysis:

- The sales volume required to breakeven.
- The sales volume necessary to earn a desired profit.
- The effect of changes in selling price, variable cost, fixed cost, and output on profit.
- The selling price that should be charged.
- The desired variable cost per unit or fixed costs.
- Breakeven analysis is important when beginning a new activity like starting a new product, or a new line of business, or expanding an existing business.
- Financial managers use breakeven to determine the feasibility of a proposed investment. Would the lower interest payments over the life of a new loan cover the costs of refinancing the existing higher interest loan?
- Management executives: Have the company's breakeven possibilities improved or deteriorated?
- Marketing managers: Will a new marketing campaign generate sufficient sales to cover the costs of the campaign? Would the introduction of a new product add to the company's profitability?
- Production managers: Would modernization of production facilities pay for themselves in cost savings.

The literature in business and marketing (Grewal and Levy, 2011; Shim, and Siegel, 2000; Siegel, Shim, and Hartman, 1992; McBryde-Foster, 2005) presents the following assumptions for breakeven and profitability analyses and all these authors utilize the unstable perfectly competitive firm's breakeven analysis:

- Sales price per unit is constant during the period of analysis.
- Variable costs per unit are constant during the period of analysis.
- Total fixed costs are constant during the period of analysis.
- Everything produced is sold so there is no inventory.
- The company sells one product or a constant mix of products.

Scheuing (1989) utilizes the unstable perfectly competitive firm's breakeven formula to select among alternative marketing mixes for a firm. These marketing mixes include alternative advertising budgets and alternative distribution cost allowances. He goes on to discuss breakeven management by using three tools: (1) increasing revenues by increasing price; (2) reducing fixed costs; and (3) reducing variable costs. He notices that "Raising the price of the new product appears to be the "quick fix" for the break-even problem, increasing revenue in a hurry without much effort." He then discusses the price elasticity of demand and states that increasing price should be linked with the price elasticity of the demand curve and that a firm may use price differentiation to increase its revenue. He provides an excellent example of the use of price discrimination to increase sales revenue and profit for an imperfectly competitive firm. While he utilizes a downward sloping demand curve for this

illustration, he uses the breakeven formula for the unstable perfectly competitive firm. Further, his illustration generates a quadratic TR curve that produces two breakeven points that he does not relate to his earlier discussion of breakeven analysis that used an unstable perfectly competitive firm. This is another example of the confusion created by using the incorrect breakeven formula for an imperfectly completive firm.

Perreault, Cannon, and McCarthy (2011) discuss the uses and limitations of breakeven analysis for an unstable perfectly competitive firm (described in Table 1, below).

- They note that it is popular, easy to use, and helpful in evaluating alternatives.
- They note that each price has its own breakeven point (BEP), so it is helpful to calculate a BEP for each of several possible prices and compare the BEP for each price to the likely demand at that price and reject those BEPs for which the demand is expected to be far below the BEP.
- They provide a formula for calculating BEP in dollars, so TR = TC, and a breakeven formula for target profit that is added to the fixed costs to obtain the target sales volume.
- They note that managers can lower BEP by reducing the fixed costs or by lowering the variable cost per unit so the profits would start faster.

They do not discuss that this particular breakeven formula describes a price-taker firm that can sell all of its output at the given market price so it is faced with only one price level. However, they do recognize the following:

- Breakeven analysis is too often misunderstood.
- Beyond the BEP the profits grow indefinitely.
- The straight-line TR indicates that TR grows indefinitely so any quantity can be sold at the assumed price but this is usually not true. Most managers do not have TR curves that increase indefinitely.
- The straight-line TR curve means that the firm has a perfectly horizontal demand curve at that price, but most managers face a downward sloping demand curve.
- Breakeven analysis is useful for analyzing costs and evaluating what might happen to profits in different market environments.
- It is a cost-oriented approach that does not consider the effect of price on the quantity that consumers will want, i.e., the demand curve.
- To identify the most profitable price, marketers should estimate the demand curve and then use marginal analysis to equate MR = MC.
- They present the behavior of an imperfectly competitive firm (described in Table 1, below) under their heading "Marginal Analysis Considers Both Costs and Demand".
- Their marginal analysis includes two breakeven points as is expected with a quadratic profit function, however, they do not link it with their earlier discussion of breakeven analysis.
- They emphasize that managers should not strive to identify the precise price that will maximize profit but to get an estimate of how profit might vary across a range of relevant prices. They present many approaches to help managers understand the likely shape of their demand curve for a target market.

While books in economics are typically very clear about a typical firm's goal as profit maximization and how to achieve it under various market conditions, few books in business or marketing successfully clarify the basic assumptions and limitations of breakeven and profitability analyses even though there is significant business and marketing literature on elaborate pricing and profitability models. Typically, the books in marketing and business do not specifically identify the various types of market structure and their marginal conditions (MR = MC) to achieve maximum profit. Their formulas and graphs for breakeven analysis typically consider the case of an unstable perfectly competitive firm and apply them to the most common situation of an imperfectly competitive firm. Such authors typically ignore that two breakeven points exist for a quadratic profit curve. The appropriate breakeven formula for a perfectly competitive firm and for an imperfectly competitive firm would be their quadratic profit set equal to zero. This quadratic equation will produce two breakeven points (roots) within which would lie the maximum profit and the corresponding price, quantity and other marketing mix elements. Some authors seem to associate marginal analysis (MR = MC) for profit maximization with imperfectly competitive firms only that face a downward sloping demand curve although a perfectly competitive firm would also use marginal analysis to maximize its profit at MR = MC; it would have a horizontal demand curve.

2. Methodology

This paper utilizes a general framework for profit maximization for a firm in a perfectly competitive market and another firm in an imperfectly competitive market. Breakeven analyses are utilized in both cases. Various situations involving different shapes of relevant functions and their implications on profit and marketing mix decisions are analyzed. In addition, several examples involving data and graphs are provided to illustrate important concepts.

R software is used here for analysis; this software is free and can be downloaded and installed from http://cran.r-project.org/. Six manuals on R are available in PDF format at this website and also in R itself in the pull down menu under Help. In addition to the base R software, a lot of free packages can be downloaded from the Internet and installed in R software. R can be run from the command line interactively or in a batch mode; it can be run on the web, or in a GUI with the R Commander (Rcmdr) package. R is a good substitute for SAS, SPSS, Maple, MATLAB, and Mathematica. R software can solve optimization problems usingcalculus or the more general numerical analysis methods. We present several numerical examples to illustrate the models and provide the essential commands in R software and its output.

3. General framework

Business organizations must earn some target profit if they are to survive and prosper. Marketing managers operate within the general framework of the corporate/business plans to carefully develop and implement marketing plans for organizational success. The general framework of the marketing management process can be summarized as follows (Kotler, 1999):

R → STP → MM → I → C,

where R is the research (i.e., market research), STP is the segmentation, targeting, and positioning, MM is the marketing mix (or 4 Ps, i.e., product, price, place, and promotion), I is the implementation, and C is the control (getting feedback, evaluating results, and revising or implementing STP strategy and MM tactics).

3.1. Product differentiation and promotion. To better compete in their imperfectly competitive markets, firms often strive to differentiate their products from those of their competitors by using quality improvements and promotions; they establish well-known brand names to increase their price control. The steepers are their downward sloping demand curves, the higher is their ability to charge higher prices for their products. Further, successful firms typically segment their markets and offer a different marketing mix to each market segment while they position/reposition their products appropriately to offer intended value propositions.

3.2. Price discrimination. Firms in an imperfectly competitive market with product differentiation and market segmentation may use price discrimination (Scheuing, 1989) to maximize their revenues or profits. Consider the airlines industry that is highly competitive and many airlines have gone bankrupt or merged with other airlines in order to survive. These airlines typically do not charge the same price for a seat to all passengers; they utilize yield management concepts and software (Smith, Leimkuhler, and Darrow, 1992) to maximize their revenues by charging different prices to different passengers based on the time and day of a reservation, weekend stay at the destination, nonstop flight, senior citizen status, child status, stay exceeding 45 days at the destination, electronic versus paper ticket, first class versus coach class, etc. Similarly, universities typically charge different prices based on in-state versus out-of-state status, online versus in-class courses, summer versus fall/spring courses, scholarship versus no scholarship status, tuition waiver versus no tuition waiver status, graduate versus undergraduate status, etc.

3.3. Market structure and profitability analyses. It is important to select the correct market structure in which a firm competes. A firm's market structure could be any of these types: perfect competition, monopolistic competition, oligopoly, or monopoly. Although, there are significant differences among these market structures, generally, the firm would achieve equilibrium, i.e., maximum profit at MR = MC. Firms in each market structure would have quadratic profit functions and two breakeven points. However, an unstable perfectly competitive firm would have only one breakeven point which is the most frequently used situation in business and marketing literature and incorrectly utilized across all other market structures. Table 1 summarizes the behavior of an unstable perfectly competitive firm and that of an imperfectly competitive firm.

Table 1. Breakeven and profitability analyses

	An unstable perfectly competitive firm (profits increase indefinitely)	Imperfectly competitive firm
Demand curve	Horizontal demand curve at the market price.	Downward sloping to the right $P = a + bQ$, $a > 0$, b <0, where a is the maximum price and b is the change in price per unit sold.
Pricing power	No pricing power.	The firm has some pricing power.
Information	Perfect information, so no need for promotion.	The firm promotes its product.
TR	$TR = PQ$	$TR = PQ = aQ + bQ^2$
$TC = F + VC$	$TC = F + vQ$.	$TC = F + vQ$,
$\Pi = TR - TC$	$\Pi = (P-v)Q - F$ Π is a positively sloped function that intersects the vertical axis at $-F$ and intersects the horizontal axis at $F/(P-v)$ and increases indefinitely as sales increase.	$\Pi = aQ + bQ^2 - F - vQ$, or $\Pi = bQ^2 + (a-v)Q - F$ Let $A = b$, $B = a-v$ and $C = -F$, then $\Pi = AQ^2 + BQ + C$. Π is a quadratic function that reaches a maximum point then begins to decline for additional sales. Maximum profit is at $MR = MC$ which lies between the two breakeven points

Table 1 (cont.). Breakeven and profitability analyses

	An unstable perfectly competitive firm (profits increase indefinitely)	Imperfectly competitive firm
Breakeven quantity	*Set* $\Pi = (P\text{-}v)Q - F = 0$ to obtain $Q_B = F/(P\text{-}v)$	Set $\Pi = AQ^2 + BQ + C = 0$ to obtain its roots (breakeven points) $Q_{Bi} = [-B \pm \sqrt{(B^2 - 4AC)}]/2A$, where, $i = 1, 2$; $A = b$, $B = a\text{-}v$ and $C = -F$.
Target profit quantity, Q_T.	$Q_T = (F + \text{Target Profit})/(P\text{-}v)$	Target $\Pi = AQ^2 + BQ + C$ will imply Q_{Ti}, where, $i = 1, 2$; $A = b$, $B = a\text{-}v$ and $C = -F$.

Table 2. Demand schedule and corresponding costs, revenues, and profit

P ($)	QE (units)	$TR = P^*QE$	F = $100	$VC = 2QE$	$TC = 100+2QE$	$Profit = TR\text{-}TC$
7	0	0	100	0	100	-100
7	20	140	100	40	140	0
7	40	280	100	80	180	100

The behaviors of two types of perfectly competitive firms and an imperfectly competitive firm.

Case 1. Unstable perfectly competitive firm – horizontal demand and linear profit curves.

Consider a hypothetical firm that produces and sells widgets. Here, P is the price in dollars per unit, Q is the number of units produced and sold (no inventory), TR is the total revenue, F is the fixed cost, v is the variable cost per unit, VC is the variable cost, $P - v$ is the contribution margin per unit, TC is the total cost, and Π is the profit. In this situation, the company first receives the order then produces the product so there is no inventory. This situation could be approximated by some web-based companies that produce their products after they receive customer orders. In this case, TR and TC curves are linear so the profit curve is also linear and there is only one breakeven point at the intersection of TR and TC curves which corresponds to the point where the profit curve intersects the horizontal axis. This is a special case of a perfectly competitive firm since TC is linear which along with linear TR curve generates a linear profit curve that increases indefinitely as production and sales increase. This is an unstable situation with no equilibrium.

Let price, P = \$7/unit, variable cost, v = \$2/unit, fixed costs, F = \$100, and target profit = \$100. Q = QE is expected sales volume.

The first two columns in Table 2 show the demand schedule where the price per unit is fixed thus giving rise to a horizontal demand curve at price = \$7/unit, where units are plotted on the horizontal axis and price on the vertical axis. This horizontal demand curve describes the behavior of a typical small "price-taker" firm in a perfectly competitive market selling an undifferentiated product at the given market price to customers who have perfect information about the market so no promotion is required. This firm can sell its entire production at the market price and would be unable to sell at a higher price and there is no incentive to sell below the market price.

The following relationships exist among price (P), average revenue (AR), and marginal revenue (MR) functions of this firm. P = \$7; AR = \$7, $Q \neq 0$; and MR = \$7. Therefore, $P = AR = MR$ = \$7. The information in Table 2 is used to produce Figure 1 where quantity demanded is measured on the horizontal axis and TR, TC and profit are measured in dollars on the vertical axis. The breakeven point is reached at Q_B = 20 units and the target profit of \$100 is obtained at Q = 40 units. These plots are produced by using the following R code and are given in Figure 1 below:

P < -7; v < -2; F < -100; Q < -c(0,100); TR < -function(Q){P*Q}; TC < -function (Q) {v*Q + F}; PI < -function (Q){(P - v)*Q - 100}; curve(TR, -10, 80, *ann* = FALSE, *las* = 1, *cex* = 1); curve (TC, -10, 80, add = T); curve (PI, -10, 80, add = T); abline (h = 0, v = 0); QB < -F/(P - v); QB; abline (h = 140, v = 20, *lty* = 2); *title* (*xlab* = "*Quantity* (*units*)", *ylab* = "*TR*, *TC*, *Profit* (\$)", main = "Figure 1. Breakeven Analysis: Unstable Perfectly Competitive Firm", cex.main = 1, sub = Case 1: Horizontal Demand Curve (not shown) and Linear Profit Curve.", *cex.sub* = 1); text (40,300, substitute (TR)); text (40,200, substitute (TC)); text (40,100, substitute (Profit)); text (15,155, substitute ("BE pt")); text (15,15, substitute ("BE pt"));

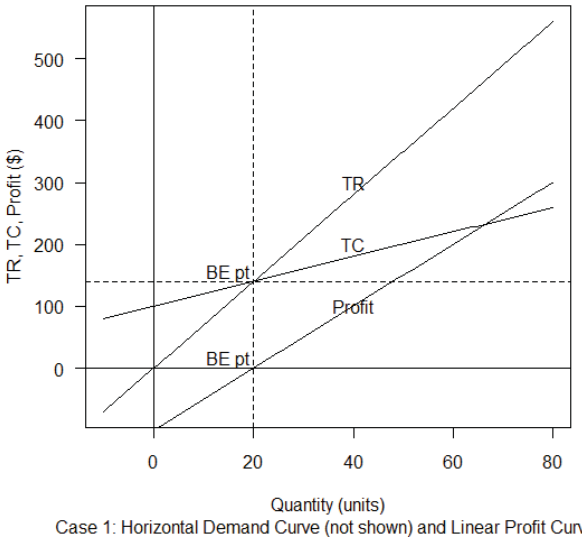


Fig. 1. Breakeven analysis: unstable perfectly competitive firm

3.4. Perfectly competitive firm – downward sloping demand & quadratic profit curves. A firm in a perfectly competitive market faces a horizontal demand curve. This implies no pricing power since the firm is a price-taker and it can sell its entire (homogeneous) production at the market price thus its TR is a positively sloped straight line that increases indefinitely. A more typical perfectly competitive firm will have a U-shaped (or inverted S-shaped) TC curve that along with a linear TR curve would generate a quadratic profit curve and two breakeven points. The quadratic formula can be used to solve for the two roots or breakeven points of the profit function. These roots may or may not be distinct, and they may or may not be real. If the expression under the square root, called the discriminant, is zero then there is only one real root. If the discriminant is negative then there are two distinct non-real or complex roots. If the discriminant is positive then there are two real roots. Therefore, this analysis is restricted to those situations where the discriminant is zero or positive so one or two distinct real roots are obtained. In this stable situation, an equilibrium between supply and demand is achieved since the profit function is quadratic and MC slopes upward to intersect MR = AR curve, indicating maximum profit (MR = MC) that lies between two breakeven points. The linear TR curve is based on a horizontal demand curve at a given price level that is determined by the market supply and demand.

Case 2. Imperfectly competitive firm – downward sloping demand curve.

An imperfectly competitive firm has some control over its price and is faced with a downward sloping demand curve, a quadratic profit function that implies two breakeven points, so the maximum profit (MR = MC) and target profit would lie between these two breakeven points. Most real world markets are imperfect where buyers and sellers do not have perfect information, firms use promotion to inform and persuade customers, products are differentiated, and firms compete on price as well as on non-price variables of the marketing mix. In fact, much of marketing management is about customer satisfaction and customer relationship management in an imperfectly competitive market.

This example is adapted from Scheuing (1989) and Kotler (1967). Consider a company that produces and sells widgets and faces a downward sloping linear demand curve, $Q = 100 - 10\ P$, the inverse of which (with Q on the horizontal axis and P on the vertical axis) is $P = 10 - 0.10\ Q$, so $a = 10$ and $b = -0.10$. If $v = 2$, and $F = 100$, then

$$\Pi = AQ^2 + BQ + C,$$

where $A = b$, $B = a - v$ and $C = -F$, or $A = b = -0.10$, $B = a - v = 10 - 2 = 8$, and $C = -F = -100$. The quadratic formula provides the two roots (breakeven points) for this profit function.

$$Q_{Bi} = [-B \pm \sqrt{(B^2 - 4AC)}]/2A,$$

where, $i = 1, 2$; $A = b$, $B = a\text{-}v$ and $C = -F$.

Enter these values in the breakeven formula (quadratic formula) to obtain the following:

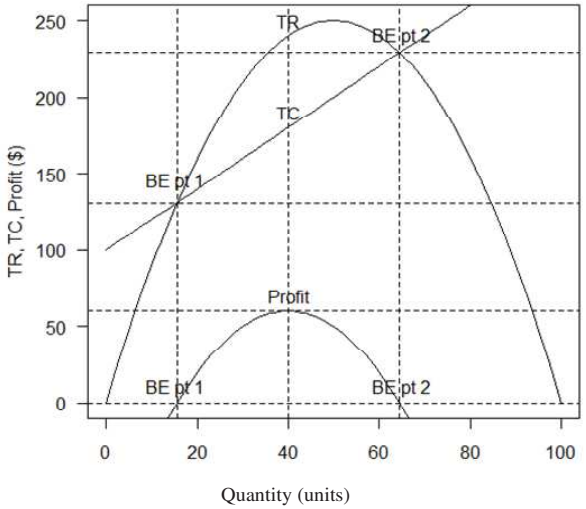
$Q_{Bi} = [-8 \pm \sqrt{(8^2 - 4\ (-0.10)(-100))}]/2(-0.10)$, or $Q_{Bi} = = [-8 \pm \sqrt{24}]/-0.20$, therefore the two breakeven points (roots) of the profit curve are as follows:

$Q_{B1} = 15.51$ units and $Q_{B2} = 64.50$ units.

By entering these quantities in the demand curve, the two corresponding prices are obtained as follows:

$P_{B1} = \$8.45$ and $P_{B2} = \$3.55$.

This situation can be depicted by plotting the quadratic profit curve that intersects the horizontal axis at the two breakeven points: $Q_{B1} = 15.51$ units and $Q_{B2} = 64.50$ units. At these two breakeven points, the TR = TC.



Case 2: Downward sloping demand curve and Quadratic Profit Curve.

Fig. 2. Breakeven analysis: an imperfectly competitive firm

In Figure 2, the quantity is plotted on the horizontal axis while TR, TC and profit are plotted on the vertical axis. Both breakeven points and the maximum profit (at MR = MC) are also shown. Figure 2 and the two breakeven points or roots of the profit function were obtained by using this R code:

v < -2; *F* < -100; *P* < -*c*(0,10); *Q* < -*c*(0,100); *P* < -function (*Q*){10-0.10*Q}; *TR* < -function(*Q*){(10 - 0.10*Q)*Q}; *TC* < -function(*Q*) {*v*Q + F}; *PI* < -function(*Q*){-0.10*Q^2 + 8*Q -100}; curve(*TR*, 0, 100, *ann* = FALSE, *las* = 1, *cex* = 1); curve (*TC*, 0, 100, *add* = *T*); curve (*PI*, 0, 100, *add* = *T*); polyroot (*c*(-100,8,-0.10)); abline (*h* = 0, *lty* = 2); abline (*h* = 131.018, *v* = 15.5051, *lty* = 2); abline (*h* = 228.9569, *v* = 64.4949, *lty* = 2); abline (*h* = 60, *v* = 40, *lty* = 2); title (xlab = "Quantity (units)", ylab = "TR, TC, Profit ($)",

main = "Figure 2. Breakeven Analysis: An Imperfectly Competitive Firm", cex.main = 1, sub = ("Case 2: Downward sloping demand curve and Quadratic Profit Curve."), *cex.sub* = 1); text (40,250, substitute (TR)); text (40,190, substitute (*TC*)); text (40,70, substitute (Profit)); text (15,145, substitute ("BE pt 1")); text (65,240, substitute ("BE pt 2")); text (15,10, substitute ("BE pt 1")); text (65,10, substitute ("BE pt 2")).

These examples utilized linear TC curves but a more general TC curve would be U-shaped (or inverted S-shaped) indicating economies of scale and diseconomies of scale. A more complete analysis of the demand and cost curves for profitability and breakeven analyses should include all relevant factors and their impact on demand and cost. Such factors include prices of complementary and substitute products; buyer's income; promotional expenditures and other expenses including those for market research, training of marketing personnel, product quality, distribution decisions, etc.

4. Marketing mix selection and demand estimation

4.1. Marketing mix selection. Consider the following example that is adapted from Kotler (1967) and Scheuing (1989) to illustrate estimation of demand curve, and use of breakeven and profitability analyses to determine an appropriate marketing mix for a company in an imperfectly competitive market.

The expected quantity demanded for the company's product (QE) is forecasted based on its price (P), advertising budget (A), and distribution budget (D) using historical data and input from salespeople and management. Here, the three marketing mix elements have these levels: P = \$16, \$20 or \$24; A = \$10,000 or \$50,000; and D = \$10,000 or \$50,000.

The following R software code for regression analysis and the data in Table 3 produced these results:

Multiple linear regression example

fit < -lm(*QE* ~ *P* + *A* + *D*, *data* = *mktmix*)

summary (*fit*) # *show results*

$QE = a + b\,A + c\,D + d\,P$

$QE = 33{,}360 + 0.0059\,A + 0.1054\,D - 1{,}403\,P$

(0.00001) (0.001) (0.01) (0.05)

Here, multiple $R^2 = 0.8567$, adjusted $R^2 = 0.803$, F-statistic = 15.95 with 3 and 8 degrees of freedom, and p-value = 0.0009756. Thus the regression model is statistically significant and it is the best prediction model based on these three independent variables. The p-values are given in parentheses below the regression estimates; the regression coefficients for the three marketing mix elements are statistically significant and have correct signs. However, the intercept shows that the mean effect of excluded variables is positive and significant implying that the model is probably misspecified. In order to properly specify this model, this company should search for some important excluded variables and include them in this model. Regression analysis using dummy variables is another alternative method that should be explored.

Table 3. Marketing mix selection using breakeven and profitability analyses

No.	P (\$)	A (\$)	D (\$)	QE (units)
1	16	10000	10000	12400
2	16	10000	50000	18500
3	16	50000	10000	15100
4	16	50000	50000	22600
5	20	10000	10000	5500
6	20	10000	50000	8200
7	20	50000	10000	6700
8	20	50000	50000	10000
9	24	10000	10000	3500
10	24	10000	50000	6200
11	24	50000	10000	5500
12	24	50000	50000	8500

Let the variable costs per unit, v = \$10, overhead costs allocated to the product under consideration, O = \$38,000, so the fixed costs are $F = O+A+D$. Using the quadratic formula, the two breakeven points (roots) can be calculated for the profit function. The first breakeven point will be of higher significance than the second breakeven point because it would be achieved first. The marketing mix that is associated with the first breakeven point should serve as a point of departure in marketing mix selection. This initial decision could be improved by exploring the marketing mix that would maximize profitat at MR = MC.

4.2. Estimating product demand and market shares. The above example about marketing mix selection did not specifically consider the product element of the marketing mix decision. It can be assumed here that the product under consideration was a mid-level product based on its features. A more comprehensive marketing mix decision would consider product aspects as well. The discussion below elaborates upon the product aspects using Apple's iPad and iPad 2 as an illustration.

Table 4 shows that Apple's most basic iPad 2 (16 GB+Wi-Fi) sells for \$499 and it includes all features except higher storage and 3G that cost more. All iPad and iPad 2 have Wi-Fi.

Table 4. Prices and features of iPad and iPad 2

No.	iPad specs	iPad prices in 2010 at its introduction	iPad prices in 2011 at iPad 2 introduction	iPad 2 specs	iPad 2 prices in 2011 at its introduction
1	16 GB+Wi-Fi	$499	$399	16 GB+Wi-Fi	$499
2	16 GB+Wi-Fi, 3G AT&T	$629	$529	16 GB+Wi-Fi, 3G (AT&T or Verizon)	$629
3	32 GB+W-Fi	$599	$499	32 GB+W-Fi	$599
4	32 GB+W-Fi, 3G AT&T	$729	$629	32 GB+W-Fi, 3G (AT&T or Verizon)	$729
5	64 GB+W-Fi	$699	$599	64 GB+W-Fi	$699
6	64 GB+Wi-Fi, 3G AT&T	$829	$729	64 GB+W-Fi, 3G (AT&T or Verizon)	$829

iPad was a breakthrough product that, upon its introduction in 2010, created or redefined tablets, a new product category, it sold 15 million units during that year. iPad 2 includes many more and better features than iPad but is offered for sale in 2011 at the iPad prices of 2010, while each iPad now sells at $100 less than its introductory price in 2010. Apple also sells software through its App Store, accessory products, and upgrades that generate additional revenue for the company for this product category.

iPad offered six product variations based on three levels of capacity (16 GB, 32 GB, 64 GB), one type of color (black), and two Internet options (Wi-Fi, 3G-AT&T). iPad 2, however, offers eighteen product variations based on three levels of storage (16 GB, 32 GB, 64 GB), two types of color (black or silver), and three Internet options (Wi-Fi, 3G-AT&T, 3G-Verizon). The choice of 3G carrier adds $130 to the equivalent Wi-Fi only version. The charges for signing up with the wireless carrier are separate and are paid to the carrier.

It is estimated that the bill of materials and the cost of manufacturing the 32 GB+Wi-Fi+3G iPad for Apple is estimated to be between $270 to $320 (Murphy, 2011). Since this product sells for $729, it would leave $409 to $459 for Apple to cover its marketing and management costs and the rest would be its profit margin, so it is very profitable for Apple (Snell, 2011).

Lancaster (1971) presented a new approach to estimate demand based on product characteristics instead of consumer preferences as used in the traditional approach to demand estimation in economics. So for iPad 2, instead of considering 16 different demand curves, we may consider only storage, wireless carrier, and price to estimate demand. This approach would be much more efficient and revealing than the traditional approach to estimating demand based on consumer preferences for all product choices. Understanding product demand based on properties or characteristics of products is closer to conjoint analysis, which is a frequently used method in marketing theory and practice. To develop a characteristics-based demand curve, products from Apple and those from its competitors would be analyzed. The consumer buyer would typically choose only one tablet so this is a discrete choice problem. Following Lancaster (1971), information from *Consumer Reports* can be used to construct expected market shares for various brands by using rank data. Williams (2011) reported that *Consumer Reports* compared several iPad tablets with those of its competitors (Dell, Archos, Samsung, Motorola and View Sonic) on 17 criteria and concluded: "The Apple iPad 2 with Wi-Fi plus 3G (32G), $730, topped the Ratings, scoring Excellent in nearly every category." Such information, along with actual sales data, and their own marketing plans can help marketers determine their present/future market shares and estimate demand for their products compared with those of their competitor's.

Conclusion and future research

This paper presented a general framework for selecting marketing mix elements for profit maximization and utilized R software for analyses. Managers can utilize a similar approach to facilitate their own marketing mix decisions. Breakeven and profitability analyses are powerful tools for managerial decision making; however, often these tools are not properly used. Firms compete under different market structures and their optimal decisions vary according to their market structures and other relevant environmental factors. The simplest case is that of an unstable perfectly competitive firm that has an indefinitely increasing profit curve that generates indefinite profit after the breakeven point. Unfortunately, this simple case has been applied across the business and marketing literature for situations involving imperfectly competitive firms where a different profitability and breakeven analysis would be more suitable. A perfectly competitive firm faces a horizontal demand curve at the given market price and would not need a marketing manager; there is no need for promotion since the customers have perfect information; and the firm can sell its entire production at the market price. A horizontal demand curve is valid for perfectly competitive firms that sell homogeneous products; it is invalid for imperfectly competitive firms that sell differentiated products, compete with small or large competitors, and their customers have less than perfect information about the products and firms so the firms promote their products to influence customer demand.

A proper understanding of customer demand is a critical aspect of marketing management since customer satisfaction is critical for marketing success. So to make the most profitable price, quantity, and marketing mix decisions, managers should estimate their demand curve. Even a rough estimate of the demand curve is better than no estimate at all. Typically the demand curve would be downward sloping to the right. Price sensitivity or price elasticity of demand should be considered in setting the right price for a product. A company that plans to introduce a new or a modified product to the market must answer some important questions regarding its breakeven and profitability analyses. Many factors, like its marketing mix, market demand, competition, economy, government regulations, technological changes, etc., would influence these decisions.

Many firms sell more than one product and for a multiproduct firm, its one product may be in a monopoly situation, while another may be in an oligopoly, and still another may be in a monopolistic or perfectly competitive market. Over time, the market conditions may change so a monopoly situation may become an oligopoly, a monopolistic competition, or a perfect competition as other companies enter or leave the market and the market structure changes. Enlightened marketing managers should consider the social costs and benefits of their decisions as well.

References

1. Grewal, D., and M. Levy. M Marketing. – IL: McGraw-Hill Irwin, 2011.
2. Harris, C.C. The Break-even Handbook: Techniques for Profit Planning and Control. – NJ: Prentice-Hall, Inc., 1978.
3. Kotler, P. Kotler on Marketing: How to Create, Win, and Dominate Markets. – NY: The Free Press, 1999.
4. Kotler, P. Marketing Management. – NJ: Prentice-Hall, 1967. – pp. 326-328.
5. Lancaster, Kelvin. Consumer Demand: A New Approach. – New York: Columbia University Press, 1971.
6. McBryde-Foster, Merry J. "Break-Even Analysis in a Nurse-Managed Center // *Nursing Economics*, 2005. – № 23, Vol. 1. – pp. 31-34.
7. Murphy, D.D. iPad 2 Teardowns: UBM Estimates Total Parts Cost at $270, iSuppli Says $320, PC Magazine (2011). Retrieved on May 16, 2011 from http://www.pcmag.com/article2/0,2817,2381890,00.asp.
8. Palda, K.S. Economic Analysis for Marketing Decisions. – NJ: Prentice-Hall, 1969.
9. Palda, K.S. Pricing Decisions and Marketing Policy. – NJ: Prentice-Hall, 1971.
10. Perreault, W., J. Cannon, and E.J. McCarthy. Basic Marketing: A Marketing Strategy Planning Approach. – NY: McGraw-Hill Irwin, 2011.
11. Scheuing, E.E. New Product Management. – Ohio: Merrill Publishing Company, 1989.
12. Shim, J.K. and J.G. Siegel. Financial Management. – NY: Barron's Educational Series, 2,000.
13. Siegel, J.G., J.K. Shim, and S.W. Hartman Schaum's Quick guide to Business Formulas: 201 Decision-Making Tools for Business, Finance, and Accounting Students. – NY: McGraw-Hill, 1998.
14. Smith, B., J.F. Leimkuhler, and R.M. Darrow. Yield Management at American Airlines // *Interfaces*, 1992. – Vol. 22, No. 1. − pp. 8-31.
15. Snell, J. Review: iPad 2 Apple's Updated Tablet is Faster, Lighter, Thinner // Macworld, 2011. – May. – pp. 30-36.
16. Spiro, R., G. Rich, and W. Stanton. Management of a Sales Force. – NY: McGraw-Hill Irwin, 2008.
17. Williams, B. iPad Tops Consumer Reports Ratings, 2011, April 6. Retrieved on May 16, 2011 from http://www.ipad-apps-review-online.com/ipad-news/ipad-tops-consumer-reports-ratings/.