“Applying Modern Portfolio Theory to Municipal Financial and Capital Budgeting Decisions”

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Applying modern portfolio theory to municipal financial and capital budgeting decisions

Abstract

In this paper, the authors propose that the modern portfolio theory well known in investment literature may be applied to local public financial and capital budgeting decision making. Municipalities can maximize the welfare of their communities in a similar manner as investors maximize their returns from their investment portfolios. The authors propose that the local projects should not be evaluated in isolation of each other. Rather, they should be evaluated collectively as a portfolio of individual investments so as to ensure that the overall welfare is maximized for the community. Such maximization process enables decision-makers to see the big picture and make more rational budgeting decision that will better serve the local residents.

Key words: municipal finance, modern portfolio theory, capital budgeting, and public investment decisions.
JEL Classification: G11, G13, H43, H53, H54, H72.

Introduction

Capital budgeting is one of the most important financial decision processes in both private and public sectors. It involves asking the important investment question as to what long-term investments should the organization make. For local government and municipalities, capital budgeting is a very important planning tool as it allows them to provide for the necessary infrastructure to maintain or enhance future service levels (*Municipal Capital Budgeting Handbook*, 2007). Local government and municipalities often spend hundreds millions of dollars on capital investments each fiscal year and capital expenditures constitute a large proportion of municipalities' total annual spending budget.

However, despite its importance, capital budgeting at the municipality and local government level was virtually unknown in the United States until the 1990s (Marquette, 1986). Municipal governance in the United States was seen as the "most corrupt in Christendom" (Marquette, 1986). During the early period, research on capital budgeting for municipalities and local government was virtually ignored and any scholarship that did examine the municipal capital budgeting was more concerned with debt finance, underwriters, and insurers than the process of capital budgeting, investment project selection and prioritization (Mullins & Pagano, 2005). While the capital budget was understudied, most states and local governments employed dual budgeting systems in which both an operating budget and a capital budget existed in uneasy harmony (Mullins & Pagano, 2005). Indeed, Mullins and Pagano (2005)'s survey finds that the public capital budgeting did not receive serious coverage as an accepted area of study until a few years after the publication of Peterson's plea to not ignore the sunk public capital costs of infrastructure in the nation's older cities (Peterson, 1978). Chan (2004) conducted a survey of capital budgeting practices of Canadian municipal governments and found that only a minority of Canadian municipalities used capital budgeting techniques, and that payback period dominated over discounted cash flow analysis in evaluating capital investments. The common pitfalls associated with the payback period technique are two-folds: (1) it ignores the time value of money; and (2) it sets an arbitrary cut-off date. Even if municipalities adopt discounted cash flow analysis which is conceptually considered as a superior investment decision criteria, the main challenge in its practical implementation is finding the appropriate discount rate. More importantly, municipalities tend to overlook the correlations and co-variances among multiple capital investment projects.

It is under this research context that we are motivated to introduce an alternative approach, i.e., applying the modern portfolio theory to municipalities' capital budgeting decisions process. Since the pioneering work of Markowitz (1952), modern portfolio theory has developed to a sophisticated area of research and has commonly been applied to the selection of stocks and bonds. However, there is scanty literature on how to apply modern portfolio theory to state and municipal governments' financial planning and regional economic development. Municipalities have been slow in catching on the concept of value maximization, which has been adopted by the corporate sector for several decades, as seen in share price maximization. This may be partially due to the fact that decision makers view value or wealth more as an objective for private entities than for the public sector. However, value maximization may be an important pursuit for enhancing social welfare on a long-term basis.

In this paper, we introduce the modern portfolio theory and illustrate how states and municipalities can apply this theory for optimal financial planning

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and economic development. The paper is organized as follows. Section 1 introduces the modern portfolio theory (MPT). Section 2 illustrates the application of MPT to regional financial planning by state and municipal governments. The final section concludes the paper.

1. Modern portfolio theory

Modern portfolio theory (MPT) was first introduced by the pioneering work of Markowtiz (1952). The theory shows that risk-averse investors can construct investment portfolios to optimize expected returns for a given level of risk. That is, an investment portfolio can be optimized by maximizing expected returns while minimizing the volatility of returns, measured by the standard deviation of returns. A brief description of the MPT model is as follows:

Suppose that we have formed a portfolio that consists of two investments, a bond fund and a stock fund. The expected portfolio return is calculated as follows:

$$E(r_p)=w_s E(r_s)+w_b E(r_b), \quad (1)$$

where w_s is the weight or proportion of stocks invested and w_b is the weight of the bonds invested. r_s is the return on stocks and r_b is the return on bonds.

The standard deviation of the portfolio return, which is a measure of the level of volatility and the degree of uncertainty over the actual return, is calculated as follows:

$$\sigma_p=\left(w_s^2\sigma_s^2 + w_b^2\sigma_b^2 + 2w_s w_b \rho_{sb}\sigma_s\sigma_b\right)^5, \quad (2)$$

where w_s is the weight or proportion of stocks invested and w_b is the weight of the bonds invested. σ_s is the standard deviation on stocks and σ_b is the standard deviation on bonds. ρ_{sb} is the correlation coefficient of returns on stocks and bonds.

MPT shows that the optimal combinations of various securities can result in a minimum level of risk for a given return. The optimal trade-off between risk and return is represented by the *efficient frontier* which is depicted in Figure 1 as follows.

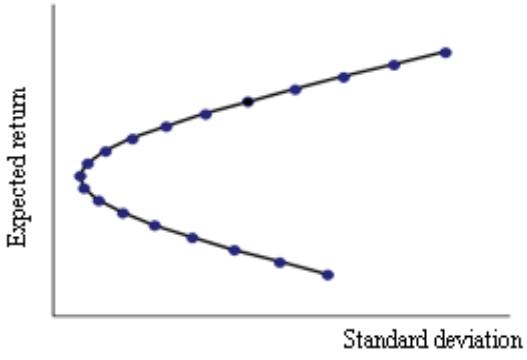

Fig. 1. Efficient frontier

If we extend this model to include a riskless asset, then the optimal portfolio becomes a linear set which is called the Capital Allocation Line (CAL). Figure 2 depicts that a single *P*-combination of risky and riskless assets will dominate all other asset allocation portfolios. Portfolio *P* is the optimal portfolio since it has the best risk-return trade-off. The CAL for portfolio *P* dominates other lines for it has the best risk/return or the steepest slope. The slope indicates the risk-adjusted expected excess return over risk-free rate as follows:

$$Slope = (E(R) - R_f)/\sigma, \quad (3)$$

where $E(R)$ is the expected return on a risky asset, R_f is the return for riskless asset, σ is the standard deviation. The slope of line tangent to *P* is greater than the slope of any other line:

$$[E(R_P) - R_f)/\sigma_P] > [E(R_A) - R_f)/\sigma_A]. \quad (4)$$

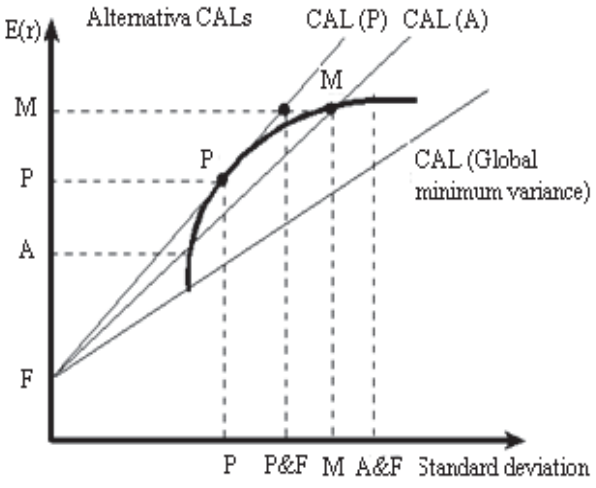

Fig. 2. The optimal portfolio as indicated by portfolio P

2. Applying modern portfolio theory to municipal capital budgeting decisions

Capital budgeting, the economics of project evaluation, is relatively well-developed and commonly practiced in the corporate world. However, it is still at a rudimentary stage in the area of public project evaluation (Thomassen, 1990). There is a great need for the development and improvement in the application of capital budgeting at the municipal level (Bunch, 1996). All public project decisions must begin with estimating future social benefits. Unlike benefits as defined in corporate finance, social benefits should include non-monetary benefits and should be calculated on a pre-tax basis.

Expected return for social investment can be estimated in similar fashion as is done in corporate finance. The expected return of a public project is the internal rate of return that renders the present value of future social benefits equals to the cost of the projects.

The calculated internal rate of return is only the projected or expected return. The actual return may turn out differently due to measurement errors of benefits and changes in benefits due to economic changes. Because of the reality of uncertainty, all returns are stochastic and thus have means (expected returns) and standard deviations (σ).

As long as the expected return and standard deviation remain the same with respect to the amount of investment, the portfolio theory may be applied. It is reasonable to assume that the expected and standard deviations for the projects under consideration are constant within the range of expenditure level being considered.

As we attempt to apply the modern portfolio theory to municipal finance, one should be aware of certain factors that are peculiar to public investment.

The major difference between corporate finance and municipal finance is that benefit and cost are measured in social terms rather than in private terms. Cost of projects to be considered should include not only the budgetary or monetary cost, but also non-monetary cost. Both non-monetary cost and benefit associated with public investments are not easily measured in economic terms. In certain cases, normative judgments might need to be exercised in order to estimate the value of such benefits. For example, the economic benefit of job training can be measured by the increase productivity and that the economic benefit of highway construction can be measured by the reduction in travel time, which is a factor of production. However, the valuation of non-economic benefits, such as the value of improved citizenry associated with job training and the value of time gained for spending with families associated with highway construction are not easily measured in monetary terms and thus require normative evaluation. Such issues inherent to public investment make municipal finance more challenging. Finally, managers making good investment decision for a corporation might receive feedbacks rather quickly if the market response favorably to the investment decision by raising the stock price. However, the lack of market signal for municipal financial decision means that public servants often receive less timely feedback for their decisions. For long-term investments, it is possible that the public servant leaves office before the project comes to fruition. Thus, the incentive for the public servant to maximize value for the community needs to transcend financial or political reward. This reinforces the importance of servant hood as an important virtue for those seeking to serve in the public office.

Despite the aforementioned differences between corporate and municipal finance, MPT and traditional public cost-benefit analysis are not mutually exclusive. Application of MPT still requires traditional cost-benefit analysis to measure the value of cost and benefits in order to calculate the expected return and standard deviation. The advantage of integrating MPT into the traditional public cost-benefit analysis is that MPT takes the entire process one level higher by taking a more comprehensive and more value-conscious approach than the traditional approach which entails evaluating internal rate of returns and the benefit to cost ratios for the individual projects in isolation to each other.

To illustrate how the modern portfolio theory may be applied to public capital budgeting decisions, we use a simple two-project case where a municipality needs to decide how the total budget should be allocated between job training and highway improvement. Note that there are many portfolios that can be created from these two projects since a portfolio simply represents a particular mix of the two projects. To obtain the optimal mix for these two investments, we first need to estimate the expected returns for these projects. For example, returns for public projects and programs are measured in terms of social benefits rather than cash flows.

Benefits for job training include the increase in labor productivity as measured by incremental gross wages. Other benefits include cost savings in law enforcement and correction programs due to reduced crime. Benefits for highway improvement include the value of incremental output in the private sector due to time saved in commuting. Other benefits would be cost savings in vehicle repairs and social cost savings in terms of reduction in traffic mishaps.

Returns on project can be estimated by the internal rate of return method, which discounts future benefits with a rate such that the PV of benefits equals cost. Such a rate is the return of the project. Risk is measured by standard deviation of return. A numerical example showing how expected returns, standard deviation of returns, and correlation between returns may be estimated is described in the Appendix. Table1 shows the expected returns, standard deviations, and correlation coefficient between the two projects.

Table 1. Asset allocation: two project investment decisions

	Expected return	Standard deviation	Corr. coeff$_{H,J}$
Highway	0.12	0.15	0.40
Job training	0.20	0.30	
Risk-free rate	0.07	0	

Note that the job training project has higher expected return (20 percent) than the highway project but also higher risk as measured by the standard deviation (30 percent). This may be due to the fact that the potential contribution of well-trained workers to local econo-

my may be great. But the risk is great because it is difficult to predict how successful the trained workers will be and whether they will contribute to the local economy or move out after they are trained.

Should the local economy weaken in the future, the benefits derived from both projects will most likely be reduced, but they may not be reduced by the same degree. Thus, the correlation between the two projects is unlikely to be one. The closer their returns move together, the higher would be the correlation coefficient and vice versa. Given the dissimilar characteristics of these two projects, our hypothetical correlation coefficient for the returns of these two projects is 0.40. Such relatively low correlation indicates that the portfolio consisting of these two projects has greater potential for risk reduction. The points on the frontier in this case represent the return and risk of the portfolio (two projects combined) at various weights given that the correlation is 0.40.

To maximize social welfare, the municipality may not necessarily allocate the entire budget to the job-training project even though it has higher expected return because of the associated risk. To figure out the optimal allocation between these two projects, we apply the portfolio theory as explained to locate the best point on the frontier. Figure 3 depicts graphically how the optimal budget allocation between the highway and job training projects is determined.

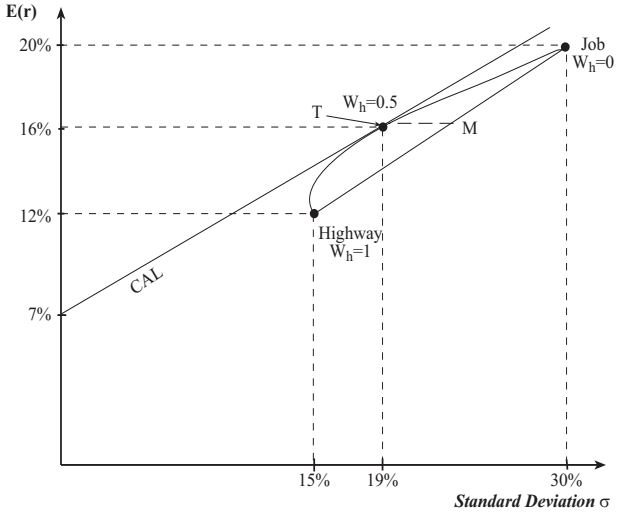

Fig. 3. Optimal portfolio risk and return

In Figure 3, the straight line Highway-Job represents the opportunity set of a portfolio that consists of both projects with a correlation equal to 1. The midpoint *M* of the straight line connecting the two projects represents a portfolio with a weight of 0.5, which means 50 percent of the total budget is allocated to the highway project. In this case, the standard deviation of the portfolio *M*, which represents the 50-50 mix of highway and job training, is simply the average standard deviation for highway and job training. Since the actual correlation is 0.4, the actual standard deviation of portfolio with 50-50 mix is less than the average of the two standard deviations for the two projects. Graphically, the standard deviation of such portfolio (with $w = 0.5$) would be to the left of *M*. Such is the case for any other value of weight. This means that for any other portfolio mix, the standard deviation of the portfolio is below the weighted average of the individual standard deviations for the two projects. Thus, the frontier is a concave curve to the left of the straight line connecting the two projects, as indicated in Figure 3.

Assume that the yield of municipal bond is 5 percent. This may not be the true cost of borrowing to society because the costs of borrowing for municipalities are artificially lowered by the tax exemption status. In order to make rational social decision, we need to estimate the true social cost of borrowing, that would prevail had the municipal bonds been taxable. This can be obtained by looking at the yields of corporate bonds with the same rating and similar credit risk. Let's say we found one at 7 percent. Then we can use 7 percent as a proxy for the social cost of borrowing for the municipality.

Tham (2000) proposed a way to estimate return of projects from the viewpoint of equity holders rather than from the viewpoint of the firm as a whole. Such approach takes into consideration the risk added by taking on debt (Tham, 2000). The points on the efficient frontier represent returns to the municipality without debt, whereas the *CAL* with *Y*-intercept at 7 percent represents the opportunity set of expected returns if the municipality finances the optimal project portfolio with debt that cost 7 percent per year. Thus, the *CAL* in this case is analogous to the return of equity for corporate investors. Assume that the interest rate is fixed. Then a riskless project with zero standard deviation would need to yield a minimum return of 7% in order to be considered. Thus, 7 percent would be the *Y*-intercept on the graph in Figure 3. A straight line can be created by connecting the *Y*-intercept at 7 percent to any point on the frontier. The slope of this line is $(\Delta Y/\Delta X)$, where ΔY can be interpreted as the reward as defined by the expected return of the portfolio in excess of the cost of borrowing, and ΔX is the risk differential as defined by the standard deviation of the same portfolio in excess of the zero standard deviation associated with the fixed cost of borrowing at 7 percent.

In other words, reward is the expected social return for the project portfolio in excess of the social cost of borrowing, and the risk is simply standard deviation of return at that particular mix minus the standard deviation of the cost of borrowing, which is assumed to be zero. Dividing the reward of the portfolio by the risk of the portfolio at that same weight, we get the slope of the line which we call Ratio.

$$Ratio = \frac{Expected\ Return\ of\ Portfolio\ Interest\ Rate}{Standard\ Deviation\ of\ Portfolio} \quad (5)$$

where *Ratio* represents the reward-risk ratio at a given weight or mix. Using (1) for the expected return of portfolio and (2) for the standard deviation of portfolio, we can express the ratio as:

$$Ratio = \frac{w_h E(R_h) + (w_j) E(R_j) - Interest\ Rate}{(w_j{}^2 \sigma_j{}^2 + w_h{}^2 \sigma_h{}^2 + 2 w_j w_h \rho_{jh} \sigma_j \sigma_h)^{0.5}}. \quad (6)$$

Using similar notations as previous, we choose W_h to represent the optimal weight for the highway project, and W_j for the job training, and $E(R_h)$ for the expected return for the highway project, $E(R_j)$ for the expected return for the job training, and R_f for the risk-free rate, which is the social cost of borrowing for the municipality; σ_h for standard deviation for the highway project; σ_j for standard deviation for the job training, and ρ for correlation coefficient of returns.

Sharpe (1994) used the reward-risk ratio to evaluate investment fund performance. Funds with the highest ratio are considered to have the best performance. This concept is applied to public budget allocation decision in that the optimal mix is found by maximizing the reward risk ratio, which is the slope of the straight line connecting the *Y*-intercept to the frontier. It can be seen that the slope attains the highest value at *T*, the point of tangency to the frontier. The point of tangency represents the optimal mix for the two projects. We will show in the next part that the optimal weight for the highway is 0.52 at the point of tangency. This means that the municipality should allocate 52 percent of its budget to highway and 48 percent to job training. How much debt to take depends on how much risk the municipality is willing to tolerate. Whatever it may be, welfare is maximized at the given level of debt if 52 percent of the debt is applied to highway and 48 percent to job training.

In real life, the solution for the optimal mix should be derived mathematically rather than graphically. To solve for the optimal weight for the highway project mathematically, we maximize the reward to risk ratio, with respect to the weight. Setting the derivative of the reward-risk ratio with respect to W_h to zero and solve for W_h, we get:

$$W_h^* = \frac{N}{D}, \quad (7)$$

where

$$N = \left[E(R_h) - R_f\right]\sigma_j^2 - \left[E(R_j) - R_f\right]\sigma_h \sigma_j \rho,$$
$$D = \left[E(R_h) - R_f\right]\sigma_j^2 + \left[E(R_j) - R_f\right]\sigma_h^2 - \left[E(R_h) - R_f + E(R_j) - R_f\right]\sigma_h \sigma_j \rho,$$

where $\rho = \rho_{hj}$.

Plugging in the numbers from Table 1 above, we get the optimal weight for the highway as follows:

$$W_b = \frac{(.12 - .07)(.30^2) - (.20 - .07)(.15)(.30)(.40)}{(.12 - .07)(.30^2) + (.20 - .07)(.15^2) - [(.12 - .07) + (.20 - .07)](.15)(.30)(.40)} = 0.52. \quad (8)$$

The optimal capital allocation for the highway is found to be 0.52 and the optimal capital allocation for the job training project is (1-.52) = 0.48. Thus, the expected return for this optimal portfolio is:

$$E(R_p) = W_h(E(R_h) + [1 - W_h(E(R_j)] = (0.516)(0.12) + (1 - 0.516)(0.20) = 0.16 \text{ or } 16\%. \quad (9)$$

The standard deviation for optimal portfolio is:

$$\sigma_p = (W_j^2\ \sigma_j^2 + W_h^2\ \sigma_h^2 + 2 W_j\ W_h\ \rho_{jh}\ \sigma_j\ \sigma_h) = [(0.484^2)(.30^2) + (0.516^2)(.15^2) +$$
$$+2\ (0.484)(0.516)(.40)(.30)(.12)]^5 = 0.19 \text{ or } 19\%.$$

Table 2 summarizes the results of this optimal risky portfolio.

Table 2. Optimal budget allocation to projects

Weight highway	0.52
Weight job training	0.48
Expected return	0.16
Standard deviation	0.19
Reward to risk	0.467

The results highlighted in Table 2 indicates that allocating 52 percent of the capital budget to highway and 48 percent to training will yield the highest risk adjusted social return for the municipality. The expected social return for such portfolio is 16 percent and its risk as measured by standard deviation is 19 percent. This budget allocation will yield the highest expected social value in the context of the reality that future benefits are uncertain.

3. Comparing portfolio theory to utility theory

Traditionally, social welfare function is often based on utility theory. Social utility are often specified as functions of units of public goods provided (such as highway and job training) and can be expressed in terms of indifference curve. Working with indifference curve requires that a subjective judgment regarding the tradeoff between the two goods be made. For example, if the slope of the indifference curve is 1.2, then it would take 1.2 units of highway to yield the same social welfare as 1 unit of job training. Such tradeoff requires normative judgment that is beyond the scope of positive economics.

On the one hand, portfolio theory operates in the realm of returns and risks. And no normative judgment needs to be made regarding the tradeoff between expected return and risk. The rule simply maximizes the expected return at a given level of risk. Utility theory lacks practical appeals because the social utility function is unobservable and does not lend itself to be estimated accurately. Even if it can be estimated, the utility function may change over time. On the other hand, the efficiency frontier being utilized by the portfolio theory is based on a straightforward mathematical relationship which does not require estimation.

The optimal point on the frontier yields the highest return to risk ratio which maximizes social value. Note that the budget allocation as implied by such point is optimal regardless of social preference. This is a great advantage of the portfolio approach.

Furthermore, the portfolio approach takes into consideration the uncertainty of benefits generated by the proposed projects, whereas the utility approach often ignores the effect of uncertainty of future benefits to be generated by the proposed projects. In the portfolio approach, risk or uncertainty is often measured by standard deviation of return and correlation between returns. The estimates for these parameters will become more accurate with experience as more historical data for projects are gathered over time.

In a nutshell, the main difference between the traditional utility approach and the proposed portfolio approach is that traditional approach is based on utility theory which seeks to maximize a normative variable called social welfare, whereas the portfolio approach proposed in this paper seeks to maximize a positive (non-normative) variable, which is a ratio of social return to social risk. The maximization of such ratio leads to the maximization of social value.

Conclusion

The criteria used by municipalities for public capital budgeting are often *ad-hoc* and lack concrete objective that can be quantified or measured. An alternative approach is proposed for the purpose of achieving social value maximization, which entails an objective that can be quantified or measured. Because portfolio theory entails a quantifiable objective, application of the portfolio approach may reduce some of the political wrangling that often plagues public budgeting decision process tying down important projects for years. Reducing the lag time between proposal and approval will facilitate urgent projects to be made available sooner to the people.

In conclusion, the advantages of the proposed portfolio approach, as compared to the traditional approach, are as follows:

1. Maximizing social value with given resources.
2. Attaining an optimal budget allocation without making the normative judgment regarding tradeoffs between competing projects.
3. Portfolio approach may reduce political debates by providing objective that is quantifiable and thus allowing critical projects to be approved sooner for the benefit of the people.

Although having long been utilized in the private sector, the concept of value maximization has neither been fully recognized nor understood in the public sector. This paper contributes to the bridging of this gap. The amount of potential benefits to be reaped by states and municipalities from applying the principle of value maximization to capital budgeting for social projects and programs may be mind-boggling.

Thus, we propose that municipality capital budgeting projects should not be evaluated in isolation of each other. Rather, they should be evaluated collectively as a portfolio of individual investments so as to ensure that the overall welfare is maximized for the community. Such maximization process enables decision-makers to see the big picture and make more rational budgeting decision that will better serve the local residents.

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Appendix

The purpose for this Appendix is to provide a numerical example for illustrating how returns and standard deviations for returns may be estimated in practice. Let us assume the two projects being considered are $PROJECT_j$ (job training) and $PROJECT_h$ (highway improvement) in a municipality. The initial cost of the job training program is $20M and that of the highway is $50M. Both projects have a 10-year horizon. The actual and projected benefits for both of these projects are indicated below.

Table 1A. Actual benefits and actual returns for job training and highway improvement

Year	Ba_j	Ba_h
0	-20,000	-50,000
1	5,000	9,800
2	4,200	10,100
3	4,180	7,800
4	3,800	9,200
5	3,100	8,100
6	4,000	8,700
7	2,800	7,500
8	3,800	9,700
9	5,000	9,800
10	3,500	9,400
Return (IRR)	15.4%	12.5%

Notes: Ba_j are the actual (*ex-post*) benefits generated by $PROJECT_j$ (job training); Ba_h are the actual (*ex-post*) benefits generated by $PROJECT_h$ (highway improvement).

After ten years, the actual benefits would be known and thus the actual return can be calculated with the internal rate of return method. The difference between actual return and the original projected return made at the time when the project was approved would be the error term for the return on this particular project.

For example, based on the figures in Table 1A, the actual return generated by job training (as defined by internal rate of return) is 15.4 percent. Projected return generated by job training (as defined by internal rate of return) is 14.6 percent. Thus, the error for return on job training in this case would be 15.4 percent − 14.6 percent = .8 percent.

Similarly, the actual return generated by highway (as defined by internal rate of return) is 13.2 percent. Projected return generated by highway (as defined by internal rate of return) is 12.9 percent. Thus, the error for return on highway in this case would be 13.2 percent − 12.9 percent = 3 percent.

The standard error for the return on job training can be estimated by summing such errors squared from past projects divided by $n - 1$, where n represents the number of job training projects in the past. Similarly, the correlation coefficient can be estimated by using past returns on job training and on highway improvement.

This is practical as long as records on benefits are kept for previous projects. Estimates will improve as time progresses since more data and more projects would have been undertaken over a longer period of time. However, if there are insufficient data available for returns on prior projects (such as having only one job training project prior to the current job training project being considered) needed for computing the standard error for returns and correlation coefficients, we propose to use error in projected benefits as proxy for error in returns. That is possible because the data for one project would yield sufficient errors in benefits for calculating standard deviation and correlation (a 10-year project would yield 10 errors in benefits which enable us to calculate standard deviation for benefits).

Let us assume that we use the benefit data from a prior project in job training and a prior project in highway improvement (as given in Table 1A) as proxies for estimating standard deviation and correlation for returns. First, the errors in the projected benefits can be calculated in the table as follows.

Table 2A. Errors in projected benefits for $PROJECT_j$ and $PROJECT_h$

Year	Ba_j (000)	Bp_j (000)	e_j	Ba_h (000)	Bp_h (000)	e_h (000)
0	-20,000	-20,000		-50,000	-50,000	
1	5,000	4,000	0.200	9,800	9,500	0.031
2	4,200	3,980	0.052	10,100	9,500	0.060
3	4,180	3,960	0.053	7,800	9,500	-0.218
4	3,800	3,940	-0.037	9,200	9,500	-0.033
5	3,100	3,920	-0.265	8,100	9,500	-0.173
6	4,000	3,900	0.025	8,700	9,500	-0.092
7	2,800	3,880	-0.386	7,500	9,500	-0.267
8	3,800	3,860	-0.016	9,700	9,500	0.021
9	5,000	3,840	0.232	9,800	9,500	0.031
10	3,500	3,820	-0.091	9,400	9,500	-0.011
Return (IRR)	15.4%	14.6%		12.5%	13.8%	
Std error			15.1%			11.6%
Correlation						0.70

Notes: Bp_j are the projected annual benefits for $PROJECT_j$ (at time of project approval); Bp_h are the projected annual benefits for $PROJECT_h$ for highway improvement (at time of project approval); $e_j = (Ba_j - Bp_j)/ Ba_j$, where e_j represents errors in projected benefits for job training; $e_h = (Ba_h - Bp_h)/ Ba_h$, where e_h represent errors in projected benefits for highway improvement; $\sigma_j = 15.1$ percent , which represents the standard error of e_j; $\sigma_h = 11.6$ percent, which represents standard error of e_h; correlation coefficient $\rho = Cov\ (e_j\ \ e_h)\ /\sigma_j\ \sigma_h$.

We can consider such alternative approach of using benefit errors as an indirect method for estimating standard deviations for returns and correlation for returns. As more data on project returns are gathered over time, we may switch to the direct method of estimating standard deviation and correlation with past return data.

If we lack sufficient data for estimating correlation between returns accurately, we can derive the optimal weight or allocation percentage for a range of correlation coefficient values rather than for one specific correlation value.

Although a two-project case is used as illustration in this paper, the portfolio theory can be extended and be applied to a portfolio with any number of investments. Thus, it can be used for determining the optimal weights for multi-project cases as well.