"Investigating implied asset correlation and capital requirements: empirical evidence from the Italian banking system"

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Investigating implied asset correlation and capital requirements: empirical evidence from the Italian banking system

Abstract

The Basel Committee's reform to strengthen the global capital framework, known as Basel III, takes into account a series of measures to address procyclicality and, consequently, make banks' capital requirements more stable during the different phases of the economic cycle. The range of possible approaches that Supervisory Authorities could follow to address this issue includes measures such as the use of through-the-cycle probability of default (PD) estimates and/or the calibration of the other risk parameters, i.e., the confidence level and the relation between PD and asset correlation, in an anti-cyclical way.

Particularly, this paper aims at detecting further the relation between PD and asset correlation, based on Italian banking system empirical loss data. The authors test the regulatory asset value correlation assumptions through a measure of implied asset correlation that they get by equalling the empirically observed unexpected loss with the regulatory capital requirements.

This research sheds more light on the inverse relation between PD and asset correlation, which is one of the main hypotheses the internal ratings based approach is built on, and that has not been modified by the Basel III reform. The paper demonstrates that the sign of this relation depends on the combination of two opposite effects: the "PD effect", which is consistent with the inverse relation hypothesis and the "PD volatility effect", which has been neglected by prior literature. According to the provided evidence, if a certain change in the PD comes along with a change in the volatility of the default rate distribution, the inverse relation doesn't hold.

Keywords: asset correlation, banks, Basel II, risk management, regulation.
JEL Classification: G21, G28, G32.

Introduction

Asset correlation plays a crucial role in the Basel II capital accord, being a critical driver of the amount of the regulatory capital generated across the different asset classes set by the Basel Committee (e.g., corporates, commercial mortgages, residential mortgages, credit cards and consumer lending). Besides, modeling correlation within a bank loan portfolio is still under active discussion from both an academic and a practitioner perspective. Together with the probability of default (PD), asset correlation is one of the key factors for credit risk models since value-at-risk (VaR) calculations are very sensitive to changes in these two parameters.

While approaches for estimating default probabilities have been considerably improved during the past decades, the analysis of co-movements between borrowers was considered to be still in its infancy by the Basel Committee when it released its proposal for the new version of the first, 1988 capital accord, known as Basel I. That's why the Committee adopted standard specifications for correlations, which can be interpreted as conservative guidelines for the unknown magnitudes of this important risk parameter[1].

This paper provides an empirical study of the Basel II asset value correlation assumptions within the context of the internal ratings-based (IRB) approach. We want to study how the risk factors impact on a bank's capital requirement based on the analysis of the implied asset correlation that we get by equaling the empirically observed unexpected loss with the regulatory capital requirements.

The paper is organized as follows. Section 1 describes the treatment of asset value correlation within the Basel II regulatory framework. Section 2 presents a review of the prior research dealing with the issues related to the regulatory treatment of asset correlation within the capital accord. Section 3 depicts the methodology we use in our study, the dataset we take into account and the impact of Basel II risk drivers on the estimated empirical correlation. Section 4 shows our main findings and the interpretation of the these results based on the comparison of two main effects: the PD effect and the PD volatility effect; the last Section concludes.

1. Asset correlation and Basel II capital requirements

Basel II capital accord represents an important advance in measuring and managing risk-based regulatory capital. Nevertheless, the internal ratings-based (IRB) approach, its cornerstone, is based on some fundamental assumptions, both in its "advanced" version and in the "foundation" one. Some of these assumptions concern the correlation parameter that banks must use to feed the risk weight formulas set by the regulators to calculate the capital requirement (k) associated with the asset classes in which a

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[1] In depth, the Committee decided not to allow banks to use their correlation estimates because of the following reasons: (1) the derivation of correlation parameters is still an area of evolving empirical and theoretical research; (2) giving to banks the possibility to derive their own estimates of correlation would make it difficult for supervisors to systematically compare risk-based capital ratios across banks; (3) adjusting the correlation values is a policy instrument that regulators did not want to lose.

bank's loan portfolio has been segmented. In fact, regulators have calibrated and set predetermined values for the correlation parameter within each of these formulas.

Particularly, the formula we will refer to in the paper is the one used for calculating the capital requirement (k) for a 1-euro loan granted to a corporate counterparty:

$$K = \underbrace{1.06}_{\substack{SCALING \\ FACTOR}} \cdot \left\{ LGD \cdot N \left[\frac{1}{\sqrt{(1-R)}} \cdot G(PD) + \sqrt{\frac{R}{1-R}} \cdot G(x) \right] - PD \cdot LGD \right\} \cdot \underbrace{\frac{1+(M-2.5)\cdot(\alpha - \beta \cdot \ln PD)^2}{1 - 1.5 \cdot (\alpha - \beta \cdot \ln PD)^2}}_{\substack{MATURITY \\ ADJUSTMEN}}, \quad (1)$$

where:

♦ *PD* is the probability of default per rating grade, i.e., the average percentage of obligors that default in this rating grade in the course of one year;

♦ *LGD* is the loss given default, which gives the percentage of exposure a bank may lose in case of borrowers default. These losses are usually shown as a percentage of the exposure at default (*EAD*), and depend, amongst others, on the type and amount of collateral as well as the type of borrower and the expected proceeds from the work-out of assets;

♦ *R* denotes the asset correlation among borrowers, and is defined as the sensibility to a common risk factor;

♦ *N* indicates the cumulative distribution function for a standard normal random variable (i.e., the probability that a normal random variable with a zero-mean and variance equal to one is less or equal to x);

♦ *x* indicates the percentage of the losses that supervisors are willing to accept. The smaller is x, the stricter is the model, which leads to higher capital requirements. The Basel Committee decided to set x equal to 0.1%;

♦ *G(z)* denotes the inverse cumulative distribution function for a standard normal random variable (i.e., the value of x such that $N(x) = z$);

♦ The scaling factor of 1.06 has been settled by the Committee in order to avoid that shift from the old standards of Basel I to the internal ratings method may cause ad excessive decrease in the banks' capital.

The maturity adjustment highlighted in the formula is function of the maturity (*M*) and *PD*, in order to take into account the downgrading risk. The two parameters α and β are equal to 11.852% and 5.478%, respectively. Note that in the IRB Foundation approach *M* is set equal to 2.5, whereas in the IRB Advanced approach it must be comprised between 1 and 5.

In January 2001 the Basel Committee proposed a measure of regulatory capital based on ratings internally developed by banks, and presented the first version of the above "closed" formula to assess the regulatory capital. That formula aimed at replicating

the results of the portfolio models developed by major investment banks and consulting firms in the second half of the last decade of the 20th century, and the Basel Committee initially assumed asset correlations to take a value of 20% for all bank obligors[1]. Later that year, in response to feedbacks from practitioners, and ensuing the results of its quantitative impact studies, the international supervisory authority proposed an alternative formula for capital calculation, where asset correlation would be a decreasing function of the firm's probability of default.

In particular, it assumed that asset correlation would have been 10% for the highest PD and 20% for the lowest PD[2]. The risk weight formulas initially proposed by the Committee in the new version of the capital accord, were subsequently modified in order to provide a solution for a question raised by some national supervisory authorities with regard to a calibration problem in measuring SMEs' credit risk. The point was that the original risk weight formulas were considered too steep and too high, thus determining excessive capital charges for this category of borrowers. That's why the Committee proposed two main changes: first, it introduced two different formulas for large companies and SMEs; second, the Committee assumed that asset correlation declines with a firm's probability of default.

As to the former, the Committee added an adjustment for firms with a turnover between €5 and €50 million. Furthermore, banks are allowed to correct the correlation formula based on the logic that correlation decreases with company size, and to use even more favourable risk weights for retail exposures (i.e., exposures versus very small firms with a turnover between €1 and €5 million), provided that total exposure to anyone remains below €1 million. As to the latter, this rule is expected to smooth capital requirements for risky small businesses during recession, based on the assumption that small and medium companies are more sensitive to economic

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[1] See BCBS (2001a). Using the Committee's jargon, within the asymptotic single-risk factor (ASRF) model adopted to determine the regulatory capital requirements, the asset values of every obligor were assumed to have a "factor loading" of 0.20 with the common risk factor.
[2] See BCBS (2001b).

downturns than larger firms. If this assumption doesn't hold, less risky small firms are going to pay for more risky ones, being charged with higher capital requirements.

In the final version of the new capital accord, asset correlations for sovereign, corporate and bank exposures were assumed to range between 12% and 24%, again inversely related to the probability of default, according to the following formula, where we also show the downward firm-size adjustment for small and medium-sized corporates, highlighted by the dotted line[1]:

$$R = 0.12 \cdot \left[1 + e^{(-50 \cdot PD)}\right] - 0.04 \cdot \left[1 - \left(\frac{S-5}{45}\right)\right], \quad (2)$$

where R is the correlation coefficient for the i-th loan, PD is the borrower's probability of default and S is the borrower's turnover.

Within the Asymptotic single-risk factor (ASRF) model, which is at the basis of the new Basel II IRB framework, borrowers' asset values are all correlated with a single systematic risk factor (broadly speaking, "the state of the economy", or financial market forces affecting the performance of all companies). This means that, on the one hand, low correlation debtors experience credit problems that can be traced back to borrower-specific problems. On the other hand, high correlation entails a higher sensitivity to the common systematic risk factor.

As it is said, under Basel II, the level of asset correlation depends on the borrower's credit quality. Based on empirical studies conducted by the Committee, the higher is the quality of the assets, the higher is the sensitivity and the correlation with market events. This finding is consistent with the financial theory that a larger part of economic loss on high-quality credits is related to the systematic risk. By contrast, the performance of low-quality assets tends to be less correlated with market events, and more driven by borrower-specific characteristics.

2. A review of the related literature

Despite the great importance of asset correlation in both the measurement of portfolio credit risk and the calibration of the Basel II risk weight formulas, few papers had dealt with the issues related to this subject at the time of the first Basel II consultative document. For example, to our knowledge, the only paper referred to the Italian banking system is Sironi and Zazzara (2003). Based on the mortality rates

published by the Bank of Italy, they estimate the average default correlation ratio for Italian bank loans granted to non-financial companies and family businesses, grouped according to geographic area and size of the drawn loan facilities. Then, based on a two-state Merton type model, they derive the average asset return correlation compatible with the previous default correlation ratio. Their estimated asset return correlations are consistently lower than the 20%-value the Committee proposed at the time their paper was written.

In the following years an increasing attention has been given by both academics and practitioners, particularly detecting, on the one hand, the existence of an inverse relationship between asset correlation and probability of default, and, on the other hand, between asset correlation and firm size.

As to the first point, using data referred to different asset classes for the UK and the US banking systems, and adopting the same methodology we implement in this study, Fitch Ratings (2008) finds that the Basel II asset value correlation assumptions are generally more conservative than the empirical correlation they derive, and that empirically derived asset correlations tend to vary geographically. Contrary to the Basel II assumptions for certain asset classes, there generally doesn't appear to be a uniform statistical relationship between asset correlations and PDs.

Düllman and Scheule (2003) argue that if sectors which are highly cyclical, such as manufacturing and construction, are dominated by large firms, whereas in less cyclical sectors, such as transport and communication services, SMEs prevail, one might expect that systematic risk and asset correlation overall increase with firm size. Another reason to theoretically justify the positive relationship between asset correlation and firm size, is that large firms can take advantage of diversification opportunities more than small firms, thus minimizing the idiosyncratic component of their risk.

As to the inverse relationship between asset correlation and probability of default, prior literature has pointed out two main theoretical arguments: first, the assumption is supported only if the credit risk of a firm increases, this can be attributed to firm-specific factors and not to the business cycle; second, firms more vulnerable to the business cycle can choose a safer capital structure, thus reducing their probability of default. Subsequent studies provided different results, depending, among the other things, on whether these studies used default-based data or asset-based data. In fact, the availability of the necessary bank-level data for the analysis of credit portfolio correlations remains an important, practical issue to produce stable and reliable estimates.

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[1] In fact, for small and medium-sized corporates the Committee allows for an extra downward firm-size adjustment up to 4%, thus bringing the range of corporate asset correlation between 8% and 20%.

Düllman et al. (2007) estimate asset correlations from the monthly time series of asset returns of circa 2,000 European firms, based on the KMV model for the period of 1996-2004, and apply these estimates in a value-at-risk (VaR) analysis. The authors compare the time-varying individual borrower correlation estimates in a market model and sector-specific estimates in a sector model, and analyze their impact on the economic capital required for credit portfolio risk. They also apply the Basel II IRB model, finding that the VaR fluctuates substantially over time for the market and the sector model, due to both changes in the expected default frequencies and the asset correlations, whereas the VaR of the IRB model is more stable over time, mainly due to the smoothing effect of the hard-wired negative dependency of asset correlations on the probability of default. Distinguishing between heterogeneous and homogeneous portfolios in terms of exposure size, they observe that modeling individual asset correlations has a strong impact on VaR for credit portfolios of heterogeneous borrower size, suggesting that the omission of individual dependencies can substantially reduce the VaR estimate. Furthermore, for portfolios with heterogeneous exposure size, the IRB model matches the models with sector-dependent correlations reasonably well in terms of VaR, whereas in the case of a more fine grained portfolio with homogeneous exposure size, it produces overall more conservative risk estimates, even if at the peaks of credit risk in the observation period the distance from the other models disappears.

Focusing on a sample of large corporate or quoted firms, Lopez (2004) empirically detects the relationship between portfolio average asset correlations, firms' PDs and asset size. As in Düllman et al. (2007), the analysis is based on the KMV's methodology for determining credit risk capital charges, that he implemented on more international portfolios of the US, Japanese and European firms. His results confirm the existence of an inverse link between asset correlation and PD, thus suggesting that firms with higher default probabilities are more subject to their own specific risk than to systematic risk. Besides, he finds that asset correlation is an increasing function of firm size, meaning that larger companies are more correlated with the general economic environment and the common risk factor. Furthermore, the asset correlations he finds generally match those based on the formula presented in the November 2001 BCBS' proposal for a new regulatory capital standard.

Dietsch and Petey (2004) provide estimates of asset correlation in two samples of around 440,000 French SMEs and circa 280,000 German SMEs, by using default data and adopting a single-factor risk model, over the years of 1995-2001 in France and 1997-2001 in Germany, respectively. Their results firstly prove that the sensitivity to one systematic risk factor is quite low, with average values of asset correlations being around 1% for both French and German SMEs. Then, the asset correlations decrease on average with the size of their sample SMEs, showing that the SMEs' credit risk is less sensitive to the systematic risk factor as a firm's size increases. Finally, they do not find a negative relation between asset correlation and PD: in fact, this relationship is U-shaped in France and positive in Germany. Based on their findings, the authors argue that: (1) large SMEs should receive more favorable treatment than large firms because they seem to be less sensitive to systematic risk; (2) even if on an individual basis riskier than the large SMEs, bank loans to smaller SMEs should be treated as retail exposures, due to their weak sensitivity to systematic risk and the benefits of large portfolio diversification.

Based on the one-factor model which is at the basis of the Basel II IRB approach, Düllmann and Scheule (2003) compute asset correlation in a database that includes more than 50,000 German corporate obligors, observed for a 10-year period, from 1991 until 2000. They do not find an unambiguous support for the Basel Committee's assumption that asset correlation decreases with the probability of default. By contrast, the modification that asset correlation increases with size, which has been introduced into the risk weights by the Committee but is restricted to SMEs, is overall supported in their analysis. This relation holds for all the obligors' PD categories that they take into account, and seems to be stronger for lower credit quality obligors.

De Servigny and Renault (2002) and Gordy and Heitfield (2002) use rating agencies' grades and provide estimates for large companies. Focusing on a US sample over the period of 1981-2001, the former research uses Standard&Poor's database to investigate the properties of default correlation. They study the performance of different measures of correlations derived from actual default data, showing that no single measure outperforms the others for all levels of correlation and sample size. Finally, they also address the issue of the consistency of equity correlation as a good proxy for asset correlation by comparing average equity correlation and default correlation for their 21-year sample period. Finally, Gordy and Heitfield (2002) show that the slight positive relationship between credit quality and asset implied correlation is not statistically significant and that there is no real value in terms of accrual precision to reject the hypothesis of constant implied correlation across rating classes.

3. Estimating empirical implied asset correlation in the Italian banking system

3.1. Data and methodology. This study analyzes the mean and the volatility of empirical loss data to enable the estimation of both the expected and the unexpected loss (EL and UL, respectively), which in turn can be used to derive the correlation value that would generate that same level of unexpected loss within the IRB formulas.

The initial step is to source a robust time series of pooled loss observations. In our empirical analysis we will refer to the quarterly statistics on Italian banks' loan portfolio quality, publicly available at the Bank of Italy's website. Nevertheless, we must be aware that our data consistency with the Basel II asset class definitions is not complete due to the regulatory treatment of loans issued to small and medium enterprises. On the other hand, in using these data we benefit from the size of the dataset (more exposures in the pool and a low probability of single-obligor concentrations), and the high frequency of observations. Finally, we have to point out that our loss statistics reflect only the bad debts and are not based on the larger category of the impaired loans, which includes restructured loans, substandard loans and the overdue/overdrawn loans.

Particularly, we are interested in the flow of adjusted bad debts and the outstanding amount of loan facilities excluding adjusted bad debts. These pooled data have the advantage of representing the exposures held on the Italian banks' balance sheets. The time period we analyze stretches from the first quarter of 1990 to the first quarter of 2010. It is important that data capture a period of market stress, since empirical analysis focused only on benign market conditions would understate correlations. We consider data on non-financial corporations, segmented by customer location (i.e., the geographical area), customer sectors (such as building & construction, transport & communication services, etc.) and total credit used.

Following the methodology adopted in Fitch Ratings (2008), in order to estimate the empirically derived correlation through the IRB regulatory formula, we adopt a four-step procedure that can be summarized as follows:

1. Since the regulatory formula is calibrated over a 1-year time horizon, based on the quarterly default rate that we draw from the Bank of Italy's dataset, we calculate the annualized default rate for each loan facility (ADRLF) as the ratio of the quarterly flow of adjusted bad debts (QFABD), multiplied by four, to the outstanding amount of loan facilities (LF) excluding adjusted bad debts. In symbols:

$$ADRLF = \frac{QFABD \cdot 4}{LF}. \tag{3}$$

The ADRLF is assumed to be equivalent to the annualized PD and the mean value of its distribution can be used to feed the regulatory formula.

2. For each quarter, we calculate the corresponding loss rate by multiplying the ADRLF by a 45%-LGD (the value decided by the Committee for senior unsecured claims on corporate, sovereigns and banks within the IRB-Foundation approach) and then we calculate the mean (μ) and standard deviation (σ) of the loss rates across the given time series. These two statistics are essential to deriving correlation values from the IRB formulas.

3. Since it provided the best fit to the mean and the standard deviation of the empirical loss rates distribution, we use the beta distribution to calculate the total empirical losses (EL + UL). The beta distribution is defined on the interval (0,1) and is completely characterized by two positive shape parameters, that we denoted by α and β, which are easily obtained from the mean and standard deviation of the losses themselves as follows:

$$\alpha = \mu \cdot \left(\frac{\mu \cdot (1 - \mu)}{\sigma^2} - 1 \right), \tag{4}$$

$$\beta = (1 - \mu) \cdot \left(\frac{\mu \cdot (1 - \mu)}{\sigma^2} - 1 \right), \tag{5}$$

If α and β are known, the probability density function for a beta distribution is:

$$f(x) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1} (1 - x)^{\beta-1}, \ \alpha > 0, \beta > 0, \tag{6}$$

where:

$$B(\alpha, \beta) = \int_0^1 x^{\alpha-1} (1 - x)^{\beta-1} dx.$$

Consequently, the total loss (EL + UL) is simply the value of x when P(x) = 99.9% (chosen to comply with the Basel II standards). In order to obtain the unexpected loss we must subtract from the total loss the expected one.

4. The empirical asset correlation may be derived by calculating that correlation value which equates the Basel II unexpected loss, obtained through the regulatory formula shown in Section 1, and fed with the mean value of our PD distribution and a 45%-LGD, to the empirical unexpected loss, based on the beta distribution.

3.2. The impact of Basel II risk drivers on estimated empirical correlation. In order to better understand our results, it is important to detect how

Basel II risk drivers (PD, LGD, M and S) impact on the estimated empirical correlation. As concerns PD, starting from a situation of equilibrium (i.e., the empirical unexpected loss equates the regulatory capital), ceteris paribus, an increase in the probability of default determines two results, both in the direction of a decrease in the implied asset correlation coefficient. First, holding other things constant, an increase in the PD triggers a rise in capital requirement, entailing that the implied correlation coefficient has to decrease to make the capital requirement equal to the empirical UL. Second, when the PD goes up EL grows and, given a certain amount of total loss (EL + UL), UL shrinks, and the coefficient of implied asset correlation has to further decrease in order to make the regulatory capital equal to the empirical UL.

As highlighted in the previous section, given a certain level of PD, we calculate the expected loss using a 45%-LGD. It is possible to show that different values of LGD have no significant impact on our estimates: LGD levels are used not only to calculate the empirical unexpected loss but also to feed the regulatory formula. Particularly, for a given increase in LGD the empirical unexpected loss and the capital requirement calculated according to the regulatory formula approximately increase by the same magnitude. Consequently, a given increase in LGD has no significant impact on the empirically derived correlation.

On the contrary, different values of the maturity (M) impact on the estimated empirical correlation: the higher is the maturity, the lower are the correlation estimates. M values are used only to feed the regulatory formula and have no impact on the empirical unexpected loss. Particularly, for a given increase in M the capital requirement increases while the empirical unexpected loss doesn't change. As a result, one needs a lower level of empirically derived correlation to match the new value of capital requirement with the empirical unexpected loss.

Finally, a firm's turnover (S) has no impact on the estimated empirical correlation. Turnover is used only to calculate the Basel II correlation for a given PD so as to feed the regulatory formula. Higher values of S will result in higher capital requirements but there isn't any impact on our estimates because this effect is neutralized by the iterative process we adopt to calculate our measure of implied asset correlation.

Following the above described methodology, we empirically derive asset correlation values under three different hypotheses concerning the loan maturity (M): its upper and lower regulatory bounds

according to the IRB-Advanced approach (5 years and 1 year, respectively), and the value of 2.5 years, which is used in the IRB-Foundation approach for senior unsecured claims on corporate, sovereign and banks. Furthermore, the LGD is set equal to 45%, as proposed by the Basel Committee in the IRB-Foundation approach for senior unsecured claims on corporate, sovereigns and banks.

4. Empirical evidence and main findings: PD effect *vs.* PD volatility effect

Our main findings are reported in the following tables. Table 1 shows the empirically derived, implied asset correlation, distinguishing by customer location and total credit used. The second column reports the average probabilities of default, based on the Bank of Italy's historical data. The third column displays the volatility of the PD time series, whereas the asset correlation values, calculated according to the regulatory formula, as function of each PD level, are presented in the fourth column. The last three columns highlight the asset correlation estimates that we empirically derive, given the three referred-to-maturity hypotheses.

Our estimates provide support to the conservatism of Basel II correlation coefficients: implied correlations are lower than those we calculate based on the inverse relation between PD and LGD assumed within the regulatory framework. This conservatism can be explained by the global scope of Basel II and the consequent need to take into account potential differences in risk profile across banks active in different regions and with heterogeneous credit portfolios. Put in other words, overestimating the value of the empirical asset correlation the Committee aims at damping down on the model risk. The Basel II conservatism is also appropriate given that our empirically derived correlations are based on long-term data, primarily consisting of normal market conditions. Co-movements among asset values and defaults raise dramatically when financial markets are stressed as assets tend to behave more uniformly and defaults tend to cluster during market downturns, thus making more likely an underestimation of correlation patterns under stressed conditions.

Furthermore, empirically derived correlations vary according to the total credit used. Assuming that the total credit used by a borrower can be considered as a proxy for firm size, we can see that both for the country as a whole and for all geographical areas our implied correlation increases with the total credit used, and this is consistent with the idea of larger firms being more sensitive to the common systematic risk factor.

Table 1. Implied correlation – distribution by customer location (geographical area) and total credit used

	PD	σ	Basel II correlation	Implied correlation		
				M = 5	M = 2.5	M = 1
Italy						
Less than €125,000	1.83%	0.52%	16.80%	0.52%	0.79%	1.06%
From €125,000 to €500,000	2.46%	0.71%	15.50%	0.64%	0.94%	1.22%
€500,000 and more	2.62%	0.99%	15.24%	1.12%	1.58%	2.03%
North West regions						
Less than €125,000	1.57%	0.40%	17.48%	0.38%	0.59%	0.82%
From €125,000 to €500,000	2.02%	0.51%	16.36%	0.44%	0.66%	0.89%
€500,000 and more	2.00%	0.63%	16.41%	0.68%	1.01%	1.34%
North East regions						
Less than €125,000	1.29%	0.37%	18.31%	0.46%	0.72%	1.00%
From €125,000 to €500,000	1.70%	0.50%	17.12%	0.53%	0.80%	1.09%
€500,000 and more	1.89%	0.78%	16.67%	1.13%	1.65%	2.16%
Central regions						
Less than €125,000	2.15%	0.62%	16.09%	0.59%	0.87%	1.15%
From €125,000 to €500,000	2.93%	0.95%	14.77%	0.89%	1.26%	1.60%
€500,000 and more	3.33%	1.33%	14.27%	1.44%	1.99%	2.48%
Southern regions						
Less than €125,000	2.74%	0.87%	15.05%	0.81%	1.16%	1.49%
From €125,000 to €500,000	3.92%	1.29%	13.69%	1.08%	1.48%	1.84%
€500,000 and more	4.51%	2.11%	13.26%	2.34%	3.10%	3.76%
Islands						
Less than €125,000	2.84%	1.14%	14.91%	1.32%	1.85%	2.34%
From €125,000 to €500,000	4.26%	1.65%	13.49%	1.56%	2.10%	2.58%
€500,000 and more	4.56%	2.36%	13.23%	2.88%	3.79%	4.57%

Source: Our elaborations on data from Bank of Italy (statistical database on-line).

With regard to the relation between PD and asset correlation, as seen in Section 2, in the prior literature there doesn't appear to be conclusive empirical evidence of what the exact statistical relationship is. In order to shed more light on this aspect, in the following Table 2 we divide our time horizon into three different sub-periods: (1) March 1990-March 2000; (2) March 1995-March 2005; (3) March 2000-March 2010, and in Table 3 we calculate our implied correlation for the different borrowers' industrial sectors.

Particularly, in Table 2 we describe how the asset correlation changed during the above mentioned time horizons for the class of total credit used equal to €500,000 (the results for the other categories of total credit used, not shown in the paper but available upon request, do show the same evidence). Though the default rate goes systematically downward, asset correlation does not follow a monotonic trend, due to what we'll later define the "PD volatility effect".

Table 2. Implied correlation for three different time periods (total credit used equal to €500.000 and more) – distribution by customer location (geographical area)

	PD	σ	Basel II correlation	Implied correlation		
				M = 5	M = 2.5	M = 1
Italy						
March 90-March 00	3.31%	0.85%	14.29%	0.60%	0.85%	1.08%
March 95-March 05	2.49%	0.79%	15.45%	0.78%	1.12%	1.45%
March 00-March 10	1.91%	0.46%	16.61%	0.39%	0.59%	0.79%
North West regions						
March 90-March 00	2.33%	0.58%	15.74%	0.46%	0.68%	0.91%
March 95-March 05	1.85%	0.37%	16.76%	0.27%	0.41%	0.56%
March 00-March 10	1.67%	0.48%	17.22%	0.52%	0.79%	1.07%
North East regions						
March 90-March 00	2.30%	0.79%	15.80%	0.88%	1.27%	1.65%
March 95-March 05	1.55%	0.49%	17.52%	0.60%	0.92%	1.25%
March 00-March 10	1.46%	0.48%	17.78%	0.61%	0.94%	1.28%

Table 2 (cont.). Implied correlation for three different time periods (total credit used equal to €500.000 and more) – distribution by customer location (geographical area)

	PD	σ	Basel II correlation	Implied correlation		PD
				M = 5		
Central regions						
March 90-March 00	4.31%	1.13%	13.39%	0.73%	1.00%	1.25%
March 95-March 05	3.30%	1.00%	14.30%	0.82%	1.15%	1.46%
March 00-March 10	2.34%	0.55%	15.72%	0.40%	0.60%	0.79%
Southern regions						
March 90-March 00	5.98%	1.95%	12.60%	1.38%	1.82%	2.19%
March 95-March 05	4.97%	2.11%	13.00%	2.06%	2.72%	3.29%
March 00-March 10	3.04%	0.81%	14.63%	0.61%	0.87%	1.11%
Islands						
March 90-March 00	6.20%	2.25%	12.54%	1.76%	2.29%	2.74%
March 95-March 05	4.91%	2.68%	13.03%	3.33%	4.34%	5.20%
March 00-March 10	2.93%	0.77%	14.77%	0.58%	0.83%	1.07%

Source: Our elaborations on data from Bank of Italy (statistical database on-line).

The evidence we provide for the Italian banking system shows that implied, empirically derived asset correlations vary across different geographic areas and different borrower industries (see the following Table 3), entailing that the Basel II assumption of applying the same correlation value, irrespective of potential differences in risk profile in borrowers belonging to different industries or located in different geographical areas, might not appropriately differentiate the relative risk profile of bank assets.

Table 3. Implied correlation – distribution by borrowers' industrial sectors

	PD	σ	Basel II correlation	Implied correlation		
				M = 5	M = 2.5	M = 1
Agricultural, forestry and fishery products	2.48%	1.00%	15.47%	1.24%	1.76%	2.26%
Metal products, expect transport equipment	1.80%	0.65%	16.87%	0.85%	1.25%	1.67%
Food and tobacco products	2.54%	0.62%	15.38%	0.46%	0.67%	0.88%
Textiles, clothing and foot wear	3.01%	0.74%	14.66%	0.51%	0.74%	0.95%
Paper and paper products	2.32%	0.79%	15.76%	0.85%	1.23%	1.61%
Building and construction	2.67%	1.02%	15.16%	1.17%	1.65%	2.11%
Wholesale and retail trade services, recovery and repair service	2.36%	0.71%	15.68%	0.67%	0.97%	1.27%
Lodging and catering services	2.31%	0.85%	15.78%	1.01%	1.45%	1.88%
Other market services	1.86%	0.72%	16.73%	0.97%	1.42%	1.88%
Fuel, power products and chemical products	1.54%	0.53%	17.57%	0.72%	1.08%	1.46%
Ores and metals and non-metallic minerals and products	2.08%	0.67%	16.25%	0.73%	1.07%	1.42%
Agricultural and industrial non-metallic minerals and products	2.06%	0.74%	16.29%	0.89%	1.29%	1.70%
Electrical goods, office and data processing machine, etc…	2.26%	0.68%	15.88%	0.67%	0.97%	1.28%
Other manufactured products (including rubber and plastic products)	2.23%	0.70%	15.94%	0.71%	1.04%	1.37%
Transport and communication services	2.53%	0.75%	15.39%	0.52%	0.76%	0.99%

Our empirical evidence shows that the relation between PD and asset correlation is ambiguous because, a certain change in the PD among different geographic areas and total credit used (Table 1), among different geographical areas and time horizons (Table 2), or among different industrial sectors (Table 3) comes along with a certain change in the PD volatility. We find that the resulting effects of a change in the PD and in its volatility, which we name as "PD effect" and "PD volatility effect", respectively, are opposite.

The PD effect has been described in Section 3. With regard to the "PD volatility effect", if an increase in the PD comes along with a rise in the volatility of the PD distribution, the empirical UL rises. Consequently, the coefficient of the implied correlation has to move upwards in order to make the capital requirement equal to the empirical UL.

If we consider the only "PD effect", the inverse relationship between PD and asset correlation seems to hold, but if we take the "PD volatility effect" into account, it depends on the respective magnitude: particularly, when the former is overcome by the second, the inverse relationship is not confirmed. The following Table 4 summarizes how an increase in the main risk factors affects the empirical unex-

pected loss, the capital requirement and the final effect on the implied, empirically derived asset correlation.

Table 4. The relation between an increase of risk factors and implied correlation

Risk factor	Empirical UL	Capital requirement	Implied correlation
PD	(–)	(+)	(–)
PD volatility	(+)	=	(+)
LGD	(+)	(+)	=
M	=	(+)	(–)

Concluding remarks and further developments of the research

Our results suggest that Basel II correlations are higher than those derived from the actual loss data we use, thus confirming the conservatism of the regulatory coefficients, and support the idea that asset correlation varies geographically and across different industries. Our findings want to shed more light on the inverse relation between probability of default and asset correlation. Based on our methodology, we show that, in explaining this relation, the volatility of the empirical probability of default plays a decisive role: in fact, if an increase in the PD causes a reduction in our implied asset correlation ("PD effect") due to the combination of the positive impact on the capital requirements and the negative impact on the empirical UL, when the PD volatility rises, implied correlation gets higher ("PD volatility effect"). If the latter overcomes the former, the asset correlation can decrease even if PD goes up.

The inverse relation between PD and the asset correlation coefficient, which is one of the main hypotheses the IRB approach is built on, has not been modified by the Basel III reform, that takes into account a series of measures to address procyclicality and, consequently, make banks' capital requirements more stable during the different phases of the economic cycle.

The range of possible approaches that Supervisory Authorities could follow to address this issue includes also the calibration of the risk parameters of the regulatory formula, i.e., the PD, the confidence level and the relation between PD and asset correlation in an anti-cyclical way. These solutions could be designed within the boundaries of the existing regulatory framework without any introduction of new supervisory tools.

In details, the asset correlation values used in the regulatory formula should not be kept fixed any longer or related to the PD, depending on the specific regulatory portfolio, but should vary over time according to the different phases of the economic cycle. Particularly, the size of the asset correlation parameter should decrease in a recession (when PDs increase) and increase during expansionary phases (when PDs decrease), thus strengthening the inverse relation between asset correlation and probability of default set by the Basel Committee. Further investigations could help to understand how asset correlation values can be calibrated in an anti-cyclical way. Furthermore, as made for the concentration risk, due to the importance that it has within the banking risk management, an adjustment accounting for the different values of geographic correlation could also be part of the Pillar 2 framework.

Our study shows the important role played by the PD volatility which has not been taken into account by the regulatory formula. Particularly, as concerns the PD values used to feed the regulatory formula, the Basel Committee states that PD must be estimated by banks as a long run average of a one year default rate. In order to further reduce the cyclicality of the PD estimates, these could be adjusted ex post by considering their volatility over time. For instance, the adjustment could reflect the gap between current PDs and PDs corresponding to recessions. According to this last issue some possible approaches have already been developed by FSA (2008) and CEBS (2009).

Finally, some issues still call for further research. First, the behavior of asset correlations under periods of financial market stress. While the Basel II assumptions appear conservative relative to "through-the-cycle" analysis we run in this paper, it is unclear to what extent these static assumptions would sufficiently capture an increase in correlations during market crises. It also would be interesting to investigate the relationship between correlations and firm size because our results don't bring conclusive evidence.

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