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A note on cost-benefit analysis, the marginal cost of public funds, and the marginal excess burden of taxes

Abstract

This study discusses how to treat taxes in a cost-benefit analysis (CBA). In particular the authors relate the shadow price of taxes in CBA to the concepts of the marginal cost of public funds (MCPF) and the marginal excess burden (MEB) of taxes. The paper demonstrates that the MCPF is equal to one plus the MEB for a marginal increase in a distortionary tax.

Keywords: cost-benefit analysis, marginal cost of public funds, excess burden, taxation.
JEL Classification: H21, H41, H43, Q51.

Introduction

In a cost-benefit analysis of a public sector program one has to address the question how to treat taxes. In a sense this seems straightforward although possibly very complicated to handle in the real world due to a lack of data and estimates of relevant price and income elasticities. There is also a huge literature on closely related issues like the marginal cost of public funds (MCPF) and the marginal excess burden (MEB) of taxes. The MCPF measures the monetary welfare cost of raising an additional euro in the presence of distortionary taxation. The MEB is another kind of experiment where typically a hypothetical lump-sum payment is introduced. This payment keeps the individual on the same utility level as with a proposed increase in the income tax. According to Ballard and Fullerton (1992) one can speak of a Harberger-Pigou-Browning tradition or a MEB-tradition in which the marginal cost of public funds is always larger than unity and a Dasgupta-Stiglitz-Atkinson-Stern tradition or MCPF-tradition in which it may be larger or lower than one[1].

Many recent studies have focused on redistributive issues, see, for example, Sandmo (1998) and Gahvari (2006) while others have focused on the provision of a public good in the presence of an income tax[2]. If the tax is optimally chosen a typical result is that the studies confirm the Samuelson's (1954) rule that the sum of consumers marginal willingnesses to pay for a public good should be equal to the marginal cost of providing the good. There is also an emerging literature on the MCPF concept in environmental economics where environmental taxation is analyzed in the presence of distortionary taxation (Bovenberg and van der Ploeg, 1994). A recent contribution by Gahvari (2006) addresses the MCPF concept within a Mirrlees (1971) second-best framework with heterogenous agents. For a full treatment of the concept of the MCPF see Dahlby (2008).

The purpose of this paper is modest. Within a simple general equilibrium model of a single individual economy we derive simple cost-benefit rules that can be used to assess small increases in the provision of a public good under alternative tax regimes (lump sum, ad valorem, and income taxes). It is demonstrated that these rules can be designed so as to resemble the MCPF, at least when producer prices remain unchanged by the considered marginal projects. On the other hand, the concept of a MEB of taxes seems more difficult to relate to a traditional cost-benefit analysis. However, we are able to show that for a *marginal* change in an arbitrary tax (one plus) the marginal MEB is equal to the MCPF. This is the main contribution of this study since it opens up the possibility to use computable general equilibrium models to estimate the MCPF for different distortionary taxes. We also propose a slightly different design of cost-benefit rules that we believe are easier to estimate than the one drawing on the MCPF concept. Finally, we claim that the MCPF concept becomes 'polluted' and difficult to estimate if producer prices are allowed to adjust so as to maintain general equilibrium. An Appendix addresses the treatment of taxes in a multi-individual society.

1. Some simple cost-benefit rules for a tax-distorted economy

Consider a society consisting of a number, here normalized to unity, of individuals with identical preferences and incomes[3]. The social welfare function of this society is written as[4]:

$$W = v\cdot(1, p\cdot(1+t), w\cdot(1-t_w), T, g) = \\ = V(p\cdot(1+t), w\cdot(1-t_w), T, g), \tag{1}$$

[1] The literature is huge but important contributions include Diamond and Mirrlees (1971), Harberger (1971), Stiglitz and Dasgupta (1971), Atkinson and Stern (1974), Hatta (1977), Mayshar (1991), Snow and Warren (1996), Liu (2004), and Slemrod and Yitzhaki (2001).

[2] The early contributions assumed a linear income tax (see, for example, Christiansen (1981), Hansson (1984), Stuart (1984), Fullerton (1991), and Ballard and Fullerton (1992)).

[3] The multi-individual case is considered in the Appendix.

[4] We assume for simplicity that utility functions are well-behaved (see, for example, Boadway and Bruce (1984) or Myles (1995) for definitions of a well-behaved utility function).

where v (.) is the indirect utility function of the representative individual, the first commodity serves as numeraire so its price is normalized to unity, p is the producer price of a commodity that is subject to an ad valorem tax t, w is the wage rate, t_w is a proportional wage tax, T is a lump-sum income, and g is a public good or environmental quality which is used here to generate a cost-benefit rule for a tax-distorted economy. In order to simplify the exposition we will assume that producer prices remain unchanged and that firms earn zero profits. These seemingly strong assumptions will not affect the results presented in this note. This is so because the effects caused by small induced price adjustments will net out from general equilibrium cost-benefit rules. We leave the proof for the last Section.

The government's budget constraint can be stated as follows:

$$T = t \cdot p \cdot x + t_w \cdot w \cdot l - w \cdot l_g, \qquad (2)$$

where x is a commodity, l is labor supply, and labor l_g is the sole input used in producing the public good.

Consider next a small increase in the provision of the public good keeping the tax rates t and t_w constant, i.e., letting T act as a residual "balancing" the government's budget. The associated change in social welfare is:

$$dW = V_g \cdot dg + V_T \cdot dT = V_g \cdot dg + \lambda \cdot dT, \qquad (3)$$

where a subscript $T(g)$ refers to a partial derivative with respect to $T(g)$, and λ is the marginal utility of lump-sum income.

The associated change in the government's budget is:

$$\begin{aligned} dT &= [p \cdot t \cdot \frac{\partial x}{\partial T} + t_w \cdot w \cdot \frac{\partial l}{\partial T}] \cdot dT + \\ &+ [p \cdot t \cdot \frac{\partial x}{\partial g} + t_w \cdot w \cdot \frac{\partial l}{\partial g}] \cdot dg - \\ &- w \cdot dl^g = \alpha \cdot dT + \beta \cdot dg - w \cdot dl^g, \end{aligned} \qquad (4)$$

where the first (second) expression within brackets is denoted $\alpha\,(\beta)$. Thus changes in the lump-sum tax ($dT < 0$) impacts on deadweight losses of distortionary taxes as is it seen from the first expression within brackets. The second expression within brackets shows that also a change in the provision of the public good might affect demands and supplies of tax distorted commodities. Next, using equation (4) to eliminate dT from equation (3)[1] and multiplying through by $1/\lambda$ yields:

$$\frac{dW}{\lambda} = \left[\frac{V_g}{\lambda} + \frac{\beta}{1+\alpha}\right] \cdot dg - \left[\frac{1}{1+\alpha}\right] \cdot w \cdot dl^g, \qquad (5)$$

where we have reversed the sign in front of α in order for the expression to reflect a tax increase. Multiplying through by $1/\lambda$ converts the expression from (unobservable) units of utility to monetary units and yields our basic cost-benefit rule. The direct benefits expression $(V_g/\lambda) \cdot dg$ is the willingness-to-pay for the considered small project. However, there is also an indirect effect equal to $\beta/(1-\alpha) \cdot dg$ through the impact of the project's output on deadweight losses[2]. The direct cost of the project should be multiplied by a factor $k = 1/(1+\alpha)$ reflecting the impact of the project's costs on deadweight losses. Later on we will relate these 'adjustment' factors to the concepts of the marginal cost of public funds and the marginal excess burden of taxes. A small project for which $dW/\lambda > 0$ is socially profitable, i.e., its benefits exceeds its costs.

Let us next consider a Ramsey variation (see Ramsey, 1927), according to which the project is financed through a change in the ad valorem tax (holding T constant). Proceeding in the same way as above the cost-benefit rule reads:

$$\frac{dW}{\lambda} = \left[\frac{V_g}{\lambda} + \frac{\beta}{1+\alpha}\right] \cdot dg - \left[\frac{1}{1+\alpha}\right] \cdot w \cdot dl^g, \qquad (6)$$

where $\alpha^t = \left[\left(\frac{\partial x}{\partial q}\right) \cdot t \cdot p \cdot x^{-1} + t_w \cdot w \cdot \frac{\partial l}{\partial q} \cdot x^{-1}\right]$

with $q = p \cdot (1 + t)$, i.e., the consumer price, and β contains the same terms as in equation (5). The reader should note that we could alternatively interpret x as a vector of goods that are subject to a value-added tax (VAT). Then t is interpreted as the common VAT rate.

It is important to stress that in equation (6) we consider a "pure" Ramsey variation where lump-sum taxation is not available. Thus the project is fully financed by a change in t, i.e. one can consider t as endogenous. If t is considered to be exogenous one must allow lump-sum transfers to balance the budget.

Finally, if the project is fully financed through an increase in the wage tax rate one arrives at the following cost-benefit rule for a small or marginal project:

$$\frac{dW}{\lambda} = \left[\frac{V_g}{\lambda} + \frac{\beta}{1+\alpha^{t_w}}\right] \cdot dg - \left[\frac{1}{1+\alpha^{t_w}}\right] \cdot w \cdot dl^g, \qquad (7)$$

[1] Lundholm (2005) derives a few rules using this approach but based on a different definition of the MCPF or what he terms the "social MCPF". In equation (14) we show that our rules incorporate one definition of the MCPF.

[2] Using the production function $g = g\,(l^g)$ this indirect effect can be transferred to the cost expression.

where $\alpha^{t_w} = \left[p \cdot t \frac{\partial x}{\partial w_n} \cdot l^{-1} + t_w \cdot w \cdot l^{-1} \cdot \frac{\partial l}{\partial w_n} \right]$ with w_n denoting the net or after-tax wage, and β once again contains the same terms as in equation (5). If preferences are weakly separable in g the β-term vanishes[1] and we obtain a simple rule according to which one should compare the marginal willingness-to-pay for the project with its direct cost multiplied by a factor reflecting the impact of the project on marginal deadweight losses. Thus the cost-benefit rule reduces to:

$$\frac{dW}{\lambda} = \frac{V_g}{\lambda} \cdot dg - \left[\frac{1}{1+\alpha^i} \right] \cdot w \cdot dl^g , \tag{8}$$

where a superscript i refers to the particular tax instrument used to finance the project. Later we will suggest an alternative decomposition of changes in tax revenues that might be more straightforward to estimate than cost-benefit rules involving a^i (for $i = T, t, t_w$) and β. However, let us now turn to a brief discussion of the marginal cost of public goods.

2. On the marginal cost of public funds

There are many different definitions of the marginal cost of public funds, see, for example, Jones (2005) and Dahlby (2008) for details. However, in this study we focus on just one variation based on the following Lagrangian:

$$L = V(.) + \mu \cdot N(T, t, t_w, g), \tag{9}$$

where $N(.) = T - t \cdot p \cdot x - w - t_w \cdot l + w \cdot l^g$, and μ is a Lagrange multiplier. The aim here is to maximize social welfare subject to the government's budget constraint. However, before taking a look at the (first-order) conditions for a second-best optimum we provide a definition of the concept of the marginal cost of public goods, *MCPF*. Let us consider a project financed by adjusting the ad valorem tax t. Then, following Gahvari (2006) *MCPF* is defined as:

$$MCPF^t = \frac{1}{\lambda} \cdot \frac{\partial V / \partial t}{\partial N / \partial t}. \tag{10}$$

Thus MCPF measures the monetary welfare cost of raising an additional euro in taxes. Similarly, one might define the marginal benefit of spending an additional euro on the public good, *MBPG*, as follows:

$$MBPG = \frac{1}{\lambda} \cdot \frac{\partial V / \partial g}{\partial N / \partial g}. \tag{11}$$

[1] A utility function is weakly separable in g if it can be written as: $U = U[f(x, l), g]$.

In order to shed some further light on these concepts, let us introduce a couple of first-order conditions for a second-best optimum:

$$\frac{\partial L}{\partial t} = \frac{\partial V}{\partial t} + \mu \cdot \frac{\partial N}{\partial t} = 0,$$

$$\frac{\partial L}{\partial g} = \frac{\partial V}{\partial g} + \mu \cdot \frac{\partial N}{\partial g} = 0. \tag{12}$$

Thus at a second-best optimum it holds that:

$$\frac{V_g}{\lambda} = MCPF^t \cdot \frac{\partial N}{\partial g}. \tag{13}$$

It might be noted that if lump-sum taxation is available and $t = t_w = 0$, then the simple rule suggested by Samuelson (1954) applies, i.e., $MCPF = 1$ so that the (sum of individuals') willingness-to-pay for the public good is equal to the marginal cost of providing the good.

The question arises how *MCPF* is related to our cost-benefit rules. Using equation (10), one finds after straightforward calculations that:

$$MCPF^t = \frac{1}{1+\alpha^t}, \tag{14}$$

where α^t is defined in equation (6). Thus the concept of the *MCPF* (as defined here) is relevant also for cost-benefit analysis. However, the reader should note that unless preferences are weakly separable in the public good, the cost-benefit rule will contain an additional "correction" factor (i.e., β) reflecting the public good's impact on tax wedges, just like the MBPG concept.

3. On the marginal excess burden of taxes

Measures of the marginal excess burden of taxes are typically formulated in terms of an equivalent variation, see, for example, Fullerton (1991). The *EV* is the maximum amount of money the individual is willing to pay in order to avoid that distortionary tax increase. *MEB* is then defined as following: $MEB = (EV - \Delta N) \cdot N^{-1}$, where ΔN is the change in tax revenue. A more general approach would allow prices to adjust following the change in the tax. Such general equilibrium measures can be estimated if a computable general equilibrium (CGE) model is available. If the *MEB* concept is applied in a cost-benefit analysis the relevant approach is to multiply direct project costs by one plus the *MEB*. According to Gahvari (2006) and Auerbach and Hines (2002), in general, $MCPF \neq 1 + MEB$. The reason is the fact that the concepts refer to distinctly different thought experiments. The MEB refers to hypothetical lump sum payments/compensations that allows the individual to remain at a particular utility level. The *MCPF*, on the other hand, aims at capturing the actual changes in deadweight losses that a project causes.

However, the concept of the *MCPF* refers to *marginal* changes in a tax. Therefore, let us consider the case of the *marginal MEB* (denoted MEB^{t_i}). This measure is now defined as $MEB^{t_i} = \left(dEV^{t_i} - dN^{t_i}\right)\cdot\left(dN^{t_i}\right)^{-1}$, where $dEV^{t_i} = \lambda^{-1}\cdot\left(\partial V/\partial t_i\right)\cdot\partial t_i$ is the willingness-to-pay for escaping a small increase in t_i, and $dN^{t_i} = \left(\partial N/\partial t_i\right)\cdot\partial t_i$ is the change in total tax revenue caused by a ceteris paribus marginal increase in tax t_i. Then we arrive at the following result:

$$MEB^{t_i} = dEV^{t_i}/dN^{t_i} = \frac{1}{\lambda}\cdot\frac{\partial V/\partial t_i}{\partial N/\partial t_i} = MCPF^{t_i}, \quad (15)$$

where $MCPF^{t_i}$ is defined as in equation (10) with t replaced by t_i. Thus, our result is completely general. It follows that if a CGE model is available one could use a reasonably small change in tax rate i to obtain a rough estimate of $1+MEB^{t_i}$, i.e., $MCPF^{t_i}$, holding all other tax rates constant. Repeating the procedure for the other taxes one obtains a vector of estimated values that could be applied in empirical studies.

Still, there might be induced effects also on the benefit side that a properly undertaken CBA must account for as the β-factor in e.g. equation (7) indicates.

4. An alternative way of handling taxes in a CBA

An alternative approach to the one outlined in Section 2 would be to relate the taxes to what production or factor uses are crowded out by the considered project. For example, laborers that are drawn from other production activities are associated with an opportunity cost equal to $w\cdot(1+t)$ since this is the amount consumers ultimately are willing to pay for the commodities produced by the marginal worker. Similarly, laborers that would otherwise stay outside the labor force are now valued at their reservation wage, i.e., $w\cdot(1-t_w)$.

In order to further illustrate this approach we use the social welfare function in equation (1) and assume, for simplicity, lump sum taxation, although the cost-benefit rule will contain the same terms regardless the chosen tax instrument[1]. Then the cost-benefit rule will read as follows:

$$\frac{dW}{\lambda} = \frac{V_g}{\lambda}\cdot dg + p\cdot t\cdot dx + w\cdot t_w\cdot dl - w\cdot dl^g, \quad (16)$$

where dx and dl refer to the combined effects of changes in g and the chosen tax instrument. Now if labor supply remains constant, i.e. $dl = 0$, we can assume that the laborers needed for the production of the public good are drawn from production of commodity x. Then it holds that $p\cdot t\cdot dx = -t\cdot w\cdot dl^g$, where we have used the first-order condition for profit maximization $p\cdot\partial f(l)/\partial l = w$ and the fact that $dl^d = -dl^g$, i.e., the loss in private sector employment is equal to the gain in public sector employment since, by assumption, $dl = 0$. The difference between consumers marginal willingness to pay (WTP) and the real marginal cost of crowded-out goods is equal to $p\cdot t$. This difference is an additional cost of the considered project. Thus, the cost-benefit rule reads:

$$\frac{dW^{UBC}}{\lambda} = \frac{V_g}{\lambda}\cdot dg - w(1+t)\cdot dl^g, \quad (17)$$

where a superscript *UBC* refers to an upper bound for the project's costs. This simple rule generalizes to the case with many different commodities and a value added tax since the value of the marginal product is equal to w in all sectors (assuming homogenous labor and perfect competition).

On the other hand if $dl = dl^g$ then leisure is crowded out so $dx = 0$. In this case we should value laborers at their after-tax (reservation) wage rate since $dl = dl^g$ in equation (16). Thus, this lower bound rule reads:

$$\frac{dW^{LBC}}{\lambda} = \frac{V_g}{\lambda}\cdot dg - w\cdot\left(1-t_w\right)\cdot dl^g, \quad (18)$$

where a superscript *LBC* refers to a lower bound for the project's costs.

These are simple "rule of thumbs" and can easily be applied so as to obtain upper and lower bounds (ceteris paribus) for a project's societal costs[2]. They provide a possibly fruitful approach when estimates that α and β are not available. However, the approach obviously becomes more difficult to apply when there are many different inputs.

5. Flexible prices

In order to illustrate the effects of endogenous producer prices and pure profits the social welfare function is written as:

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[1] However, the induced effects of the project on consumption and employment in equation (16) might depend on how the project is financed. This is obvious from the expressions for α, α^t and α^{t_w} in Section 2.

[2] However, it cannot be ruled out that tax wedges are changed in such a way that the societal cost exceeds our upper bound or falls short of our lower bound.

$$W = V(p \cdot (1+t), w \cdot (1-t_w), T + \pi, g), \qquad (19)$$

where $\pi = \pi(p,w)$ is the, by assumption well-behaved, profit function of the representative firm supplying x^S using labor l^d as the only input (while it for simplicity is assumed that the firm producing the numeraire earns zero profits). For this Robinson-Crusoe economy it is assumed that the individual and sole owner of firms treat profit income as a lump-sum income although profits are endogenous from the point of view of society.

In addition to effects captured by equation (16) we have the following effects:

$$\begin{aligned} &\lambda \cdot \left(x^s - x^d\right) \cdot dp + \left(l^s - l^d - l^g\right) \cdot dw - \\ &-\lambda \cdot t \cdot x^d \cdot dp - \lambda \cdot t_w \cdot l^s \cdot dw + \lambda \cdot t \cdot x^d \cdot dp + \qquad (20) \\ &+\lambda \cdot t_w \cdot l^s \cdot dw = 0, \end{aligned}$$

where x^d denotes demand for x, l^s denotes supply of labor, and dp and dw refer to the total changes caused by the change in g plus the change in the chosen tax instrument[1]. The two first terms vanish since prices, by assumption, clear markets. The consumer is affected by tax changes as p and w change (shown in the middle row) but these effects are exactly offset by the same effects but with opposite signs through the government's budget (shown in the final row[2]). However, the concept of the *MCPF* becomes 'polluted' in the sense that induced price changes show up in the denominator as well as the numerator. Moreover these changes are *partial* in the sense that we evaluate $\partial V/\partial t$ and $\partial N/\partial t$ *ceteris paribus* while equation (20) accounts for the general equilibrium adjustments. Therefore, it seems as if the concept of the *MCPF* is less fruitful in contexts where producer prices change and firms earn pure profits. In addition it becomes extremely involved to estimate *MCPF*.

Conclusion

In this study we have investigated whether the concept of the marginal cost of public funds is suitable for use in a conventional cost-benefit analysis. Indeed if all producer prices are assumed to be constant and preferences are weakly separable in the public good it is legitimate to multiply a small project's direct costs by a factor reflecting the *MCPF*. However, if the separability condition is not satisfied one must in addition account for the impact of the public good on the magnitude of the tax wedges. Moreover, if producer prices adjust – as they typically do also for a small project in a general equilibrium context – the *MCPF* will be extremely complicated to estimate since it now also contains effects in both numerator and denominator of the price adjustments caused by the project. On the other hand, in a "conventionally" formulated cost-benefit rule these induced effects net out as it is seen from equation (20).

Finally, we have shown that for a *marginal* change in the wage tax, the *MEB* concept is relevant for a cost-benefit analysis. Our result opens up the possibility to use computable general equilibrium models to compute the *MCPF* for small changes in different taxes. However, it remains to be shown that the concept is relevant also if it is calculated from "large" tax changes in the way conventional definitions suggest.

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[1] In this case, dx and dl in equation (16) refer to the combined efects of changes in g, the chosen tax instrument, p and w.

[2] We have moved the term $\lambda \cdot l^g \cdot dw$ from the final ("government") row to the first row in equation (20).

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Appendix

In this Appendix we briefly consider an economy consisting of $H > 1$ different individuals. Assume that the project under consideration is financed by uniform lump-sum taxation and that the social welfare function is utilitarian. The welfare differential is written as follows:

$$dW = \sum_h \left[V_g^h \cdot dg + \lambda^h \cdot dT\right] = \sum_h V_g^h \cdot dg + \bar{\lambda} \cdot \left[t \cdot p \cdot dx^H + t_w \cdot w \cdot dl^H - w \cdot dl^g\right], \tag{A.1}$$

where $\bar{\lambda}$ is the expected or mean marginal utility of lump-sum income, a superscript (h) refers to an aggregate or total quantity (individual h), and the same decomposition is used as in section 5[1]. Rearranging and multiplying through by the expected marginal utility of income, equation (1) can be stated as:

$$\frac{dW}{\bar{\lambda}} = \sum_h \frac{\lambda^h}{\bar{\lambda}} \cdot WTP^h + t \cdot p \cdot dx^H + t_w \cdot w \cdot dl^H - w \cdot dl^g, \tag{A.2}$$

where $WTP^h = \left[V_g^h / \lambda^h\right] \cdot dg$ is individual *h's* willingness to pay for the considered change in the provision of the public good. This *WTP* can be estimated using survey techniques like contingent valuation or choice experiments (conjoint analysis) or market based approaches like the travel cost method and the property value method. However, the problem is that the willingness-to-pay of each individual must be weighed using the individual's own marginal utility of lump-sum income relative to the average marginal utility of income. Unless the distribution of marginal utilities of income is relatively even across individuals, the sum of WTP^h will be a poor predictor of society's valuation of the project in question, even in the special case of a utilitarian social welfare function. It should be noted that with the exception of the $\lambda^h / \bar{\lambda}$ -term, equation (A.2) contains the same terms as equation (16).

If a commodity tax or income tax is used to finance the project it is not possible to factor out the marginal utility of income in the way that is done in equation (A.1) since consumption and labor supply levels vary across individuals, in general. Therefore evaluation of the project's benefits and costs becomes very involved unless it is assumed that λ^h is evenly distributed across individuals or x^h is the same for all in the case of $dt > 0$ or l^h is the same for all in the case of $dt_w > 0$. In the last two cases, one can factor out $\bar{\lambda}$ in the same way as in equations (A.1) and (A.2). Still, in these cases one faces the problem in valuing benefits discussed above.

[1] Alternatively the cost-benefit rule can be expressed in terms of α and β as in Section 2.