“Are Spanish commercial banks rationing credit? The dynamics of the loan-deposit gap”

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Are Spanish commercial banks rationing credit? The dynamics of the loan-deposit gap

Abstract

Are banks denying credit to firms and households in Spain? A positive answer to this question seems to be a well installed presumption between analysts and politicians who demand and motivate government intervention in several forms, including direct public finance, publicly loan guarantee schemes and even interest rates subsidization. This paper presents a brief overview of the theoretical and empirical arguments for or against credit rationing and of the Spanish Bank System practices as a previous step to provide new empirical evidence of the commercial bank system practices during and before the current crisis in Spain, analyzing the long-run relationship between loans and deposits. Our results suggest that this relationship is time-varying, which supports the view that a new commercial bank practice emerged a few quarters before the starting of the current crisis leading it, a new phenomenon with negative potential effects over firms and households.

Keywords: SME, credit rationing, loan, credits, nonlinear models, Spain.

Jel Classification: G18, G21, M13, C22.

Introduction

Credit rationing can be defined as a situation where some loan applicants deny a loan altogether, despite being willing to pay more than bank’s quoted interest rates in order to obtain one, and being observationally indistinguishable from borrowers who do receive a loan or even the case where loan applicants receive a smaller loan than they desire at the quoted interest rate (Keaton, 1979). In Spain, during the current crisis the availability of credit is at the heart of the debate over SMEs. In that sense, the debate is centered around a simple question: Whether Spanish Bank System is supplying an amount in accordance with the current chance that those who owe money to the bank will not repay it.

Between 2001 and 2007, Spanish commercial banks lived one of the most brilliant periods in the recent history, after suffering an intense process of transformations due to the need to make radical changes in order to face up to the globalization of financial markets, new regulations and financial innovations. Following the work of Piñeiro (2009), this period was characterized, at least by the following facts: i) Spanish banks reached a return on assets which doubled the European average return, and a return on equity 72% higher than the average in the rest of Europe; ii) As from 2004, Spanish households experienced a negative balance: mortgage loans were extraordinarily high, with growth rates between the 30 and 40 percent. On the other hand, an acceleration in the mortgage securitization and in other capital raising instruments led to an increase in the liquidity gap; and iii) a high liquidity led to a great dynamism in the investment process, with easy securities and other debt assets in foreign markets joint to low risk-premium associated to credits.

In this context, the credit was increasing by 30% annually during eight years whereas deposits increased at a rate between 6-7%, enhancing the loan-deposits gap, probably due to the expansion in the mortgage credit. However, the subprime crisis in the US – rumors began in 2006 – has led to the alarm and financial collapse during the third quarter of 2006. From this moment, a growing mismatch between deposits and credits can be observed. As a result, and using data from the Spanish Central Bank, the difference between credits and deposits has grown by 21 percentage points between 2000 and 2007. On the other hand, the credit supply has become more restrictive given the current crisis and the financial turbulences. As a consequence, this situation has led to the expected effects on the amount of the loan funds and on the collaterals (2009).

As it is well-known, the financial crisis has motivated the Central banks’ and Government interventions in several forms, including liquidity injections, direct public finance, publicly loan guarantee schemes, and, even interest rates subsidization. However, the effectiveness of these interventions may be put in doubt.

In this general framework, the study of the current crisis impact on the commercial banks practices must be crucial, analyzing the evolution of the difference between credits and deposits. In fact, this differential represents the Bank’s financial intermediation margin. The aim of this article is: to analyze the developments in the loan-deposit gap, in order to determine how the financial intermediation in the Spanish Financial System has evolved and what its consequences can be over the credit rationing problem, and its relation with the business cycle.

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1 See Carbó and Rodríguez (2005).

2 Another closed definition of this gap could be considered: the “corrected gap” in which the difference between credits and deposits is corrected by means of the default rate (See Beltrán et al., 2009).

51
To do this we will employ a vector error-correction model (VECM). Contrary to earlier studies we will test whether or not the relationship is time-dependent (in particular, dependent on the business cycle). If the statistical test indicates that the relation is not time-dependent, linear cointegration techniques are sufficient. Otherwise, non-linear techniques should be used. Therefore, we extend earlier analyses in two ways: i) analyzing the relationship between loans and deposits in a VECM linear model, where the error-correction term can be interpreted as the gap, and given the relationship between them, interpreting the gap adjustment process when a shock occurs; and ii) testing the possible existence of a non-linear relationship, as a way to verify if the long-term relationship is time-varying.

In sum, this paper aims at investigating the interactions between credits and deposits, in the framework of a VECM model, using Spanish quarterly data during the period of 1980:1-2009:3. In addition, in an attempt to explore the robustness of the results obtained by means of the traditional approach, i.e. analysis of a linear VECM, we will test if the relationships under investigation are time-dependent, using threshold cointegration model.

The rest of this article is organized as follows: The empirical methodology is outlined in the next section, whereas empirical tests are performed in section 2. The main conclusions are summarized in the last section.

1. Econometric methodology

As mentioned above, before employing non-linear econometric methodology we estimate a linear VECM using the maximum likelihood technique. The data used in the empirical analysis are quarterly observations drawn from the Banco de España, in millions of Euros. The sample period ranges from 1980:1 to 2009:3.

The benchmark linear model is a finite-order VAR of the following form:

$$x_t = c + \sum_{i=1}^{k} A_i x_{t-i} + \epsilon_t.$$  (1)

In the above model, $x_t = [c_t, d_t]$ is a vector of non-stationary variables containing the credit ($c_t$) and deposit ($d_t$), $A$ is a 2x2 matrix of parameters, and $\epsilon_t$ is a 2x1 vector of residuals. Cointegration requires that all the variables have the same order of integration. Before estimating a linear cointegration model we have tested for the order of integration of the two series. To this end, we have used the modified version of the Dickey-Fuller and Phillips-Perron tests proposed by Ng and Perron (2001). According to these results, $c_t$ and $d_t$ would be $I(1)$. See Appendix A, Table A1, for more details.

In order to characterize the long-run dynamic adjustments, we can rewrite the equilibrium VAR model as a vector error-correction model (VECM). The VAR($k$) model can be rewritten in its VECM representation by subtracting $x_{t-1}$ from the left and right hand sides:

$$\Delta x_t = c + (A_1 - I)x_{t-1} + \ldots + A_k x_{t-k} + \epsilon_t = c + (A_1 - I)x_{t-1} - (A_1 - I)x_{t-2} + (A_1 - I)x_{t-2} + \ldots + A_k x_{t-k} + \epsilon_t.$$

Hence,

$$\Delta x_t = c + \sum_{i=1}^{k-1} \Gamma_i \Delta x_{t-i} + \Pi x_{t-k} + \epsilon_t,$$  (2)

where $\Gamma_i = \left[ I - \sum_{j=1}^{i-1} A_j \right]$ and $\Pi = \left[ I - \sum_{j=1}^{k} A_j \right]$.

Another decomposition of (1) is given by:

$$\Delta x_t = c + \sum_{i=1}^{k-1} \Gamma^*_i \Delta x_{t-i} + \Pi x_{t-i} + \epsilon_t,$$  (2')

where $\Gamma^*_i = \left[ \sum_{j=i}^{k} A_{j+1} \right]$ and $\Pi = \left[ I - \sum_{j=1}^{k} A_j \right]$.

The matrix $\Pi$ is usually decomposed as:

$$\Pi = \alpha \beta^T,$$  (3)

where $\alpha$ and $\beta$ are nxr matrices containing the adjustment coefficients and the cointegrating vector, respectively, $n$ is the number of variables, $r$ is the number of cointegrating relationships (one, in our case). The symbol $\Delta$ in equation (2) is the first difference operator. In this form all terms in equation (2) are stationary, that is, integrated of order zero, denoted $I(0)$.

The lagged residuals from the cointegrating vector $\beta' x_{t-1}$ act as an error-correction term. This term captures the extent of disequilibrium for the system of variables with respect to the long-run relation between all variables in the system. The $\alpha$ parameters on the error-correction terms in each individual equation indicate the speed of adjustment of this variable back to its long-run value. A significant error correction term (i.e., a significant $\alpha$ parameter) implies long-run causality from the explanatory variables to the dependent variable under consideration.

---

1 Let us define the loan-deposit gap ($g_t$) as the difference between credits ($c_t$) and deposits ($d_t$).
In our application the system can be written as:

\[
[\Delta c_t] = \Gamma(L) [\Delta c_{t-1}] + \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} (c_{t-1} - \beta d_{t-1}) + \begin{bmatrix} \varepsilon_{c_t}^* \\ \varepsilon_{d_t}^* \end{bmatrix},
\]

where \( \alpha_1 \) and \( \alpha_2 \) indicate the speed of adjustment of each variable back to its long-run value.

We estimated this model using the maximum likelihood procedure developed by Johansen (1988, 1991). Importantly, we can observe that if \( \beta \) is close to 1, the error-correction term equals \( c_{t-1} - d_{t-1} = g_{t-1} \), i.e., the error correction term is equal to the gap. This is very important in order to facilitate the interpretation of our results. The estimation results (linear VECM) are reported in Table 1 (for fixed and nonfixed beta), while results obtained from applying the Johansen reduced rank regression approach are reported in Table A3.

The results suggest that the hypothesis of non-cointegration (\( r=0 \)) cannot be rejected at the 5% level (can be rejected at 10% level). Both in the credit equation and in the deposit equation the error-correction terms are not significant. As the \( \alpha \)'s are not statistically different from zero, both series are said to be long-run weakly exogenous with respect to the long-run equilibrium.

However, the non-significance of the \( \alpha \) parameters (and the rejection of linear cointegration) could be due to the presence of nonlinearity in the relation – i.e., the relation could be time-dependent. In particular, the relation could vary according to different stages of the business cycle. We will account for nonlinearity by applying a two-regime threshold cointegration model, proposed by Hansen and Seo (2002).

### Table 1. Linear VECM estimates – Credit-deposit

<table>
<thead>
<tr>
<th>Variable</th>
<th>Non fixed ( \beta )</th>
<th>( \hat{\beta} = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c )</td>
<td>325.27 (1421.89)</td>
<td>4556.24* (1578.18)</td>
</tr>
<tr>
<td>( \Delta c_{t-1} )</td>
<td>1.182* (0.112)</td>
<td>0.699* (0.138)</td>
</tr>
<tr>
<td>( \Delta d_{t-2} )</td>
<td>-0.288* (0.135)</td>
<td>-0.236* (0.161)</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>-0.032 (0.020)</td>
<td>0.014 (0.021)</td>
</tr>
</tbody>
</table>

Notes: Standard errors are in parentheses. **, * Significant at the 10%, 5% levels, respectively.

1 Johansen’s approach is based on MLE of the VECM, by step-wise concentrating the parameters out, i.e., maximizing the likelihood function over a subset of parameters, treating the other parameters as known, and giving the number \( r \) of cointegrating vectors, with the matrix \( \beta \) is the last to be concentrated out.

### 2. Modelling non-linearity

We then account for non-linearity by applying a threshold cointegration method. The concept of threshold cointegration characterizes a discrete adjustment, in a way in which the system will reach the long-run equilibrium only when it exceeds or does not reach a critical threshold.

Hansen and Seo (2002) provide a vector error-correction model (VECM) in which a cointegration relationship exists between two variables and a threshold effect as an error correction term. As an extension of model (4), a two-regime threshold cointegration model takes the form:

\[
[\Delta c_t] = \Gamma(L) [\Delta c_{t-1}] + \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} (c_{t-1} - \beta d_{t-1}) + \begin{bmatrix} \varepsilon_{c_t}^* \\ \varepsilon_{d_t}^* \end{bmatrix}
\]

with \( (c_{t-1} - \beta d_{t-1}) \leq \gamma \),

\[
[\Delta c_t] = \Gamma(L) [\Delta c_{t-1}] + \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} (c_{t-1} - \beta d_{t-1}) + \begin{bmatrix} v_{c_t}' \\ v_{d_t}' \end{bmatrix}
\]

with \( (c_{t-1} - \beta d_{t-1}) > \gamma \).

Hansen and Seo (2002) proposed a heteroskedastic-consistent LM test where the null hypothesis of linear cointegration (i.e., there is no threshold effect) is tested against the alternative of threshold cointegration. The test assumes a fixed value of \( \beta \) (1, in our case) or nonfixed beta. Application of the test for our model reveals that the null hypothesis of linear cointegration is indeed rejected in favor of threshold cointegration. We refer to Appendix A for details (see Table A4).

The estimated threshold – estimated gap – is \( \hat{\gamma} = 30.822,4 \) (with \( \hat{\beta} = 1 \)) and \( \hat{\gamma} = 166.272 \) (with nonfixed \( \beta \)). Hence, the first regime would occur when the gap is below 30822,4 (166.272 with nonfixed beta). This is the relatively usual regime, including 88.03% (87.18%, when \( \beta \) is not fixed) of the observations. By contrast, the unusual regime, including 11.97% (12.82%) of the observations would occur when the gap is above 30822.4 (166.272).

The estimated two-regime threshold VAR is reported in Table 2, where significant error-correction effects appear in the unusual regime (the estimated \( \alpha \) parameters are significant) but not in the usual regime (with beta fixed).

For the unusual regime, the adjustments coefficients (\( \alpha \)) are significantly different from zero when the gap is above 116.272, meaning that a value of the gap above 116.272 in one quarter produces downward pressure on the credit in the subsequent quarter to restore the long-run equilibrium and an upward pressure on the deposit. By contrast, when the gap is below 116.272, both variables (credit and deposit) suffer an upward pressure.
The economic interpretation of the above findings is as follows. When the gap is very high (period from 2006:1 to 2009:3), or put differently, when the loan-deposit gap is very high, this phenomenon in itself causes downward pressure on credits, whereas deposits grew.

**Table 2. Threshold VECM estimates (Hansen and Seo approach)**

<table>
<thead>
<tr>
<th>Threshold</th>
<th>(c_{t-1} - d_{t-1} \leq 38022.4)</th>
<th>(c_{t-1} - d_{t-1} &gt; 38022.4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c)</td>
<td>(\Delta c_c) 4.38022 (2080.57)</td>
<td>(\Delta d_d) 11.423* (0.191)</td>
</tr>
<tr>
<td>(\Delta c_{c,t-1})</td>
<td>0.8151* (0.2051)</td>
<td>1.0183* (0.1508)</td>
</tr>
<tr>
<td>(\Delta d_{d,t-2})</td>
<td>-0.1168 (0.2163)</td>
<td>0.060 (0.2722)</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.0178 (0.048)</td>
<td>-0.1793* (0.064)</td>
</tr>
<tr>
<td>Observations percentage</td>
<td>88.03 %</td>
<td>11.97 %</td>
</tr>
</tbody>
</table>

Notes: Standard errors are in parentheses. **, * Significant at the 10% and 5% levels, respectively.

**Fig. 1. Response of credit and deposit to error correction \((c_{t-1}-d_{t-1})\)**

Figure 1 plots the error-correction effect, i.e., the estimated response of (changes in) loans \((\Delta c_c)\) and deposits \((\Delta d_d)\) to the discrepancy between them (i.e., to the gap) in the previous period, holding the other variables constant. As we can see, for a “high” gap (i.e., above the threshold, greater than 166.272 millions of Euros), the response of credits is negative, suggesting the existence of a credit rationing problem. However, for a “low” gap, the response of both series (loans and deposits) is positive.
In sum, according to our results, the null hypothesis of linear cointegration is rejected in favor of a two-regime threshold cointegration model. Consequently, a system of two regimes would seem to characterize the discontinuous adjustment of loan-deposit gap towards a long-run equilibrium. The new regime, or the relatively unusual regime in the Spanish economy (with 12.82% of the observations), is coincident with the recent period including the current crisis, as we can see in Figure 2. This figure shows the threshold is a few quarters before the starting date of the current crisis which indicates how Spanish bank system arrived at the end of an expansive phase and the beginning of the recession.

![Quarterly GDP growth, 1980:1-2009:3](image)


**Concluding remarks**

The relationship between credit and deposit reveals, in an indirect way, the magnitude and extension of the current credit rationing. Our paper contributes to a better understanding of this relation in Spain. We have shown that the relation has changed with the current phase of the business cycle, revealing that whereas in the previous recessions the gap did not reach the threshold, in the current crisis a new regime and, therefore, a new nature in the cointegration relationship between credits and deposits emerges. This is an important difference between the previous and the current crisis, and an indicator of its financial character.

Given the current international credit crunch, the regime of high gap confirms the diagnosis which demands a public effort to face up to the firms and household credit rationing.

**References**


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1 In the figure, the shadowed area is the second regime.
First, we show results from unit root tests to see whether the variables from our model are stationary or not. Second, we present results from several statistical tests which guided us throughout our empirical analysis. When using time series data, it is often assumed that the data are non-stationary and, thus, that a stationary relationship needs to be found in order to avoid the problem of spurious regression. For these reasons, we begin with examining the time-series properties of the series. We use a modified version of the Dickey and Fuller.
(1979, 1981) test (DF) and a modified version of the Philips and Perron (1988) tests (PP) proposed by Ng and Perron (2001) for the null of a unit root, in order to solve the traditional problems associated with conventional unit root tests. Ng and Perron (2001) propose a class of modified tests, $\overline{M}$, with GLS detrending of the data and using the modified Akaike information criterion to select the autoregressive truncation lag.

Table A1 reports the results of Ng-Perron tests, $\overline{MZ}_{t}^{GLS}$, $\overline{MZ}_{t}^{GLS}$, $\overline{MSB}^{GLS}$, $\overline{MPT}^{GLS}$ and ADF tests. All test statistics formally examine the unit root null hypothesis against the alternative of stationarity. The null hypothesis of non-stationarity for series in level $c$ and $d$ cannot be rejected, regardless of the test. Accordingly, these two series would be $I(1)$.

Table A1. Unit root Ng-Perron tests

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\overline{MZ}_{t}^{GLS}$</th>
<th>$\overline{MZ}_{t}^{GLS}$</th>
<th>$\overline{MSB}^{GLS}$</th>
<th>$\overline{MPT}^{GLS}$</th>
<th>Lags ADF</th>
<th>Lags</th>
</tr>
</thead>
<tbody>
<tr>
<td>Credits</td>
<td>-1.427</td>
<td>-0.445</td>
<td>0.312</td>
<td>9.775</td>
<td>5</td>
<td>0.256</td>
</tr>
<tr>
<td>Deposits</td>
<td>-4.090</td>
<td>-1.038</td>
<td>0.254</td>
<td>6.441</td>
<td>3</td>
<td>-0.031</td>
</tr>
</tbody>
</table>

Notes: * Rejects null hypothesis at 1% significance level. ** Rejects null hypothesis at 5% significance level. *** Rejects null hypothesis at 10% significance level.

The critical values are tabulated in Ng & Perron (2001).

Testing for the lag length

Cointegration analysis requires the model to have a common lag length. To select the lag length of the VAR we have used the Akaike information criterion (AIC), the Schwarz information criterion (SC), and the Hannan-Quinn (HQ) criterion. The choice of $k$ based on the information criterion suggests that $k = 2$ is to be preferred. Hence, since the VECM variables are in first-differences, our estimates (see Tables 1 and 2 in the text) incorporate one lag.

Table A2. Results for choosing the lag length of the VAR model based on the AIC, SC and HQ criteria

<table>
<thead>
<tr>
<th>Lag</th>
<th>AIC</th>
<th>SC</th>
<th>HQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>53.62290</td>
<td>53.67012</td>
<td>53.64207</td>
</tr>
<tr>
<td>1</td>
<td>43.31793</td>
<td>43.45958</td>
<td>43.37544</td>
</tr>
<tr>
<td>2</td>
<td>41.79345*</td>
<td>42.02961*</td>
<td>41.88937*</td>
</tr>
</tbody>
</table>

Testing for cointegration

The results obtained from applying the Johansen reduced rank regression approach to our model are given in Table A3. The two hypotheses tested, from no cointegration $r = 0$ (alternatively $n - r = 2$) to the presence of one cointegration vector ($r = 1$) are presented in the two first columns. The eigenvalues associated with the combinations of the $I(1)$ levels of $x_t$ are in column 3. Next the $\lambda_{\text{max}}$ statistics comes that tests whether $r = 0$ against $r = 1$. That is, a test of the significance of the largest $\lambda_{\text{max}}$ is performed. The results suggest that the hypothesis of no cointegration ($r = 0$) can be non-rejected at the 5% level (with the 5% critical value given in column 5). The $\lambda_{\text{trace}}$ statistics tests the null that $r = q$, where $q = 0, 1$ against the unrestrictive alternative that $r = 2$. On the basis of this test the null hypothesis is non-rejected. Hence, following the tests for cointegration rank suggests the non-rejection of the null hypothesis of no cointegration.

Table A3. Johansen cointegration test

<table>
<thead>
<tr>
<th>$H_0 : r$</th>
<th>$n - r$</th>
<th>$\lambda_{\text{max}}$</th>
<th>$\lambda_{\text{max}} (0.95)$</th>
<th>$\lambda_{\text{trace}}$</th>
<th>$\lambda_{\text{trace}} (0.95)$</th>
<th>Lags</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td>0.0815</td>
<td>9.9426</td>
<td>14.2646</td>
<td>14.7471</td>
<td>15.4947</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0.0402</td>
<td>4.8045</td>
<td>3.8415</td>
<td>4.8044</td>
<td>3.8415</td>
</tr>
</tbody>
</table>

Note: * Denotes rejection at the 5% significance level.

Testing for nonlinearity

Hansen and Seo (2002) proposed a heteroskedastic-consistent LM test, namely, sup LM $^0$ (for a fixed $\beta$; $\beta = 1$ in our case) and supLM (for nonfixed $\beta$) for the null hypothesis of linear cointegration (i.e., there is no threshold effect) against the alternative of threshold cointegration. For the test, the p-value is calculated using a parametric bootstrap
method (with 5000 simulations replications), as proposed by Hansen and Seo (2002)\(^1\). Therefore, according to Table A4, threshold cointegration appears at the 2.6\% significance level for the sup \(LM^0\) test and 0.8\% when \(\beta\) is nonfixed, so that the null hypothesis of linear cointegration would be strongly rejected.

| Table A4. Hansen-Seo tests of threshold cointegration |
|-----------------|-----------------|
| **Test statistic value** | sup \(LM^0\) | sup \(LM\) |
| Calculated p-values (fixed regressor) | 0.026 | 0.008 |
| Calculated p-values (bootstrap) | 0.026 | 0.012 |
| Threshold parameter | 3802.24 | 16627.2 |
| Estimate of the cointegrating vector | 1.00 | 0.898 |

**Appendix B. Error-correction term interpretation**

This appendix shows for our application that the residual in the VECM can be interpreted as the credit-deposit gap. Our benchmark model is given by the following expression:

\[ c_t = \mu + \beta d_t + e_t. \]

In order to contribute to a correct interpretation of the error-correction term, observe that, the error-correction mechanism is derived from the relationship in the first differences:

\[
\begin{align*}
\Delta c_t &= \gamma_0^c + \gamma_1^c \Delta c_{t-1} + \gamma_2^c \Delta d_{t-1} + \alpha^c e_{t-1}, \\
\Delta d_t &= \gamma_0^d + \gamma_1^d \Delta c_{t-1} + \gamma_2^d \Delta d_{t-1} + \alpha^d e_{t-1},
\end{align*}
\]

If \( \beta = 1 \), then

\[ c_t = \mu + (1) d_t + e_t \Rightarrow e_t = c_t - \mu - d_t. \]

Hence,

\[
\begin{align*}
\Delta c_t &= \gamma_0^c + \gamma_1^c \Delta c_{t-1} + \gamma_2^c \Delta d_{t-1} + \alpha^c (c_t - \mu - d_t), \\
\Delta d_t &= \gamma_0^d + \gamma_1^d \Delta c_{t-1} + \gamma_2^d \Delta d_{t-1} + \alpha^d (c_t - \mu - d_t),
\end{align*}
\]

As \( e_t = c_t - d_t \),

\[
\begin{align*}
\Delta c_t &= \delta_0^c + \gamma_1^c \Delta c_{t-1} + \gamma_2^c \Delta d_{t-1} + \alpha^c e_{t-1}, \\
\Delta d_t &= \delta_0^d + \gamma_1^d \Delta c_{t-1} + \gamma_2^d \Delta d_{t-1} + \alpha^d e_{t-1},
\end{align*}
\]

where

\[
\begin{align*}
\delta_0^c &= \gamma_0^c - \mu \alpha^c, \\
\delta_0^d &= \gamma_0^d - \mu \alpha^d.
\end{align*}
\]

\(^1\) The test is denoted by \( \text{sup} \, LM^0 = \sup_{\beta, \gamma} LM(\beta, \gamma) \), where \( \beta \) is the known value of \( \beta \) (in our case \( \beta = 1 \)). The sup \( LM^0 \) is a heteroskedastic-consistent LM test statistic for the null hypothesis of linear cointegration against the alternative of threshold cointegration. We have used the bootstrap method developed by Hansen and Seo (2002) to calculate asymptotical critical and p-values.