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Value-at-Risk-based risk management on exchange traded funds: the Taiwanese experience

Abstract

This study investigates the daily Value-at-Risk (VaR) for 0050-ETFs returns of the Taiwan Stock Exchange from 2003 to 2007. The essential source of performance improvements between distributional assumption and volatility specification is identified utilizing symmetric (GARCH) and asymmetric (GJR-GARCH) volatility models under alternative distributions through two-stage models selection criterion. Empirical results indicate that the roles of distributional assumption and asymmetric volatility specification achieve their superiority at different confidence levels. Additionally, different fat-tailed distributions should be considered at different confidence levels. Eventually, we encourage in that GJR-*t*/GARCH-*HT* model is a useful technique for conservative/aggressive risk managers against market uncertainty in volatile ETFs markets.

Keywords: exchange traded funds, Value-at-Risk, market risk, GARCH, Taiwan.
JEL Classification: C52, C53, G15.

Introduction

Since the bankruptcy or near bankruptcy of various financial institutions[1] has occurred after they incurred huge losses through exposure to unforeseen market moves during the past 15 years, financial economists, regulators and practitioners have paid considerable attention to efficient market risk management. Additionally, the introduction of the first "Basel Accord" in 1996 allowed banks to use internal market risk management models to fulfill their requirements regarding capital adequacy. With this framework, the Value-at-Risk (VaR) methodology has become a popular first line of defense against downside risk in financial positions among financial institutions, and a popular tool for risk management among regulators and risk managers. VaR provides financial institutions with a sense of the minimum expected loss with a small probability (α_1) cover given time horizon k (usually 1- or 10-days). Alternatively, VaR refers to the maximum potential loss that will occur over a given time horizon at a given confidence level ($1-\alpha_1$).

Recently, international iShares[2] exchange-traded funds, ETFs, have become extensively adopted investment instruments among global investors because they represent diversified portfolios of securities that combine the best qualities of both closed- and open-end mutual funds. Similar to closed-end mutual funds, iShares ETFs can be traded throughout the trading day by their net-asset value (NAV); like open-end mutual funds, iShares ETFs allow for the creation and redemption of securities, but have lower associated expenses, and are more tax efficient. In the early 1990s, the American Stock Exchange (AMEX) introduced Standard & Poor's Depositary Receipts (SPDRs), which are backed by a stock portfolio that closely tracks the underlying index. The success of SPDRs has subsequently led to a wave of issues of similar products elsewhere, including in Taiwan[3]. The first issue of ETFs in Taiwan, the Polaris Taiwan Top 50 Tracker Fund (hereafter, 0050-ETFs), was successfully launched in 2003, opening a new era for financial markets in Taiwan. Table 1 presents the daily volume of various ETFs traded between September 2006 and December 2007. The 0050-ETFs is the most actively traded securities among the various iShares listed in the Taiwan Stock Exchange (hereafter, TAIEX) in terms of daily volume of shares traded. To the best of our knowledge, the existing literature dealing with ETFs has focused on price discovery (Chu and Hsieh, 2002; Tse and Martinez, 2007), hedge (Alexander and Barbosa, 2008), international cointegration between ETFs and country funds (Olienyk et al., 1999), as well as risk-and-return performance between ETFs available for foreign markets and closed-end country funds (Harper et al., 2006). In contrast, this study assesses market risk in the ETFs market from the perspective of VaR analysis. No previous study has attempted this and this study seeks to provide some further

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[1] See, e.g., the bankruptcy of Baring's Bank in 1995, Asian financial crises in 1997, near bankruptcy of Long-Term Capital Management in 1998, American subprime mortgage catastrophe in 2007, and Devastating losses of more than $7 billion for Societe Generale's bank in 2008.

[2] iShares were originally created by Barclays Global Investors in March 1996 under the name of World Equity Benchmark Shares (WEBS).

[3] Recently, emerging markets, such as Taiwan, have attracted significant interest from foreign institutional investors, who have been attracted by the high rates of return offered. Investment in the Taiwan Stock Exchange (TSE) by Qualified Foreign Institutional Investors (QFII) was deregulated in 1991, and restrictions were further loosened in 2003. Taiwan thus has attracted growing numbers of foreign investors during the past decade.

insights into risk management for volatile ETFs markets in Taiwan.

Table 1. Average daily trading volume of various ETFs listed in the TAIEX

Item	Daily share volume
(0050) Polaris Taiwan Top 50 Tracker Fund (Since 2003/6/30)	6,171
(0051) Polaris Taiwan Mid-Cap 100 Tracker Fund (Since 2006/8/31)	927
(0052) Fubon Taiwan Technology Tracker Fund (Since 2006/9/12)	593

Note: This table presents the daily volume (in thousand of shares) of ETFs traded between September 12, 2006 and December 31, 2007.

Since the risk management group at J.P. Morgan developed the RiskMetrics model for measuring VaR in 1994, RiskMetrics has become a benchmark with practitioners for quantifying market risk. The RiskMetrics model assumes that asset returns are normally distributed, with zero mean, and with variance being expressed as an exponentially weighted moving average of historical squared returns. This model has been criticized for having at least two drawbacks. First, it is widely documented that the distribution of financial returns is leptokurtic and fat-tailed, and thus it is assumed that conditional normality may result in substantial bias in VaR forecasts. Second, returns volatility is often characterized by a number of stylized facts, including time-varying volatility and asymmetric volatility (or leverage effect). Such common temporal dependencies of financial returns were found to have significant impact on the forecasting accuracy of VaR (Alexander and Leigh, 1997; Brooks and Persand, 2003).

The vast literature related to VaR applications has demonstrated an improvement in VaR estimations associated with generalized autoregressive heteroskedasticity (GARCH) models with returns innovations that allow fat-tailed distributions. For example, So and Yu (2006) studied GARCH models, including RiskMetrics and two long memory GARCH (IGARCH, FIGARCH) models, in Value-at-Risk estimation, and found evidence that *t*-error models are superior to normal-error models in determining an appropriate value of VaR for long position at the 99% confidence level. Bams et al. (2005) reached similar conclusions, arguing that the GARCH(1,1)-*t* model is adequate for correctly assessing extreme losses for exchange rate positions. On the other hand, Hung et al. (2008) found evidence that the proposed GARCH-HT model-based VaR approach achieves good accuracy and efficiency at both low and high confidence levels for alternative energy commodities when asset returns exhibit leptokurtic and fat-tailed features.

Notably, Brooks and Persand (2003) asserted that asymmetry is an important issue in the VaR framework, and therefore must be modeled in the volatility specification. To this end, Angelidis et al. (2004) evaluate the performance of an extensive family of ARCH models with three distributional assumptions (normal, student-*t* and GED) in modeling the daily Value-at-Risk of five stock indices. Based on the proposed quantile loss function, there was strong evidence that the combination of the student *t*-distribution with EGARCH models produces the most adequate VaR forecasts for the majority of stock market data. However, Angelidis and Degiannakis (2005) indicated that models with a normal distribution produce adequate daily VaR forecasts at the 95% confidence level, while models that take account of the leverage effect for the conditional variance, the leptokurtosis, and the asymmetry of the data, accurately forecast the VaR at the 99% confidence level. Additionally, Huang and Lin (2004) used the RiskMetrics, APARCH-*N* and APARCH-*t* models to analyze the accuracy and efficiency of each model for stock index futures prices at low and high confidence levels. Their analytical results suggested that APARCH-*N* performs better at lower confidence levels whereas APARCH-*t* is more accurate than alternative at higher ones. Ané (2006) also provided supportive evidence that the additional flexibility brought by the APGARCH model provides little, if any, improvements for accurate VaR forecasts.

However, despite an extensive literature on VaR forecasting, none of them discuss whether both distributional assumption and volatility asymmetry are essential for improving VaR performance at different confidence levels. At low confidence levels (e.g., 90%), a risk manager that considers both the above factors is likely to overestimate the true VaR. Such a risk manager may impose higher capital charges than necessary, imposing excessive and impractical opportunity costs in relation to capital[1]. Accordingly, it is interesting to investigate whether both the distributional assumption and volatility specification can affect the measurement of market risk in the context of VaR at different confidence levels.

This study differs from previous research in three major dimensions. First, we implement symmetric (GARCH) and asymmetric (GJR-GARCH) volatility models using three distributional assumptions (normal, student-*t* and heavy-tailed (*HT*) distributions) to estimate the 90% and 99% one-day-ahead VaR[2] for

[1] For the case of high confidence level (e.g., 99%), a risk manager that does not consider either fat-tails or asymmetric volatility may underestimate the true VaR, resulting in the VaR being insufficient to cover huge losses arising from market risk.

[2] VaR forecasts are considered over a daily horizon because this horizon is considered relevant for trading purposes, and is therefore believed to

0050-ETF returns, whereas no previous research has addressed market risk in the ETFs market from the perspective of VaR analysis. Second, particular emphasis has been placed on the predictive content of two different possible sources of performance improvements, asymmetry in the volatility process, and distributional assumption. The discussion carried out in this study enables us to recognize the essential source of performance improvements between distributional assumption and volatility specification when forecasting the downside risk of financial positions at different confidence levels. Finally, a two-stage models selection criterion (Sarma et al., 2003) is then employed for comparative evaluation of the predictive performance of these VaR models in risk management. In the first stage, the models are back-tested to determine their predictive accuracy by simultaneously using the unconditional coverage (LR_{uc})[1] test (Kupiec, 1995) and the conditional coverage (LR_{cc})[2] test (Christoffersen, 1998). In the second stage, the firm utility-based loss function (FLF) is defined and used to further evaluate models that have met the prerequisites of both back-testing criteria. Under this framework, the one-sided sign tests advocated by Diebold and Mariano (1995) are employed to further examine the competing models in terms of loss function which can reveal the superiority of one model to another. Thus, it facilitates the selection of the optimal VaR technique among the various candidates available to risk managers.

This study investigates the one-day-ahead VaR forecasting performance of the various models for the 0050-ETFs returns over the period from June 30, 2003 to July 12, 2007. Empirical results indicate that the fat-tailed distributional assumption significantly influences VaR predictions at lower confidence level while volatility asymmetry does not. On the other hand, both the fat-tailed distributional assumption and the asymmetric volatility specification are essential for improving VaR performance at higher confidence level. In addition, our results also suggest that conservative/aggressive risk managers can adopt the proposed GJR-*t*/GARCH-*HT* model as a useful technique in protecting against market uncertainty in volatile ETFs markets.

The remainder of this study is organized as follows. Section 1 describes the econometric framework under consideration, including the various VaR models and the evaluation criteria applied to VaR estimates. Section 2 then presents the data description and model estimates, while Section 3 details a comparative analysis of the VaR performance of competing models. Finally, the last section concludes.

1. Econometric framework

1.1. The GARCH genre of volatility models. Let $r_t = 100(\ln p_t - \ln p_{t-1})$ denote the continuously compounded rate of returns from time $t-1$ to t, where p_t is the price level of underlying assets at time t, and denotes the information set of all observed returns up to time $t-1$ by Ω_{t-1}. The symmetric GARCH(1,1) model with a basic mean[3] can be formulated as follows:

$$r_t = \mu + \varepsilon_t,\ \varepsilon_t = \sigma_t z_t,\ z_t \mid \Omega_{t\_1} \sim F(0,1), \tag{1}$$

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2, \tag{2}$$

where μ and σ_t^2 denote the conditional mean and variance of returns, respectively. ε_t is the innovation process, while F(0,1) is a density function with a mean of zero and a unit variance. Furthermore, ω, α and β are nonnegative parameters with the restriction of $\alpha + \beta < 1$ to ensure the positive of conditional variance and stationarity as well.

A simple class of GARCH-type models that can cope with asymmetric volatility in response to asymmetric shocks is the GJR-GARCH model advocated by Glosten et al. (1993)[4]. The GJR-GARCH model differs from model (2) by:

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \gamma d_{t-1}\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2, \tag{3}$$

where the indicator function d_{t-1} takes the value of unity if $\varepsilon_{t-1} < 0$, and 0 otherwise. The indicator variable differentiates between positive and negative shocks, so that asymmetric effects in the data are captured by γ. Thus, in the GJR-GARCH model, positive news has an impact of α, and negative news has an impact of ($\alpha + \gamma$), with negative (positive) news having a greater effect on volatility if $\gamma > 0$ ($\gamma < 0$). Besides, ω, α and β are nonnegative parameters with the restriction of $\alpha + \beta + 0.5\gamma < 1$, whereas the estimate of the sum $\alpha + 0.5\gamma$ should still be positive (Ling and McAleer, 2002).

be interesting to academics, regulatory bodies and practitioners who engaged in risk management.

[1] The LR_{uc} test can reject a model having either too high or too low failures. However, it has been criticized for its inability in response to volatility clustering.

[2] The LR_{cc} test enables the rejection of models that generate either too many or too few clustered VaR violations.

[3] We do not focus on the specification of the conditional mean since Angelidis et al. (2004) have indicated that it is indifferent for one-step-ahead VaR forecasts.

[4] As pointed out by Franses and Dijk (1996), although the EGARCH model (Nelson, 1991) is an alternative candidate for asymmetric model specification, it was found not to be quite beneficial for repeated forecasting exercises parameter estimation of the EGARCH model can be tedious.

1.2. Distributional assumptions. From the seminal work by Engle (1982), the density function of z_t was considered as the standard normal distribution as follows:

$$F(z_t) = \frac{1}{\sqrt{2\pi}} \exp(-0.5 z_t^2). \quad (4)$$

Another common feature of many financial returns is that their sample kurtosis is quite large, implying fat tails in their empirical distributions. Hence, when estimating the GARCH model for such data, researchers have adopted the student-*t* distribution (Bollerslev, 1987; Huang and Lin, 2004; So and Yu, 2006, among others) or the heavy-tailed (*HT*) distribution (Politis, 2004; Hung et al., 2008). If a student-*t* distribution with υ degrees of freedom is assumed, the probability density function (pdf) of z_t takes the following form:

$$F(z_t, \upsilon) = \frac{\Gamma((\upsilon+1)/2)}{\Gamma(\upsilon/2)\sqrt{\pi(\upsilon-2)}} \left(1 + \frac{z_t^2}{\upsilon - 2}\right)^{-(\upsilon+1)/2}, \quad \text{for } \upsilon > 2 \quad (5)$$

where $\Gamma(\bullet)$ is the gamma function and υ is the degree-of-freedom (or shape) parameter. For large values of υ, its density converges to that of the standard normal.

Instead, with an HT distribution the pdf of the innovations becomes:

$$F(z_t, a_0, 1) = \frac{(1 + a_0 z_t^2)^{-1.5} \exp\left(-\frac{z_t^2}{2(1+a_0 z_t^2)}\right)}{\sqrt{2\pi(\Phi(a_0^{-0.5}) - \Phi(-a_0^{-0.5}))}}, \quad (6)$$

where 1 denotes the standard deviation of z_t, and Φ denotes the cumulative probability density function of the standard normal distribution. The shape parameter, a_0, reflects the degree of the heavy tails with constraint $0 < a_0 < 1$. When $a_0 \to 0$, the HT will reduce to a standard normal distribution, while the distribution has a thicker tails than the normal when $a_0 \to 1$ (For a detailed description on HT distribution consult Politis, 2004).

Accordingly, we construct six competing model specifications in modeling volatility of the ETFs returns in our comparative analysis: GARCH-*N*, GJR-*N*, GARCH-*t*, GJR-*t*, GARCH-*HT* and GJR-*HT* models. The parameter vector $\Theta \equiv [\mu, \omega, \alpha, \beta, \cdots]$ is obtained from the maximization of the sample log-likelihood function, using QMLE (Quasi maximum likelihood estimation, QMLE) as follows:

$$LL(\Theta) = \sum_{t=1} \ln F(\Theta) \quad (7)$$

where $F(\bullet)$ is the likelihood function of the GARCH models with various distributional assumptions.

1.3. Value-at-Risk and evaluation criteria of the model-based VaR. Under the framework of the parametric techniques (Jorion, 2000), the conditional VaR estimate for a one-day holding period is obtained as follows:

$$VaR_{t+1} = F(z_t; \alpha_1) \cdot \hat{\sigma}_t + \mu, \quad (8)$$

Where $F(z_t; \alpha_1)$ denotes the corresponding quantile of the z_t distribution[1], while $\hat{\sigma}_t$ is the volatility forecast generated from (2) or (3).

*1.3.1. LR test for unconditional coverage (LR_{uc})***.** To backtest the VaR results, this study first employs a likelihood-ratio test by Kupiec (1995) to examine whether the true failure rate is statistically consistent with the VaR model's theoretical failure rate. The null hypothesis of the failure rate P is tested against the alternative hypothesis that the failure rate is different from P, in which statistics is given by:

$$LR_{uc} = 2\ln\left[\frac{\hat{\pi}^{n1}(1-\pi)^{n_o}}{P^{n_1}(1-P)^{n_o}}\right] \sim \chi^2(1), \quad (9)$$

where $\hat{\pi} = n_1/(n_0+n_1)$ is the maximum likelihood estimate of *P*, and n_1 denotes a Bernoulli random variable representing the total number of VaR violations[2].

1.3.2. LR test for conditional coverage (LR_{cc}). Christoffersen (1998) developed a conditional coverage test (LR_{cc}) that jointly investigates whether the total number of failures is equal to the expected one, and the VaR exceptions are independently distributed. Given the realizations of the return series r_t and the set of VaR estimates, the indicator variable I_t can be defined as follows:

$$I_t \begin{cases} 1 \text{ if } r_{t+1} < VaR_t \\ 0 \text{ if } r_{t+1} \geq VaR_t \end{cases}. \quad (10)$$

Since accurate VaR estimates display the property of correct conditional coverage, the I_t series must exhibit both correct unconditional coverage and serial independence. The LR_{cc} test is a joint test of these two properties, and the corresponding test

[1] Hung et al. (2008) derived the analytical quantile-operator for the HT distribution. It facilitates convenient estimation of the out-of-sample VaR with HT distribution.

[2] If the predicted VaR cannot cover the realized dollar loss, this is termed as a violation.

statistics is $LR_{cc} = LR_{uc} + LR_{ind}$ as we condition on the first observation. Consequently, under the null hypothesis that the failure process is independent and the expected proportion of exceptions equals P, the appropriate likelihood ratio is represented as follows:

$$LR_{cc} = -2\ln\frac{(1-P)^{n_0}P^{n_1}}{(1-\hat{\pi}_{01})^{n_{00}}\hat{\pi}_{01}^{n_{01}}(1-\hat{\pi}_{11})^{n_{10}}\hat{\pi}_{11}^{n_{11}}} \sim \chi^2(2), \quad (11)$$

where $n_{i,j}$ is the the number of observations with value i followed by value j (i, $j=0$, 1), $\pi_{ij} = P\{I_t = j \mid I_{t-1} = i\}$ (i, $j=0$, 1), $\hat{\pi}_{01} = n_{01}/(n_{00}+n_{01})$, $\hat{\pi}_{11} = n_{11}/(n_{10}+n_{11})$.

1.3.3. Risk management loss function and superiority test. For those models which can pass these coverage tests, this study follows a two-step model selection criterion of Sarma et al. (2003) by further selecting one model among the various candidates through utility-based loss function which are much closer to the real risk manager's utilities. The firm loss function (FLF)[1] reflecting the utility of a firm is given by:

$$L_{t=1}^{F} = (r_{t+1} - VaR_t)^2 \cdot I_{\{r_{t+1}<VaR_t\}} - \delta \cdot VaR_t \cdot I_{\{r_{t+1}>VaR_t\}}, \quad (12)$$

where δ measures the opportunity cost of capital. According to Marcucci (2005), the opportunity cost of capital can be linked the risk-free interest rate, and thus, we also set δ=1.5% in our empirical illustrations.

To address the superiority among the various candidates, this study employs the same one-sided sign tests as in Diebold and Mariano (1995) and Sarma et al. (2003) to further examine the competing models in terms of FLF: To consider two VaR models, model i and model j, define the loss differential between model i and j as $X_t = L_{i,t} - L_{j,t}$ (Negative values of X_t indicate a superiority of model i over j). Thus, the null hypothesis of a zero-median loss differential is tested against the alternative hypothesis of a negative median, with a studentized version of the sign test given by:

$$\hat{S}_{ij} = (S_{ij} - 0.5T)(0.25T)^{-0.5}, \quad (13)$$

where $S_{ij} = \sum_{t=1}^{T} I_{\{X_t>0\}}$, $I_{\{\}}$ is the indicator function, and T denotes the evaluation period. Under the null, $\hat{S}_{ij}$ is asymptotically distributed as a standard normal. The null hypothesis is rejected at the 5% significance level if $\hat{S}_{ij} < -1.645$. Rejections of $\hat{S}_{ij}$ ($\hat{S}_{ij}$) would imply that model i (j) is significantly superior to model j (i).

2. Data description and model estimates

2.1. Data. The data examined in this paper are daily closed price of the Polaris Taiwan Top 50 Tracker Fund (0050-ETFs) obtained from Bloomberg database. The data set for 0050-ETFs covers the period from June 30, 2003 to July 12, 2007 for a total of 1001 observations.

Preliminary analysis of daily returns of 0050-ETFs for the whole sample period is presented in Table 2. From Panel A, the average daily returns are positive and very small compared with the variable standard deviation. The 0050-ETFs returns display significant evidence of skewness and kurtosis. Namely, the returns series is skewed towards the left, indicating that there are more negative than positive outlying returns in ETFs markets of Taiwan, while the returns series is characterized by a distribution with tails that are significantly thicker than for a normal distribution. J-B test statistic further confirms that the daily 0050-ETFs return is non-normally distributed. Moreover, the Q^2 and LM-test statistics display linear dependence of squared returns and strong ARCH effects[2]. Consequently, these preliminary analyses of the data encourage the adoption of a sophisticated distribution, which embody fat-tailed features, and of conditional models to allow for time-varying volatility. To avoid the spurious results, Panel B reports the Phillips and Perron (1988) (PP) unit root tests and KPSS (Kwiatkowski, Phillips, Schmidt and Shin, 1992) unit root tests. The test results indicate no evidence of non-stationarity in the 0050-ETFs returns.

Table 2. Preliminary analysis of 0050-ETFs daily returns

Panel A. Summary statistics						
Mean %	Std. dev.	Skewness	Kurtosis	J-B	$Q^2(12)$	LM(12)
0.061	1.204	-0.460*	5.613*	1348.510*	99.490*	76.786*
Panel B. Unit root tests						
PP	Bandwidth		KPSS	Bandwidth		
-30.718*	7		0.122	6		

Notes: 1. * denotes significance at the 1% level. 2. J-B represents the statistics of Jarque and Bera (1987)'s normal distribution test. 3. $Q^2(12)$ denotes the Ljung-Box Q test for 12th order serial correlation of the squared returns. 4. LM test also examines for autocorrelation of the squared returns. 5. PP and KPSS are the test statistics for stationarity of return series. The PP-test rejects the null hypothesis of non-stationarity if the test statistic is negative and the absolute value of the test statistic exceeds the critical value of the respective significance level: 1%: -3.436; 5%: -2.864; 10%: -2.568. The KPSS-test

[1] Under risk management loss functions, taking into account the opportunity cost of capital along with the magnitude of the VaR violations is crucial to select the best model (Sarma et al., 2003; Marcucci, 2005; Hung et al., 2008).

[2] To save space, we do not report the descriptive graphs of 0050-ETFs returns. Note that: 1) The 0050-ETFs in level depicts a bull market during 2007. 2) Volatility clustering is obviously observed from the graph of 005-ETFs daily returns. 3) The density and the QQ-plot against the normal distribution show that both returns distribution exhibits fat tails.

rejects the null hypothesis of stationarity if the test statistic exceeds the critical value of the respective significance level: 1%: 0.739; 5%: 0.463; 10%: 0.347.

2.2. Model estimates. In order to perform the VaR analysis, the GARCH-type models with three alternative distributions are estimated in this section. All GARCH models are estimated with 750 daily returns, and the estimation period is then rolled forward by one new day ahead and dropping the most distant day. Through this procedure, the remaining 250 observations are taken as the out-of-sample for estimating VaR.

In this study, the parameters are estimated by quasi maximum likelihood estimation (QMLE) in terms of the BFGS optimization algorithm using the econometric package of WinRATS 7.0. Model estimates and diagnostic tests for 0050-ETFs returns during the in-sample period are provided in Table 3.

As shown in Table 3, the parameters, ω, α, β and γ in the conditional variance equations are all positive and found to be highly significant (at least at the 5% level). Meanwhile, the symmetric GARCH component exhibits the existence of strong volatility persistence in the ETFs returns, as $\alpha+\beta\approx 1$. A notable point in Table 3 is that the parameter γ of the conditional volatility equation in each GJR-type model is positive and highly significant, implying that negative shocks (bad news) exert larger impact on ETFs volatility than positive shocks (good news) of the same magnitude. The estimated values for the shape parameters υ and a_0, which range from 3.350 to 3.405 and 0.124 to 0.123, are statistically significant at the 1% level, confirming the presence of fat tails in the returns series. Moreover, the fat tail is reduced, but not eliminated, when the variance equation is modeled using a non-linear GARCH specification.

Turning the discussion to diagnostic tests, the Ljung-Box Q statistic indicates that the linear (GARCH) or non-linear GARCH (GJR) specifications in these models are sufficient to correct the serial correlation of the returns series in the conditional variance equation. Subsequently, a comprehensive evaluation of the predictive performance of the competing VaR models will be carried out in the next section.

Table 3. Estimation results

Model	μ	ω	α	β	γ	υ	a_0	$Q^2(12)$	LL
GARCH-*N*	0.084[b] [0.041]	0.265[c] [0.013]	0.141[c] [0.010]	0.693[c] [0.010]	-	-	-	14.729	-1198
GJR-*N*	0.062 [0.039]	0.293[c] [0.013]	0.070[c] [0.009]	0.676[c] [0.010]	0.133[c] [0.022]	-	-	13.819	-1195
GARCH-*t*	0.064[a] [0.033]	0.061[c] [0.009]	0.070[c] [0.013]	0.909[c] [0.005]	-	3.350[c] [0.481]	-	13.380	-1148
GJR-*t*	0.058[a] [0.035]	0.097[c] [0.012]	0.049[b] [0.020]	0.878[c] [0.007]	0.062[c] [0.023]	3.405[c] [0.437]	-	11.619	-1147
GARCH-*HT*	0.065[b] [0.031]	0.025[c] [0.004]	0.033[c] [0.007]	0.919[c] [0.007]	-	-	0.124[c] [0.026]	15.308	-1150
GJR-*HT*	0.060[a] [0.036]	0.034[c] [0.005]	0.026[c] [0.004]	0.904[c] [0.006]	0.019[b] [0.008]	-	0.123[c] [0.028]	14.352	-1149

Notes: 1. a, b and c denote significance at the 10%, 5% and 1% level, respectively. 2. υ and a_0 respectively denote specific parameters of the t-distribution and HT-distribution, where υ and a_0 are positive shape parameters governing the fat tails of the densities with constraints $\upsilon>2$ and $0<a_0<1$. 3. $Q^2(12)$ is the Ljung-Box Q test for serial correlation in the squared standardized residuals with 12 lags. 4. LL refers to the log-likelihood value. 5. Standard errors are in parentheses.

3. Evaluating the VaR performance

3.1. Unconditional and conditional coverage tests results. To assess the forecasting performance of VaR models, Table 4 presents a range of out-of-sample Value-at-Risk summary statistics under low (90%)[1] and high (99%) confidence levels.

From Panel A of Table 4, the GJR-*N* model yields the highest average value of VaR estimates, and then followed by the GARCH-*N* model. However, either GARCH-*N* or GJR-*N* model fails to pass the LR_{uc} and LR_{cc} tests, indicating that models with normal error tend to overestimate model-based VaR. On the other hand, the remaining models all pass the coverage tests, suggesting that fat-tailed distributional assumptions are able to produce adequate daily VaR forecasts. This reveals that the distributional assumption plays a significant role for VaR estimates at lower confidence level rather than the specification of volatility asymmetry. Interestingly, the GARCH-*HT* and GJR-*HT* models not only pass the LR_{uc}/LR_{cc} test but also their empirical failure rates are much closer to the prescribed ones. Thus, the adoption of

[1] Notably, the VaR results at the 95% confidence level are very similar to those obtained at the 90% confidence level and are not reported here.

HT-distribution is superior to that of *t*-distribution for daily VaR forecasts at low confidence level, regarding predictive accuracy.

For the case of 99% confidence level, we observe that GARCH-*N*, GJR-*N*, GARCH-*t* and GJR-*t* models can pass the unconditional coverage test, indicating that the sample point estimate is statistically consistent with the prescribed confidence level of these four VaR models. Turning to the column of LR_{cc} statistic, we find that the aforesaid four models also can pass the conditional coverage test, indicating that these models' performance is quite stable over time and does not deteriorate in turbulent markets. However, neither GARCH-*HT* nor GJR-*HT* models can offer adequate VaR forecasts according to LR_{uc} statistic. Given that for these cases, the models have been rejected by the LR_{cc} test, indicating that clustered violations were generated. Namely, GARCH-*HT* and GJR-*HT* models are very slow at updating the VaR value when market volatility changes rapidly. There is evidence that as a superior performance of a model with the *HT* distribution at lower confidence level does not guarantee its predominance at a higher one. In addition, the empirical failure rate generated by the GJR-*t* model is closest to the prescribed one. Arguably, the GJR model with the *t*-distribution is superior to those with normal- and *HT*-distributions for daily VaR forecasts at high confidence level.

However, for both low and high confidence levels situation, a risk manager cannot select a unique VaR technique when there is more than one model that have passed these coverage tests. Consequently, a two-step model selection procedure is recommended for further selecting one model among the various candidates through the utility-based firm loss function in terms of different confidence levels.

Table 4. Out-of-sample Value-at-Risk summary statistics

Panel A. VaR results at the 90% confidence level					
	Mean VaR	Violations	Failure rate	LR_{uc}	LR_{cc}
GARCH-*N*	-1.4006	12	0.0480	9.1217*	11.6200*
GARCH-*t*	-1.1930	20	0.0800	1.1845	1.2916
GARCH-*HT*	-1.0840	27	0.1080	0.1737	0.5798
GJR-*N*	-1.4277	13	0.0520	7.6268*	9.6089*
GJR-*t*	-1.2080	20	0.0800	1.1845	1.2916
GJR-*HT*	-1.1099	23	0.0920	0.1821	0.1911
Panel B. VaR results at the 99% confidence level					
	Mean VaR	Violations	Failure rate	LR_{uc}	LR_{cc}
GARCH-*N*	-2.5857	5	0.0200	1.9568	4.3871
GARCH-*t*	-3.1482	4	0.0160	0.7691	3.9306
GARCH-*HT*	-2.1486	6	0.0240	3.5553*	5.4073*
GJR-*N*	-2.6198	5	0.0200	1.9568	4.3871
GJR-*t*	-3.1269	3	0.0120	0.0949	4.2096
GJR-*HT*	-2.2021	7	0.0280	5.4969*	7.3421*

Notes: 1. The critical values of the LR_{uc} and LR_{cc} statistics at the 10% significance level are 2.71 and 4.61, respectively. 2. * indicates significance at the 10% level.

3.2. Model selection based on Diebold and Mariano's sign test. For those models which can meet the prerequisite of the appropriate coverage tests, this study applies the one-sided sign tests by further evaluating the remaining competing models in terms of firm loss function which can assess the superiority from one model to another. The models that are chosen for the second-stage model selection are based on those models with the best and second best unconditional/conditional coverage in the first step.

Table 5 reports the summary results of the standardized sign tests at low and high confidence levels. Panel A of the table lists the average values of loss function obtained by the various VaR models according to the selection procedure discussed above. These values indicate that the GJR-*HT* (GARCH-*t*) model produces lower economic loss than the GARCH-*HT* (GJR-*t*) at the 90% (99%) confidence level. Nevertheless, a lower average value of FLF does not necessarily imply the superiority of that model among its competitors. Accordingly, sing test in Diebold and Mariano (1995) is then implemented for further examining the superiority among these remaining candidates.

The standardized sign test statistic is reported in Panel B of Table 5. The sign test applied to these models with respect to the FLF shows that the GARCH-*HT* model significantly outperforms the GJR-*HT* model at low confidence level. Such evidence suggests that the specification of a fat-tailed distribution (*HT*-error) is much important than volatility asymmetry when estimating daily VaRs at

a lower confidence level. On the other hand, the GJR-*t* model is significantly superior to the GARCH-*t* model at high confidence level. Consequently, it would be construed as strong evidence that both the asymmetric volatility specification and *t*-error distributional assumption are essential for improving VaR predictions when using a higher confidence level.

Table 5. Superiority tests in terms of FLF at various confidence levels

Panel A. Average values				
90% confidence level			99% confidence level	
Model	FLF		Model	FLF
GARCH-*HT*	0.24397		GARCH-*t*	0.11326
GJR-*HT*	0.23674		GJR-*t*	0.11588
Panel B. Sign tests				
$\hat{S}_{G\text{-}HT,\ GJR\text{-}HT}$	-2.6563*		$\hat{S}_{G\text{-}t,\ GJR\text{-}t}$	1.8973
$\hat{S}_{GJR\text{-}HT,\ G\text{-}HT}$	2.6563		$\hat{S}_{GJR\text{-}t,\ G\text{-}t}$	-1.8973*

Notes: 1. The critical value of the $\hat{S}_{ij}$ ($\hat{S}_{ji}$) statistics at the 5% significance level is -1.645. 2. * denotes significance at the 5% level. 3. A rejection of $\hat{S}_{ij}$ ($\hat{S}_{ji}$) means that model i (j) is significantly superior to model j (i).

To sum up all results obtained in 3.1 and 3.2, three main consequences have emerged from these empirical findings. Above all, the distributional assumption plays a significant role for VaR predictions at lower confidence level rather than the specification of volatility asymmetry. Besides, both the distributional assumption and the asymmetric volatility specification are essential for improving VaR performance at higher confidence level. Finally, the *HT*-distribution is preferred at lower confidence level, while the *t*-distribution is an ascendant alternative at higher confidence level.

Conclusion

Exchange-traded funds (ETFs) have recently become prevalent investment instruments among global investors. The extensive literature associated with ETFs has mainly focused on price discovery, hedge, international cointegration, and risk-and-return performance between ETFs and closed-end country funds. In contrast with previous works, this study contributes to the literature on modeling and quantifying market risk in the ETFs market from the perspective of Value-at-Risk analysis. Such attempt has never before been undertaken and thus this study seems to provide some further insights into risk management for the volatile ETFs markets.

This study empirically investigates the one-day-ahead VaR forecasting performance of six Value-at-Risk models for the 0050-ETFs of TSE over the period from June 30, 2003 to July 12, 2007. Particular emphasis has been given to the predictive content of two different possible sources of performance improvements: asymmetry in the volatility process and distributional assumption. Performance is evaluated using a two-stage models selection criterion that addresses the accuracy of each model. Consequently, this current study has produced some solid evidence that should be considered by risk managers seeking to calculate VaR accurately. First, at lower confidence level, the GARCH/GJR model associated with returns innovations that follow normal distribution tends to overestimate model-based VaR. In contrast, models with student-*t* or *HT* errors can produce adequate VaR forecasts, indicating that the fat-tailed distributional assumption significantly influences VaR predictions at lower confidence level while volatility asymmetry does not. Second, at higher confidence level, it is not only the fat-tailed distribution that plays a crucial role in VaR predictions, but also the specification of volatility asymmetry. Finally, the applicability of a fat-tailed distribution in modeling VaRs is not necessarily constant at different confidence levels since our results suggest that *HT*-distribution is preferred at lower confidence level, while the *t*-distribution is an ascendant alternative at higher confidence level.

Overall, the empirical results are encouraging in that conservative/aggressive risk managers can adopt the proposed GJR-*t*/GARCH-*HT* model as a useful downside risk measure in protecting against market uncertainty in volatile ETFs markets.

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