“Mean variance optimization via factor models in the emerging markets: evidence on the Istanbul Stock Exchange”

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Mean variance optimization via factor models in the emerging markets: evidence on the Istanbul Stock Exchange

Abstract

Markowitz's mean-variance analysis, a well known financial optimization technique, has a crucial role for the financial decision makers. This quadratic programming method determines the optimal portfolios within the risk-return perspective. Estimation of the expected returns and the covariances for the financial assets has a significant importance in quantitative portfolio management. The famous financial models used in estimating the input parameters are CAPM, Three Factor Model and Characteristic Model. The goal of this study is to investigate the significance of asset pricing models in the Markowitz's mean-variance optimization technique for the different Turkish benchmark indices. The optimized risky financial assets have demonstrated higher portfolio risks rather than risky portfolios with risk-free assets. Portfolio risk is found lower for CAPM, Three Factor Model and Characteristics Model, however higher for naive returns. The performances of optimized CAPM portfolios are higher than multi-factor models. Consequently, asset pricing models have significant role in the Markowitz's mean-variance optimization technique since they provide higher portfolio performances with lower risks than the optimized portfolios of naive returns.

Keywords: financial optimization, mean-variance optimization, asset pricing models, Turkey.
JEL Classification: C60, C61, G11, G12, G10, C22, C32, C13.

Introduction

The quantitative investment analysis has widely studied a portfolio theory. For the last 50 years the financial decision makers have interested in the portfolio risk, asset return and the optimal way of combining the risky and risk-free assets in a particular portfolio or fund (DeFusco et al., 2007).

Optimization procedure has a great deal of importance in financial engineering and financial decision making since this technique harmonizes the financial modeling with the finance theory in the framework of risk and return. However, the optimization is mainly engaged with the mathematics and models so that, in this respect the optimal portfolios are investigated for the individual and institutional investors (Zenios, 2007).

Markowitz (1952) had proposed a widely used method[1] in quantitative portfolio management, the mean-variance optimization. At first, mean-variance analysis could not be able to appeal a great interest, however the financial practitioners have applied this technique after a time. The goal of the mean-variance analysis is to determine the efficient set and to reveal the portfolio structure of optimal portfolios within the framework of risk versus return. Therefore, the technique employed in this analysis is the quadratic programming approach.

Besides, mean-variance analysis has a deep influence in the practical portfolio management since this well-known optimization technique provides a risk-return framework for the most of financial managers in their asset selection procedures. Currently, the investment firms are exercising with the mean-variance optimization theory throughout their optimal portfolio selection procedure.

Estimating the inputs of the mean-variance optimization is of great importance. However, in the literature there are considerable approaches to determine the input parameters before the financial optimization. The main approaches for input estimations are made according to the historical means, variances, and correlation, and to the historical beta estimation using market models (DeFusco et al., 2007).

In the finance literature, there are competing financial asset pricing models since these models can be relevant alternatives in the input estimation for the mean-variance optimization. The Capital Asset Pricing Model *(CAPM)*, a widely used financial model in finance for calculating the cost of capital and portfolio performance, identifies the riskiness of the assets. The CAPM is developed by Sharpe (1964), Lintner (1965) and Mossin (1966) and the model has achieved a considerable popularity in pricing the risky financial assets. In this model, the asset risk is measured by means of beta coefficient *(market beta)* and the risk premium per unit of riskiness is constant across the entire assets. Besides, the CAPM has a linear relationship between expected risk premium of the assets and their market beta that can be defined as systematic risk. And the CAPM affirms that the expected returns on assets may vary since the market beta values of the assets are not similar.

Author would like to thank referees for their suggestions to make this paper more comprehensible. All evaluations and mistakes, if ever, belong to the author.

[1] In 1990, H. Markowitz, M. Miller and W. Sharpe were awarded the Nobel Prize for their frontier studies regarding the quantitative finance.

Besides, there exist some studies which criticize the CAPM approach. Jagannathan and Wang (1996) argued that in many studies, attempting to find out the performance of the CAPM, this method was not entirely explaining the returns on the assets. In this respect, Fama and French (1992) and Fama and French (1996) underlined the critical evidence with regard to the insufficiencies of CAPM, so that they tested CAPM on the basis of return data of assets and observed a non-linear relationship between average return and the beta coefficient.

In further, Fama and French (1993, 1996) had criticized that the CAPM was insufficient to explain the stock returns and they developed a multifactor asset pricing model which is called Three Factor Model. Fama and French (1992) found out that neither beta nor the earnings-to-price ratio give additional explanatory power to a pricing model for security returns which utilizes only the size and BE/ME ratio as the explanatory variables (Fant and Peterson, 1995).

The Three Factor Model is an improved version of conventional static CAPM. The main advantage of these factor models is their simplicity of interpreting the factor values on the basis of time series. However, both the CAPM and the Three Factor Model depended upon the linear regression which can be established among the excess stock returns and a single factor of the set of factors. Fama and French (1993, 1996) argued that the Three-Factor Model is superior to CAPM in explaining the variability of the portfolio or stock returns according to their regression results. Besides, Gaunt (2004) affirms that the Three Factor Model can give more significant results in comparison to CAPM applications.

Daniel and Titman (1997) proposed an alternative financial model to the Fama and French's Three Factor Model which is called the "Characteristics Model". However, Daniel and Titman refused the factor risk assumption and they defined a new book-to-market ratio. According to their approach, apart from the risk the characteristics of the large firms may produce low financial asset return. In their characteristics-based approach, Daniel and Titman (1997) found out that company characteristics for behavioral reasons may result in higher returns for small and value stocks, and they investigated this hypothesis by characteristic-balanced[1] portfolios.

[1] These are zero-investment portfolios with long *(short)* securities with high *(low)* weights on each of the three factors, market excess return, SMB, and HML while having similar stock characteristics in the long and short weights.

Asset pricing theory has a critical role in estimating the returns of financial assets. Single-factor and multi-factor asset pricing models such as CAPM, Three Factor Model and Characteristics Model have become significant in determining the security returns for a given period. Besides, estimating return inputs for mean-variance analysis is a critical factor for the financial optimization process. However, asset pricing models are successful in estimating the mentioned return inputs.

Within this framework, the goal of this study is to analyze the effects of asset pricing models onto the Markowitz's mean-variance optimization. In this respect, it has given the approaches regarding the mean-variance optimization. And then, the results of the optimization studies under asset pricing return estimations and the performance measurements have been evaluated.

1. Literature

1.1. Mean-variance analysis technique in modern portfolio theory. Markowitz's mean-variance optimization has a great deal of importance in determining the optimal portfolio weights. The optimal portfolio weights have significant effects on the financial decision making since they can provide rational investment opportunity set to the investor.

The mean-variance portfolio theory is mainly based upon the notion of measuring the investment opportunities in expected return and variance of asset return. The assumptions of the mean-variance analysis are based on the following issues (DeFusco et al., 2007):

- all investors are risk averse so that they prefer less risk to more for the same level of expected return;
- investors have the information regarding the expected returns, variances and covariances of all assets;
- the investors need only to know the expected returns, variances and the covariances of returns to determine optimal portfolios; and
- there exist no transaction costs or taxes limitation.

According to Zenios (2007), mean-variance analysis is considered as a positive and a normative tool. On this basis, for the positive tool approach, this technique supports the hypotheses regarding how the financial markets and investors behave. The CAPM is the most famous outcome of positive mean-variance analysis. Besides, as a normative tool mean-variance analysis investigates the framework for developing suggestions on how investors should behave. However, the efficient portfolios[1] of the financial assets in mean-variance analysis are regarded as a prosperous application of normative tool for optimization models. The mean-variance analysis supports the decision making of financial engineers.

1.2. Mathematical models of the mean-variance analysis. The main issue for the basic mean-variance optimization models is the optimal proportional allocation "x_i" to the $i^{th.}$ financial asset. The sum of the particular weights of the financial assets in a portfolio corresponds to "1". Within the Markowitz's Mean-Variance analysis, there are two identical optimization formulations regarding the two equivalent statements for the efficient portfolios in the literature. These approaches are the maximization of the portfolio expected return and the minimization of the portfolio risk which is measured by variance of the expected returns (Zenios, 2007; DeFusco et al., 2007; Cornuejols and Tütüncü, 2007).

1.2.1. Maximization of the expected returns of the financial asset. The expected return for a particular portfolio can be expressed as:

$$\sum_{i=1}^{n} x_i = 1, \tag{1}$$

besides, the portfolio has an expected return which is calculated as follows:

$$R(x : \bar{r}) = \sum_{i=1}^{n} \bar{r}_i x_i, \tag{2}$$

however, the risk level of the portfolio or the portfolio variance can be defined as:

$$\sigma^2(x) = \varepsilon\left[\left[R(x : \bar{r}) - R(x : \bar{r})\right]^2\right], \tag{3}$$

$$\sigma^2(x) = \sum_{i=1}^{n}\sum_{i=1}^{n} \sigma_{ij} x_i x_j . \tag{4}$$

In order to achieve a portfolio which has maximum expected return for a given upper bound on risk level, Markowitz (1952) proposed the following optimization model.

$$\textit{Maximize } R(x : \bar{r}) \tag{5}$$

$$\textit{subject to } \sigma^2(x) \le \omega \tag{6}$$

$$\sum_{i=1}^{n} x_i = 1. \tag{7}$$

In this maximization problem, ω corresponds to the upper allowed level of the portfolio risk where there is nonlinear constraint in the model since the variance of a given portfolio has a quadratic function of the vector x.

1.2.2. Minimization of the portfolio risk of the financial assets. According to Zenios (2007), formulating nonlinearity constraint for the optimization problems is not recommended due to the difficulties in solving procedure and then, the variance minimization approach has been advised for the formulation of the optimization.

$$\textit{Minimize } \frac{1}{2}\sigma^2(x), \tag{8}$$

$$\textit{subject to } R(x : \bar{r}) = \mu , \tag{9}$$

$$\sum_{i=1}^{n} x_i = 1. \tag{10}$$

The minimization problem follows a classical quadratic programming of an objective function[2] under linear constraints. Whereas, μ is the desired level of expected return for the financial assets. Large number of variables can be involved in this minimization model. On the other hand, the constraint regarding the expected return may be modified as $R(x : \bar{r}) \ge \mu$ if μ is considered as the lower bound for the portfolio return.

1.2.3. Minimization of the portfolio risk with risk-free asset. The investors may prefer to hold risky securities and risk-free asset[3] in their portfolios since the standard deviation of this asset corresponds to null. As we denote x_f as the weight of the risk-free asset within the portfolio, and by inserting $x_f = 1 - \sum_{i=1}^{n} x_i$ into the variance minimization model given in Equations (8), (9) and (10) then the model may be expressed as follows (Zenios, 2007; DeFusco et al., 2007):

$$\textit{Minimize } \frac{1}{2}\sigma^2(x), \tag{11}$$

$$\textit{subject to } \sum_{i=1}^{n} (\bar{r}_i - r_f) x_i = \mu - r_f . \tag{12}$$

[1] The efficient portfolio can be considered within two dimensions. The portfolio which has maximal expected asset return for a given upper bound on risk or the portfolio that has a minimal risk for a given expected rate of return on assets.

[2] In order to make a simplification in the calculation of derivatives, the scaling factor is taken as ½ within the objective function.

[3] Risk-free asset may be considered as the secure financial assets *(Government's securities)* which do not carry any interest rate risk such as Treasury bills.

Besides, risk-free asset is an effective option in order to minimize the risk level of the portfolio. However, risk-free asset may facilitate to diminish potential residual or can be utilized to finance purchases throughout borrowing.

1.3. Estimating return inputs for mean-variance optimization. The Markowitz's model provides an optimal portfolio assumed that we have perfect information on the expected returns and covariances for the assets under consideration. For this reason, an important practical issue is the estimation of the expected returns and the covariances (Cornuejols and Tütüncü, 2007).

Determining the inputs for the mean-variance optimization has a major importance. There are several approaches regarding the input estimation for this optimization procedure (DeFusco et al., 2007).

It is worthy to examine the economic importance of financial asset pricing models from an investor's portfolio optimization perspective. It is in contrast to the conventional cross-sectional analysis of stock returns that focuses on the comparison of expected returns of the financial assets. Chou et al. (2004) investigated the asset pricing anomalies on the variances and the covariances of the expected returns for Japanese market. Besides, Chan et al. (1999) had focused only on estimating the covariances of the stock returns for the optimization.

1.3.1. Historical estimation of the optimization inputs. The approach involves calculating means, variances, and correlations of the financial asset returns from the historical data perspective. This method needs to estimate a considerable number of parameters as we optimize larger portfolios which contain numerous financial assets. The quantity of estimates needed and the quality of historical estimates of inputs are the main limitations for the historical estimation approach. The required quantity of estimates can easily be in huge number so that, the number of covariances increases in the square of the number of securities (Defusco et al., 2007; Wolf, 2004; Chan et al., 1999).

According to Chan et al. (1999), the second constraint for the historical estimation approach is that historical estimates of portfolio return parameters typically have substantial estimation error and this problem is least severe for the estimates of variances. The problem is due to historical estimation of the mean portfolio returns since the variability of risky asset returns is high in comparison to mean, and this problem is unable to be solved by increasing the observation number. Besides, estimation error is a considerable issue within historical estimates of covariances.

1.3.2. Market model estimations for the optimization inputs. The historical beta estimations are more smoother in computing the variances and covariances of the financial asset returns since these returns are assumed to correlate with a limited set of independent variables or factors. The market models[1] are used to estimate the variances and covariances of the asset returns.

After having linear regression for each financial asset, the intercepts (a_i) and the beta values (β_i) can be estimated in the framework of historical data on asset returns and market returns. However, these estimates are then utilized to compute the expected returns and the variances and covariances of those returns for the mean-variance optimization. Besides, it is beneficial for the investor to select the appropriate market index to represent the financial market in the beta estimation procedure (Defusco et al., 2007).

2. Data and methodology

The aim of this study is to investigate the significance of estimated asset returns of asset pricing models *(CAPM, Three Factor Model, and Characteristics Model)* in comparison to the naive returns onto the Markowitz's mean-variance optimization.

Secondarily, the fund performances of the optimized portfolios are found out via risk-adjusted performance evaluation measures in order to analyze the effects of the factor models on the optimization process. In this respect, the stocks having continuous returns within last 390 weekly period *(January 2nd, 2001- June 30th, 2008)* data in İstanbul Stock Exchange *(ISE)*-National 100, 50 and 30 indices have been selected.

However, daily data were used in the input return estimation studies which were carried out by asset pricing models. The daily returns of the portfolios and securities were evaluated in harmony with the equation given below.

$$R_p = \frac{V_t - V_{t-1}}{V_{t-1}}. \qquad (13)$$

The parameters for the portfolio and security returns are as follows:

R_p = Daily return (%) on portfolio (security) p at t period;

V_t = Daily closing value (YTL) of the portfolio/security p at t period;

[1] Market models may be evaluated as the asset pricing models such as CAPM, Three Factor Model and Characteristics Model.

V_{t-1}= Daily closing value (YTL) of the portfolio/security *p* at *t*-1 period.

The Turkish Government Internal Loan (GIL) Performance Index *(30 day based)* data have been used for the risk free assets.

Empirical study includes three sequential processes. Firstly, the returns of the stocks within the ISE indices with asset pricing factor models have been estimated. In this step, the naive, and factor model returns are determined.

Afterwards, the variance-covariance matrices have been prepared for the naive *(original)* returns and the returns estimated by the factor models. The mean-variance optimization studies are then carried out via the variance-covariance and the time series output data of the mentioned asset pricing models. In the last section, the fund performances of the optimized portfolios according to the naive, and the asset pricing models have been determined by the risk-adjusted performance measurement approaches.

2.1. The market models used in the estimations for optimization inputs. The returns according to factor models have been estimated in order to apply mean-variance optimization procedure on the asset pricing model data. In single and multi-factor asset pricing model tests, the returns of ISE indices' securities have been estimated according to Equation (1) so that, the variance-covariance matrices have been established by the estimated CAPM, Three Factor Model and CAR portfolio returns.

The static CAPM model, introduced by Sharpe (1964), Lintner (1965) and Mossin (1966), has been applied in single factor estimations for pricing the risky financial assets.

$$E(R_i) = R_f + \beta_i\,[R_m - R_f], \tag{14}$$

where R_f – risk-free rate of return (GIL performance index); $E(R_i)$ – expected return of asset *i*; $E(R_m)$ – expected return on the value-weight market portfolio; β_i – beta coefficient of asset *i*.

Initially, in the tests regarding CAPM, the expected returns for each ISE index have been estimated according to Equation (14). However, I have forecasted the beta coefficients ($\hat{\beta}_j$) by single serial regressions. Then, the returns inputs have been calculated according to the estimated CAPM returns.

Besides, Fama and French (1993, 1996) developed the Three Factor Model for estimating the expected returns by multi-factor asset pricing. In this model, expected returns are explained by their sensitivities to the excess market rate of return, the difference between the return on a portfolio of small stocks and the return on a portfolio of large stocks *(SMB)* and the difference between the return on a portfolio of high BE/ME stocks and the return on a portfolio of low BE/ME *(HML)*.

The Three Factor Model has been considered as a notable approach in estimating the returns of the financial assets. The parameters of this model are given below.

$$E(R_i) - R_f = \beta_{im}\,[E(R_m) - R_f] + \beta_{is}\,E(SMB) + + \beta_{ih}\,E(HML), \tag{15}$$

where $E(R_i)$-R_f – expected excess return of the stock over riskless interest rate; $E(R_m)$-R_f – expected excess return of the market over riskless interest rate; *SML* – difference of the returns on small and big stocks; *HML* – difference of the returns on high and low *BE*/*ME* ratio stocks; β_{im} – sensitivity of portfolios' excess return onto the market's excess return; β_{is} – sensitivity of excess return of the portfolio onto the *SMB* returns; β_{ih} – sensitivity of excess return of the portfolio onto the *HML* returns.

And then, in order to establish the Three Factor Model construction, the SMB and HML portfolios have been primarily composed in accordance with the portfolio performing approach developed by Fama and French (1993, 1996). After performing the SML and HML portfolios according to the approach developed by Fama and French (1993, 1996), the rate of returns of these portfolios was calculated as stated in subsection 2.2.

However, in Three Factor Model, the beta coefficients are the slopes that have been estimated by applying the multiple regression of $E(R_{it})$ - R_{ft} on $E(R_{mt})$ - R_{ft} and SML_t and HML_t. In other words, for each ISE index securities, the beta coefficients were estimated by serial regressions pursuant to Equation (15) and the betas $\left(\hat{\beta}_j, \hat{s}_j, \hat{h}_j\right)$ according to Three Factor Model were forecasted.

The Characteristics Model is an alternative financial asset pricing model either for CAPM or the Three Factor Model. On this basis, Daniel et al. (2001) and Daniel and Titman (1997) notify that the Characteristics Model can provide better results to the Three Factor Model in explaining the cross-sectional variations in expected returns.

Daniel and Titman assumed expected returns are a function of the observable, slowly varying firm characteristic which is defined as $\tilde{\theta}_{it}$.

$$E\,(R_{it}) = a + b\;\tilde{\theta}_{it}\,, \tag{16}$$

where $E(R_{it})$ – expected return of the stock *i* for *t* period; *a* – intercept value for the linear regression; *b* – sensitivity of return of the portfolio onto the

Model, the changes in the returns on the stock and in firm characteristics $\tilde{\theta}$ are negatively correlated and there is no direct relation between firm characteristics $\tilde{\theta}$ and loadings on the distressed factors.

To establish the CAR benchmark portfolio, the CAR portfolios have been composed in accordance with the portfolio performing approach proposed by Daniel and Titman (1997) and Daniel et al. (1997). The rate of returns of CAR portfolios were calculated as stated in subsection 2.3.

Besides, after performing the CAR portfolios for each ISE indices' securities, the sensitivity of return of the portfolio onto the returns of the firm characteristics portfolio *(factor b)* has been estimated by serial regressions pursuant to Equation (16) for the Characteristics Model. The rate of returns of the CAR portfolios have been estimated in accordance with Daniel and Titman (1997) and Daniel et al.. (1997).

2.2. Composing the benchmark portfolios for the Three Factor Model. The Three Factor Model portfolios have been composed according to the approach proposed by Fama and French (1993, 1996).

2.2.1. Portfolios composed according to the market values (SMB) for Three Factor Model. The *SMB* portfolios in ISE are composed primarily for the subsequent multiple regression analyzes. In this respect, the securities having the largest market equity *(ME)* in ISE were selected on the basis of portfolio size and then, these assets were classified in the four different subsequent portfolios within the criterion as explained below.

- *B* composed of 23 security that has *ME* > 1,000 million *YTL*;
- BS_1 composed of the first 12 security that has 1,000 > *ME* > 580 million *YTL*;
- BS_2 composed of the last 12 security that has 1,000 > ME > 580 million *YTL*;
- *S* composed of 24 security that has *ME* < 580 million *YTL*;

Thereafter, the daily returns of the SMB portfolio were calculated as:

$$SMB = ½(S+BS_2)-½(BS_1+B). \quad (17)$$

In other words, the returns of *SMB* portfolio were determined by subtracting the average returns of the securities that have small *ME*'s from the securities having big *ME*'s.

2.2.2. Portfolios composed according to the market value/book value (HML) for Three Factor Model. Another input for multiple regression operation is the daily HML portfolio returns. In this case, the securities that have highest *BE/ME* ratio were selected and then separated into four subsequent portfolios according to the criterion as given:

- *H* composed of 18 security that has *BE/ME* > 1.9;
- HL_1 composed of the first 18 security that has 1.9 > *BE/ME* > 0.6;
- HL_2 composed of the last 18 security that has 1.9 > *BE/ME* > 0.6;
- *L* composed of 17 security that has *BE/ME* < 0.6;

Besides, the daily returns of HML portfolios were determined according to the following equation.

$$HML = ½(H+HL_1) - ½ (HL_2 + L). \quad (18)$$

On the other side, the *HML* portfolio returns have been calculated as deducting the average returns of the securities that had high *BE/ME* ratios from the securities having small *BE/ME* ratios.

2.3. Composing the benchmark model for the Characteristics Model. The Characteristics Model (CAR) portfolios have been performed with respect to the approach developed by Daniel and Titman (1997) and Daniel et al.. (1997).

2.3.1. Portfolios composed according to the size values and book-to-market ratios for Characteristics Model. The CAR portfolios in ISE indices were performed to carry out the multiple regression analyzes. In this respect, a sequential process has been followed. Initially, I made a classification according to the size of the securities and then separated the portfolios into three sub-portfolios. Afterwards, nine sub-portfolios have been selected in respect to *BE/ME* ratios of the securities.

Finally, these nine portfolios are divided into 27 sub-portfolios according to the return performances for last 12 months. Finally, after having completed the mentioned grouping operation, the value-weighted returns for the subsequent 12 months for each of 27 sub-portfolios have been calculated. And then, Characteristics portfolios have been performed and the expected returns for a particular stock in a certain time have been calculated as the average of the last 390 weeks' returns of Characteristics portfolio to which this security has involved.

2.4. Optimization and performance measurement. In the optimization studies, the conventional mean-variance optimization approach developed by Markowitz (1952) both for the risky portfolios and risky portfolios with risk free assets has been followed. The data for the optimization are gathered by the estimated returns of the CAPM, the Three Factor Model and the Characteristics Model. Besides, the naive returns of the financial assets have also been considered within the optimization data.

The naive returns and the estimated returns of the mentioned factor models have been used in order to construct variance-covariance matrix for each ISE index. On this data set, the minimized portfolio variances by mean-variance optimization technique have been calculated pursuant to Equation (8), (9), (10), (11) and (12). In this way, the efficient sets for each model have been drawn. In order to analyze the performance of the optimized portfolios, the risk-adjusted performance measurement approaches have been applied.

3. Findings

The input estimation for the mean-variance optimization depends upon the different market models. However, the historical betas have been estimated using static CAPM as single index model as well as Three Factor Model and Characteristics Model as multiple index market models. The variance and covariance matrices of asset returns have been computed according to each market model so that, the mean-variance optimizations were applied upon these input data including the naive returns of the assets.

The findings regarding the optimization studies for the risky portfolios and risky portfolios with riskless assets were given. Subsequently, the performance measurement scores were calculated for the optimized portfolios.

3.1. The optimization results of the risky portfolios. Figure 1 illustrates the optimization results for ISE-100 index.

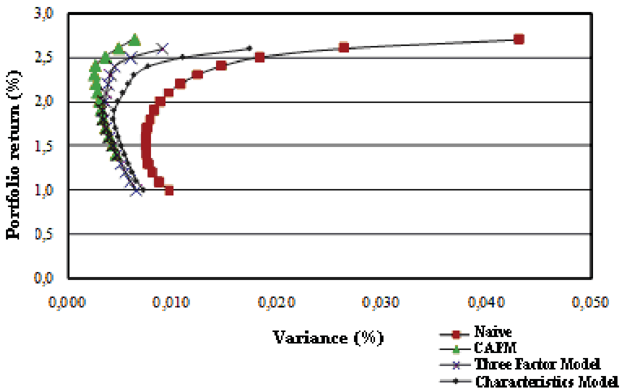

Fig. 1. Efficient sets for ISE-100 index in 2001-2008 (2nd quarter) period

Figure 1 points out that the efficient sets determined by the mean-variance optimizations of the naive and factor models have different characteristics. However, the efficient set of the naive portfolio has the highest portfolio variance as we compare with the factor models. The lowest variance value for the efficient set belongs to CAPM.

Three Factor Model and Characteristics Model have higher portfolio variances than CAPM. Besides, lower portfolio variances within all factor models in comparison to portfolios with naive returns have been determined. Besides, the factor models give parallel risk structure, since it has determined a sharp increase in risk above ~2.5% return level. Figure 2 shows the efficient sets for the optimization of ISE-50 index.

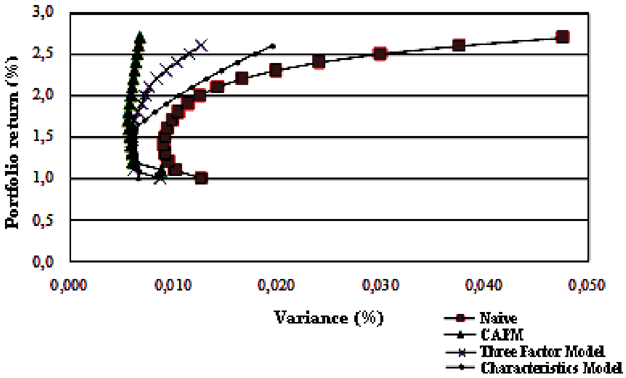

Fig. 2. Efficient sets for ISE-50 index in 2001-2008 (2nd quarter) period

In Figure 2, the variance level for the efficient set of the naive portfolio has the highest portfolio variance in comparison to the factor models. The CAPM estimates reveal smaller portfolio variances than Three Factor Model and Characteristics Model.

On the other side, the efficient sets of Three Factor Model, Characteristics Model and naive portfolio have the similar structure since their portfolio variance level decreases to a minimum point as the returns increase *(above ~1.5%)* whereas these variance values increase afterwards. However, on the contrary, the variance level of the efficient set of the CAPM returns are quite stable and do not increase in significant manner.

Figure 3 illustrates the efficient sets for the optimization of ISE-30 index as risky portfolios.

As can be seen in Figure 3, exactly the same efficient sets for the CAPM and the Characteristics Model for ISE-30 index have been determined. The naive portfolio performs a sharp increase in portfolio risk above ~1.0% return level. The CAPM, Characteristics and Three Factor Model have the similar efficient set up to ~1.6%, however above this level the portfolio risk for the Three Factor Model increases significantly.

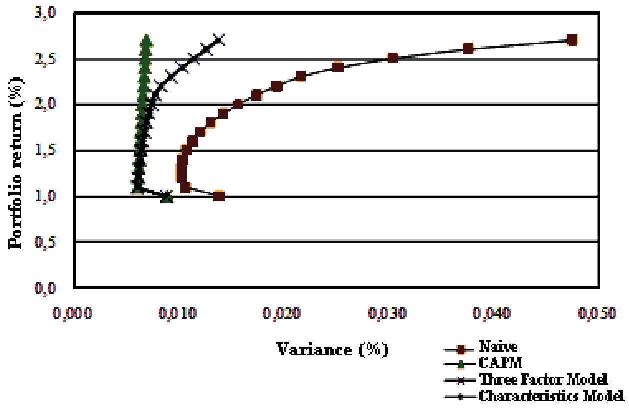

Fig. 3. Efficient sets for ISE-30 index in 2001-2008 (2nd quarter) period

3.2. The optimization results of the risky portfolios with risk-free asset. Figure 4 illustrates the optimization results for ISE-100 index with risk-free asset.

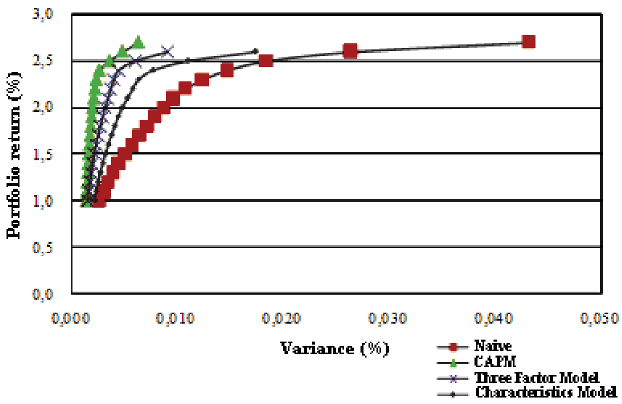

Fig. 4. Efficient sets for ISE-100 index with risk-free asset in 2001-2008 (2nd quarter) period

Figure 4 shows that the portfolio risk levels for factor models and naive portfolio have diminished due to the involvement of the risk-free asset into the asset mix. The variance level of CAPM is smaller than Three Factor Model, Characteristics Model and the naive returns.

However, the factor models demonstrate a parallel risk structure and above 2.5% return level a significant increase in portfolio variance has been determined. Figure 5 demonstrates the efficient sets for the optimization of ISE-50 index with risky assets.

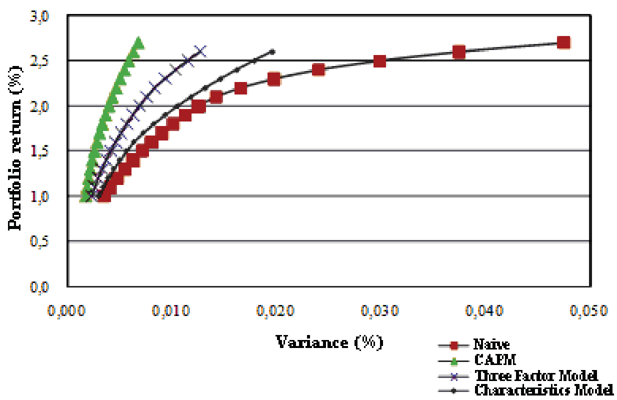

Fig. 5. Efficient sets for ISE-50 index with risk-free asset in 2001- 2008 (2nd quarter) period

Figure 5 demonstrates that all portfolio risks have been stabilized and diminished since the risk-free asset was inserted into the asset mix. However, the portfolio risk of the efficient set for the CAPM is smaller than the Three Factor Model, the Characteristics Model and the naive returns.

Besides, the factor models follow a parallel structure in terms of risk increase up to %2.5 return level, a significant increase in portfolio variance has been determined. The optimization results under ISE-30 index with risk-free asset are given in Figure 6.

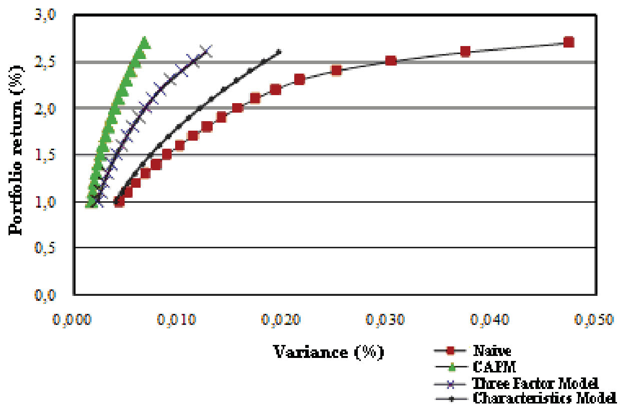

Fig. 6. Efficient sets for ISE-30 index with risk-free in 2001-2008 (2nd quarter) period

In Figure 6, the portfolio risk levels have been stabilized and diminished due to the risk-free asset. The portfolio variance of the efficient set for the CAPM is smaller than Three Factor Model, Characteristics Model and the naive returns. The risk level increases dramatically above ~1.5% return level not only for the factor models but also for the naive portfolio.

3.3. The performance measurement results of the optimized risky portfolios. The results regarding the risk-adjusted performance appraisal measures of the optimized risky portfolios for the ISE indices are illustrated in Table 1.

Table 1. The results of the performance measurement for optimized risky portfolios in 2001-2008 (2nd quarter) period

		Portfolio performance scores		
Portfolio	Model	Sharpe ratio	Treynor measure	Jensen's alpha
ISE - 100	Naive	-0.0265	-0.0005	-0.0001
	CAPM	5.2456	0.0545	0.0225
	Three Factor Model	3.3587	0.0368	0.0184
	Characteristics Model	3.3962	0.0562	0.0173
ISE - 50	Naïve	0.1066	0.0022	0.0009
	CAPM	2.1920	0.0226	0.0153
	Three Factor Model	1.9527	0.0196	0.0132
	Characteristics Model	2.1695	0.0251	0.0142

Table 1 (cont.). The results of the performance measurement for optimized risky portfolios in 2001-2008 (2nd quarter) period

		Portfolio performance scores		
Portfolio	Model	Sharpe ratio	Treynor measure	Jensen's alpha
ISE - 30	Naive	-0.0277	-0.0006	-0.0002
	CAPM	1.3967	0.0144	0.0102
	Three Factor Model	1.3920	0.0144	0.0102
	Characteristics Model	1.4143	0.0178	0.0102

In Table 1, the optimized CAPM portfolios have the highest performances for ISE-100 and ISE-50 according to the Sharpe Ratio, whereas Characteristics Model is superior to other models for ISE-30 portfolio.

In terms of Treynor measure, Characteristics Model demonstrates high performance for each portfolio. On the other hand, in the framework of Jensen's alpha figures, the CAPM portfolios have demonstrated higher performances in comparison to the other asset pricing models. However, CAPM, Characteristics and Three Factor Models have the same Jensen's alpha value for ISE-30.

3.4. The performance measurement results of the optimized risky portfolios with risk-free asset. The results for the risk-adjusted performance appraisal measures of the optimized risky portfolios with risk-free asset are illustrated in Table 2.

Table 2. The results of the performance measurement for optimized risky portfolios with risk-free assets in 2001-2008 (2nd quarter) period

		Portfolio performance scores		
Portfolio	Model	Sharpe ratio	Treynor measure	Jensen's alpha
ISE - 100	Naive	0.0107	0.2302	0.0001
	CAPM	5.4844	0.1654	0.0097
	Three Factor Model	3.6308	0.2476	0.0096
	Characteristics Model	3.3925	0.1489	0.0096
ISE - 50	Naive	0.1461	0.2108	0.0008
	CAPM	3.3590	0.2717	0.0096
	Three Factor Model	2.6783	0.3501	0.0095
	Characteristics Model	2.3372	0.3660	0.0094
ISE - 30	Naive	0.0628	0.1988	0.0003
	CAPM	3.3590	0.2717	0.0096
	Three Factor Model	2.6784	0.3501	0.0095
	Characteristics Model	2.2219	0.1373	0.0094

In Table 2, the CAPM has the best performance for ISE-100 and ISE-50 on the basis of Sharpe Ratio. According to Treynor measure, Three Factor Model demonstrates higher performance for each portfolio. Besides, in terms of Jensen's alpha figures, the CAPM has higher performance than the other models. However, CAPM, Characteristics and Three Factor Models have the similar Jensen's alpha value for each ISE portfolio.

Discussion and conclusion

This paper investigates the significance of the single and multi-factor asset pricing models on the Markowitz's (1952) well-known mean-variance optimization technique for the different basic Turkish benchmark indices. In order to implement Markowitz's mean-variance optimization, the stock returns which were considered as inputs, have been estimated by CAPM, Three Factor Model and Characteristics Model. The variance-covariance matrices have been established according to the results of the multi factor model estimations for the Turkish asset prices.

The optimization results have revealed that the risky assets constructed from ISE indices have higher portfolio risks when compared to risky portfolios with risk-free assets. It has been evaluated that the risk-free assets have assisted to neutralize the total portfolio risk. Besides, either in the risky portfolios or in the risky portfolios with risk-free assets, similar risk ranking for the naive returns and the single and multi-factor asset pricing models has been realized.

After implementing the optimization procedures for both portfolios, it has been found out that the portfolio risk would increase as in the following sequence: the CAPM, the Three Factor Model, the Characteristics Model and the naive model. In other words, inputs estimated by the CAPM could give lower portfolio variances than the mentioned multi-factor asset pricing models.

For the risky portfolios, significant increases in the portfolio risk above ~2.5%, ~1.5% and ~1% return levels for ISE-100, ISE-50 and ISE-30 indices have been determined respectively. Besides, distinctive portfolio risk increases above ~2.5% return level for ISE-100 and above ~2.0% return level for ISE-50 and ISE-30 indices have been recorded for the risky portfolios with risk-free assets.

Moreover, as the number of the financial assets in the portfolio increases, the mean-variance optimization could give more significant results and better mean-variance frontier since it is able to achieve smaller portfolio risk with the diversification.

As we evaluate the portfolio performances for the optimization procedure, the optimized CAPM portfolio could demonstrate higher performance than the optimized portfolios of the multi-factor models. Besides, it has also been realized that as the assets number in the portfolio increases, the portfolio performance could reach higher values. We may also state that the performances of Three Factor Model and Characteristics Model are found similar for each ISE indices. Further, the naive returns have given higher portfolio risks during the mean-variance optimization and the optimized naive portfolio has also exposed worse portfolio performances.

The main finding of the empirical study is the mentioned asset pricing models that have significant role in the Markowitz's mean variance optimization technique since they can provide higher portfolio performances than the optimized portfolios of naive returns. In other words, estimating the expected returns and the covariances of the financial assets by means of single and multi-factor asset pricing models have substantial importance within the financial optimization process.

Finally, according to the optimization results and portfolio performance figures for the basic ISE indices, we may conclude that the asset pricing models have a positive impact on the mean-variance optimization process. In this framework, we not only might diminish the portfolio risk levels and reach better mean-variance frontiers but also raise the portfolio performances by applying the mentioned asset pricing models in the Turkish stock markets.

References

1. Arshanapalli, B., Coggin, T.D., Doukas, J., 1998, "Multifactor Asset Pricing Analysis of International Value Investment Strategies", *Journal of Portfolio Management,* Summer 10-23.
2. Chan, L.K.L., Karceski, J., Lakonishoki J., 1999, "On Portfolio Optimization: Forecasting Covariances and Choosing the Risk Model", *Review of Financial Studies*, Vol. 12, 937-974.
3. Chou, P., Li, W, Zhou, G., 2004, "Portfolio Optimization under Asset Pricing Anomalies", *Japan and the World Economy*, Vol. 18, 121-142.
4. Cornuejols, G., Tütüncü, R., 2007, Optimization Methods in Finance, Cambridge University Press, 1st. Eddition 138-168, UK.
5. Daniel, K., Grinblatt, M., Titman, S., Wermers, R., 1997, "Measuring Fund Performance with Characteristic-Based Benchmarks", *The Journal of Finance*, Vol. LII, No.1, 1035-1058.
6. Daniel, K., Titman, S., 1997, "Evidence on the Characteristics of the Cross Sectional Variation in Stock Returns", *The Journal of Finance*, Vol. LII, No.1, 1-33.
7. Daniel, K., Titman, S., 1997, "Evidence on the Characteristics of Cross Sectional Variation in Stock Returns", *Journal of Finance,* 52: 1-31.
8. Daniel, K., Titman, S., Wei, J., 2001, "Explaining the Stock Returns in Japan: Factors or Characteristics?", *The Journal of Finance*, Vol. LVI, No. 2, 743-766.
9. Davis, J., Fama, E., French, K., 2000, "Characteristics, Covariances, and Average Returns: 1929 to 1997", *The Journal of Finance*, Vol. LV, No.1, 389-406.
10. Davis, J.L., Fama, E.F., French, K.R., 2000, "Characteristics and Average Returns: 1929 to 1997", *Journal of Finance*, 56: 389-405.
11. Defusco, R.A., McLeavey, D.W., Pinto, J.E., Runkle, D.E., Quantitative Investment Analysis, 2nd Eddition, CFA Investment Series, John Wiley and Sons Inc., 429-510, UK.
12. Fama, E.F., French, K.R., 1992, "The Cross-Section of Expected Stocks Returns", *Journal of Finance*, 47: 427-465.
13. Fama, E.F., French, K.R., 1993, "Common Risk Factors in the Returns on Stocks and Bonds", *Journal of Financial Economics*, 33:3-56.
14. Fama, E.F., French, K.R., 1995, "Size and Book-to-Market Factors in Earnings and Returns", *Journal of Finance*, 50 /1: 131-155.
15. Fama, E.F., French, K.R., 1996, "Multifactor Explanations of Asset-Pricing Anomalies", *Journal of Finance*, 51/1: 55-84.
16. Fama, E.F., French, K.R., 1997, "Industry Costs of Equity", *Journal of Financial Economics*, 43:153-193.
17. Fant, L.F., Peterson, D.R., 1995, "The Effect of Size, Book-to-Market Equity, Prior Returns, and Beta on Stock Returns: January versus the Remainder of the Year", *Journal of Financial Research*, 18/2: 129-142.
18. Gaunt, C., 2004, "Size and Book to Market Effects and the Fama-French Three Factor Asset Pricing Model: Evidence from the Australian Stock Market", *Accounting and Finance*, 44: 27-44.
19. Jackson, M., Staunton, M., 2001, Advanced Modelling in Finance, John Wiley&Sons, 103-153, UK.
20. Jagannathan, R., Wang, Z., 1996, "The Conditional CAPM and the Cross Section of Expected Returns", *Journal of Finance*, Vol. 11, No. 1, 3-6.
21. Jarjir, S.L., 2007, "Explaining the Cross-section of Stock Returns in France: Characteristics or Risk Factors?", *The European Journal of Finance*, Vol. 13, No. 2, 145-158.
22. Lam, K., 2005, "Is the Fama-French Three-Factor Model Better Than CAPM?", *MA Project* (Canada: Simon Fraser University).
23. Lee, E., Liu, W., Strong, N., 2007, "UK Evidence on the Characteristics versus Covariance Debate", *European Financial Management*, Vol. 13, No. 4, 742-756.

24. Lewellen, J., 1999, "The Time Series Relations among Expected Return, Risk, and Book to Market", *The Journal of Financial Economics*, Vol. 54, 5-43.
25. Maggin, J.L., Tuttle, D.L., Pinto, J.E., McLeavey, D.W., 2007, Managing Investment Portfolios: A Dynamic Process, 3rd Eddition, CFA Investment Series, John Wiley&Sons Inc., 257-295, 717-769, UK.
26. Markowitz, H., 1952, "Portfolio Selection", *Journal of Finance*, Vol. 48, 77-91.
27. Mossin, J., 1966, "Equilibrium in a Capital Market", Econometrica, 34,768-783.
28. Sharpe, W.F., 1964, "Capital Asset Prices: A Theory of Market Equilibrium Under Condition of Risk", *Journal of Finance*, Vol. 19, 425-442.
29. Shefrin, H., Statman, M., 1995, "Making Sense of Beta, Size, and Book-to-Market", *Journal of Portfolio Management*, Winter: 26-33.
30. Wang, K., 2001, "Multifactor Model of Growth and Z Score for Projecting Stock Return and Evaluating Risk", *SSRN Working Paper*.
31. Zenios, S.T., 2007, Practical Financial Optimization: Decision Making for Financial Engineers, 1st Eddition, BlackWell Publishing, 73-93, UK.