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IS THERE A NON-LINEAR RELATIONSHIP BETWEEN BRANCH NETWORK SIZE AND BANK PROFITABILITY? EVIDENCE FROM VIETNAMESE COMMERCIAL BANKS

Abstract

Vietnamese commercial banks enlarged their physical distribution networks throughout the past two decades, at a time when banks in most developed markets were closing outlets. How large a network can become before additional outlets stop paying for themselves has not been settled, partly because existing studies infer non-linearity from the sign of a squared term instead of testing it. This study aims to examine whether a non-linear relationship exists between branch network size and bank profitability in Vietnamese commercial banks and to identify the corresponding turning points. The analysis applies a two-step system generalized method of moments estimator to an unbalanced panel of 28 commercial banks observed from 2008 to 2024 and tests the shape of the estimated relationship formally, requiring the turning point to fall inside the observed range and the slope to change sign across it. Profitability rises with network size up to a threshold and declines beyond it, and monotonicity is rejected at the 10 percent level in every specification. Measured by total transaction points, the threshold is 172 outlets when profitability is assessed against assets and 261 outlets when it is assessed against equity, a gap consistent with the network raising equity returns partly through leverage. About 51 percent of bank-year observations already lie beyond the asset threshold. A bank that judges its expansion capacity by return on equity alone will therefore read it too generously, and slightly more than half of the networks observed here have passed the size at which further outlets add to profitability.

Keywords

bank, branch, profitability, non-linearity, threshold, Vietnam

JEL Classification

G21, L25, O33

INTRODUCTION

Physical distribution networks remain the largest fixed asset that a commercial bank controls, and decisions about their size commit resources for years. A branch or a transaction office requires premises, staff, security arrangements and compliance capacity, and these costs are incurred whether or not the outlet attracts enough deposits and loans to cover them. In banking systems where deposit gathering still depends on face-to-face contact, the network is also the channel through which a bank builds its funding base. The size of the network therefore sits at the center of a trade-off that every bank management must resolve.

Vietnam offers an unusual setting for examining that trade-off. Between 2008 and 2024 the commercial banks in our sample expanded from a median of 88 transaction points in 2008 to a median of 191 in 2024, with the largest networks exceeding 1,100 outlets. Over a comparable period, 2003 to 2020, banks in the Netherlands, Lithuania and the United Kingdom reduced their branch counts by more than



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40% (Murinde et al., 2022). The Vietnamese banking system moved in the opposite direction to the systems that most of the existing evidence describes, and it did so while the number of financial technology firms operating in the country rose from 16 to 137.

The scientific problem addressed here concerns the functional form of the relationship between network size and profitability, and the consequences of getting that form wrong. If the relationship is monotonically increasing, then expansion remains a sound use of capital and the question of an optimal size does not arise. If instead the relationship rises and then falls, there exists a network size beyond which each additional outlet is associated with lower returns, and banks operating past that point are allocating capital to assets that reduce their profitability. These two possibilities carry opposite implications for bank strategy and for the supervisory treatment of network expansion plans, yet they cannot be distinguished without an explicit test of the shape of the relationship.

Resolving the question is harder than it appears. A quadratic specification with a negative squared term is frequently read as evidence of an inverted U, but a negative squared term is compatible with a relationship that declines monotonically over the entire range of the data, or with one whose maximum lies far outside that range. Establishing that a turning point exists inside the observed data requires the slope at the lower bound of the data to be positive and the slope at the upper bound to be negative, and both conditions must be tested jointly. Without that test, an estimated turning point cannot be interpreted as a profitability-maximizing network size, and the policy conclusion drawn from it may point in the wrong direction.

A second difficulty concerns measurement. Vietnamese banks operate two distinct types of outlets: full branches, which carry the wider set of permissions, and transaction offices, which are smaller and more numerous. Studies that aggregate the two into a single count implicitly assume that the profitability consequences of adding a branch and adding a transaction office are the same. Whether the two behave alike is worth testing, but the test carries a limitation that this literature has not stated. The two counts sum to the aggregate by construction, so a regression on one is close to a rescaling of a regression on the other, and what such regressions identify is the scale of the network rather than the relative contribution of each outlet type.

1. LITERATURE REVIEW AND HYPOTHESIS

The theoretical treatment of bank branch networks rests on two forces that operate in opposite directions as a network grows. On the cost side, a larger network spreads fixed overheads over a wider deposit and lending base, and this scale effect reduces the average cost of intermediation. Zardkoohi and Kolari (1994) documented declining operating costs per branch among Finnish savings banks, and found that most of the available savings were realized within the first five outlets. On the revenue side, a denser network shortens the distance between the bank and its depositors, which supports deposit gathering and relationship lending. Petersen and Rajan (2002) showed that distance between lender and borrower shapes the information available to the lender, although they

also documented that improvements in information technology have loosened that constraint over time.

The second force works against expansion. As a network extends geographically, the distance between headquarters and peripheral outlets grows, and managerial oversight becomes harder to exercise. Berger and DeYoung (2001) found that the efficiency of a bank declines with the distance between its headquarters and its affiliates, and that the decline is steeper for banks with weaker managerial capacity. Deng and Elyasiani (2008) reported that geographic diversification raised bank holding company value by 7.55% on average, while greater headquarters-to-branch distance reduced value by 11.11% and raised risk by 3.08%. Goetz et al. (2016) used a gravity-deregulation instrument and found that geographic expansion reduces

risk through portfolio diversification rather than through improved loan quality. Taken together, these results describe a setting in which the benefits of expansion accrue early and the costs accumulate later, which is the structure that generates a concave relationship.

Empirical work on the size of the relationship has produced estimates that vary considerably across banking systems. Berger et al. (1997) estimated that roughly 80% of United States bank branches operated below efficient scale, and that banks maintained about twice the number of outlets that would minimize costs. Hirtle (2007) found that mid-sized networks in the United States, those with 101 to 500 branches, recorded the lowest deposits per branch relative to both smaller and larger networks. Kondo (2018) reported that Japanese regional banks with more than 99 branches experienced declining returns on assets and on equity and located the best-performing group at 50 to 99 branches. Kusi et al. (2022) applied dynamic panel methods to 35 Ghanaian banks and found an inverted U-shaped effect of branches on financial stability, with turning points at 191 branches in a model built around the deposit-intermediation channel and 173 branches in a model built around the loan-intermediation channel. Hossain et al. (2021) identified a level of branching in Bangladesh beyond which further outlets no longer improved the performance of small and medium enterprises. Bernini and Brighi (2018) found that branch expansion raised cost inefficiency among Italian cooperative banks, and that functional distance between outlets and headquarters worsened the effect.

The dispersion of these estimates, from roughly 50 outlets in Japan to nearly 200 in Ghana, indicates that the turning point is not a universal constant but a feature of the banking system being studied. It also indicates that estimates obtained in one system transfer poorly to another, which limits what Vietnamese banks can infer from the international evidence.

A methodological concern runs through much of this literature. The standard practice of reading an inverted U from the sign and significance of a squared term is not sufficient, because that pattern is also produced by a relationship that declines throughout the observed range. Lind and Mehlum

(2010) derived the appropriate test, which requires that the slope at the lower bound of the data be positive, that the slope at the upper bound be negative, and that both conditions hold jointly. Haans et al. (2016) surveyed strategy research and found that a large share of published inverted U claims did not satisfy these conditions. Simonsohn (2018) reached a similar conclusion and proposed an alternative procedure based on fitting two separate lines. Studies of bank branch networks have rarely applied any of these tests, so the evidence for an interior optimum is weaker than the volume of published turning points suggests.

Evidence from Vietnam is thin and points in conflicting directions. Pham et al. (2022) examined 22 large Vietnamese commercial banks over 2012 to 2019 using a random effects model. They reported a positive overall association between branch count and return on equity, together with a negative and significant squared branch term, while subgroup results indicated a positive branch effect for medium-network banks and a negative effect for the largest-network banks. However, the implied turning point was not tested formally. Le (2021) applied system GMM to the relationship between geographic expansion and bank stability, found that expansion improves stability, and detected no significant quadratic effect. Le et al. (2020) reported that medium-sized Vietnamese banks were more efficient than either large or small banks, and that expansion exceeding a bank's controllable capacity was associated with deteriorating performance. Batten and Vo (2019) found bank size to be negatively associated with returns on assets and with net interest margins in Vietnam, which they attributed to rapid expansion pursued ahead of profitability.

A second strand concerns banks' competitive position relative to financial technology firms, which matters here because the value of a physical network depends on whether customers still use it. The substitution view holds that these firms divert customers from branch-based channels and compress bank margins (Boot et al., 2021; Buchak et al., 2018; Thakor, 2020). Empirical support comes from Phan et al. (2020) for Indonesia and Ben Naceur et al. (2025) for a cross-country sample. The complementarity view holds that the relationship can raise bank efficiency and expand the mar-

ket, and finds support in Ky et al. (2024) for mobile money partnerships in East Africa and in Li et al. (2017) for United States retail banks. Nguyen (2019) provided a distinct piece of evidence by showing that branch closings in the United States produced persistent declines in local small business lending, which indicates that physical outlets retain functions that digital channels have not replaced.

Three conclusions follow from this body of work. First, theory predicts a concave relationship between network size and profitability, but the published turning points differ by a factor of four across banking systems, so the Vietnamese optimum cannot be inferred from evidence gathered elsewhere. Second, existing Vietnamese studies impose either a linear specification or a quadratic one without testing whether the implied turning point lies inside the observed data, so they do not establish that an interior optimum exists. Third, studies of bank branch networks report a single outlet count without stating what that count contains. Vietnamese banks operate two types of outlets, full branches and transaction offices, which differ in cost structure and in the range of services they may provide and which sum to the aggregate. Whether the estimated relationship is sensitive to which of the three counts is used has not been examined.

This study aims to examine whether a non-linear relationship exists between branch network size and bank profitability in Vietnamese commercial banks and to identify the corresponding turning points.

The following hypothesis is tested:

H1: The relationship between the size of a bank's transaction network and its profitability is inverted U-shaped, with a turning point located inside the observed range of network sizes.

2. METHODOLOGY

The analysis uses an unbalanced panel of 28 Vietnamese commercial banks observed annually from 2008 to 2024, with the earliest year used to form the initial lag so that the estimation sample covers 2009 to 2024. Bank-level financial data were

obtained from FiinPro-X, a commercial database, and cross-checked against the audited annual financial statements of each bank. Network data, comprising the number of full branches and the number of transaction offices held by each bank at each year-end, were compiled from each bank's annual reports. Macroeconomic series were taken from the World Development Indicators of the World Bank. The count of financial technology firms operating in Vietnam was compiled from industry reports. Depending on the specification, the estimation sample comprises 389 to 392 bank-year observations; 389 of these enter all six specifications and are the basis of the descriptive statistics reported below. Agribank is excluded because it is wholly state-owned and operates under an agricultural and rural development mandate, so the size of its network is determined by policy assignment rather than by profitability considerations.

Profitability is measured by return on assets and by return on equity, both entered in levels. Network size is measured in three ways: the total number of transaction points, defined as full branches plus transaction offices; the number of full branches alone; and the number of transaction offices alone. Each measure is entered as a natural logarithm and is centered on its sample mean, so that the estimated turning point can be converted back to a number of outlets by exponentiating the sum of the turning point and the centering constant. Centering does not alter the coefficient on the squared term or the outcome of the shape test, and we do not claim that it addresses collinearity, which Echambadi and Hess (2007) showed it does not. The squared term of each centered measure captures the curvature of the relationship.

The three measures are not independent of one another, and we report them as such. Total transaction points equal branches plus transaction offices by construction, and transaction offices make up a median 77% of the total with a standard deviation of 8 percentage points, so the logarithm of the total and the logarithm of transaction offices correlate at 0.99. Branches correlate with the total at 0.94 and therefore carry the most independent variation of the three. The three specifications should accordingly be read as one relationship estimated at three scalings of the same network, not as three independent tests of it.

The control variables follow the bank profitability literature (Athanasoglou et al., 2008; Dietrich & Wanzenried, 2014). Credit risk is the logarithm of the ratio of non-performing loans to total loans. Intermediation intensity is the logarithm of the loan-to-deposit ratio. Funding cost advantage is the logarithm of the ratio of current and savings account balances to total deposits. Two macroeconomic controls, the logarithms of gross domestic product growth and of the inflation rate, absorb common conditions. Competitive pressure from financial technology is the logarithm of the number of such firms operating in the market.

Bank profitability persists over time, because competitive advantages, regulatory barriers and relationship capital carry across periods. A static specification that omits the lagged dependent variable is therefore misspecified, while including it in a fixed effects specification produces the bias described by Nickell (1981). The model is estimated as a dynamic panel:

$$\begin{aligned} Performance_{i,t} = & \alpha_0 + \alpha_1 \cdot Performance_{i,t-1} \\ & + \beta_1 \cdot NET_{i,t} + \beta_2 \cdot NET_{i,t}^2 + \gamma' \cdot X_{i,t} + \eta_i + \varepsilon_{i,t}, \end{aligned} \quad (1)$$

where *NET* denotes one of the three centered network measures, *X* is the vector of controls, η_i captures time-invariant bank characteristics, and $\varepsilon_{i,t}$ is the idiosyncratic error.

Estimation uses the two-step system generalized method of moments estimator of Arellano and Bover (1995) and Blundell and Bond (1998), with the finite-sample correction of Windmeijer (2005). Following standard practice for dynamic panel estimation, the lagged dependent variable is instrumented by its own deeper lags, with the depth set separately for each dependent variable so that the Arellano-Bond conditions are satisfied. The same GMM-style treatment is applied to the network terms, credit risk, and the financial technology measure, while the loan-to-deposit ratio, funding cost advantage, and macroeconomic controls enter as IV-style instruments. All instrument sets are collapsed to limit their proliferation, following Roodman (2009). Specification validity is assessed through the Arellano-Bond tests for first-order and second-order serial correlation in the differenced residuals, which require rejection of the null at first

order and non-rejection at second order, through the Hansen test of overidentifying restrictions, and through the requirement that the instrument count remain below the number of cross-sectional units. Full year effects are not included in the preferred specification because the panel contains only 28 banks and 15 estimation years. Adding a complete set of annual indicators to a system-GMM model that also instruments the lagged dependent variable and the network terms brings the instrument count close to the cross-sectional dimension, increasing the risk of instrument proliferation and weakening the informativeness of the Hansen test. We therefore use a parsimonious set of observed common-time controls and assess sensitivity to alternative controls for secular and economy-wide variation. This choice involves a trade-off: the preferred specification preserves a restricted instrument set but cannot absorb every unobserved year-specific shock. Accordingly, the estimates are interpreted as conditional associations rather than causal effects.

The shape of the estimated relationship is tested by the procedure of Lind and Mehlum (2010), which builds on Sasabuchi (1980). The procedure evaluates the estimated slope at the lower and the upper bound of the observed data and tests the joint null that the relationship is either monotonic or U-shaped against the alternative that it is inverted U-shaped. An inverted U requires a positive slope at the lower bound, a negative slope at the upper bound, and an extremum located between the two bounds. The test statistic is the smaller of the two one-sided t-statistics. We report the extremum, both bound slopes, the p-value of the joint test, and the turning point converted into a number of outlets.

The analytical dataset used for the empirical tests in this study was assembled specifically for this research and has not been used in any previously published work.

3. RESULTS

Table 1 reports the descriptive statistics of the estimation sample.

Table 1 shows how wide the variation in network size is within this sample. The smallest network comprises 20 transaction points and the largest

Table 1. Descriptive statistics for the estimation sample, 28 Vietnamese commercial banks, 2009–2024

Variable	Obs.	Mean	Std. dev.	Min.	P25	Median	P75	Max.
Return on assets	389	0.011	0.008	−0.048	0.005	0.009	0.016	0.056
Return on equity	389	0.120	0.092	−0.917	0.062	0.114	0.180	0.303
Transaction points	389	267.9	259.8	20	98	175	311	1,152
Branches	389	57.6	40.1	7	30	44	71	191
Transaction offices	389	210.3	222.2	12	72	130	245	1,000
Credit risk	389	−4.026	0.569	−5.878	−4.351	−3.969	−3.677	−1.633
Loan-to-deposit ratio	389	−0.114	0.205	−1.013	−0.229	−0.101	0.014	0.682
Funding cost advantage	389	−2.150	0.748	−4.678	−2.526	−1.992	−1.670	−0.529
Financial technology firms	389	4.064	0.714	2.890	3.401	4.127	4.828	4.920
GDP growth	389	−2.844	0.330	−3.668	−2.919	−2.745	−2.646	−2.461
Inflation	389	−3.289	0.712	−5.065	−3.577	−3.347	−2.719	−1.678

Notes: Credit risk, the loan-to-deposit ratio, the funding cost advantage, the financial technology count, GDP growth, and inflation are expressed in natural logarithms. Transaction points equal branches plus transaction offices. The table covers the 389 bank-year observations that enter all six specifications reported in Tables 3 and 5.

1,152, a ratio of nearly sixty to one. The distribution is right-skewed, with a mean of 268 against a median of 175, so a small number of very large networks sit well above the bulk of the sample. Transaction offices outnumber branches by roughly three and a half to one at the mean, and account for a median 77% of all outlets with a standard deviation of 8 percentage points. The composition of the network is therefore stable across banks and over time, which is why the aggregate count and the transaction-office count carry nearly the same information. Return on assets averages 1.1% and return on equity 12.0%, both of which fall within the range reported for Vietnamese banks over comparable periods.

Table 2 reports the correlation matrix.

Table 2 shows that network size correlates more strongly with return on equity (0.35) than with return on assets (0.08), a gap that reappears in the estimation results. The three network measures correlate closely with one another, and the correlation between total transaction points and transaction offices reaches 0.99, which follows from transaction offices accounting for about three quarters of all outlets. Branches are the least collinear of the three with the aggregate measure, at 0.94, so the branch specification carries the most independent information. The three measures never enter the same equation. Correlations among the con-

Table 2. Pairwise correlations among network measures, profitability and control variables, 28 Vietnamese commercial banks, 2009–2024

	ROA	ROE	NET	NET ²	BR	BR ²	TO	TO ²	CR	LDR	CASA	FT
ROA	1.00											
ROE	0.81	1.00										
NET	0.08	0.35	1.00									
NET ²	0.00	0.02	0.22	1.00								
BR	0.15	0.40	0.94	0.18	1.00							
BR ²	−0.03	−0.06	0.04	0.84	−0.07	1.00						
TO	0.06	0.32	0.99	0.21	0.90	0.06	1.00					
TO ²	0.01	0.03	0.21	0.98	0.21	0.75	0.18	1.00				
CR	−0.27	−0.37	−0.19	−0.18	−0.22	−0.08	−0.18	−0.20	1.00			
LDR	0.40	0.32	0.12	0.15	0.12	0.16	0.13	0.12	−0.07	1.00		
CASA	0.36	0.34	0.41	0.12	0.48	0.04	0.37	0.14	−0.04	0.04	1.00	
FT	0.04	0.13	0.22	−0.12	0.25	−0.17	0.20	−0.11	−0.06	0.28	0.07	1.00

Notes: NET, BR and TO denote the centered logarithms of total transaction points, branches and transaction offices. CR is credit risk, LDR the loan-to-deposit ratio, CASA the funding cost advantage, FT the financial technology count. The matrix covers the 389 bank-year observations that enter all six specifications reported in Table 3.

Table 3. Two-step system GMM estimates of the association between network size and profitability under three measures of network size, 28 Vietnamese commercial banks, 2009–2024

Variables	(1) ROA	(2) ROE	(3) ROA	(4) ROE	(5) ROA	(6) ROE
	Transaction points		Branches		Transaction offices	
Lagged dependent	0.623*** (0.116)	0.571*** (0.144)	0.584*** (0.132)	0.573*** (0.129)	0.635*** (0.111)	0.573*** (0.143)
Network size	–0.0002 (0.0004)	0.0054 (0.0061)	–0.0001 (0.0007)	0.0034 (0.0069)	–0.0002 (0.0004)	0.0047 (0.0047)
Network size squared	–0.0008** (0.0004)	–0.0085* (0.0050)	–0.0013* (0.0007)	–0.0087* (0.0051)	–0.0007** (0.0003)	–0.0070* (0.0037)
Credit risk	–0.0009 (0.0016)	–0.0384* (0.0213)	–0.0006 (0.0016)	–0.0414** (0.0194)	–0.0009 (0.0017)	–0.0394* (0.0220)
Loan-to-deposit ratio	0.0075*** (0.0021)	0.0538** (0.0239)	0.0083*** (0.0024)	0.0527** (0.0224)	0.0071*** (0.0020)	0.0507** (0.0221)
Funding cost advantage	0.0018*** (0.0004)	0.0170*** (0.0042)	0.0018*** (0.0005)	0.0175*** (0.0043)	0.0018*** (0.0004)	0.0180*** (0.0042)
GDP growth	0.0002 (0.0006)	0.0060 (0.0062)	0.0002 (0.0006)	0.0056 (0.0060)	0.0002 (0.0006)	0.0062 (0.0062)
Inflation	0.0007 (0.0005)	0.0117*** (0.0042)	0.0006 (0.0005)	0.0117*** (0.0038)	0.0006 (0.0005)	0.0121*** (0.0044)
Financial technology firms	–0.0003 (0.0006)	0.0118 (0.0077)	–0.0005 (0.0006)	0.0130* (0.0072)	–0.0002 (0.0006)	0.0130* (0.0076)
Constant	0.0094 (0.0057)	–0.0471 (0.0705)	0.0120* (0.0065)	–0.0664 (0.0675)	0.0086 (0.0060)	–0.0532 (0.0724)
Observations	390	392	389	391	389	391
Banks	28	28	28	28	28	28
Instruments	17	17	17	17	17	17
AR(1) p-value	0.001	0.007	0.002	0.005	0.001	0.007
AR(2) p-value	0.265	0.554	0.261	0.552	0.269	0.549
Hansen p-value	0.371	0.458	0.481	0.600	0.335	0.433

Notes: Windmeijer-corrected standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Total transaction points equal branches plus transaction offices by construction; the logarithms of the total and of transaction offices correlate at 0.99, and those of the total and of branches at 0.94 (Table 2). The three pairs of columns are therefore alternative scalings of one network measure rather than independent tests.

control variables remain modest, with none exceeding 0.28 in absolute value.

Table 3 reports the estimation results for the six specifications.

All six specifications satisfy the conditions required of a dynamic panel estimator. The first-order serial correlation test rejects its null at the 1% level in every column, as it should when the residuals are differenced, while the second-order test does not reject at any conventional level. The Hansen test does not reject the validity of the instrument set, with p-values between 0.335 and 0.600, and the instrument count of 17 remains well below the 28 cross-sectional units.

The coefficient on the lagged dependent variable lies between 0.571 and 0.635 and is significant at the 1%

level throughout, which indicates that profitability persists strongly and that a static specification would be inappropriate. The squared network term is negative and statistically significant in all six columns, at the 5% level in two of them and at the 10% level in the remaining four. The linear term is positive in the return-on-equity columns and negative in the return-on-assets columns, but does not reach significance in any specification. A negative squared term is on its own insufficient to establish an interior maximum, since the same pattern is produced by a relationship that declines throughout the observed range, and the formal shape test is therefore reported separately below.

Among the controls, the funding cost advantage carries the most consistent effect, entering positively at the 1% level in all six columns. The loan-to-deposit ratio is positive and significant through-

Table 4. Lind and Mehlum (2010) test for an inverted U-shaped relationship between network size and profitability, 28 Vietnamese commercial banks, 2009–2024

Network measure	Dependent variable	Extremum	Data range	Slope at lower bound	Slope at upper bound	p-value	Turning point (outlets)
Transaction points	ROA	-0.099	[-2.848, 1.804]	0.0045	-0.0031	0.032	172
Transaction points	ROE	0.318	[-2.848, 1.804]	0.0541	-0.0254	0.056	261
Branches	ROA	-0.028	[-2.268, 1.375]	0.0058	-0.0036	0.039	47
Branches	ROE	0.196	[-2.268, 1.375]	0.0429	-0.0205	0.096	59
Transaction offices	ROA	-0.148	[-3.139, 1.977]	0.0041	-0.0029	0.018	119
Transaction offices	ROE	0.333	[-3.139, 1.977]	0.0489	-0.0232	0.036	193

Notes: The null hypothesis is that the relationship is monotonic or U-shaped; the alternative is that it is inverted U-shaped. Extremum and data range are expressed in the centered logarithmic scale, with the range taken from the estimation sample of the corresponding specification. Turning points are obtained by exponentiating the sum of the extremum and the centering constant of the corresponding measure. The three network measures are arithmetically related, so the six rows report one relationship at three scalings rather than six independent tests.

out, which is consistent with lending activity generating the bulk of interest income. Credit risk is negatively associated with return on equity at conventional levels but shows no measurable association with return on assets. Inflation is positively associated with return on equity, which may reflect the ability of banks to reprice assets faster than liabilities.

Table 4 reports the shape test. Table 4 establishes the result on which the hypothesis turns. In all six cases, the extremum falls inside the observed data range, the slope at the lower bound is positive, and the slope at the upper bound is negative. These are the three conditions that an inverted U requires, and they hold jointly at the 10% level in every specification, with p-values from 0.018 to 0.096. The hypothesis is supported.

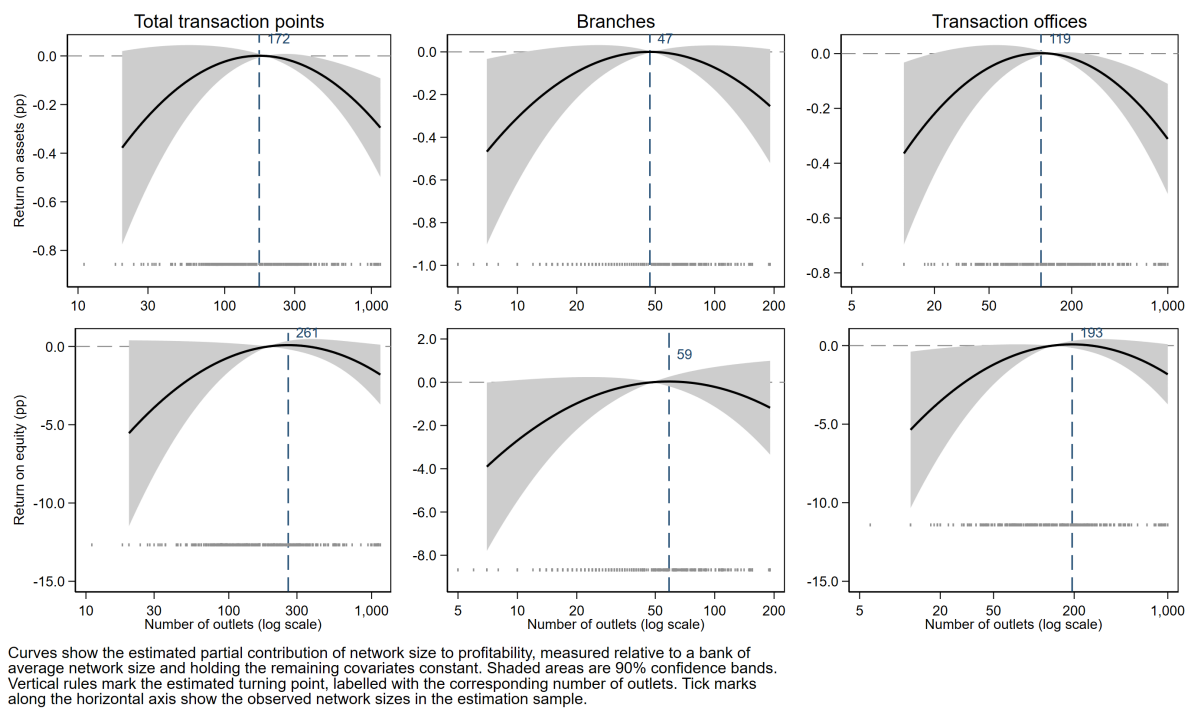
The estimated turning points differ systematically by dependent variable. Measured by total transaction points, profitability measured against assets peaks at 172 outlets, while profitability measured against equity peaks at 261 outlets. The same ordering appears for branches (47 against 59) and for transaction offices (119 against 193), giving ratios of 1.52, 1.26, and 1.62. Because the total and the transaction-office count are near-collinear, the informative comparison is between the total and branches, and the ordering they share indicates a systematic feature of the data rather than estimation noise.

Placing these estimates against the distribution in Table 1 shows how many banks are affected. About 49% of the 389 bank-year obser-

vations in the estimation sample record fewer than 172 transaction points and therefore sit on the rising segment of the return-on-assets relationship; the remaining 51% sit on the declining segment. Against the return-on-equity turning point of 261 outlets, about 65% remain on the rising segment. The median network of 175 outlets sits just above the asset threshold and well below the equity threshold, so the typical bank in this sample has passed the point at which additional outlets are associated with higher returns on assets but not the corresponding point for returns on equity. Applying the branch and transaction-office thresholds to their respective counts places 51% and 46% of observations below the turning point, close to the 49% obtained from the aggregate measure, as the arithmetic relationship among the three counts would lead one to expect.

Figure 1 plots the six estimated relationships against the number of outlets.

Figure 1 makes two features visible that the tables report only as numbers. The curvature is modest around the middle of the observed range and becomes steeper toward the upper end, indicating that the estimated profitability decline becomes more pronounced as network size moves further above the turning point. The confidence bands also widen at both extremes, which reflects the smaller number of banks operating with very few or very many outlets and indicates that the turning points are estimated most precisely in the region where most banks actually sit.



Notes: Curves show the estimated partial contribution of network size to profitability, measured relative to a bank of average network size and holding the remaining covariates constant. Shaded areas are 90% confidence bands. Vertical rules mark the estimated turning point, labelled with the corresponding number of outlets. Tick marks along the horizontal axis show the observed network sizes in the estimation sample.

Figure 1. Estimated relationship between network size and profitability under three measures of network size, 28 Vietnamese commercial banks, 2009–2024

The count of financial technology firms enters negatively in the return-on-assets specifications and positively in the return-on-equity specifications, reaching the 10% level in two of the latter. Because the count takes a single value in each year and is common to every bank, it acts as a control for common time variation rather than as a measure of the competitive pressure facing any individual bank, and its coefficient is not interpreted

as evidence on that question.

As a robustness check, we re-estimate all six specifications using a more conservative instrument classification. In this alternative specification, GMM-style instrumentation is extended to the loan-to-deposit ratio and funding cost advantage, in addition to the lagged dependent variable, network terms, credit risk, and financial technol-

Table 5. Two-step system GMM estimates under a more conservative instrument classification

Variables	(1) ROA	(2) ROE	(3) ROA	(4) ROE	(5) ROA	(6) ROE
	Transaction points		Branches		Transaction offices	
Lagged dependent	0.5782** (0.232)	0.4388*** (0.1234)	0.5772*** (0.0922)	0.5431*** (0.0968)	0.5375** (0.2417)	0.4907*** (0.1408)
Network size	-0.0004 (0.0016)	0.0136 (0.0165)	-0.0005 (0.0006)	-0.0094 (0.0154)	-0.0003 (0.0012)	0.0095 (0.0138)
Network size squared	-0.0012* (0.0007)	-0.0138** (0.0061)	-0.0013** (0.0006)	-0.0134* (0.0069)	-0.0011** (0.0004)	-0.0103* (0.0058)
Credit risk	-0.0047** (0.0021)	-0.0645*** (0.0182)	0.0004 (0.002)	-0.0454*** (0.0171)	-0.0045** (0.0023)	-0.0622** (0.0275)
Loan-to-deposit ratio	0.0062** (0.0025)	0.0663*** (0.0234)	0.0087*** (0.0023)	0.0417** (0.0196)	0.0064** (0.0027)	0.0589** (0.0228)
Funding cost advantage	0.0012 (0.0039)	0.0076 (0.0323)	0.0020*** (0.0006)	0.0396 (0.0291)	0.001 (0.0038)	0.0091 (0.034)

Table 5 (cont.). Two-step system GMM estimates under a more conservative instrument classification

Variables	(1) ROA	(2) ROE	(3) ROA	(4) ROE	(5) ROA	(6) ROE
	Transaction points		Branches		Transaction offices	
GDP growth	0.0005 (0.0009)	0.0028 (0.0078)	0.0004 (0.0006)	0.008 (0.0083)	0.0004 (0.0009)	0.0022 (0.0078)
Inflation	0.0008* (0.0004)	0.0107** (0.0041)	0.0003 (0.0005)	0.0092* (0.0047)	0.0008* (0.0004)	0.0099* (0.0052)
Financial technology firms	0.0007 (0.0005)	0.0043 (0.0062)	-0.0003 (0.0008)	0.0109 (0.0073)	0.0008 (0.0006)	0.0076 (0.0073)
Constant	-0.0089 (0.0187)	-0.1319 (0.0975)	0.015 (0.0096)	-0.0215 (0.1092)	-0.0088 (0.02)	-0.1453 (0.1482)
Observations	390	392	389	391	389	391
Banks	28	28	28	28	28	28
Instruments	20	20	19	25	20	22
AR(1) p-value	0.013	0.006	0.001	0.002	0.02	0.005
AR(2) p-value	0.267	0.691	0.283	0.452	0.277	0.682
Hansen p-value	0.562	0.672	0.364	0.541	0.529	0.618

Notes: Windmeijer-corrected standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

ogy measure, while only the macroeconomic controls enter as IV-style instruments. The instrument sets remain collapsed to restrict instrument proliferation.

Table 5 reports the resulting system GMM estimates. The principal finding is preserved under the more conservative classification. The squared network term remains negative and statistically significant in all six specifications, while the AR(2) tests continue to show no evidence of second-order serial correlation and the Hansen tests do not reject the validity of the instrument sets. Instrument counts range from 19 to 25 and remain below the 28 cross-sectional units.

Table 6 reports the corresponding Lind and Mehlum (2010) shape tests. The inverted U-shaped relationship remains supported at the 10% level in all six specifications, and the estimated turning points remain of the same order as the baseline estimates. The principal conclusion regarding the

shape of the network–profitability relationship is therefore robust to the more conservative instrument classification.

4. DISCUSSION

The turning points estimated here place Vietnamese commercial banking within the range that the international evidence describes, but towards its lower end. Kondo (2018) located the best-performing Japanese regional banks at 50 to 99 branches and found declining returns beyond 99; our branch-based estimate of 59 outlets for return on equity falls inside that band, and the corresponding estimate of 47 outlets for return on assets sits just below it. Kusi et al. (2022) reported turning points of 173 and 191 branches for Ghana, which is roughly three to four times our branch estimate. The gap is large, and the most plausible reading is that it reflects what an outlet does in each system. Ghanaian branches in that study

Table 6. Lind and Mehlum (2010) test for an inverted U-shaped relationship under a more conservative instrument classification

Network measure	Dependent variable	Extremum	Data range	Slope at lower bound	Slope at upper bound	p-value	Turning point (outlets)
Transaction points	ROA	-0.185	[-2.848, 1.804]	0.0065	-0.0048	0.099	158
Transaction points	ROE	0.495	[-2.848, 1.804]	0.092	-0.0360	0.053	311
Branches	ROA	-0.187	[-2.268, 1.375]	0.0054	-0.0041	0.037	40
Branches	ROE	-0.350	[-2.268, 1.375]	0.0513	-0.0462	0.07	34
Transaction offices	ROA	-0.144	[-3.139, 1.977]	0.0064	-0.0045	0.031	120
Transaction offices	ROE	0.464	[-3.139, 1.977]	0.074	-0.0311	0.059	220

were assessed against financial stability, estimated separately through models built around the deposit and loan channels of intermediation, a criterion for which a wider network can keep paying off even after it stops adding to profitability. Our estimates are assessed on realized profitability, a tighter criterion that nets out the operating costs a stability measure need not fully capture.

The comparison with Berger et al. (1997) is instructive in the opposite direction. Those authors concluded that United States banks operated roughly twice the number of outlets that would minimize costs, which is a statement about over-branching in a mature system. Our finding that 51% of Vietnamese bank-year observations sit beyond the return-on-assets turning point points in a similar direction, and does so in a system that was still expanding rather than consolidating. The two results are consistent with a common mechanism operating at different stages of network development.

The finding is broadly consistent with Pham et al. (2022), who used a random effects model and reported a positive overall association between branch count and return on equity together with a negative and significant squared branch term for their full sample; their subgroup estimates showed a positive branch coefficient for medium-network banks and a negative one for the largest-network banks. Two differences separate the two studies. Their estimator is static, so it does not accommodate the profit persistence that we estimate at 0.57 to 0.63; and their inference rests on the sign and significance of the squared term rather than on a formal test of where the turning point falls, so an interior optimum is implied rather than established. Our results are also closer to Le et al. (2020), who found medium-sized Vietnamese banks to be more efficient than large ones, and to Batten and Vo (2019), who reported a negative association between bank size and returns.

The systematic gap between the two turning points deserves attention, because it is a feature that studies using a single profitability measure would not detect. Return on equity equals return on assets divided by the equity-to-assets ratio, so the two measures diverge exactly when leverage varies with the variable of interest. In our sample, net-

work size is associated with the equity-to-assets ratio in a non-linear pattern, which means that part of what the network does to equity returns operates through the funding structure rather than through asset productivity. A bank that expands its network gathers more deposits, and deposit funding raises leverage. The turning point for equity returns therefore sits further to the right than the one for asset returns, and the ratio of 1.26 to 1.62, which holds for the branch measure as well as for the aggregate, is what that mechanism would produce. A practical implication follows: a bank that judges its network solely by return on equity will conclude that it has room to expand when the asset-side evidence indicates otherwise.

The separate treatment of branches and transaction offices invites an operational reading that these estimates do not support, and the point is worth making explicitly because the distinction is absent from the studies reviewed above. Turning points for the two outlet types differ by roughly a factor of two and a half, at 47 branches against 119 transaction offices for return on assets, which might be taken to mean that a bank past its branch threshold could still gain from adding the lighter format. That inference does not follow from separate regressions. The two counts are components of one total rather than competing alternatives, they are never estimated jointly here, and the logarithms of the total and of transaction offices correlate at 0.99, so the two columns are close to the same regression estimated twice. What the three columns establish is that the shape result does not depend on which count is used; what they cannot establish is the marginal contribution of one outlet type relative to the other.

The count of financial technology firms carries no interpretable competition effect in these estimates, and the design explains why. The measure counts firms operating nationally, so it takes one value in each year and is common to all 28 banks. It has no cross-sectional variation with which to identify how exposed any individual bank is to competition from such firms, and whatever it does absorb cannot be separated from the other developments that moved with the same years — credit growth, the digitalization of payments, and successive changes in prudential regulation. Read as what it is, a control for common time variation, its coef-

ficient cannot adjudicate the substitution mechanism that the literature debates. Phan et al. (2020) reported that each additional such firm reduced Indonesian bank returns on equity by 2.07% of the sample mean and Ben Naceur et al. (2025) reported margin compression across countries, while Ky et al. (2024) documented that mobile money partnerships improved bank performance in East Africa and Li et al. (2017) found a positive though insignificant association for United States retail banks. Settling that question requires a measure of each bank's own exposure, which the available data do not provide. We retain the variable because it absorbs common time variation, not because it tests competitive pressure.

Four further limitations bear on how far the estimates travel. First, system GMM relies on internal instruments, so it mitigates concerns about

the simultaneous determination of network size and profitability but does not deliver clean identification; the estimates are conditional associations and should not be read as causal effects. Second, the exclusion of Agribank restricts the sample to banks whose network decisions respond to profitability, which is appropriate for the question asked but means the results do not describe the system as a whole. Third, the turning points are point estimates from a quadratic approximation, and the shape test establishes that a maximum exists within the data rather than that it lies precisely at the estimated value. Fourth, common time variation is controlled through observed macroeconomic and technological factors rather than year dummies, which the instrument count cannot accommodate. These controls absorb most but not all of that variation, so the estimates are conditional on the residual being uncorrelated with network size.

CONCLUSION

This study aims to examine whether a non-linear relationship exists between branch network size and bank profitability in Vietnamese commercial banks and to identify the corresponding turning points. The dynamic panel estimates, formally tested for shape, show that profitability increases with network size only up to a point, after which further outlets are associated with lower returns. Under the total transaction points measure, the asset-based threshold sits at 172 outlets against an equity-based threshold of 261; the branch-only measure places these figures at 47 and 59, and the transaction-office-only measure at 119 and 193.

Three conclusions follow. The size of a bank's transaction network is a non-linear correlate of profitability with an estimated interior turning point rather than an asset whose expansion always adds value, and slightly more than half of the bank-year observations lie beyond the estimated ROA turning point on an asset-return basis. Because the equity-return threshold consistently exceeds the asset-return threshold, a bank that monitors only return on equity will read its expansion capacity too generously, and the two measures should be assessed together. Finally, what these estimates identify is the scale of a network rather than the format of its outlets. These conclusions describe associations estimated from a dynamic panel that excludes a state-owned bank operating under a policy mandate, and they should be read as a description of the profit-oriented segment of Vietnamese banking rather than as causal effects applicable to the system as a whole.

AUTHOR CONTRIBUTIONS

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 Resources: Thi Yen Nhi Nguyen.

Software: Thi Yen Nhi Nguyen.

Supervision: Quang Minh Nguyen.

Writing – reviewing & editing: Thi Yen Nhi Nguyen, Quang Minh Nguyen.

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