“Ownership concentration and competition in banking markets”

| AUTHORS      | Alexandra Lai  
|              | Raphael Solomon |
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Ownership concentration and competition in banking markets

Abstract

Many countries prohibit large shareholdings in their domestic banks. The authors examine whether such a restriction restrains competition in a duopolistic loan market. Large shareholders (blockholders) may influence managers’ output decisions by choosing capital structure, as in Brander and Lewis (1986). For the blockholder, debt has an additional benefit: it “disciplines” a manager by reducing the amount of free cash flow from which the manager can divert funds. The authors show that an economy with blockholders often leads to a more competitive banking sector. Hence, a restriction on the size of blockholdings can have anti-competitive results.

Keywords: ownership structure, capital structure, efficiency, banking.

JEL Classification: G21, G28, G32, L10.

Introduction

Do restrictions on the ownership structure of banks have competitive implications? The question is relevant to more than 50 countries, which either prohibit individuals and corporations from holding more than a given fraction of a bank’s shares or require that large shareholdings be reviewed by the government or the central bank. Figure 1 shows a histogram of the various shareholding restrictions.

![Figure 1. Bank shareholding restrictions](image)

Tables 1 through 3 provide a full list of countries with this type of banking regulation as of 2003, created from the database of the World Bank Survey of Bank Regulation and Supervision. Our paper examines whether this type of restriction restrains competition in an oligopolistic model of the loan market.

Table 1. Bank shareholding restrictions: high income countries

<table>
<thead>
<tr>
<th>Country</th>
<th>No. of banks</th>
<th>Share Limits</th>
<th>Share Limits</th>
<th>Exceed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aruba</td>
<td>5</td>
<td>5%</td>
<td>5%</td>
<td>CB</td>
</tr>
<tr>
<td>Australia</td>
<td>52</td>
<td>15%</td>
<td>15%</td>
<td>TR</td>
</tr>
<tr>
<td>Br. Virgin Is.</td>
<td>5</td>
<td>a</td>
<td>a</td>
<td>No</td>
</tr>
<tr>
<td>Canada</td>
<td>64</td>
<td>20%</td>
<td>20%</td>
<td>No</td>
</tr>
<tr>
<td>Cyprus</td>
<td>12</td>
<td>50%</td>
<td>50%</td>
<td>No</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>189</td>
<td>a</td>
<td>a</td>
<td>No</td>
</tr>
<tr>
<td>Norway</td>
<td>15</td>
<td>10%</td>
<td>10%</td>
<td>No</td>
</tr>
<tr>
<td>Qatar</td>
<td>15</td>
<td>10%</td>
<td>10%</td>
<td>No</td>
</tr>
<tr>
<td>Singapore</td>
<td>128</td>
<td>5%</td>
<td>5%</td>
<td>CB</td>
</tr>
<tr>
<td>Slovenia</td>
<td>21</td>
<td>20%</td>
<td>20%</td>
<td>CB</td>
</tr>
<tr>
<td>Taiwan</td>
<td>44</td>
<td>5%</td>
<td>5%</td>
<td>No</td>
</tr>
<tr>
<td>Turks and Caicos</td>
<td>8</td>
<td>49%</td>
<td>49%</td>
<td>No</td>
</tr>
</tbody>
</table>

Notes: a – must have at least two shareholders; CB – Central bank; Comm. – commission; Exceed – the organization that may permit exceeding the limits; MF – Ministry of Finance; No – if none exists; Pres. – President; TR – Treasury.

Table 2. Bank shareholding restrictions: medium income countries

<table>
<thead>
<tr>
<th>Country</th>
<th>No. of banks</th>
<th>Share Limits</th>
<th>Share Limits</th>
<th>Exceed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Columbia</td>
<td>29</td>
<td>95%</td>
<td>95%</td>
<td>No</td>
</tr>
<tr>
<td>Costa Rica</td>
<td>21</td>
<td>a</td>
<td>a</td>
<td>No</td>
</tr>
<tr>
<td>China*</td>
<td>105</td>
<td>10%</td>
<td>10%</td>
<td>No</td>
</tr>
<tr>
<td>Egypt</td>
<td>53</td>
<td>10%</td>
<td>10%</td>
<td>CB</td>
</tr>
<tr>
<td>Guyana</td>
<td>7</td>
<td>20%</td>
<td>20%</td>
<td>No</td>
</tr>
<tr>
<td>Malaysia</td>
<td>25</td>
<td>10%</td>
<td>20%</td>
<td>No</td>
</tr>
<tr>
<td>Malta</td>
<td>15</td>
<td>5%</td>
<td>5%</td>
<td>CB</td>
</tr>
<tr>
<td>Mauritius</td>
<td>10</td>
<td>15%</td>
<td>15%</td>
<td>No</td>
</tr>
<tr>
<td>Mexico</td>
<td>32</td>
<td>20%</td>
<td>20%</td>
<td>No</td>
</tr>
</tbody>
</table>

Notes: a – must have at least two shareholders; CB – Central bank; Comm. – commission; Exceed – the organization that may permit exceeding the limits; MF – Ministry of Finance; No – if none exists; Pres. – President; TR – Treasury.


1 Although most OECD countries, apart from Australia, Canada, Luxembourg, and Norway, do not have formal restrictions on bank shareholding, some countries (including the United States, the United Kingdom, and Japan) exhibit widely held shareholding patterns for their largest publicly traded banks. This may suggest that norms in these countries constrain bank ownership concentration.

2 See Barth, Caprio, and Levine (2001) for more details.
Table 2 (cont.). Bank shareholding restrictions: medium income countries

<table>
<thead>
<tr>
<th>Country</th>
<th>No. of banks</th>
<th>Share Limits</th>
<th>Share Limits</th>
<th>Exceed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dec-01</td>
<td>People</td>
<td>Firms</td>
<td></td>
</tr>
<tr>
<td>Montserrat</td>
<td>2</td>
<td>20%</td>
<td>20%</td>
<td>MF/CB</td>
</tr>
<tr>
<td>Nicaragua</td>
<td>6</td>
<td>20%</td>
<td>20%</td>
<td>No</td>
</tr>
<tr>
<td>Oman</td>
<td>15</td>
<td>15%</td>
<td>25-35%</td>
<td>No</td>
</tr>
<tr>
<td>Philippines</td>
<td>42</td>
<td>40%</td>
<td>40%</td>
<td>Pres.</td>
</tr>
<tr>
<td>Puerto Rico</td>
<td>17</td>
<td>5%</td>
<td>5%</td>
<td>Comm.</td>
</tr>
<tr>
<td>Sri Lanka</td>
<td>25</td>
<td>10%</td>
<td>10%</td>
<td>CB</td>
</tr>
<tr>
<td>St. Kitt's/Nevis</td>
<td>6</td>
<td>20%</td>
<td>20%</td>
<td>No</td>
</tr>
<tr>
<td>St. Lucia</td>
<td>7</td>
<td>20%</td>
<td>20%</td>
<td>MF/CB</td>
</tr>
<tr>
<td>St. Vincent</td>
<td>2</td>
<td>20%</td>
<td>20%</td>
<td>MF/CB</td>
</tr>
<tr>
<td>Thailand</td>
<td>31</td>
<td>5%</td>
<td>5%</td>
<td>No</td>
</tr>
<tr>
<td>Turkmenistan</td>
<td>13</td>
<td>35%</td>
<td>35%</td>
<td>No</td>
</tr>
<tr>
<td>Ukraine</td>
<td>152</td>
<td>a</td>
<td>a</td>
<td>No</td>
</tr>
<tr>
<td>W. Samoa</td>
<td>3</td>
<td>20%</td>
<td>20%</td>
<td>No</td>
</tr>
</tbody>
</table>

Notes: a – must have at least two shareholders; CB – Central bank; Comm. – commission; Exceed – the organization that may permit exceeding the limits; MF – Ministry of Finance; No – if none exists; Pres. – President; TR – Treasury.

Table 3. Bank shareholding restrictions: low income countries

<table>
<thead>
<tr>
<th>Country</th>
<th>No. of banks</th>
<th>Share Limits</th>
<th>Share Limits</th>
<th>Exceed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dec-01</td>
<td>People</td>
<td>Firms</td>
<td></td>
</tr>
<tr>
<td>Bangladesh*</td>
<td>50</td>
<td>10%</td>
<td>10%</td>
<td>No</td>
</tr>
<tr>
<td>Bhutan</td>
<td>2</td>
<td>20%</td>
<td>20%</td>
<td>No</td>
</tr>
<tr>
<td>Burundi</td>
<td>7</td>
<td>20%</td>
<td>20%</td>
<td>No</td>
</tr>
<tr>
<td>Fiji</td>
<td>6</td>
<td>15%</td>
<td>15%</td>
<td>No</td>
</tr>
<tr>
<td>Gambia</td>
<td>6</td>
<td>10%</td>
<td>10%</td>
<td>No</td>
</tr>
<tr>
<td>Georgia*</td>
<td>29</td>
<td>25%</td>
<td>25%</td>
<td>No</td>
</tr>
<tr>
<td>Grenada</td>
<td>5</td>
<td>20%</td>
<td>20%</td>
<td>MF</td>
</tr>
<tr>
<td>India</td>
<td>97</td>
<td>60%</td>
<td>60%</td>
<td>No</td>
</tr>
<tr>
<td>Kenya</td>
<td>46</td>
<td>25%</td>
<td>25%</td>
<td>No</td>
</tr>
<tr>
<td>Kyrgyzstan</td>
<td>20</td>
<td>15%</td>
<td>15%</td>
<td>No</td>
</tr>
<tr>
<td>Nepal*</td>
<td>13</td>
<td>49%</td>
<td>49%</td>
<td>No</td>
</tr>
<tr>
<td>Serbia/Montenegro</td>
<td>49</td>
<td>a</td>
<td>a</td>
<td>No</td>
</tr>
<tr>
<td>Sudan</td>
<td>25</td>
<td>10%</td>
<td>10%</td>
<td>No</td>
</tr>
<tr>
<td>Swaziland</td>
<td>4</td>
<td>25%</td>
<td>25%</td>
<td>No</td>
</tr>
<tr>
<td>Turkmenistan</td>
<td>13</td>
<td>35%</td>
<td>35%</td>
<td>No</td>
</tr>
<tr>
<td>Vietnam*</td>
<td>48</td>
<td>5%</td>
<td>5%</td>
<td>No</td>
</tr>
<tr>
<td>Zambia*</td>
<td>16</td>
<td>25%</td>
<td>25%</td>
<td>No</td>
</tr>
<tr>
<td>Zimbabwe</td>
<td>24</td>
<td>10%</td>
<td>25%</td>
<td>No</td>
</tr>
</tbody>
</table>

Notes: a – must have at least two shareholders; CB – Central bank; Comm. – commission; Exceed – the organization that may permit exceeding the limits; MF – Ministry of Finance; No – if none exists; Pres. – President; TR – Treasury.

Rules requiring dispersed shareholdings can cause several problems. They may deter foreign entry. They may also act as a poison pill, a mechanism to prevent hostile takeovers, without which banks might have access to cheaper capital\(^1\). Finally, they may increase agency costs. This paper focuses on the latter problem.

In an environment without shareholding restrictions, large shareholders may obtain control of banks to discipline management and to minimize agency costs. In so doing, they make the banking system more competitive (lower prices, higher output) and thus more efficient. In our model, large shareholders achieve this goal by issuing bank debt (taking uninsured deposits).

In our game-theoretic model of two competing banks, managers make daily operating decisions (represented by the choice of loan output), but also divert a fraction of the bank’s residual cash flow. Either the manager or the controlling blockholder may choose the bank’s capital structure. To obtain control, the blockholder must engage in costly monitoring. Monitoring does not guarantee control; rather, it yields the blockholder control with probability (less than one) increasing in the number of shares held. The timing of the game is as follows: (i) potential blockholders simultaneously decide whether to acquire a controlling share of a bank and monitor management, (ii) the manager or the controlling blockholder chooses the capital structure of the bank, and (iii) managers compete in the market for bank loans.

From a blockholder’s perspective, debt has two consequences. First, it “disciplines” a manager by reducing the amount of free cash flow from which the manager can divert funds. Second, it has a strategic effect vis-à-vis the other bank. Specifically, holding fixed the amount of debt at the rival bank, a unilateral increase in one bank’s debt increases its own output while reducing that of the other bank\(^2\). This raises the more indebted bank’s market share and profits at the expense of the other bank, since industry profits decline.

In a symmetric Nash equilibrium, where both banks issue debt, each bank incurs lower profits from making loans than they would under coordinated actions. In our model, however, an increase in debt at both banks may increase bank value, even as profits fall, because debt transfers payoffs from the manager to the shareholders. Moreover, industry output is higher. We show that, since managers issue less debt than blockholders, the presence of controlling blockholders increases both firm value and competition in the loans market. Hence, if shareholding

\(^1\) See Gouvin (2001), Malatesta and Walkling (1988), and Ryngaert (1988).

\(^2\) Brander and Lewis (1986) demonstrate that an increase in debt causes profit-maximizing managers to compete more aggressively in the output market relative to the pure (debt-free) Cournot outcome.
restrictions prevent the existence of blockholders, both firm value and competition in the loans market decrease, binding rule restricting ownership concentration creates two possibilities. In the first, blockholdings never exist; in the second, blockholders exist but do not monitor and never gain control. The latter case is quite interesting, since it may prove challenging for banking regulators to determine that it is occurring, yet it may have the same effects on competition as if the blockholders did not exist!

Our model is related to three distinct strands of literature. One strand relates capital structure to output. The key paper is by Brander and Lewis (1986), on which we draw extensively. Maksimovic (1988) models a repeated game in an oligopoly setting, in which collusive outcomes can occur. Debt holding can destroy the sustainability of collusion, leading to more competitive outcomes. Bolton and Scharfstein (1990) also relate debt financing to the aggressiveness of competition in a theory of predation. Das-gupta and Titman (1998) link pricing (and hence market share) decisions to capital structure through the effect of capital structure on the rate by which a firm discounts future profits. Campello (2003) finds capital structure empirically significant for explaining product market outcomes.

A second strand of literature models ownership structure and/or capital structure as responses to agency problems. Within this strand, only Zhang (1998) links capital structure and ownership structure in a model with managerial risk aversion and inside ownership. Agency problems between management and shareholders can also take the form of empire building and diversion of perquisites. Jensen (1986) notes that debt is a good antidote to managerial empire building and diversion of perquisites. In these models where capital structure is seen as alleviating agency problems, it also has an impact on firm value.

A third strand relates ownership structure to firm value. Our results are consistent with the main message of this literature: a large blockholder increases firm value. Burkart and Panunzi’s (2001) model has a manager, a large shareholder, and some dispersed shareholders, where shareholders need to monitor to prevent managerial diversion of resources. Despite the conflict of interest between dispersed shareholders and the blockholder, it is always value-enhancing for the blockholder to win effective control of the firm, because this aligns the interests of shareholders and the manager. Burkart, Gromb, and Panunzi (1997) and Bolton and Von Thadden (1998) consider the optimal ownership concentration as a response to agency problems between management and shareholders. Barclay and Holderness (1990) show empirically that the value of the firm increases if there is a blockholder, but that the increase is limited if the blockholder does not exercise control. In particular, actions that Barclay and Holderness (1990) interpret as monitoring, such as changing the composition of the board or replacing the management, yield the highest benefits in terms of firm value. Caprio, Laeven, and Levine (2004) provide empirical evidence of a positive relationship between ownership concentration and value for a sample of 244 publicly traded banks across 44 countries.

Our model links ownership structure to output through the choice of an optimal capital structure under agency problems, uniting these three strands of literature. A related paper examines the link between ownership structure and the incentive to acquire other firms: Allen and Cebenoyan (1991) allow for interaction between concentrated insider ownership and concentrated outsider ownership (the blockholder). In their sample of 58 American bank holding companies, they find that banks with a blockholder and no large inside shareholdings tend to be less acquisitive. To the extent that more competition results from fewer acquisitions, the blockholder without the inside shareholder may be said to be the most competitive ownership structure.

We abstract from private benefits of control and focus exclusively on the existence of shared benefits of control. Barclay and Holderness (1992) and others find evidence of both shared and private benefits of control, and these generally depend on the size of the blockholding. Holderness and Sheehan (1998), however, report evidence from the United States that large blockholders are constrained from expropriating cash flows and from other actions inimical to the interests of minority shareholders. Pedersen and Thomsen (2003) also find that the type of blockholder – institutional investor, corporation, financial firm, or private individual – matters for control. They find that financial firms are most likely to assert control. In our model, we can interpret the idea of different propensities to assert control in terms of different (opportunity) costs of monitoring.

---

1 A related literature examines the effects of insider ownership, as opposed to the outsider ownership that we consider. See Stulz (1990) for a theoretical model and DeYoung, Spong, and Sullivan (2001), Morek, Shleifer, and Vishny (1988), and McConnell and Servaes (1990) for empirical evidence.

2 See also Mikkelson and Regassa (1991).

3 In particular, by citing the case of Turner Broadcasting (p. 8), the authors demonstrate that ownership does not always entail control, even when the blockholder owns a majority of the shares.

---

Footnotes:

1 As Figure 1 shows, 20 per cent is both the median and modal restriction in the sample of countries that have formal bank shareholding restrictions.

2 See Jensen and Meckling (1976) on agency problems.
This paper is organized as follows. Section 1 develops the model and summarizes the multi-stage game with a timeline of the model (Figure 2). In section 2, we solve for the subgame-perfect Nash equilibrium at all stages of the game. We draw out the policy implications of our analysis in section 3. The last section offers some conclusions.

1. The model

There are two banks in the economy, indexed by \( i = 1, 2 \). Each bank \( i \) is run by a manager who holds no equity. Each bank has a potential blockholder who purchases a fraction, \( a_i \geq 0 \), of the bank’s shares. Atomic shareholders own the remaining shares. If the potential blockholder declines to purchase shares (\( a_i = 0 \)), all shares are bought by dispersed owners. All economic agents are risk-neutral and maximize wealth.

Managers choose output levels (of loans supplied) in a non-co-operative (Cournot) game. The manager also controls the choice of debt level if there is no blockholder. A blockholder, however, can influence the capital structure choice by monitoring, at cost \( c \). A controlling blockholder chooses the level of debt issued by the bank. We assume that the proceeds of the risky debt issue are immediately distributed to shareholders as dividends.

We assume that there are no conflicts of interest between the blockholder and dispersed shareholders, and we therefore focus on conflicts of interest between owners and managers. Managers are paid a fixed salary and also divert an exogenous fraction, \( \phi \), of banks’ cash flow net of payments to debt holders (hereafter, the residual cash flow) for their own consumption. Since the proceeds of the debt issue are distributed to shareholders, debt reduces the residual cash flow from which the manager diverts. A blockholder can thus “discipline” the manager through the choice of capital structure. The bank’s debt level also affects the manager’s output decisions.

We model the output decisions of managers, capital structure decisions, and the sharing and monitoring decisions of potential blockholders in a one-shot multi-stage game. The various stages of the model are described below.

Stage 1: Blockholding and monitoring decisions

At this stage, a potential blockholder acquires a share, \( \alpha_i \in [0, \alpha^{max}] \), of the bank and simultaneously decides whether to monitor. We represent legal restrictions on ownership concentration by \( \alpha^{max} < 1 \). The decision to monitor depends on how likely monitoring leads to effective control and the benefits of control. Blockholders face uncertainty over whether their monitoring is successful. If the blockholder monitors, the blockholder wins control from the manager with a probability \( p(a_i) \), a non-decreasing function of the size of blockholding, \( \alpha_i \). A controlling blockholder determines the capital structure of the bank by choosing the face value, \( D_i \), of debt to issue. With probability \( 1 - p(a_i) \), the manager retains control and chooses the debt level.

Stage 2: Capital structure decision

At this stage, the controlling blockholder of bank \( i \) chooses the debt level, \( D_i \), to maximize the expected value of the firm, which can be decomposed into the value of the firm at stage 3 and the value of debt:

\[
V^i(D_i) = V^{DE}(D_i) + V^{DM}(D_i).
\]

Bank \( i \)’s equity and debt values depend not only on its own debt level but also on the level of debt issued by its competitor bank, \( D_j \). The manager who retains control chooses debt to maximize the expected residual cash flow, which is equivalent to maximizing the value of the firm at stage 3. We interpret debt, \( D \), as uninsured (wholesale) deposits. We also abstract from competition for deposits and the sequential-service nature of bank deposits. Whereas banks typically issue debt both to cover operational funding requirements and for strategic reasons, bank debt is purely strategic in this model. Banks issue debt to obtain an advantage in the loans market and to discipline their management.

There are four different control structures:

- **BB**: Blockholders determine debt levels in both banks, \( D = (D^{BB}, D^{BB}) \). This occurs with probability \( p(a_i)p(a_j) \).

- **MM**: Managers determine debt levels in both banks, \( D = (D^{MM}, D^{MM}) \). This occurs with probability \( [1 - p(a_i)][1 - p(a_j)] \).

- **BM**: Bank \( i \)’s blockholder and bank \( j \)’s manager choose debt levels, \( D = (D^{BM}, D^{MB}) \). This occurs with probability \( p(a_i)[1 - p(a_j)] \).

- **MB**: Bank \( i \)’s manager and bank \( j \)’s blockholder choose debt levels, \( D = (D^{MB}, D^{BM}) \). This occurs with probability \( [1 - p(a_i)]p(a_j) \).

---

2 The benefits of control are endogenously determined in our model. Following Burkart and Panunzi (2001), we differentiate the rights of control conferred by ownership, but may or may not choose to exercise control. Upon choosing to exercise control, there is some uncertainty whether they obtain effective control. This makes sense when \( \alpha^{max} < 0.5 \), so that blockholders need to obtain the right to vote the proxies of dispersed shareholders. How easily they can do this depends on the size of their shareholdings, their monitoring effort, voting rules, and (potentially) luck.

3 We assume that \( p \) is non-decreasing in \( \alpha \) but needs not be differentiable; we require \( p(0) = 0 \) for \( p(\alpha) \) to be reasonable.
We denote the debt choice by a controlling blockholder facing a blockholder in the other bank as \( D_B \), and one facing a manager as \( D_{BM} \). Likewise, the debt choice by a manager facing another manager is denoted as \( D_{MM} \), while that of a manager facing a controlling blockholder in its competitor bank is \( D_{MB} \). The likelihood of any of these four control structures depends jointly on \( a_i \) and \( a_j \).

**Stage 3: Output decision**

The managers play a Cournot game, taking \( D_i \) and \( D_j \) as given. We denote bank \( i \)'s profit, net of the manager’s salary and payments to insured retail depositors, as \( R(q_i, q_j, z_i) \), where \( q = (q_i, q_j) \) is a vector of loan quantities and \( z_i \), an independent and identically distributed state variable, is uniformly distributed over the unit interval. We let \( R_i' \) denote the derivative of \( R \) with respect to \( q_i \), and \( R_i'' \) the derivative of \( R_i' \) with respect to \( z_i \). Following Brander and Lewis (1986), we impose the following restrictions on \( R_i'' \): \( R_i'' < 0 \) (concavity), \( R_i' < 0 \), \( R_{ij} < 0 \) (loans are substitutes as well as strategic substitutes, which means that a bank’s incentive to increase loans increases when the other bank reduces its loans), \( R_i' > 0 \), and \( R_{ij} > 0 \) (so that a higher realization of \( z_i \) is beneficial for bank \( i \)).

The manager of bank \( i \) chooses loan levels, \( q_i \), to maximize expected residual cash flow, which is equivalent to maximizing the value of equity at this stage. Let \( \hat{z}_i \) be the critical value of \( z_i \) for which the bank is in default if and only if \( z_i < \hat{z}_i \), implicitly defined by \( R_i(q_i, \hat{z}_i) - D_i = 0 \). Then, bank \( i \)'s expected residual cash flow (net of debt repayment) is 

\[
\int_{z_i}^{\hat{z}_i} [R_i(q_i, q_j, z_i) - D_i] dz_i,
\]

and the value of debt is 

\[
V^{ID} = \int_{z_i}^{\hat{z}_i} [R_i(q_i, q_j, z_i)] dz_i + (1 - \hat{z}_j)D_j.
\]

It is noteworthy that \( \hat{z}_i \) is a function of \( q_i, q_j, \) and \( D_j \). Brander and Lewis (1986) show that \( \hat{z}_i \) is increasing in \( D_i \) and \( q_i \), and decreasing in \( q_j, j \neq i \).

**Stage 4: Payoffs**

Uncertainty is resolved (\( z_i \) and \( z_j \) are realized), profits are realized, and debt is repaid, if possible. If no default has taken place, the manager diverts a fraction, \( \phi \), of the bank’s residual cash flow, and shareholders obtain their share of the firm’s public value, 

\[
(1 - \phi) \max \{ R_i - D_i, 0 \}.
\]

If default occurs, debt holders get all of the bank’s profits, while the manager and shareholders get nothing (limited liability).

The value of debt at time of issue (stage 2) is \( V^{ID} \), as defined above, and it accrues to shareholders. Hence, shareholders internalize the value of debt when choosing the optimal debt level at stage 2. Bank value and the price of bank shares at stage 1 are then 

\[ E[V^{IE} + V^{ID}] \]

**2. Equilibrium**

In this section, we solve the model for subgame-perfect Nash equilibria in pure strategies. That is, we start with the last stage of the game (stage 3) to solve for equilibrium Cournot outputs, given debt levels. We then proceed to solve for equilibrium debt levels (stage 2) under the four possible control
structures. Finally, we solve the game between potential blockholders, who simultaneously choose the size of shareholding to acquire and decide whether to monitor. Our analytical results are supplemented by results from numerical examples.

2.1. Equilibrium Cournot output. The manager at bank $i$ faces the following maximization problem:

$$\max_{q_i \geq 0} \int_{z_i}^1 \left[ R_i^i(q_i, q_j, z_i) - D_i \right] dz_i.$$  

Assuming an interior solution, the first-order condition is

$$\int_{z_i}^1 \left[ R_i^i(q_i, q_j, z_i) - D_i \right] dz_i = 0.$$  

In our proposition 1, we restate propositions 1 and 2 of Brander and Lewis (1986).

**Proposition 1.** (i) For identical banks, equilibrium quantities are higher the higher are debt levels. That is, $dq_i / dD_i > 0$. (ii) For non-identical banks, $D_i \neq D_j$, a unilateral increase in bank $i$’s debt increases bank $i$’s equilibrium quantity and reduces bank $j$’s equilibrium quantity. That is, $dq_i^* / dD_i^* > 0$ and $dq_j^* / dD_j^* < 0$.

2.2. Equilibrium debt levels. Whenever a manager is in control at bank $i$ (control structures MM and MB), the debt level is chosen to maximize the expected residual cash flow. The manager’s problem is as follows:

$$\max_{D_i \geq 0} \int_{z_i}^1 \left[ R^i(q^i_*, q^j_*, z_i) - D_i \right] dz_i.$$  

Taking the derivative with respect to $D_i$ yields

$$-\left[1 - F(\tilde{z}_i)\right] + \int_{z_i}^1 R^i(q^i_*, q^j_*, z_i) dz_i \frac{dq_i^*}{dD_i}.$$  

The first term is negative and reflects the decline in residual cash flow for every dollar of debt that is repaid. The second term is positive, because both $R^i_j$ and $dq^*_j / dD^*_j$ are negative. This represents the strategic effect of debt. A higher level of debt at bank $i$ induces a decrease in the equilibrium quantity of loans at bank $j$. This increases bank $i$’s profits. The optimal debt level reflects these conflicting effects. A manager will choose a positive debt level only if the strategic effect of debt is sufficiently strong.

Whenever a blockholder is in control at bank $i$ (control structures BB and BM), debt level is chosen to maximize the value of the firm, allowing for the diversion that will happen after cash flows are realized. The blockholder’s problem is given in (5), or, equivalently, in (6):

$$\max_{D_i \geq 0} \int_{z_i}^1 \left[ R^i(q^i_*, q^j_*, z_i) - D_i \right] dz_i - \phi \int_{z_i}^1 \left[ R^i(q^i_*, q^j_*, z_i) - D_i \right] dz_i.$$  

Taking the derivative with respect to $D_i$, while substituting the manager’s first-order condition yields

$$\int_{z_i}^1 R^i(q^i_*, q^j_*, z_i) dz_i \frac{dq^*_i}{dD_i} + \int_{z_i}^1 R^i(q^i_*, q^j_*, z_i) dz_i \frac{dq^*_j}{dD_j} - \phi \int_{z_i}^1 R^i(q^i_*, q^j_*, z_i) dz_i \left[1 - F(\tilde{z}_i)\right].$$  

Since $R^i_{x_i} > 0$, the first-order condition from the output stage, equation (2), implies that $R^i_{x_i} < 0$ for $z_i \in [0, \tilde{z}_i]$. Hence, the first term of (7) is negative, because a higher level of debt, and the resulting higher output, reduce profits for low realizations of $z_i$. The second term is positive, representing the strategic effect of debt. This strategic effect is larger for shareholders than for the manager, since it increases both the value of equity and the value of debt. In our model, shareholders internalize the value of debt, whereas managers disregard it. The expression inside the brackets in the final term of (7) is simply the derivative of the manager’s objective function. We conjecture that, evaluated at the optimal debt level for the blockholder’s problem, this final term is positive. This term represents the

1 While we use many parameterizations to determine the sensitivity of our results, we do not attempt to parameterize our model with actual data. Therefore, we note where a result is the outcome of those simulations. The appendix details our numerical solution procedure.
Proposition 3 follows because Cournot reaction functions have negative slopes greater than -1, a condition that is required for a stable Nash equilibrium.

We also care about how the ownership regime affects the bank’s performance, as measured by the bank’s value. Since we are unable to obtain analytical results, we defer this discussion until section 3. It is not obvious a priori whether a control structure with higher debt levels results in a higher bank value. This is because a higher debt level, while “disciplining” managers, also leads to more competition in the loans market and the latter reduces profits, all things equal.

2.3. The blockholding and monitoring decision.

To obtain the Nash equilibrium for this stage of the game, we first derive the conditions under which a blockholder will monitor. Returns to monitoring, measured in terms of firm value, depend not only on whether the blockholder wins control, but also on whether the other bank has a controlling blockholder. Hence, if the blockholder at bank $i$ decides to monitor, expected payoffs are

$$p(\alpha_i)p(\alpha_j)|V_{BB} - [1 - p(\alpha_i)]|V_{MM} -$$

$$p(\alpha_i)[1 - p(\alpha_j)]|V_{BM} + [1 - p(\alpha_j)]p(\alpha_j)V_{MB}.$$

Otherwise, expected payoffs are

$$p(\alpha_i)|V_{MB} + [1 - p(\alpha_j)]V_{MM}.$$ The difference between the two has to be greater than $c$ to induce monitoring by the blockholder. That is, a blockholder at bank $i$ monitors if and only if

$$\alpha_i, p(\alpha_i) \left\{ p(\alpha_j)(V_{BB} - V_{MB}) + [1 - p(\alpha_j)](V_{BM} - V_{MM}) \right\} \succeq c \quad (8)$$

The derivative of the left side of (8) with respect to $\alpha_i$ is positive; thus, the condition for monitoring can be expressed in terms of a critical blockholding size (concentration):

$$\alpha_i \succeq \bar{\alpha}_i(\alpha_j). \quad (9)$$

This critical shareholding size is a function of the ownership concentration at the other bank, $\alpha_j$.

Lemma 1. (i) Given that monitoring occurs, $\alpha_i \succeq \bar{\alpha}_i(\alpha_j)$, the value of the bank is increasing in $\alpha_i$. (ii) If $V_{BB} + V_{MM} - V_{MB} - V_{BM} < 0$, the critical concentration necessary to induce monitoring is increasing in the other bank’s concentration: $\bar{\alpha}'(\alpha_j) > 0$. Otherwise, the reverse is true.

Proof. See the appendix.

Whether $V_{BB} + V_{MM} - V_{MB} - V_{BM}$ is positive or negative corresponds to whether monitoring decisions are strategic substitutes or complements. Our nu-

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1 This is true whenever the manager has an incentive to choose a positive debt level. In all our numerical simulations, we obtain interior solutions for the manager’s debt choice.
Numerical simulations suggest that this expression is often negative. Lemma 1 implies that, if monitoring takes place, the blockholder prefers the highest possible concentration (up to the point where \( p'(\alpha) = 0 \)), in order to maximize the chances of winning control. This is because firm value increases whenever the blockholder wins control and is able to determine the capital structure of the bank. The first part of Lemma 1 leads us to the following conclusion.

**Corollary 1.** A blockholder who monitors acquires a shareholding of size \( \alpha^{\text{max}} \).

Let \( \psi(\alpha, \alpha_j) \) be the value of bank \( i \) as a function of both banks’ shareholding concentrations. By the above corollary, \( \psi \) takes four values:

\[
\psi(\alpha^{\text{max}}, \alpha_j), \psi(\alpha^{\text{max}}, 0), \psi(0, \alpha^{\text{max}}), \text{ and } \psi(0, 0).
\]

The block-acquisition game has the following normal-form representation:

<table>
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<tr>
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<th>B</th>
<th>NB</th>
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<tbody>
<tr>
<td>B</td>
<td>( \psi(\alpha^{\text{max}}, \alpha_j) ), ( \psi(\alpha^{\text{max}}, 0) ), ( \psi(0, \alpha^{\text{max}}) ), and ( \psi(0, 0) ).</td>
<td></td>
</tr>
<tr>
<td>NB</td>
<td>( \psi(0, \alpha^{\text{max}}) ), ( \psi(0, 0) ), ( \psi(0, 0) ).</td>
<td></td>
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</table>

where \( B \) denotes the action to acquire a blockholding and \( NB \) denotes the action not to acquire a blockholding.

The Nash equilibrium outcome of this game depends on whether \( \alpha(\alpha) \) is increasing or decreasing in \( \alpha \), and on how binding are legal restrictions on ownership. The following cases must be considered:

I. Non-restrictive ownership constraints:

\[
\max\{\alpha(0), \alpha(\alpha^{\text{max}})\} \leq \alpha^{\text{max}}.
\]

II. Moderately restrictive ownership constraints:

(a) \( \alpha(0) \leq \alpha^{\text{max}} < \alpha(\alpha^{\text{max}}) \). This case is relevant only if \( V^{BM} + V^{MM} - V^{MB} - V^{BM} < 0 \).

(b) \( \alpha(\alpha^{\text{max}}) \leq \alpha^{\text{max}} < \alpha(0) \). This case is relevant only if \( V^{BB} + V^{MM} - V^{MB} - V^{BM} > 0 \).

III. Highly restrictive ownership constraints:

\[
\alpha^{\text{max}} < \min\{\alpha(0), \alpha(\alpha^{\text{max}})\}.
\]

**Proposition 4.** The Nash equilibrium in cases I and II(b) yields the outcome where both banks have blockholders who monitor their managers and produce a higher expected output relative to the other two cases. Case II(a) produces a Nash equilibrium in which blockholders exist at both banks but neither monitors. In case III, the legal constraints on ownership are so binding that both banks are widely held in equilibrium. Industry output is the same in cases II(a) and III. A relaxation of ownership restrictions, by increasing \( \alpha^{\text{max}} \), increases industry output only in cases II(b) and I. However, it has no effect on industry output in cases II(a) and III unless the change results in \( \alpha^{\text{max}} \geq \alpha(\alpha^{\text{max}}) \).

**Proof.** See the appendix.

### 3. Numerical results

For our numerical examples, bank profits are given by

\[
R'(q_i, q_j, z_i) = z_i(1 - q_i - \gamma q_j)q_i - e^{-z_i}q_i^2,
\]

where \( z_i(1 - q_i - \gamma q_j) \) is the (linear) inverse demand function for bank \( i \)'s loans, \( C_i(q_i, z_i) = e^{-z_i}q_i^2 \) is bank \( i \)'s (quadratic) cost function, and \( \gamma \in (0, 1] \) is a measure of substitutability between bank loans\(^1\). Loan levels are restricted to those combinations that ensure a non-negative price. This functional form satisfies the restrictions from section 1\(^2\).

#### 3.1. Equilibrium values

In this numerical example, debt levels are strategic substitutes. In accordance with Proposition 2, blockholders always issue more debt than managers. In section 2, we show that a blockholder facing a competitor bank with a manager in control issues more debt than one facing another controlling blockholder. In this specification, managers' debt choices do not differ greatly whether they are facing another manager or a blockholder.

**Result 1.** In equilibrium, \( 0 < D^{MM} \leq D^{MB} < D^{BB} < < D^{BM} \).

Furthermore, equilibrium debt issue is lower the more substitutable are bank loans. On average, a manager’s debt issue is most responsive to changes in \( \gamma \), while the debt issue by a blockholder facing a manager is the least responsive.

**Result 2.** In equilibrium,

\[
\frac{dD^{MM}}{d\gamma} > \frac{dD^{MB}}{d\gamma} > \frac{dD^{BB}}{d\gamma} > \frac{dD^{BM}}{d\gamma} < 0.
\]

We obtain the above result from regressions with data generated from the numerical simulations. Managers' debt choices are sensitive to changes in the substitutability between bank loans because this directly impacts profitability, and managers care only about profits. Blockholders are concerned with

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\(^1\) Given this specification, one interpretation of \( z \) is as a shock common both to bank revenues and to bank costs, such as an exchange rate shock or a macroeconomic shock. Since the shock affects revenues linearly and costs non-linearly, revenues and costs are imperfectly correlated.

\(^2\) \( R_i' = -z_i(1 + e^{-z_i}) < 0, R_i' = -z_iq_i < 0, R_i' = -\gamma q_i < 0, R_i' = (1 - q_i - \gamma q_j)q_i + e^{-z_i}q_i^2 > 0, R_i' = (1 - q_i - \gamma q_j) + 2e^{-z_i}q_i > 0 \)
mitigating the agency problem between managers and shareholders as well as with profitability. This second concern reduces the blockholder’s sensitivity towards changes in $\gamma$. Figure 3 plots equilibrium debt levels against $\gamma$: $D^MM$ and $D^MB$ do not coincide exactly, due to random deviations arising from the numerical solution procedure. However, we find the difference between the two values to be statistically insignificant.

Figure 4, which plots equilibrium industry output against $\gamma$, demonstrates Proposition 3.

Our numerical examples lead us to the following conclusion regarding the relationship between ownership regime and firm value. Denote the value of a B-controlled bank facing a B-controlled bank as $V^{BB}$, the value of a B-controlled bank facing an M-controlled bank as $V^{BM}$, the value of an M-controlled bank facing a B-controlled bank as $V^{MB}$, and the value of an M-controlled bank facing an M-controlled bank as $V^{MM}$.

**Result 3.** In equilibrium, $V^{MB} < V^{MM} < V^{BB} < V^{BM}$.

The result is demonstrated in Figure 5, which plots equilibrium firm values against $\gamma$. The value under a controlling blockholder is always higher due to the benefits of control; that is, the blockholder mitigates agency problems, and this is value-increasing. The blockholder who faces a manager-controlled bank benefits even more from the fact that the manager issues less debt, enabling the blockholder-controlled bank to gain market share. Likewise, the manager who faces a blockholder-controlled bank loses market share, and thus firm value is lower than it would be if the rival firm was manager-controlled.

3.2. Policy implications. Our analysis demonstrates that legal restrictions on the concentration of ownership can affect bank value and competition in the loans market. Marginally relaxing this restriction will have an effect only in cases where the restriction has not prevented blockholding and monitoring to occur in the first place (case I). If ownership restrictions are binding, so that they either prevent blockholding or they prevent monitoring even in the presence of blockholdings, then a marginal increase in the maximum shareholding will generally not have any effect on bank value or competition in the loans market. For a relaxation of bank shareholding restrictions to be beneficial, the increase in maximum shareholding may need to be substantial.

It is worth highlighting Case IIb: the shareholding restriction does not prevent the existence of blockholders, but these blockholders exist only to prevent the other blockholder from monitoring. No monitoring occurs in these situations. That is, society does not derive any benefits from ownership concentration. Hence, if there are (unmodelled) costs to ownership concentration, case IIb is associated with lower social welfare relative to case III.
in which no blockholders exist. A relaxation of shareholding restrictions that induces a shift from case III to case IIa results in a net decrease in social welfare. However, a complete abolition of shareholding restrictions, or an increase in maximum allowable shareholding that is sizable enough to ensure we obtain case I, can be socially beneficial.

Our model abstracts from other conflicts of interest between equity holders and debt holders (risk-shifting) and between blockholders and minority shareholders (self-dealing). While the problem of risk-shifting is particularly relevant to highly leveraged institutions such as banks, capital requirements and positive franchise values mitigate the problem. Moreover, this problem is associated with leverage and not concentration ownership per se.

Although the original economic justification for bank shareholding restrictions might have been to prevent self-dealing, these regulations are a relatively old phenomenon, dating back to the 1960s in some countries. Since that time, two important developments bear mention. First, in the 1980s and 1990s, there was a revolution in corporate governance in the banking sector, as well as more generally. This revolution included changes such as an increased emphasis on outside directors, new rules for electing boards, and more internal oversight. Second, in the post-Basel era, there is increased supervision of banks, particularly large multinational banks. Taken together, these phenomena vastly reduce the scope for self-dealing by blockholders. The justification for these restrictions, while fairly universal in the 1960s, largely does not exist in most industrialized countries today.

**Conclusion**

This paper examines whether a restriction on ownership concentration affects competition in the bank loan market. Our analysis demonstrates that legal restrictions on the concentration of ownership can affect bank value and competition in the loans market. Marginally relaxing this restriction will have an impact only in cases where the restriction has not prevented blockholding and monitoring from occurring in the first place. If ownership restrictions are severe enough to prevent blockholding or monitoring (even if blockholders exist), then a marginal increase in the maximum shareholding will generally not have any impact on bank value or competition in the loans market. For a relaxation of bank shareholding restrictions to be beneficial, the increase in maximum shareholding may need to be substantial.

Ownership concentration matters in our model because it provides the incentives to the blockholder to engage in costly monitoring, a necessary step to obtaining the right to set the level of unsecured debt. We show that blockholders always issue more debt than managers. Debt is socially beneficial in two ways: it alleviates the agency problem and it is pro-competitive, because a higher level of debt induces the manager to compete more aggressively in the loans market. Hence, increased ownership concentration with monitoring creates a more competitive banking industry. We do not, however, model a cost to ownership concentration. In particular, we assume that there is no conflict of interest between blockholders and atomistic shareholders. We also do not model a mechanism by which control by the blockholder may adversely affect managerial incentives. A more balanced analysis might introduce a trade-off to ownership concentration or to debt issue. Hence, we might extend our analysis in two ways. In one approach, we can introduce some self-dealing by blockholders. In the other, we can model debt as impairing managerial incentives to exert effort that may raise firm profitability. Further research on these issues may prove fruitful.

**References**


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1 This causes an increase in industry output (from $Q_{BB}$ to $Q_{MM}$) and an increase in bank value (from $V_{BB}$ to $V_{MM}$).


**Appendix**

1. **Proofs of Lemma and Propositions**

**Proof of Proposition 2.** We denote a bank’s debt capacity as $D_i = \arg \max_{D_i} V^{ID} (D_i, D_i)$, where $V^{ID}$ is the value of debt for bank $i$, given Cournot equilibrium quantities, as functions of debt levels. That is,

$$V^{ID} = \int_0^{\hat{z}_i} R(q_i^*, q_i^*) dz_i + (1-\hat{z}_i) D_i.$$

Furthermore, note the following relationship between firm value ($V^{IB}$), debt value ($V^{ID}$), and equity value ($V^{IM}$), where ($V^{IM}$) is the manager’s objective function:

$$V^{IB} = V^{ID} + (1-\phi) V^{IM}.$$

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To assume that the bank remains within its debt capacity at all optimal debt levels (for the blockholder and for the manager) is equivalent to assuming that we obtain interior solutions for both the blockholder’s and the manager’s problems. This implies that
\[
\frac{\partial V^{M}}{\partial D_{j}}|_{D^{M}(D_{j})} = \frac{\partial V^{I}}{\partial D_{j}}|_{D^{I}(D_{j})} \geq 0, \quad \frac{\partial V^{I}}{\partial D_{j}}|_{D^{I}(D_{j})} = -\frac{\partial V^{I}}{\partial D_{j}}|_{D^{I}(D_{j})} \leq 0,
\]
where \(D^{M}(D_{j})\) is the manager’s debt reaction function and \(D^{I}(D_{j})\) is the blockholder’s debt reaction function.

Hence,
\[D^{M}(D_{j}) \leq D^{I}(D_{j}).\] (11)

If debt levels are strategic substitutes and Nash solutions are stable, then
\[-1 < \frac{\partial D^{M}(D_{j})}{\partial D_{j}} < 0,
\]
\[-1 < \frac{\partial D^{I}(D_{j})}{\partial D_{j}} < 0.\] (12)

Inequalities (11)-(13) yield \(D^{M} \leq D^{MM} \leq D^{BM}\) and \(D^{M} \leq D^{BB} \leq D^{BM}\). Using the symmetric Nash equilibrium conditions, \(D^{M} = D^{M}_{B}\) and \(D^{I} = D^{I}_{B}\), with inequality (11) yields \(D^{MM} \leq D^{BB}\). This completes the proof.

**Proof of Lemma 1.** Given that monitoring takes place, the bank’s expected value is
\[p(\alpha_{i})p(\alpha_{j})W^{BB} + [1 - p(\alpha_{i})][(1 - p(\alpha_{j}))W^{MM} + p(\alpha_{j})]V^{BM} + [1 - p(\alpha_{j})]p(\alpha_{j})W^{MB} + [1 - p(\alpha_{j})]p(\alpha_{j})W^{MB}.\]

The derivative of this with respect to \(\alpha_{i}\) is positive. The derivative of the left side of (8) with respect to \(\alpha_{j}\) is \(\alpha_{j}p(\alpha_{j})p(\alpha_{j})(V^{BB} + V^{MM} - V^{MB} - V^{BM})\). If \(V^{BB} + V^{MM} - V^{MB} - V^{BM} < 0\), the left side of (8) decreases with \(\alpha_{j}\), implying that a larger \(\alpha_{j}\) is required to induce monitoring; that is, \(\alpha(\alpha_{j}) > 0\).

**Proof of Proposition 4.** In the highly restrictive situation (case III), no blockholder can be given the incentives to monitor, even by the largest block possible. Hence, the unique Nash equilibrium is \((NB, NB)\), with both banks being widely held and industry output at \(Q_{MM}\), the lowest level of all the cases we consider.

In the non-restrictive situation (case I), the unique Nash equilibrium is \((B, B)\), where each bank has a blockholder who has a share, \(\alpha_{max}\), of the bank and monitors. Industry output is a function of who (blockholder or manager) wins control. Expected industry output is \(\{p(\alpha_{max})\}^{2}Q + 2p(\alpha_{max})(1 - p(\alpha_{max}))Q + [1 - p(\alpha_{max})]^{2}Q_{MM}\). The derivative of this with respect to \(\alpha_{max}\) is positive for all \(p(\alpha_{max}) < 1\) and \(p'(\alpha_{max}) > 0\). Hence, an increase in the legally allowable ownership concentration increases competition and industry output.

In the moderately restrictive situation (case II), we first consider the case of \(V^{BB} + V^{MM} - V^{MB} - V^{BM} < 0\), since this is the more common case (for all but one) in our numerical simulations. In this case, a blockholder of size \(\alpha_{max}\) will not monitor if faced by another blockholder of the same size. That blockholder will monitor, however, whenever the other bank is widely held. Thus, the outcome \((B, B)\) involves blockholders in both banks who hold \(\alpha_{max}\) of their banks but do not monitor because this shareholding does not provide the critical ownership level to induce monitoring, \(\alpha_{max} < \alpha(\alpha_{max})\). The bank’s value is therefore the same as for widely held banks: \(V(\alpha_{max}, \alpha_{max}) = V(0, 0) = V_{MM}\). If one bank, \(i\), is widely held but the other bank has a blockholder of size \(\alpha_{max}\), that blockholder will monitor, since \(\alpha_{max} < \alpha(0)\). Hence, \(V(\alpha_{max}, 0) = p(\alpha_{max})V_{MB} + [1 - p(\alpha_{max})]V_{MM}\) and \(V(0, \alpha_{max}) = p(\alpha_{max})V_{MB} + [1 - p(\alpha_{max})]V_{MM}\). Therefore, if \(V(0, 0) - V(0, \alpha_{max}) = p(\alpha_{max})V_{MM} > 0\), the Nash equilibrium is \((B, B)\). This is indeed the case (from Proposition 5): industry output is \(Q_{MM}\) and an increase in \(\alpha_{max}\) has no effect on industry output as long as \(\alpha_{max} < \alpha(\alpha_{max})\). An increase in \(\alpha_{max}\) beyond \(\alpha(\alpha_{max})\) puts us into the non-restrictive case (I).

Suppose that \(V^{BB} + V^{MM} - V^{MB} - V^{BM} > 0\). Then, a blockholder of size \(\alpha_{max}\) will monitor if the other bank has a blockholder of the same size, but will not monitor if the other bank is widely held. Hence, \(V(\alpha_{max}, \alpha_{max}) = [p(\alpha_{max})]^{2}V_{BB} + p(\alpha_{max})(1 - p(\alpha_{max}))[V_{BM} + V_{MB}] + [1 - p(\alpha_{max})]^{2}V_{MM}\), while \(V(0, 0) = V(\alpha_{max}, 0) = V(0, \alpha_{max}) = V_{MM}\). Since \(V(\alpha_{max}, \alpha_{max}) - V(0, 0) > 0\) for all \(p(\alpha_{max}) < 1\), the Nash equilibrium is \((B, B)\), in which both banks have blockholders who monitor. Expected industry output is the same as in the non-restrictive situation (case I).
2. Algorithm to solve for the Nash equilibria of stages 2 and 3 of the game

The solution is computed for fixed values of $\phi$ and $\gamma$. The solutions of type BB, MM, and BM/MB are each computed separately, in order to take advantage of the symmetries in the first two cases. The first step is to specify a parameter space for the quantities of loans, $q_i$ and $q_j$. Since quantities cannot be negative, we use the closed interval $[0, 10]$ and partition it into 4,000 points. For each point $(q_i, q_j) \in [0, 10] \times [0, 10]$, we compute the following quantity:

$$z = \min_{z \in [0,1]} \left| z(\alpha - q_1 - \gamma q_2)q_1 - e^{-z} q_1^2 \right|.$$  

Let $\hat{z} = \frac{1}{2}(\alpha - q_1 - \gamma q_2)q_1 - e^{-\hat{z}} q_1^2$. We define the space

$$Q = \{ (q_i, q_j) \in [0, 10]^2 \mid z < 0.99 \text{ and } \hat{z} > 0.01 \}. $$

This process of trimming the endpoints of the unit interval makes the solution more efficient. We specify a space for debt, $DS$, which is usually $[0, 0.15]$ at the first stage. Next we fix a debt level, $D_i \in DS$, and compute $z_i$ for each $(q_i, q_j)$ pair. For each $q_j$, we find $q^*_j$, which maximizes the objective function $\int_{z_i}^{1} (R - D) dz_i$. The value $q^*_j$ depends on $D_i$ and $q_j$. We repeat this process for all $q_j \in [0, 10]$, and then restart the process for another $D_i$ searching over a large space for debts issued. We eliminate those triples $(q_i, q_j, D_i)$ that fail to satisfy the second-order conditions of the output-stage maximization problem.

We then examine pairs $(D_i, D_j) \in DS^2$. For each pair, we look at the Nash quantities $q^*_i(q_j, D_i)$ and search for matched pairs. For example, suppose we are looking for the Nash equilibrium in quantities associated with debt levels $(0.05, 0.03)$. We look at the sets of Nash quantities from $D_j = 0.05$, calling them $\{q^*_i(i), q^*_j(i)\}$. We label the sets of Nash quantities from $D_j = 0.03$ as $\{q^*_i(j), q^*_j(j)\}$. We select pairs $\{(q^*_i(i), q^*_j(i)), (q^*_i(j), q^*_j(j))\}$, where $q^*_i(i) = q^*_i(j)$, and look for any pair such that $q^*_i(i) = q^*_j(j)$ is a Nash equilibrium in quantities. This may seem an unusual computational approach to determine a Nash equilibrium, but it relies on the fact that both banks have the same objective function (with the variables $q_i$ and $q_j$ switched). We then have, for all $(D_i, D_j) \in DS^2$, $q^*_i(D_i, D_j)$ and $q^*_j(D_i, D_j)$. The next step is to compute the Nash equilibrium debt strategy. The computational approach depends on whether the solution is for case BB, MM, or BM/MB. Since the BB and MM cases rely on symmetries, we focus on the asymmetric equilibrium for debt choice. We compute the values of the managers’ objective functions and the blockholders’ objective functions for banks $i$ and $j$. For each level of debt, $D_i$, we determine the best response, $D^*_j(i, D_i)$, based on the objective function of bank $i$. We repeat the process and compute $D^*_j(j, D_j)$, using the other objective function. The two reaction functions may have a solution, which is a Nash equilibrium. It is likely that there is no solution in the first iteration. Since it takes such a long time to compute optimal quantities given debt levels, we do not search over many debt levels at a time. In the event that no solution appears, we fit linear regressions to the reaction functions, solve for the intersection, and continue our search for the optimal debt level in the neighborhood of that intersection. Even if we do find a solution to the two reaction functions, the solution is imprecise. We typically “zoom in” on the part of the space $DS^2$ in which the “equilibrium” appears, in order to gain more precision.

3. Algorithm to solve for the Nash equilibria of stage 1 of the game

The first step is to partition the Nash equilibria of the debt stage into two sets. Let $F = V^{BB} + V^{MM} - V^{BM} - V^{MB}$, where $V$ is the value of the firm. We determine the boundaries of the three cases for $F > 0$ and $F < 0$ separately. We need to specify a parameter space for $c$, the cost of monitoring. Based on the values of the firm computed above, we use $[0.0001, 0.0020]$, partitioned into 20 subintervals. We consider three functions for $p(\alpha)$:

$$p_1(\alpha) = \alpha, \quad p_2(\alpha) = \ln(1 + \alpha), \quad \text{and} \quad p_3 = \frac{0.996 \alpha^2}{\ln(\alpha + 0.774)}, \quad 0.3932 \leq \alpha \leq 1.$$ 

Each of these three functions has a behavioral interpretation. $p_1$ represents the idea that only sole proprietors can do what they want with their firms; even a majority shareholder may face legal opposition from determined minority shareholders. Both $p_2$ and $p_3$ have the property that $p(1) < 1$; a sole proprietor may not obtain full control if there are legislative restrictions or other random barriers. But the critical difference between $p_2$ and $p_3$ is that $p_2$ is globally concave, whereas $p_3$ is convex on the subset $[0, 0.3932]$ and concave on the subset $[0.3932, 1]$. The convex-concave function captures the idea that, if a blockholder has a small block of shares, it may be almost infeasible to win control, but there is a threshold after which control becomes likely. Note that $p_2$ is a continuous function on $[0, 1]$, although not continuously differentiable. For each function, each cost, and each $(\phi, \gamma)$ pair, there is a partition of $\alpha$-space that defines the boundaries of the three cases. We compute $\bar{\alpha}(0)$ and $\bar{\alpha}(\alpha)$ for all $\alpha \in [0, 1]$ partitioned into 1,000 subintervals. From these calculations, the boundaries are apparent.