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ASSESSING INFORMATIONAL EFFICIENCY IN LARGEST AFRICAN STOCK MARKETS BY MODELING DUAL LONG MEMORY: AN ARFIMA-FIGARCH APPROACH

Abstract

Informational efficiency is a fundamental pillar of well-functioning financial markets, as it underlies informed investment decisions, effective risk management, and broader economic stability, particularly in emerging African markets, where inefficiencies are more likely to persist. This study assesses the weak-form informational efficiency of six major African stock markets – Johannesburg, Casablanca, Botswana, Nigeria, Egypt, and the Regional Stock Exchange – through the lens of long-memory behavior in returns and volatility. This is achieved by employing four advanced models: ARFIMA-FIGARCH, ARFIMA-FIEGARCH, ARFIMA-FIAPARCH, and ARFIMA-HYGARCH. Each of these models is specifically designed to capture long memory in both the conditional mean and variance. The empirical results demonstrate that the ARFIMA-FIGARCH framework, across all four model variants, consistently outperformed alternative specifications in fitting the return and volatility dynamics of all six African stock market indices. The estimated fractional differencing parameters in both the mean (d_{ARFIMA}) and variance ($d_{FIGARCH}$) equations were highly statistically significant at the 1% level for each market, confirming the presence of persistent long-memory behavior. This strong evidence of long-range dependence implies that past return information is not fully reflected in current prices, thereby violating the assumptions of weak-form market efficiency. Consequently, these findings provide compelling and systematic evidence against the weak-form Efficient Market Hypothesis (EMH) for the markets studied, highlighting a common structural inefficiency across the African financial landscape.

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INTRODUCTION

The concept of informational efficiency is a cornerstone of financial market theory, directly influencing how accurately and swiftly asset prices reflect all available information. In an efficient market, asset prices adjust instantly to new data, making it impossible to consistently forecast future prices using historical data alone. Assessing weak-form efficiency, which considers whether past prices can predict future prices, plays a crucial role in determining the effectiveness of resource allocation, the reliability of investment strategies, and the stability of financial markets.

In the context of emerging markets, such as those in Africa, evaluating weak-form efficiency holds particular significance. African stock markets are growing rapidly and attracting increasing global attention, yet they remain understudied compared to their counterparts in developed economies. Assessing the efficiency of these markets is essential for understanding their developmental trajectory and for help-

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ing investors, regulators, and policymakers make informed decisions. Market efficiency is closely linked to investment opportunities, risk management practices, and the overall stability of financial markets, making it a critical area of study for the region's economic future.

Despite the growing importance of African markets, the existing body of research often relies on traditional models that may not fully capture the complexities of these markets. These models typically assume linear relationships and often fail to account for non-linearities, long-term dependencies, and other characteristics unique to emerging markets. This gap in the literature underscores the need for more advanced techniques that can better analyze the unique dynamics of African stock markets and provide a more accurate assessment of their efficiency.

To address this gap, the present study aims to explore the weak-form informational efficiency of African stock markets by employing advanced models capable of capturing long memory in both returns and volatilities. By leveraging these more sophisticated methodologies, the study seeks to offer deeper insights into the efficiency and behavior of these markets, providing a more nuanced understanding than what has been offered by traditional approaches.

1. LITERATURE REVIEW

A significant body of research has investigated the empirical validity of weak-form informational market efficiency worldwide, examining a wide range of asset classes, including stocks, bonds, and derivatives. These studies have analyzed the extent to which markets incorporate available information and whether asset prices exhibit random walk behavior.

A variety of statistical techniques have been employed to evaluate weak-form market efficiency and the random walk hypothesis. These methods include serial correlation tests and variance ratio tests, spectral analysis, and unit root tests. Urrutia (1995) tested the hypothesis that Latin American emerging equity market prices followed a random walk using the variance-ratio methodology. He analyzed monthly index prices in local currency from December 1975 to March 1991 for Argentina, Brazil, Chile, and Mexico. The variance-ratio test rejected the random walk hypothesis, while the runs test indicated that these markets were weak-form efficient.

Similarly, Mollah (2007) examined the weak-form market efficiency of the Botswana Stock Exchange (BSE) using a triangulation econometric approach. He analyzed daily return series from 1989 to 2005 and tested the random walk hypothesis. The results rejected the random walk model and indicated serial autocorrelation of the return series, pointing to predictability and volatility in the Botswana market. His study provided further evidence that the BSE did not adhere to weak-form efficiency, aligning with the conclusion of Urrutia (1995) that inefficiencies exist in emerging markets.

In contrast, Borges (2010) explored weak-form market efficiency in stock market indices from the UK, France, Germany, Spain, Greece, and Portugal between January 1993 and December 2007. Using runs test and joint variance ratio test, Borges found mixed evidence regarding the efficient market hypothesis (EMH). Unlike Urrutia (1995) and Mollah (2007), Borges' hypothesis was only rejected for Portugal and Greece, while for France and the UK, mean reversion in weekly data led to the rejection of EMH. Conversely, Borges concluded that tests for Germany and Spain did not support the rejection of the EMH, with Spain showing the highest degree of market efficiency.

Similarly to Urrutia (1995) and Mollah (2007), Al-Jafari and Altaee (2011) investigated the weak-form efficiency of the Egyptian stock market by testing the random walk hypothesis (RWH) through multiple approaches, including unit root, runs, and variance ratio tests. They analyzed daily price data of the EGX 30 index from January 1998 to December 2010. Their empirical results rejected the RWH, indicating that stock prices did not fully reflect historical information, further supporting the findings of Urrutia (1995) and Mollah (2007) in emerging markets.

Furthermore, Chiny and Mir (2015) examined the efficiency of the regional stock market in the BRVM using daily data from January 4, 2016, to June 30, 2022. By applying the Box and Jenkins procedure, they tested the efficiency of the BRVM in the weak sense. Consistent with the earlier studies by Urrutia (1995), Mollah (2007), and Al-Jafari and Altaee (2011), their findings confirmed that the BRVM was inefficient, thereby rejecting weak-form efficiency for the regional market.

Lastly, Dutta (2015) reassessed weak-form efficiency in major European equity markets. By employing runs test, variance ratio test, and unit root test, Dutta found that prices in the selected European markets did not follow a random walk. The analysis indicated that these markets were not weak-form efficient, providing additional evidence against the efficient market hypothesis in the context of European stock markets. In line with Urrutia (1995), Mollah (2007), and Chiny and Mir (2015), Dutta's findings highlighted inefficiencies in global markets, both emerging and developed.

Other studies examined informational efficiency by analyzing long memory in stock market indices using separate models like ARFIMA and FIGARCH. Chaker (2003) explored the volatility dynamics in the Tunisian stock market, focusing on the persistence phenomenon and the presence of long-term memory. Specifically, he aimed to assess whether long-term dependent processes could appropriately model the volatility of the Tunisian stock market. Using FIGARCH models and analyzing daily data from the IBVMT and TUNINDEX indices from 1998 to 2004, Chaker found that the long-term component of volatility had a significant impact on stock market return series. Similarly, Maheshchandra (2014) examined long memory in the volatility of Indian and Chinese stock markets using FIGARCH models. By analyzing daily returns from the BSE and SSE indices from January 2009 to June 2014, his results revealed strong evidence of long memory in the conditional variance of both indices. The long memory property was more prominent in the BSE market than in the SSE.

In a related study, Nazarian et al. (2014) applied separately ARFIMA and FIGARCH models, emphasizing their capacity to capture persistence in both the mean and volatility of financial time series. Using daily data from the Tehran Stock Exchange (TSE), their study revealed no long memory in return series but confirmed the presence of long memory in the conditional variance through the FIGARCH model. In contrast to Maheshchandra (2014), Nazarian et al. (2014) concluded that FIGARCH models were better suited for modeling volatility in financial data rather than return behavior.

Building upon this analysis of market volatility, Lamouchi (2020) assessed the market efficiency of the Saudi Arabian stock market (Tadawul All Share Index, TASI) from 1998 to 2020. By analyzing the dependence structure of returns and volatility, he provided evidence of long memory in the Saudi stock market. The ARFIMA model results further confirmed the existence of long-term dependence in historical volatility, challenging the Efficient Market Hypothesis (EMH). These findings align with those of Nazarian et al. (2014) and are consistent with the conclusions drawn by Chaker (2003).

Moreover, Falloul (2020) investigated the weak-form efficiency of the Casablanca Stock Exchange using the MASI index. After applying classical econometric tests, the study rejected the weak-form efficiency hypothesis. By calculating the Lyapunov exponent, Falloul concluded that the MASI index exhibited chaotic dynamics. The ARFIMA model indicated long memory in the MASI index, further reinforcing Lamouchi's (2020) findings of inefficiency in the stock market and long-term dependencies.

Alfred and Sivarajasingham (2020) expanded the investigation of long memory by analyzing stock price returns in Sri Lanka, focusing on the All-Share Price Index (ASPI). Using ARFIMA model data from 1985 to 2018, they found no evidence of long memory in the return series. However, consistent with previous research by Nazarian et al. (2014) and Lamouchi (2020), they confirmed long memory in the volatility series, indicating inefficiency in the Sri Lankan stock market.

Furthermore, Rahmatalla and Elbashir (2024) examined the long memory feature, focusing on the Saudi Arabian Stock Market. They analyzed

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daily closing index data from 2018 to 2022 using the ARFIMA model. Their study confirmed the presence of long memory in the conditional mean of returns by calculating the Hurst exponent and fractional differential coefficient.

Lastly, Kuttu et al. (2024) examined long memory in the volatility of foreign exchange markets in Egypt, Ghana, Kenya, Nigeria, and South Africa from 1997 to 2021. They applied the FIEGARCH model and found long memory in volatilities across all five markets.

Another research stream focused on double long memory in both returns and volatilities using joint ARFIMA-FIGARCH models. Kasman et al. (2009) discovered evidence of long memory in five out of eight Central and Eastern European (CEE) stock markets, demonstrating that the ARFIMA-FIGARCH model provided more accurate out-of-sample forecasts for both returns and volatility. Similarly, Cao et al. (2009) assessed the effectiveness of double long-memory models for Value at Risk (VaR) using the Shanghai Composite Index (SHCI) and Shenzhen Component Index (SZCI), concluding that the ARFIMA-HYGARCH model was most effective, particularly for in-sample VaRs for long positions at lower VaR levels.

Building on these findings, Ural and Kucukozmen (2011) analyzed the long-memory characteristics of five major stock markets – S&P500, FTSE100, DAX, CAC40, and ISE100 – confirming that the ARFIMA(2,d,2)-FIGARCH(1,d,1) model with a skewed Student-t distribution outperformed the others. Their positive and significant long-memory parameters indicated persistent behavior in both returns and volatility. In a related study, Boubaker and Makram (2012) explored North African stock markets (TUNINDEX, MASI, and EGX30), similarly highlighting the ARFIMA-FIGARCH model's precision in capturing long-memory dynamics.

Further extending this line of research, Turkyilmaz and Balibey (2014) examined the weak-form efficiency of the Karachi Stock Exchange, finding that while the ARFIMA model did not indicate long memory in returns, the FIGARCH model did in volatility. In parallel, Mahboob et al. (2017) analyzed the Dhaka Stock Exchange, demonstrating strong evidence of long memory in both returns and volatilities through ARFIMA-FIGARCH and ARFIMA-FIAPARCH models.

Houfi (2019), analyzing the Tunisian Stock Exchange, similarly identified long memory in both returns and volatility using the ARIMA-FIGARCH model. Subsequently, Bouchareb et al. (2021) examined four Mediterranean stock markets – Morocco, Turkey, Spain, and France – reporting significant long memory in returns and volatility in Morocco and France, while Spain and Turkey only exhibited long memory in volatility.

Odonkor et al. (2022) examined the Ghana Stock Exchange and identified long memory in both returns and volatility across seven stocks using the ARFIMA-FIGARCH model. Their findings challenge the Efficient Market Hypothesis in the Ghanaian market. Boubaker et al. (2022) further expanded the analysis to six Gulf Cooperation Council (GCC) countries, showing that the ARFIMA-HYAPARCH model effectively captured long-memory dynamics in both returns and volatility, challenging the Efficient Market Hypothesis.

In the realm of cryptocurrencies, Zhuhua et al. (2023) highlighted the impact of long memory and structural breaks on the persistence of cryptocurrency markets. Their use of structural break tests and FIGARCH models with a skewed Student-t distribution confirmed the importance of accounting for structural breaks in volatility modeling. Similarly, Sosa et al. (2023) explored Bitcoin (BTC) and Ether (ETH), finding that ARFIMA-HYGARCH was best for BTC volatility, whereas ARFIMA-FIGARCH was better suited to ETH, particularly during the COVID-19 pandemic.

Likewise, Basira et al. (2024) employed dual hybrid long-memory GARCH models to forecast commodity price volatility, revealing that model performance varied across different commodities, with implications for risk management and asset allocation.

Javier et al. (2024) provided additional insights by analyzing the volatility of the Selective Stock Price (SSP) index in Chile, showing that ARFIMA-GARCH outperformed FIGARCH in terms of volatility fit, further linking the index's behavior to global economic events.

Finally, Adewole (2024) analyzed Nigeria's Real GDP, exchange rate, and inflation rate, concluding that ARFIMA-FIGARCH models identified long memory in both returns and volatility and were most effective in forecasting inflation and exchange rates.

Despite the extensive research on financial market efficiency in emerging and developed economies, there is a notable lack of studies examining the efficiency of African stock markets. This gap in the literature warrants further investigation. Traditional models, such as ARMA and GARCH, are inadequate for capturing the complexities of emerging markets, including non-linearity, volatility clustering, and long-memory effects, which are particularly prominent in African markets. Consequently, there is a clear need for advanced models that can more effectively address these complexities and provide a deeper understanding of African market dynamics.

Building on previous research, this study conducts an in-depth examination of market efficiency in Africa's largest stock exchanges by exploring long-memory patterns in returns and volatility. By applying sophisticated dual long-memory models, it aims to deliver a deeper and more nuanced understanding of market inefficiencies across the continent. By addressing the shortcomings of conventional linear models, often used in previous studies, this research also expands the existing literature by including a wider spectrum of African markets for analysis.

2. METHODOLOGY

This section outlines the data and methods used in the analysis of market efficiency and long-memory dynamics in African stock markets. The first subsection details the data sources and sample period of the stock market indices studied. The second subsection describes the econometric models applied, focusing on ARFIMA-FIGARCH and its variants, which are used to capture long-memory behavior in both returns and volatility.

2.1. Data

The dataset used in this study comprises daily closing index prices from the six major African stock exchanges: the Johannesburg Stock Exchange (JSE), Casablanca Stock Exchange (MASI), Botswana Stock Exchange (BSE), Nigerian Exchange (NGX), Egyptian Exchange (EGX), and the Regional Stock Exchange (BRVM).

The dataset spans from January 1, 2011 to September 8, 2024, encompassing roughly 3,380 daily observations. However, the Nigeria and BRVM indices are available from January 30, 2012, and March 4, 2014, respectively. The dataset was sourced from www.investing.com. The index prices were subsequently transformed into logarithmic returns, calculated as , representing the index price at time , and is the natural logarithm.

2.2. Method

In this section, the joint models are described, based on the ARFIMA model capturing the persistence behavior of returns, and on the FIGARCH, FIEGARCH, FIPARCH, or HYGARCH models capturing the long-term dependence in volatility. All four models are designed to capture long-memory properties in both the returns and volatility, though they differ in their approach to volatility dynamics and long-memory processes. In this study, the primary focus is placed on capturing the long-memory characteristics in both returns and volatility. The use of these four models enables the identification of those that best fit the data.

2.2.1. *AutoRegressive Fractionally Integrated Moving Average model:*

$$ARFIMA\left(\overline{p}, d, \overline{q}\right)$$

$\left(X_t\right)$ is an $ARFIMA\left(\overline{p}, d, \overline{q}\right)$ process of orders $\overline{p}$, $d \in Q$ and $\overline{q}$ if there exist lag polynomials

$$\phi\left(L\right) = 1 - \sum_{i=1}^{\overline{p}} \phi_i L^i \text{ of order } \overline{p},$$

$$\psi\left(L\right) = 1 + \sum_{j=1}^{\overline{q}} \psi_j L^j \text{ of order } \overline{q},$$

where all roots lie outside the unit circle, and white noise $\left(\varepsilon_t\right)$ such that:

$$\phi\left(L\right)\left(1-L\right)^d X_t = c + \psi\left(L\right)\varepsilon_t, \quad (1)$$

where L represents the lag operator and $\phi_p \neq 0$, $\psi_q \neq 0$.

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Properties of an $ARFIMA(\overline{p},d,\overline{q})$ process:

Let (X_t) be an $ARFIMA(\overline{p},d,\overline{q})$ process. Then:

- If $-0.5<d<0.5$, then $ARFIMA(\overline{p},d,\overline{q})$ is stationary with an autocorrelation function $\rho(k)$ decreasing hyperbolically $\rho(k)C.k^{2d-1}$.
- If $0<d<0.5$ and the roots of $\phi(L)$ lie outside the unit circle, the $ARFIMA(\overline{p},d,\overline{q})$ process is stationary and exhibits persistent dependence over time. In this case, the autocorrelations remain positive and decay at a hyperbolic rate as the lag increases, reflecting the presence of long-range memory in the process.
- If $-0.5<d<0$, then $ARFIMA(\overline{p},d,\overline{q})$ is stationary and anti-persistent. Autocorrelations decrease hyperbolically towards zero.
- If $d\geq 0.5$, then $ARFIMA(\overline{p},d,\overline{q})$ is non-stationary.
- If $d=0$, then $ARFIMA(\overline{p},d,\overline{q})$ reduces to the standard $ARMA(\overline{p},\overline{q})$ process with short memory.
- If $d=1$, an $ARIMA(\overline{p},1,\overline{q})$ is obtained.

2.2.2. *Integrated Generalized Autoregressive Conditional Heteroskedasticity Model:* $IGARCH(p,d,q)$.

The $GARCH(p,q)$ process can be expressed as an $ARMA$ process for the square of the error ε_t^2:

$$\varepsilon_t^2=\omega+\sum_{k=1}^{r}(\alpha_k+\beta_k)\varepsilon_{t-i}^2+\eta_t-\sum_{j=1}^{p}\beta_j.\eta_{t-j}, \quad (2)$$

with $\alpha_k=0$ if $k>q$ and $\beta_k=0$ if $k>p$ and $r=max(p,q)$, and $\eta_t=\varepsilon_t^2-h_t^2$.

h_t^2, the forecast of ε_t^2, satisfies $Var(\varepsilon_t/I_{t-1})=h_t^2=E(\varepsilon_t^2/I_{t-1})-(E(\varepsilon_t/I_{t-1}))^2$.

Therefore, $\eta_t=\varepsilon_t^2-h_t^2$ is the error associated with this forecast. It can be deduced that η_t is a white noise. According to equation (2), ε_t^2 is an $ARMA(r,p)$ process with

$\Phi(L)=1-\sum_{k=1}^{r}(\alpha_k+\beta_k).L^k$ and

$\Psi(L)=1-\sum_{j=1}^{p}\beta_j.L^j$. If $\alpha(L)=\sum_{i=1}^{q}\alpha_i.L^i$

and $\beta(L)=\sum_{j=1}^{p}\beta_j.L^j$ are set, then equation (2)

becomes:

$$(1-\alpha(L)-\beta(L))\varepsilon_t^2=\omega+(1-\beta(L))\eta_t. \quad (3)$$

The sufficient condition for the positivity of ε_t^2 is $\omega>0$, $\alpha_k\geq 0$, and $\beta_k\geq 0$ for $1\leq k\leq r$. The process is covariance stationary if $\Phi(L)$ has all its roots located outside the unit circle, which is equivalent to the condition

$\sum_{k=1}^{r}(\alpha_k+\beta_k)<1$. The lag polynomial $\Phi(L)$

could have a unit root, which is expressed by the condition

$\sum_{k=1}^{r}(\alpha_k+\beta_k)=\sum_{i=1}^{q}\alpha_i+\sum_{j=1}^{p}\beta_j=1.$

Engle and Bollerslev (1986) introduced a model that adheres to equation (6), calling it the Integrated GARCH process, abbreviated as IGARCH. An *IGARCH*(*p*, *q*) process with an integration order of 1 is defined as follows:

$$\Phi(L)(1-L)\varepsilon_t^2=\omega+\Psi(L)\eta_t, \quad (4)$$

where the polynomials $\Phi(L)$ and $\Psi(L)$ have all their roots located outside the boundary of the unit circle.

The relationships can be deduced:

$$\mu_t=E(X_t/I_{t-1}) \quad (5)$$
$$=\psi_0+\sum_{i=1}^{\overline{p}}\psi_{t-i}.X_{t-i}+\sum_{j=1}^{\overline{q}}\phi_{t-j}.\varepsilon_{t-j},$$

$$\sigma_t^2=Var(X_t/I_{t-1})=Var(\varepsilon_t/I_{t-1})=h_t^2,$$

$$X_t=\mu_t+\varepsilon_t.$$

2.2.3. *Fractionally Integrated Autoregressive Conditional Heteroscedasticity model:*

$FIGARCH(p,d,q)$.

A defining feature of IGARCH models is the persistent influence of past squared shocks $\eta_{t-j} = \varepsilon_{t-j}^2 - h_{t-j}^2$ for $j > 0$ on the variance ε_t^2. Drawing an analogy with ARMA and ARFIMA models, Baillie et al. (1996) expanded the IGARCH framework by permitting the integration parameter d to take values within the range $[0,1]$. This led to the development of the $FIGARCH(p,d,q)$ process.

$$\Phi(L)(1-L)^d \varepsilon_t^2 = \omega + \Psi(L)\eta_t, \quad (6)$$

where $\phi(L)$ and $\Psi(L)$ possess all their roots beyond the unit circle. The equation for conditional volatility h_t^2 can be deduced:

$$h_t^2 = \omega(1-\beta(L))^{-1} + \left[1-(1-\beta(L))^{-1}\Phi(L)(1-L)^d\right]\varepsilon_t^2. \quad (7)$$

The model parameters can be estimated using either the method proposed by Baillie et al. (1996) (commonly known as the BBM approach) or the technique introduced by Chung (1999).

2.2.4. *Fractionally Integrated Exponential GARCH model:*

$FIEGARCH(p,d,q)$

To account for asymmetry in financial data, Bollerslev and Mikkelsen (1996) introduced the Fractionally Integrated Exponential $GARCH$ $(FIEGARCH)$ model, which is characterized by the following formulation:

$$\ln(h_t^2) = \omega + \Phi(L)^{-1}(1-L)^{-d} \times (1+\alpha(L))g(z_t), \quad (8)$$

where $d \in [0,1]$ and

$$g(z_t) = \theta_1 z_t + \theta_2(|z_t| - E(|z_t|)), \quad z_t = \varepsilon_t / h_t, \; E(|\varepsilon_t / h_t|) = \sqrt{2/\pi}. \quad (9)$$

In this model, the parameters θ_1 and θ_2 capture the distinct effects of the sign and magnitude of shocks on volatility. Specifically, positive news affects volatility by $(\theta_1 + \theta_2)$, while negative news impacts volatility by $(\theta_1 - \theta_2)$. When $\theta_1 > 0$ and $\theta_2 > 0$, positive shocks have a stronger impact on volatility compared to negative shocks. Conversely, if $\theta_1 < 0$ and $\theta_2 > 0$, negative shocks lead to more pronounced volatility than positive ones.

2.2.5. *Fractionally Integrated Asymmetric Power ARCH model:*

$FIAPARCH(p,d,q)$

Tse (1998) introduced the FIAPARCH(p,d,q) model, incorporating the asymmetric power ARCH framework developed by Ding et al. (1993). The $FIAPARCH(p,d,q)$ model is defined by:

$$h_t^\delta = \omega + \left[1-(1-\beta(L))^{-1}\Phi(L)(1-L)^d\right] \times (|\varepsilon_t| - \gamma\varepsilon_t)^\delta, \quad (10)$$

where $d \in [0,1]$, $\delta > 0$, $-1 < \gamma < 1$.

The parameter γ captures the model's asymmetry, where $\gamma > 0$ indicates that negative shocks exert a stronger effect on volatility than positive shocks, and the reverse is true for $\gamma < 0$. The conditional variance demonstrates long-range dependence if $0 < \delta < 1$. The $FIAPARCH(p,d,q)$ model simplifies to the standard model when $\delta = 2$ and $\gamma = 0$. Estimation of the $FIAPARCH(p,d,q)$ parameters can be performed through the methods proposed by Baillie et al. (1996) or Chung (1999).

2.2.6. *Hyperbolic GARCH model:*

$HYGARCH(p,d,q)$

Davidson (2004) introduced the Hyperbolic $GARCH$ (HYGARCH) model to address a drawback of the FIGARCH model, specifically its tendency to produce infinite variance. The $HYGARCH(p,d,q)$ model is formulated as follows:

$$\Phi(L)\left((1-\alpha)+\alpha(1-L)^d\right)\varepsilon_t^2 = \omega + (1-\beta(L))\eta_t, \quad (11)$$

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where $d \in [0,1]$, $\alpha \geq 0$, $\phi(L)$, $\beta(L)$, and η_t are defined as previously.

The equation for conditional volatility can be deduced:

$$h_t^2 = \omega + \left[\begin{array}{l} 1 - (1-\beta(L))^{-1} \\ \times \Phi(L)\left((1-\alpha)+\alpha(1-L)^d\right) \end{array} \right] \varepsilon_t^2. \quad (12)$$

The $HYGARCH(p,d,q)$ model reduces to the $GARCH(p,q)$ model when $\alpha = 0 \ \ \Leftrightarrow ln(\alpha) < 0$ or $d = 0)$ and reduces to the $FIGARCH(p,d,q)$ model when $\alpha = 1 \ \ (\Leftrightarrow ln(\alpha) = 0$. If $d = 1$, then the $HYGARCH(p,d,q)$ model reduces either to a stationary $GARCH(p,q)$ $\alpha < 1 \Leftrightarrow ln(\alpha) < 0$, to an $IGARCH(p,q)$. $\alpha = 1 \Leftrightarrow ln(\alpha) = 0$, or to a $GARCH(p,q)$ $(\alpha > 1 \Leftrightarrow ln(\alpha) > 0)$ with explosive conditional variances. The process is stationary if $0 < \alpha < 1$ d non-stationary if $\alpha > 1$.

2.2.7. Joint Models: ARFIMA- FIGARCH type models

In joint models, (X_t) follows an $ARFIMA(\overline{p}, d_{ARFIMA}, \overline{q})$ process of orders $\overline{p}$, $d_{ARFIMA} \in \mathbb{Q}$, and $\overline{q}$ such that:

$$\phi(L)(1-L)^{d_{ARFIMA}} X_t = c + \psi(L)\varepsilon_t, \quad (13)$$

with $\phi(L)$ and $\psi(L)$ being lag polynomials of orders $\overline{p}$ and $\overline{q}$, respectively, nd having all their roots located outside the unit circle, along with (ε_t) denoting white noise.

The volatility h_t follows the $FIGARCH(p, d_{FIGARCH}, q)$, $FIEGARCH(p, d_{FIEGARCH}, q)$, $FIAPARCH(p, d_{FIAPARCH}, q)$, or $HYGARCH(p, d_{HYGARCH}, q)$ processes, as described previously.

3. RESULTS

The analysis starts by presenting the Augmented Dickey-Fuller (ADF) test results.

3.1. Stationarity analysis of the six indices

The stationarity of the six-time series was previously confirmed in Benbachir's (2025) study, where the Augmented Dickey-Fuller (ADF) test results showed that the logarithmic returns of the six indices are stationary.

3.2. Analysis of double long memory of the six African stock markets

In this section, the dual long-memory behavior in both the conditional mean and variance of the six indices is modeled by estimating four combined models, namely ARFIMA-FIGARCH, ARFIMA-FIEGARCH, ARFIMA-FIAPARCH, and ARFIMA-HYGARCH. These models are evaluated under various distributional assumptions, including the Normal distribution, Student's t-distribution, Skewed Student's t-distribution, and Generalized Error Distribution (GED). Several parameter configurations were tested for each model, and only those with parameters statistically significant at the 5% level were selected. The models that provided the best fit to the data were ultimately chosen.

3.2.1. Estimation of ARFIMA-FIGARCH models for the JSE Index

Four models – ARFIMA-FIGARCH, ARFIMA-FIEGARCH, ARFIMA-FIAPARCH, and ARFIMA-HYGARCH – are applied to the JSE Index, considering different distribution assumptions. Three models are identified as the best fit for the data, and the estimations for these models are presented in Table 1.

As shown in Table 1, the models that best fit the data for the JSE index are ARFIMA(1,-0.046,1)-FIGARCH(1,0.431,1), ARFIMA(0,-0.117,1)-FIGARCH(1,0.422,1), and ARFIMA(1,-0.126,0)-FIGARCH(1,0.422,1), under the Skewed Student distribution. All parameters, including the d_{ARFIMA} and $d_{FIGARCH}$ factionary parameters, are statistically significant at the 1% level. This indicates that the three joint models effectively capture the double long memory property in both conditional mean and conditional volatility for the JSE index, thereby challenging the market effi-

Table 1. Estimation of ARFIMA-FIGARCH models for JSE index

ARFIMA(1,d,1)-FIGARCH (1,d,1) (Skewed Student distribution)			ARFIMA(0,d,1)-FIGARCH (1,d,1) (Skewed Student distribution)			ARFIMA(1,d,0)-FIGARCH (1,d,1) (Skewed Student distribution)		
	Coef.	**t-prob**		**Coef.**	**t-prob**		**Coef.**	**t-prob**
d-Arfima	–0.0462	0.001	Cst (M)	0.00041	0.000	Cst (M)	0.00040	0.000
AR(1)	–0.8179	0.000	d-Arfima	–0.1169	0.000	d-Arfima	–0.1257	0.000
MA(1)	0.8545	0.000	MA(1)	0.1032	0.001	AR(1)	0.1118	0.001
d-Figarch	0.4310	0.000	d-Figarch	0.4221	0.000	d-Figarch	0.4217	0.000
$ARCH(\varphi_1)$	0.2370	0.000	$ARCH(\varphi_1)$	0.2422	0.000	$ARCH(\varphi_1)$	0.2423	0.000
$GARCH(\beta_1)$	0.62141	0.000	$GARCH(\beta_1)$	0.6158	0.000	$GARCH(\beta_1)$	0.6157	0.000
Asymmetry	–0.1221	0.000	Asymmetry	–0.1262	0.000	Asymmetry	–0.1267	0.000
Tail	9.5657	0.000	Tail	9.3493	0.000	Tail	9.354425	0.000

ciency hypothesis. These results indicate that the Johannesburg stock market exhibits inefficiency in its weak form.

In all three models, the d_{ARFIMA} parameter falls between –0.5 and 0, indicating that the logarithmic return series is covariance stationary. This leads to a gradual, hyperbolic decline in the Autocorrelation Function (ACF), which is a hallmark of long-memory dynamics. The negative d_{ARFIMA} value suggests mean-reverting behavior, meaning deviations from the mean tend to correct over time, which is important for financial time series modeling and risk management.

For the $d_{FIGARCH}$ parameter, which is approximately 0.4 (close to 0.5) and within the range $0 < d_{FIGARCH} < 0.5$, the ACF of the volatility series decays hyperbolically rather than exponentially. This implies that past volatility influences future volatility for an extended period, although the effect diminishes gradually. A $d_{FIGARCH}$ value of 0.4 indicates that the volatility series is covariance stationary but exhibits long-memory characteristics, crucial for accurate forecasting and risk management. The models successfully capture the volatility clustering in the JSE index, where periods of elevated volatility are consistently followed by further high volatility, and periods of low volatility are followed by more tranquil market conditions.

3.2.2. Estimation of the ARFIMA-FIGARCH models for the MASI Index

Four models – ARFIMA-FIGARCH, ARFIMA-FIEGARCH, ARFIMA-FIAPARCH, and ARFIMA-HYGARCH – are applied with various distributional assumptions to the MASI index. Table 2 presents the model that provides the best fit for the MASI index data.

Table 2. Estimation of the ARFIMA-FIGARCH models for the MASI index

ARFIMA (1,d,0)-FIGARCH (1,d,1) (GED distribution)		
	Coefficient	**t-prob**
d-Arfima	0.0273	0.000
AR(1)	0.0315	0.028
d-Figarch	0.3818	0.000
$ARCH(\varphi_1)$	0.4958	0.000
$GARCH(\beta_1)$	0.6736	0.000
G.E.D.(DF)	1.1854	0.000

According to Table 2, the model that best fits the MASI index data is ARFIMA (1,0.027,0)-FIGARCH (1,0.382,1) under the GED distribution. In this model, all parameters, including d_{ARFIMA} and $d_{FIGARCH}$, are significant at the 1% confidence level, while the AR(1) coefficient is significant at the 5% level. This combined model successfully captures the dual long-range dependence in both the conditional mean and variance for the MASI index, suggesting a deviation from weak-form market efficiency. These results demonstrate that the Moroccan stock market is inefficient in its weak form.

The d_{ARFIMA} parameter for the return series is 0.028, indicating that it is covariance stationary and exhibits long-range dependence properties. The gradual, hyperbolic decline in the autocorrelation function indicates that past values have a prolonged influence on future values, although the impact weakens progressively over time. A d_{ARFIMA} close to 0 means the series reverts to its

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mean quickly, with deviations correcting faster and recent data points being more relevant for forecasting.

The $d_{FIGARCH}$ parameter is approximately 0.4, also indicating covariance stationarity and long-range depedece characteristics in the volatility series. This value, close to 0.5, means that the autocorrelation in the conditional variance decreases hyperbolically, showing a slower rate of decrease. This is essential for precise forecasting and effective risk management, as it reflects the volatility clustering effect, where phases of elevated volatility tend to be succeeded by similarly volatile periods, and low volatility periods follow one another.

3.2.3. Estimation of the ARFIMA-FIGARCH models for the BSE Index

Four models – ARFIMA-FIGARCH, ARFIMA-FIEGARCH, ARFIMA-FIAPARCH, and ARFIMA-HYGARCH – are applied, with different distribution assumptions, to the BSE index. Table 3 presents the model that provides the best fit for the BSE index data.

Table 3. Estimation of the ARFIMA-FIGARCH models for the BSE index

ARFIMA (1,d,0)-FIGARCH (1,d,0) (Student distribution)		
	Coefficient	**t-prob**
d-Arfima	0.0729	0.000
AR(1)	−0.0260	0.023
d-Figarch	0.2997	0.000
$GARCH(\beta_1)$	0.1228	0.050
Student (DF)	2.1943	0.000

Table 3 shows that the model that best fits the BSE index data is ARFIMA (1,0.073,0)-FIGARCH (1,0.3,0) under the Student distribution. All parameters, including d_{ARFIMA} and d_{ARFIMA}, are statistically significant at the 1% level, except for the coefficients of AR(1) and $GARCH(\beta_1)$, which are significant at the 5% level. This model captures the double long-range dependence property in both the conditional mean and conditional variance of the BSE index, challenging the market efficiency hypothesis. These results suggest that the Botswana stock market lacks efficiency in its weak form.

The d_{ARFIMA} parameter of 0.073 indicates that the return series is covariance stationary and exhibits long-range dependence properties. The autocorrelation function exhibits a hyperbolic decline, indicating that past values exert a prolonged influence on future values, although this influence gradually diminishes over time. A d_{ARFIMA} close to 0 implies that the return series reverts to its mean more swiftly, with deviations being corrected more rapidly and volatility spikes having a shorter-lasting effect.

The $d_{FIGARCH}$ parameter is approximately 0.3, indicating that the volatility series is covariance stationary and shows long-range dependence characteristics. The value, being near 0.5, indicates that the autocorrelations in volatility decay at a hyperbolic rate rather than an exponential one, leading to a slower decline. This suggests that past volatility exerts a prolonged influence on future volatility. The FIGARCH model successfully captures the volatility clustering observed in the BSE index, where periods of heightened volatility tend to be followed by subsequent periods of high volatility, and the same applies to periods of low volatility.

3.2.4. Estimation of the ARFIMA-FIGARCH models for the NGX index

Four models – ARFIMA-FIGARCH, ARFIMA-FIEGARCH, ARFIMA-FIAPARCH, and ARFIMA-HYGARCH – are tested, with various distribution assumptions for the NGX index. The models that best fit the NGX index data are displayed in Table 4.

As shown in Table 4, the models that best fit the data for the NGX index are ARFIMA (0,0.105,1)-FIGARCH (1,0.374,1) under a normal distribution and ARFIMA (1,0.061 ,0)-FIGARCH (1,0.396,1) under the GED distribution. All parameters, including d_{ARFIMA} and $d_{FIGARCH}$, are statistically significant at the 1% level. These two models effectively capture the double long-range dependence property in both conditional mean and conditional variance of the NGX index, challenging the market efficiency hypothesis. The results suggest that the Nigerian stock market exhibits inefficiency in its weak form.

For both models, the d_{ARFIMA} values are 0.105 or 0.061, indicating that the return series is covari-

Table 4. Estimation of the ARFIMA-FIGARCH models for NGX index

ARFIMA (0,d,1)-FIGARCH (1,d,1) (Normal distribution)			ARFIMA (1,d,0)-FIGARCH (1,d,1) (GED distribution)		
	Coefficient	t-prob		Coefficient	t-prob
d-Arfima	0.1049	0.000	d-Arfima	0.0613	0.000
MA(1)	0.1097	0.002	AR(1)	0.0785	0.000
d-Figarch	0.3740	0.000	d-Figarch	0.3964	0.000
$ARCH(\varphi_1)$	0.5277	0.000	$ARCH(\varphi_1)$	0.5130	0.000
$GARCH(\beta_1)$	0.7107	0.000	$GARCH(\beta_1)$	0.6556	0.000

ance stationary with long-memory properties. The autocorrelation function decays hyperbolically, reflecting persistence in the series and mean-reverting behavior.

The $d_{FIGARCH}$ parameter is approximately 0.4, satisfying $0 < d_{FIGARCH} < 0.5$. Since this value approaches 0.5, it indicates that the autocorrelations in volatility decay hyperbolically at a slower rate rather than exponentially. This long-memory characteristic of volatility is crucial for accurate forecasting and risk management. The FIGARCH model successfully captures volatility clustering in the *BSE* index, where periods of elevated volatility tend to be followed by similar periods of high volatility, and low volatility periods are followed by more subdued volatility.

3.2.5. *Estimation of the ARFIMA-FIGARCH models for the EGX Index*

Four models – ARFIMA-FIGARCH, ARFIMA-FIEGARCH, ARFIMA-FIAPARCH, and ARFIMA-HYGARCH – are applied, with different distribution assumptions to the EGX index. The models that best represent the *EGX* index data are outlined in Table 5.

Tabel 5 shows the models that best fit the data for the EGX index are ARFIMA(1,0.071,1)-FIAPARCH(0,0.308,1) under normal distribution and ARFIMA(1,0.071,1)-HYGARCH(0,0.261,1) under GED distribution. All the parameters, including d_{ARFIMA} and $d_{AFIGARCH}$, are statistically significant at a 1% significance level, suggesting that the two joint models allow capturing the double long-range dependence property in the conditional mean and conditional variance of the EGX index, which contradicts the market efficiency hypothesis. These findings confirm the inefficiency of the Egyptian stock market in its weak form.

For both models, the d_{ARFIMA} parameter is approximately 0.07, indicating that the return series is covariance stationary with long-range dependence characteristics. The autocorrelation function of the return series decays hyperbolically rather than exponentially, reflecting both the persistence of the series and its tendency to revert to its mean.

The $d_{FIGARCH}$ parameters are around 0.31 or 0.26, satisfying $0 < d_{FIGARCH} < 0.5$, with the values being quite close to 0.5. This suggests that the volatility series also decays hyperbolically rather than

Table 5. Estimation of the ARFIMA-FIGARCH models for the EGX index

ARFIMA (1,d,1)-FIAPARCH (0,d,1) (Normal distribution)			ARFIMA (1,d,1)-HYGARCH (0,d,1) (Normal distribution)		
	Coef.	t-prob		Coef.	t-prob
d-Arfima	0.0712	0.007	d-Arfima	0.0714	0.005
AR(1)	–0.3555	0.039	AR(1)	–0.3668	0.035
MA(1)	0.4894	0.001	MA(1)	0.5003	0.001
d-Figarch	0.3084	0.000	d-Figarch	0.2614	0.000
$ARCH(\varphi_1)$	–0.1612	0.000	$ARCH(\varphi_1)$	–0.1438	0.006
$APARCH(\gamma_1)$	0.1984	0.001	Log Alpha (HY)	0.1929	0.000
$APARCH(\delta)$	2.2352	0.000			

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exponentially. The proximity of $d_{FIGARCH}$ to 0.5 implies that autocorrelations in volatility decay more slowly, indicating long-range dependence behavior. The FIGARCH model effectively captures volatility clustering in the EGX index, where periods of elevated volatility tend to be succeeded by additional periods of heightened volatility.

3.2.6. Estimation of the ARFIMA-FIGARCH models for the BRVM Index

Four models – ARFIMA-FIGARCH, ARFIMA-FIEGARCH, ARFIMA-FIAPARCH, and ARFIMA-HYGARCH – are applied, with various distribution assumptions to the BRVM index. The models that provide the best fit for the BRVM index data are shown in Table 6.

As shown in Table 6, the models that best fit the *BRVM* index data are ARFIMA(1,0.149,0)-FIGARCH(0,0.287,1) under the normal distribution and ARFIMA(0,0.1696,1)-FIGARCH(0,0.287,1) under the GED distribution. All parameters, including d_{ARFIMA} and $d_{AFIGARCH}$, are statistically significant at the 1% level. These results suggest that both models effectively capture the double long-range dependence property in the conditional mean and conditional variance of the BRVM index, indicating a deviation from the market efficiency hypothesis. As a result, the findings indicate that the BRVM stock market is inefficient in its weak form.

For both models, the d_{ARFIMA} parameter is around 0.15 or 0.17, indicating that the return series is covariance stationary and exhibits long-range dependence properties. The autocorrelation function decays hyperbolically rather than exponentially, reflecting the series' persistence and tendency to revert to its mean.

The $d_{FIGARCH}$ parameter is approximately 0.3, which satisfies the condition $0 < d_{FIGARCH} < 0.5$, and is quite close to 0.5. This suggests that the volatility series also decays hyperbolically, with autocorrelations in the conditional variance decaying more slowly. This long-memory behavior is crucial for accurate forecasting and effective risk management. The FIGARCH model successfully captures volatility clustering in the BRVM index, where periods of high volatility are typically followed by additional high volatility.

4. DISCUSSION

Various joint models, namely ARFIMA-FIGARCH, ARFIMA-FIEGARCH, ARFIMA-FIAPARCH, and ARFIMA-HYGARCH, were employed in this study, confirming the presence of dual long-range dependence in both returns and volatility, suggesting significant inefficiencies in the six major African stock markets.

The findings of this study align with previous research, such as that of Boubaker and Makram (2012) and Odonkor et al. (2022), where long-range dependence was also observed in various African stock markets. Together, these studies highlight a critical pattern: numerous emerging markets, especially in Africa, display inefficiencies that deviate from the EMH framework.

Additionally, this study aligns closely with Yunus (2022), whose research on the MINT stock markets also identifies significant long memory in both the conditional mean and conditional variance of the Nigerian stock market. This correspondence emphasizes the inefficiencies present in the Nigerian stock market and the broader MINT context, suggesting that investors may capitalize on predict-

Table 6. Estimation of the ARFIMA-FIGARCH type models for the BRVM index

ARFIMA (1, d, 0)-FIGARCH (0, d, 1) (Normal distribution)			ARFIMA (0, d, 1)-FIGARCH (0, d, 1) (GED distribution)		
	Coefficient	**t-prob**		**Coefficient**	**t-prob**
d-Arfima	0.149099	0.000	d-Arfima	0.169613	0.000
AR(1)	–0.191529	0.000	MA(1)	–0.208370	0.000
d-Figarch	0.286625	0.000	d-Figarch	0.286986	0.000
$ARCH(\varphi_1)$	–0.165595	0.000	$ARCH(\varphi_1)$	–0.147187	0.000
			G.E.D. (DF)	1.221611	0.000

able behavior and volatility patterns within the Nigerian market.

This analysis further supports the findings of Ziky and Ouali (2021) regarding the Moroccan MASI index, which identified long memory in the Moroccan stock market, thereby reinforcing arguments against weak-form market efficiency in that context. Additionally, it contradicts Onour's (2010) findings of a short-memory process in Moroccan stock returns.

The findings of this study provide evidence of long memory in the South African market, contradicting the results of Arewa et al. (2023), who concluded that the South African stock market exhibits a short memory process, suggesting a weak-form efficiency. However, long memory in the Nigerian stock market was reported by Arewa et al. (2023), which is consistent with the findings presented here.

Moreover, the present study identifies significant long memory in both the conditional mean and conditional variance across the six major African stock markets, contrasting with the study by Turkyilmaz and Balibey (2014), where long memory was observed only in volatility.

CONCLUSION

The aim of this study is to assess the weak-form informational efficiency of the six major African stock markets – Johannesburg, Casablanca, Botswana, Nigeria, Egypt, and the Regional Stock Exchange – by modeling the dual long-range dependence properties in both conditional mean and conditional variance. Advanced econometric models, including ARFIMA-FIGARCH, ARFIMA-FIEGARCH, ARFIMA-FIAPARCH, and ARFIMA-HYGARCH, were employed to test for the presence of double long-range dependence in these markets, under various distribution assumptions, including Normal, Student's t, Skewed Student's t, and Generalized Error Distribution (GED). Utilizing these four models enables the selection of the one that best fits the data.

The empirical results for all models showed that across all six indices, an ARFIMA-FIGARCH model fits the data best, with both the d_{ARFIMA} and $d_{FIGARCH}$ fractional parameters being statistically significant at the 1% level. These models capture the long-memory dynamics in both returns and volatility, suggesting weak-form inefficiency for all indices. The d_{ARFIMA} values for all indices indicate that the return series is covariance stationary and displays long-memory characteristics. For the JSE index, the $d_{FIGARCH}$ value points to long-memory in volatility, where past volatility influences future volatility for an extended period. In the MASI index, the d_FIGARCH value demonstrates long-memory and covariance stationarity in the volatility series. Similarly, the $d_{FIGARCH}$ value for the BSE index reflects long-range dependence and covariance stationarity in volatility. For the NGX index, the $d_{FIGARCH}$ values suggest that volatility autocorrelations decay at a slower, hyperbolic rate rather than exponentially. The EGX index's $d_{FIGARCH}$ values show that volatility decays hyperbolically, reflecting persistent long-memory behavior. Similarly, for the BRVM index, the $d_{FIGARCH}$ values imply hyperbolic decay in volatility, indicating long-memory dynamics.

The findings of this study provide significant practical implications for various market participants. Investors can capitalize on long-range dependence and inefficiency in African stock markets to develop trading strategies that exploit historical price patterns. For risk managers, the observed persistent volatility underscores the need for models that incorporate long-memory effects in both returns and volatility to improve risk forecasting. The weak-form inefficiency revealed by the study indicates that markets are not fully efficient, highlighting the need for stronger regulatory measures, enhanced transparency, and better liquidity. Furthermore, the introduction of advanced financial instruments, such as derivatives, could help manage prolonged volatility risks, while foreign investors may view these inefficiencies as both a risk and an opportunity for higher returns.

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Future research could explore the underlying factors contributing to long memory in African stock markets. Investigating market microstructure, the role of institutional investors, and the impact of macroeconomic variables could provide further insights into the persistence of price behavior. Additionally, comparative studies examining long memory characteristics across regions or asset classes could deepen our understanding of market efficiency complexities.

AUTHOR CONTRIBUTIONS

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Investigation: Soufiane Benbachir.
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