“Exploring multifractality in African stock markets: A multifractal detrended fluctuation analysis approach”

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EXPLORING MULTIFRACTALITY IN AFRICAN STOCK MARKETS: A MULTIFRACTAL DETRENDED FLUCTUATION ANALYSIS APPROACH

Abstract

This paper investigates the multifractal behavior of the six largest African stock markets, including the Johannesburg, Casablanca, Botswana, Nigerian, Egyptian, and Regional Stock Exchanges. Despite the growing significance of these markets in the global economy, there is limited understanding of their underlying dynamics, particularly regarding their multifractal properties. This lack of knowledge raises concerns about the informational efficiency of these markets, as traditional models may not adequately capture the complexities of price movements. To achieve the goals of the study, the Multifractal Detrended Fluctuation Analysis (MF-DFA) method is applied to capture the multifractal dynamics, and shuffling and phase randomization techniques are performed to identify the sources of the multifractality of the six African stock markets. The empirical results, derived from the generalized Hurst exponents, Rényi exponents, and Singularity spectrum, show that all six stock markets display multifractal behavior, characterized by irregular and complex price movements that vary across different scales and timeframes. Additionally, the study finds that both long-term correlations and heavy-tailed distributions contribute to the observed multifractality. Long-term correlations lead to persistent price trends, challenging the Efficient Market Hypothesis (EMH), while heavy tails increase market unpredictability by raising the likelihood of extreme events like crashes or booms. The findings have significant practical implications for stakeholders in African stock markets, enabling investors and portfolio managers to enhance risk assessment and develop effective trading strategies while helping market regulators improve efficiency and stability through appropriate policies. Financial institutions can also refine risk management frameworks to better account for extreme events.

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INTRODUCTION

Stock markets are complex systems characterized by non-linear behaviors, long memory, and dynamics that vary across different time scales, a phenomenon known as multifractality. Empirical studies have demonstrated that financial series exhibit non-linearity, temporal dependencies over long periods, and fractal dimensions that change at different scales. Traditional analysis methods often fail to capture these complexities. By focusing on multifractal characteristics, this study addresses that gap, revealing the presence of long-term correlations and heavy-tailed distributions that standard models fail to capture. This multifractal approach is crucial for understanding the complex, non-linear dynamics of stock markets, providing deeper insights into volatility and risk across

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varying market conditions and time scales. Multifractal Detrended Fluctuation Analysis (MF-DFA) offers a more thorough grasp of market scaling properties and highlights potential inefficiencies that traditional methods might overlook.

The MF-DFA method, introduced by Kantelhardt et al. (2002), is an innovative approach that extends Detrended Fluctuation Analysis (DFA). The DFA method was introduced in the 1990s by Peng et al. (1994) to address the limitations of traditional time series analyses, which were often affected by trends or non-stationarities, making it difficult to detect long-term correlations. DFA enables the removal of local trends and measures the residual fluctuations at various scales, providing a valuable tool for analyzing complex data. While DFA focuses on identifying long-term correlations, MF-DFA enables the examination of behaviors across multiple scales, capturing the multifractal nature of certain time series.

The motivation for analyzing stock markets through the lens of multifractality stems from several important factors. First, it reveals the intricate complexity and scaling properties of market returns, offering a deeper perspective on market behavior and volatility. Additionally, it enables the identification of inefficiencies by assessing how well prices reflect available information. The multifractal approach provides valuable insights into risk and volatility across different time horizons, aiding investors in developing more effective risk management strategies, while assisting market regulators in enhancing efficiency and stability through appropriate policies.

1. LITERATURE REVIEW

This literature review delves into the growing application of the MF-DFA method in financial research, in particular its role in capturing multifractality within stock market exchanges. Over the past two decades, an increasing body of empirical research has employed MF-DFA to analyze financial markets, revealing key insights into their multifractal characteristics.

Early research in Asian stock markets, including studies by Wang et al. (2009), Gu and Huang (2019), Hong-Yong and Tong-Tong (2018), Zhu and Zhang (2018), Xinsheng et al. (2013), and Faheem et al. (2020), employed MF-DFA to explore the multifractal behavior of these markets. Specifically, Wang et al. (2009) examined the impact of price limit reforms on the Shenzhen stock market, using the daily closing price of Shenzhen Component Index (SZCI) from April 3, 1991 to December 15, 2008. Through the application of the MF-DFA technique, they determined that the level of multifractality could act as an indicator of market efficiency under certain conditions, suggesting that the efficiency of the Shenzhen stock market improved after the reforms. In a related analysis, but by using a high frequency of 5 minutes from the Shenzhen Stock Exchange (SZSE), Gu and Huang (2019) used MF-DFA to study the multifractal dynamic of the Shenzhen Stock market, which is one of the two main stock exchanges in China. They found that fluctuations are multifractal, with multifractality arising from both the probability density function's width and long-term correlations. In a different study, Hong-Yong and Tong-Tong (2018) focused on the stock, bond, and fund markets by studying the Shanghai Composite Index, Shanghai Bond Index, and Shanghai Fund Index. Using MF-DFA and multifractal spectrum analysis, they found multifractality in all three indices and noted varying fractal characteristics in cross-correlations across time scales. In another study, Zhu and Zhang (2018) used MF-DFA to investigate the multifractal properties of the CSI 800 index consisting of the CSI 500 index and CSI 300 index, aiming to reflect the performance of stocks with the large, mid and small sizes of China A-share market. They found that the returns exhibit multifractal properties, which may originate from the long-range correlations. In a comparable study analyzing the stock index futures market, Xinsheng et al. (2013) investigated the multifractal characteristics of the Chinese stock index futures market using MF-DFA and multifractal spectrum analysis on a 2942 10-minute closing prices dataset. The data focused on the IF1009 futures contract, tied to the CSI 300 index, one of the most actively traded contracts in 2010. The study identified significant multifractal behavior in price

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movements, underscoring the complex dynamics of the Chinese futures market. They identified two main sources of multifractality: long-term correlations and heavy-tailed probability distributions, with long-term correlations being the primary factor. In their study, Faheem et al. (2020) utilized MF-DFA to analyze long-term dependence and multifractal parameters in stock indices from nine MSCI emerging Asian economies. The findings revealed varying degrees of multifractality, suggesting long-term correlations consistent with the fractal market hypothesis. The Chinese and South Korean markets showed the least long-range dependence, while India and Malaysia had the highest levels. These results imply potential market inefficiencies, indicating that institutional investors may use active trading strategies to enhance portfolio profitability.

In a further analysis, Hasan and Mohammad (2015) used MF-DFA to analyze American and Asian stock markets from July 1, 2002 to December 31, 2013, across pre-crisis, crisis, and post-crisis phases. They found evidence of multifractality in the indices of all markets. They observed significant non-linearity during the 2007–2008 crisis in the U.S., Japan, Hong Kong, South Korea, and Indonesia. Outside the crisis period, the tail exponent of return distributions increased across all markets studied. By shuffling the original index return series and generating the surrogate series, they showed both the long-range correlations and fat tails distributions.

In a distinct analysis, Wang et al. (2014) and Chen and Wang (2017) were interested in investigating the American stock market. Specifically, Wang et al. (2014) analyzed daily returns of the NASDAQ Composite Index using MF-DFA and found that the return series was leptokurtic and did not follow a normal distribution. They identified long-term memory as a primary source of multifractality and showed that both the original and reordered return series exhibited multifractality. The study concluded that the stock market did not achieve weak-form efficiency. Similarly, Chen and Wang (2017) utilized MF-DFA to investigate the multifractal behavior of stock and portfolio returns based on size and book-to-market (BM) ratios. They constructed six portfolios from all companies listed on the NYSE, AMEX, and NASDAQ by categorizing them into the bottom 30%, middle 40%, and top 30% based on size and BM ratios. Their findings demonstrated significant multifractality in portfolio returns, primarily driven by long-term dependence. The study also revealed persistent cross-correlations for small fluctuations and anti-persistent cross-correlations for large fluctuations between portfolio returns and the market average return.

In another investigation of Central and Eastern European (CEE) stock markets, Milo et al. (2020) used MF-DFA to assess the market efficiency of seven CEE stock markets. Their analysis of blue-chip indices up to August 2018, following seasonal and trend decompositions, revealed long-term correlations in stock returns, indicating that these markets are not efficient and are still developing.

In a separate study, Benbachir and El Alaoui (2011a) used MF-DFA to explore the multifractal properties of the Moroccan All Shares Index (MASI) and the Moroccan Most Active Shares Index (MADEX) from the Casablanca Stock Exchange. They found that long-range temporal correlations and fat-tail distributions are key sources of multifractality. The MASI index showed more complex multifractal behavior than the MADEX index, indicating that larger indices tend to have richer multifractal characteristics.

Other authors, such as Ezgi and Gazanfer (2013), Günay (2014), and Benbachir and El Alaoui (2011b), have focused on the study of multifractality in the foreign exchange markets. Ezgi and Gazanfer (2013) evaluated the performance of MF-DFA and Wavelet Transform Modulus Maxima (WTMM) in identifying both mono- and multifractality in the USD/TRY currency pair following the 2001 crisis. Their analysis revealed a moderate degree of multifractality and indicated that WTMM was less effective in detecting it. Furthermore, they applied MF-DFA to investigate the multifractal properties of other Eastern European currencies, including the Russian ruble and the Hungarian forint. In a similar study, Günay (2014) used MF-DFA to analyze the multifractality of returns for GBP/USD, EUR/USD, USD/JPY, and USD/CHF. He found that GBP/USD returns were monofractal, while the returns for EUR/USD, USD/JPY, and USD/CHF exhibited multifractality. For

EUR/USD and USD/JPY, the multifractality was attributed to thick-tailed distributions, whereas for USD/CHF, it was linked to both long-term memory and thick tails. The study also revealed an ambiguous relationship between market liquidity and multifractality. In a comparable analysis, Benbachir and El Alaoui (2011a) applied MF-DFA to explore the multifractal properties of the Moroccan Dirham against the US Dollar from January 4, 1999 to August 18, 2010. The analysis identified long-range temporal correlations and fat-tail distributions as the main sources of multifractality, with long-range correlations being the dominant factor.

Other authors have explored sectoral stock markets, including Poojari et al. (2022) and Raza et al. (2024). Poojari et al. (2022) employed MF-DFA to evaluate the efficiency of stock markets in 37 banks across ten emerging and frontier economies. Their findings indicated significant multifractality, with frontier economies demonstrating the highest complexity due to underdeveloped infrastructure. The study highlighted that time-varying Hurst exponents are crucial for investment decisions and confirmed that most banks exhibit anti-persistent behavior. Similarly, Raza et al. (2024) analyzed the efficiency of Dow Jones conventional and Islamic sectoral stock markets from January 1, 2010, to August 1, 2022, comparing the pre-COVID (2010-2019) and COVID-19 (2020-2022) periods. Prior to the pandemic, the Islamic healthcare sector was the most efficient in the short term, while the financial sector excelled in long-term efficiency. During COVID-19, the conventional financial sector was the most efficient in the short term, whereas utilities demonstrated the highest long-term efficiency. The study included ten sectoral indices, such as utilities, telecommunications, technology, healthcare, financials, and energy.

This literature review highlights the expanding application of the MF-DFA method in financial research, emphasizing its effectiveness in revealing the multifractal characteristics of various stock markets. The studies reviewed demonstrate that multifractality is a crucial indicator of market dynamics, inefficiencies, and investor behavior across different regions and economic conditions, providing valuable insights for both researchers and practitioners.

In line with previous research, this study thoroughly examines the multifractal behavior of the six largest African stock markets. Utilizing MF-DFA, it accomplishes two primary objectives: evaluating the level of multifractality and uncovering the fundamental sources that contribute to it.

2. METHODOLOGY

Before discussing the methods applied in this paper, the data used are first described.

2.1. Data

The data for this study consist of daily closing prices of indices across the largest six African stock markets:

- Johannesburg Stock Exchange (JSE): Founded in 1887, it is the largest in Africa.
- Casablanca Stock Exchange (MASI): Founded in 1929, it is the second-largest stock exchange in Africa.
- Botswana Stock Exchange (BSE): Officially established in 1989, it is the third-largest in Africa.
- Nigerian Exchange (NGX): It was established in 1960 as the Lagos Stock Exchange, renamed the Nigerian Stock Exchange (NSE) in 1977, and later rebranded as the Nigerian Exchange Group (NGX) in 2021 following its demutualization. It is the fourth-largest stock exchange in Africa.
- Egyptian Exchange (EGX): Established in 1883, it was formed by the merger of the Alexandria and Cairo exchanges.
- Regional Stock Exchange (BRVM): Established in 1996, it serves as a regional stock exchange for eight West African countries in the West African Economic and Monetary Union (WAEMU).

The data of this study span from 01/01/2011 to 09/08/2024, comprising nearly 3,380 observations. However, the BRVM index data are only available

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from 04/03/2014. All data were downloaded from the website www.investing.com.

The index prices were then converted into logarithmic returns $r_t = \ln(P_t) - \ln(P_{t-1})$, where P_t denotes the index price, and ln corresponds to the natural logarithm.

2.2. Method

According to Kantelhardt et al. (2002), MF-DFA consists of five distinct steps. Consider a time series $X = X(k)$, $1 \le k \le N$, representing a financial series, where N is the length of the series. It is assumed that this series has a compact support, meaning that $X(k) = 0$ for only a negligible fraction of the values.

Step 1: In this first step, the study determines the profile $Y = (Y(i))_{1 \le i \le N}$ of the series X defined by:

$$Y(i) = \sum_{k=1}^{N} \left(X(k) - \bar{X} \right), \quad (1)$$

where $\bar{X}$ is the mean of the series X.

Step 2: For a given time scale s such that $2 \le s \le N/3$, the profile Y is divided into $Ns = Int(N/s)$ non-overlapping segments of the same length s, where $Int(.)$ represents the function that gives the integer part of a real number. Since N is generally not a multiple of s, a short part at the end of the profile may be neglected. To incorporate all the ignored parts of the series, the same procedure is repeated starting from the end of the profile. Thus, $2Ns$ segments are obtained.

In each segment, the Ordinary Least Squares (OLS) method is used to properly fit the data in each segment with a local trend. Let us denote by $p_v^m(i) = \alpha_0^v + \alpha_1^v.i + \cdots + \alpha_m^v.i^m$ the fitting polynomial for the v-th segment. In the empirical study, the order m of the fitting polynomial can be quadratic, cubic, or even of a higher order. Choosing an appropriate value of m can avoid overfitting the series. In this study, $m = 2$ is chosen.

Step 3: After determining $p_v^m(i)$, the variances $F^2(v,s)$ is calculated for all time scales s

- For $1 \le v \le Ns$, the variance $F^2(v,s)$ is defined by:

$$F^2(v,s) = \frac{1}{s} \sum_{i=1}^{s} \left[Y((v-1)s+i) - p_v^m(i) \right]^2 \quad (2)$$

- For $Ns+1 \le v \le 2Ns$, the variance $F^2(v,s)$ is defined by:

$$F^2(v,s) = \frac{1}{s} \sum_{i=1}^{s} \left[Y((N-v-Ns)s+i) - p_v^m(i) \right]^2 \quad (3)$$

Step 4: By averaging the variances over all segments, the fluctuation function $F_q(s)$ of order q is obtained defined by:

$$For\ q \ne 0 : F_q(s) = \left[\frac{1}{2Ns} \sum_{i=1}^{2Ns} \left(F^2(v,s) \right)^{\frac{q}{2}} \right]^{\frac{1}{q}}$$
$$For\ q = 0 : F_0(s) = exp\left[\frac{1}{4Ns} \sum_{i=1}^{2Ns} ln\left(F^2(v,s) \right) \right] \quad (4)$$

The purpose of the MF-DFA procedure is primarily to determine the behavior of the fluctuation functions $F_q(s)$ as a function of the time scale s for various values of q. To this end, steps 2 through 4 must be repeated for different time scales s.

Step 5: The multi-scale behavior of the fluctuation functions $F_q(s)$ is analyzed by estimating the slope of the log-log plots of $F_q(s)$ versus s for different values of q. If the analyzed time series exhibits long-term correlation according to a power-law, such as fractal properties, the fluctuation function $F_q(s)$ will behave, for sufficiently large values of s, according to the following power-law scaling law $F_q(s) \sim s^{H(q)}$. In general, the exponent $H(q)$ can depend on q. To estimate the values of $H(q)$ for different values of q, a semi-logarithmic regression of the time series $H(q)$ on the time series $F_q(s)$. is performed.

If stationary time series are being dealt with, only the exponent $H(2)$, is obtained which is identically equal to the standard Hurst exponent H.

Therefore, the exponent $H(q)$ generalizes the Hurst exponent H and is commonly referred to as the generalized Hurst exponent. If $H(q)=H$ for all values of q, then the studied time series is monofractal; otherwise, $H(q)$ is a monotonically decreasing function of q, and the corresponding time series is multifractal. For positive values of q, the average fluctuation function $F_q(s)$ is dominated by segments v with large variances $F^2(v,s)$. Thus, for positive values of q, the generalized Hurst exponents $H(q)$ describe the scaling properties of large fluctuations. In contrast, for negative values of q, the exponents $H(q)$ describe the scaling properties of small fluctuations.

It is well known that the generalized Hurst exponent $H(q)$ is directly related to the multifractal scaling exponent $\tau(q)$, commonly known as the Rényi exponent:

$$\tau(q)=q.H(q)-1. \tag{5}$$

Another interesting way to characterize the multifractality of time series is to use the Hölder spectrum or the singularity spectrum $f(\alpha)$ of the Hölder exponent α. The singularity spectrum $f(\alpha)$ is related to the Rényi exponent $\tau(q)$ through the Legendre transform:

$$\begin{cases} \alpha=\tau'(q) \\ f(\alpha)=q.\alpha-\tau(q) \end{cases}, \tag{6}$$

where $\tau'(q)$ is the derivative of the function $\tau(q)$.

The richness of the multifractality can be determined by the width of the spectrum, $\Delta\alpha=\alpha_{max}-\alpha_{min}$. The wider the spectrum, the richer the multifractal behavior of the analyzed time series.

2.3. Sources of multifractality

Kantelhardt et al. (2002) identified two primary sources of multifractality in a time series, long-term temporal correlations in small and large fluctuations and heavy-tailed distributions.

To determine how each source is contributed to the overall multifractality, two transformations can be used on the original return series, namely shuffling (random permutation) and surrogate (phase randomization). By permuting the return series, the distribution of different moments is preserved, but long-term correlations are eliminated. After random permutation, the data have the same distribution but no temporal correlation or memory. Phase randomization helps isolate the contribution of long-term correlations to multifractality by randomly shifting the temporal phases of the data and disrupting these correlations while preserving their overall fluctuation behavior.

This study used two shuffling techniques, referred to as "randperm" and "randi". For the phase randomization, the Inverse Fast Fourier Transform (IFFT) method (Proakis & Dimitris 1996) was applied.

3. RESULTS

Before applying the MF-DFA method, the daily prices and logarithmic returns of the indices were presented. Next, the descriptive statistics was provided for the logarithmic returns of the six indices, followed by the application of the Augmented Dickey-Fuller (ADF) test to both the daily price series and the logarithmic return series of each index.

3.1. Graphical representation of the daily prices and logarithmic returns of the indices

Figure 1 displays the series of daily prices and logarithmic returns of the six stock market indices studied.

3.2. Descriptive statistics

Table 1 presents the descriptive statistics of the logarithmic returns of the six indices.

Table 1 shows that the kurtosis values of the six indices surpass the value 3, suggesting the presence of fat tails in the distribution. It can also be seen that the six skewness are negative and different from 0, indicating that the distribution has a long tail on the left side for all the indices. The high value of the Jarque-Bera (JB) statistics for the six indices implies the rejection of the null hypothesis of normality.

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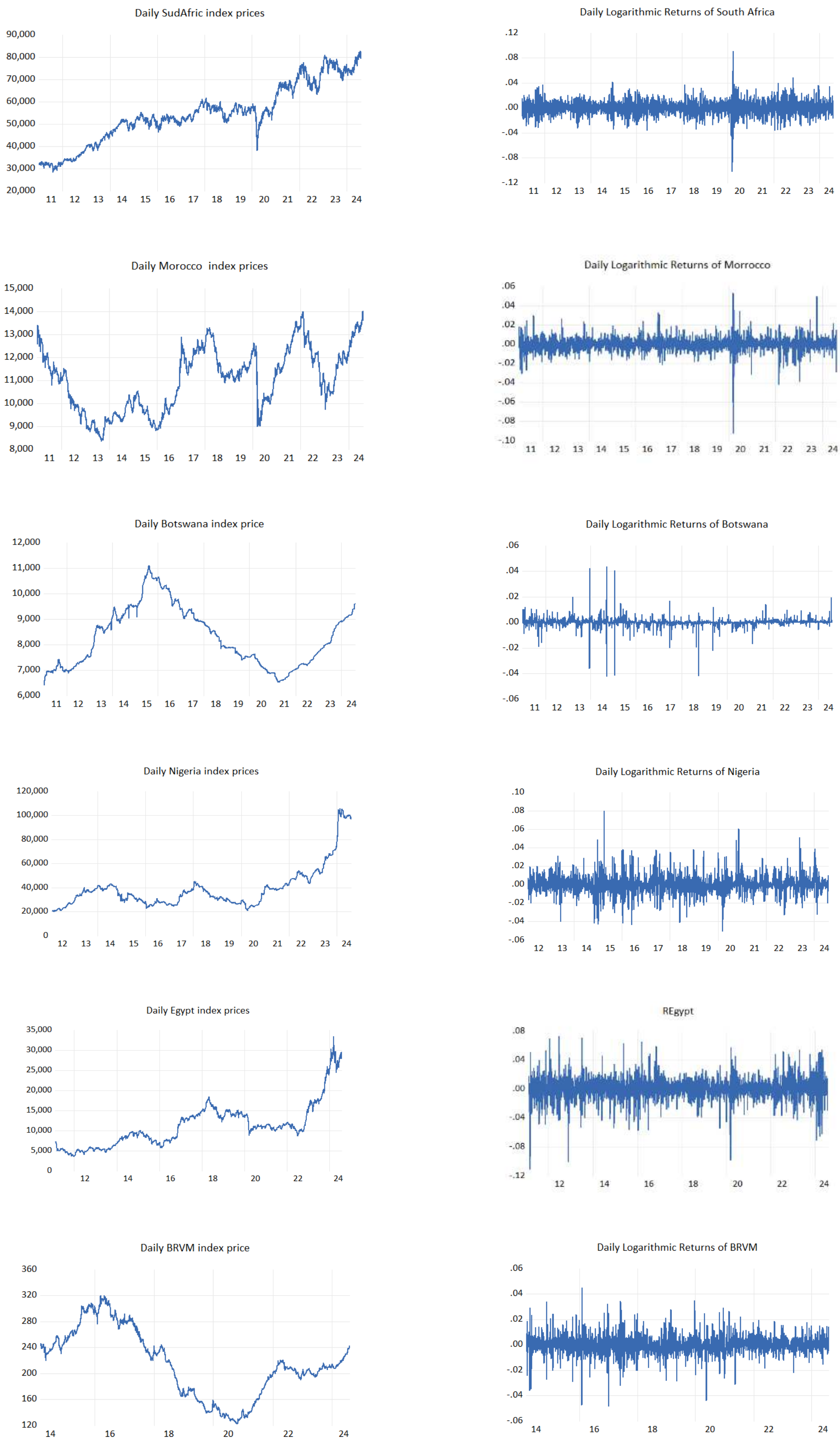

Figure 1. Graphs of the daily prices (1st column) and returns of the six indices (2nd column)

Table 1. Descriptive statistics of logarithmic returns for the six indices

Index Statistics	JSE	MASI	BSE	NGX	EGX	BRVM
Mean	0.000270	0.000027	0.000120	0.000503	0.000427	–0.000002
Median	0.000476	0.000118	0.000000	0.000111	0.000701	–0.000126
Maximum	0.090484	0.053054	0.043681	0.079848	0.073143	0.044763
Minimum	–0.102268	–0.092317	–0.041993	–0.050329	–0.111170	–0.048421
Std. Dev.	0.010793	0.006845	0.002895	0.009602	0.014752	0.007277
Skewness	–0.355662	–1.058416	–0.681120	0.359515	–0.549902	–0.054362
Kurtosis	10.61188	22.33554	95.28118	8.617204	8.349902	8.120346
Jarque–Bera	8272.651	53409.58	1191409.	4142.373	4080.634	2830.629
Probability	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Observations	3397	3388	3357	3100	3283	2590

Table 2. ADF test applied to daily prices and logarithmic returns of the six indices

Index ADF test statistics	JSE	MASI	BSE	NGX	EGX	BRVM
	t-Statistic					
ADF test statistic for Prices	–3.844	–2.894	–1.040	1.0678	–0.602	0.028
ADF test statistic for Returns	–59.727	–35.977	–18.481	–40.941	–47.320	–34.906
Test critical values	1% level: –3.961 5% level : –3.411 10% level : –3.127					

3.3. Test for non-stationarity

The Augmented Dickey-Fuller (ADF) test was applied to the daily price and the logarithmic return series of the six indices. The results are displayed in Table 2.

Table 2 shows that all the ADF test statistics for the daily prices of the six indices are greater than the critical values at the 1%, 5%, and 10% significance levels, indicating that the daily price series of the six indices are generated by non-stationary processes. It is also noteworthy that all the ADF test statistics for logarithmic returns of the six indices are lower than the critical values at the 1%, 5%, and 10% significance levels, indicating that the daily logarithmic returns series of the six indices are generated by a stationary process.

3.4. Application of the MF-DFA technique

This section applies the MF-DFA technique to analyze the multifractal properties of the series of the logarithmic returns of the six indices.

3.4.1. *Multi-scale behavior of the fluctuation functions $F_q(s)$ with respect to s*

The multi-scale behavior of the fluctuation functions $F_q(s)$ was analyzed with respect to the time scales $s \in [20:10:100, 200:100:1000]$. for values of

$$q \in \begin{bmatrix} -45:5:-5, -3.1:0.1:-0.1, \\ 0.1:0.1:3.1, 5:5:45 \end{bmatrix}.$$

An estimation of $h(q)$: is obtained by regressing $Log(s)$ on $Log(F_q(s))$.

$$Log(F_q(s)) \approx H(q).Log(s). \quad (7)$$

Figure 2 shows the logarithmic scale plots of $F_q(s)$ with respect to s for 10 values of q chosen from $\{-15, -5, -3, -0.7, 0, 0.7, 3, 5, 20\}$ for the six indices:

All these curves provide insights into the scaling properties and multifractality of the data. For each value of q, the plot exhibits a straight line, indicating power-law scaling, which is a hallmark of multifractal systems.

3.4.2. *Generalized Hurst exponents $H(q)$*

The generalized Hurst exponents $H(q)$ are given by the slopes of the lines obtained by least squares fitting $Log(s)$ on $Log(F_q(s))$.

Figure 3 shows the plot of the generalized Hurst functions $H(q)$ as a function of the variable

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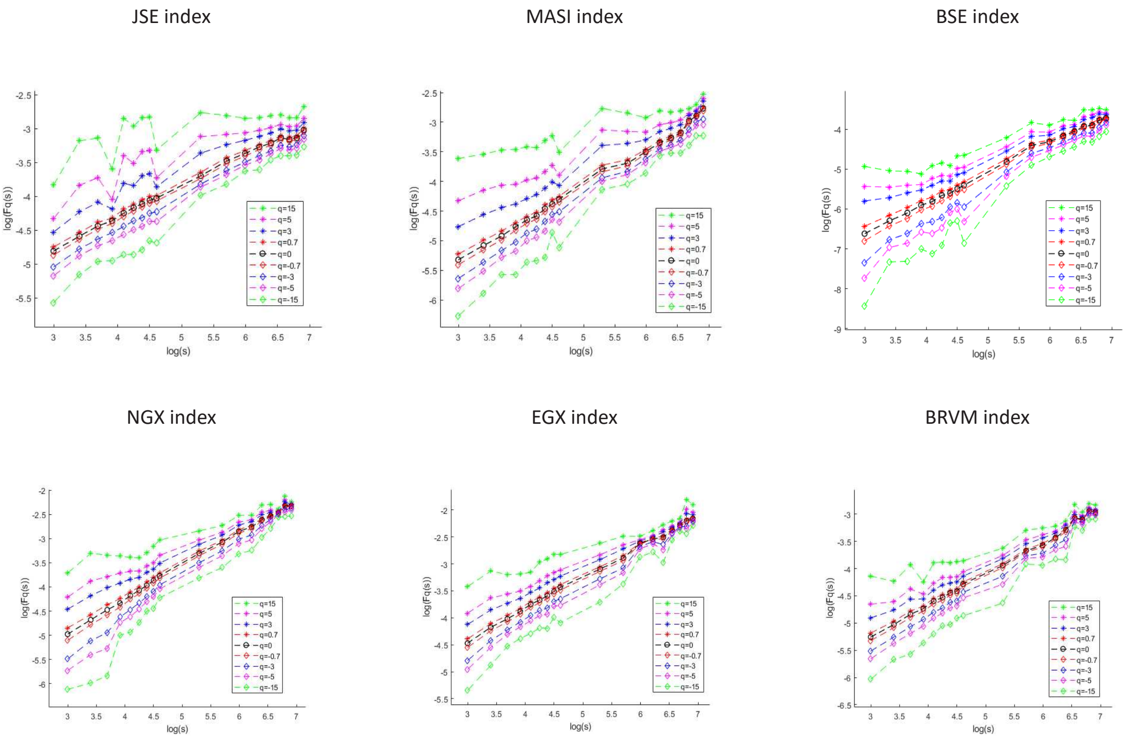

Figure 2. Plots of $Log\left(F_q\left(s\right)\right)$ vs. $Log\left(s\right)$ for the six index returns

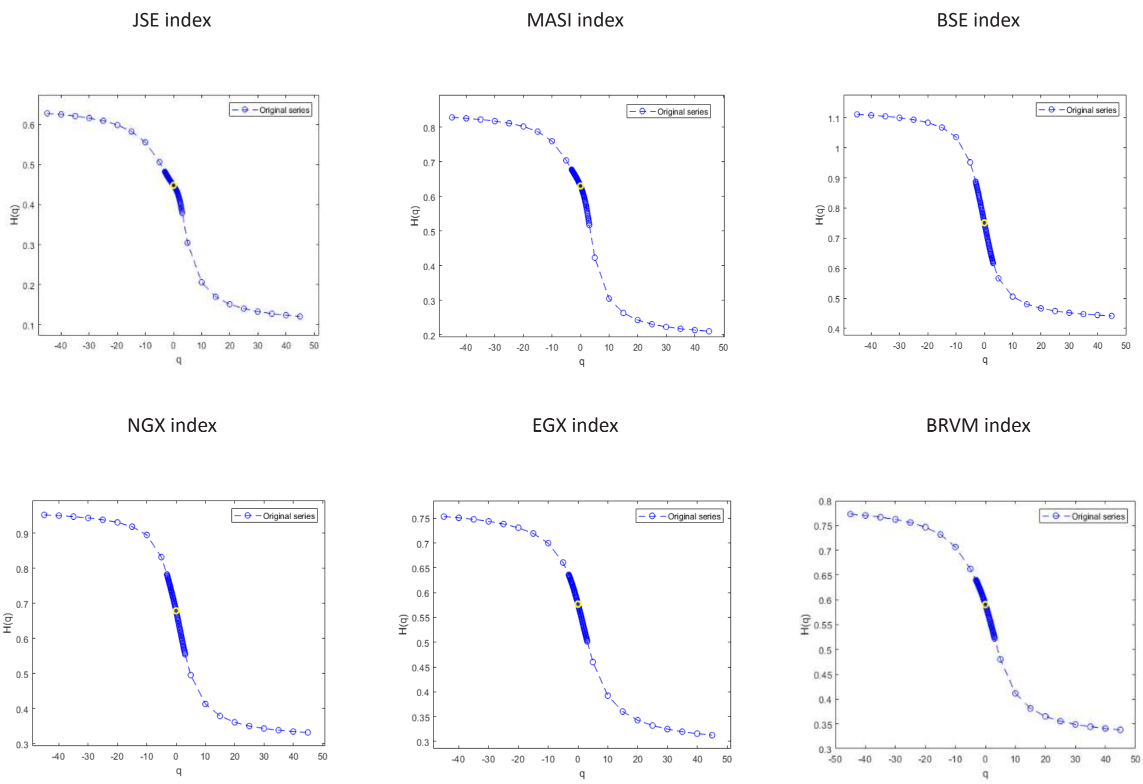

Figure 3. Plots of $H\left(q\right)$ vs. q for the six index returns

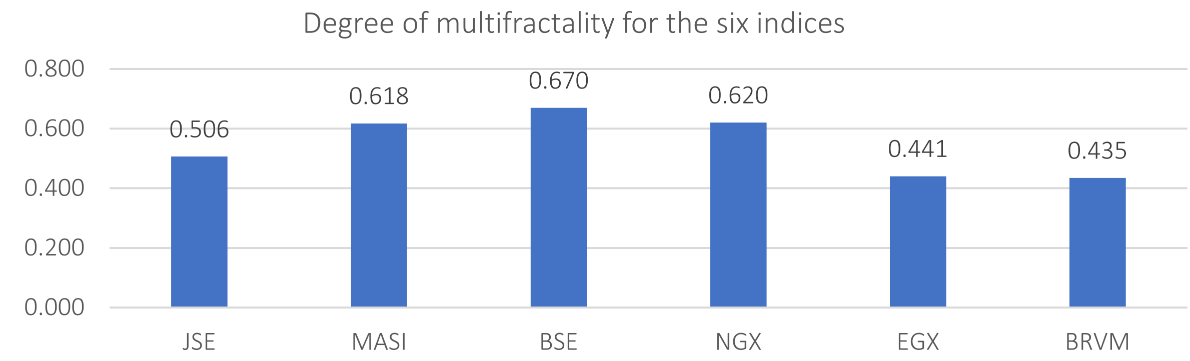

Figure 4. Degrees of multifractality based on the generalized Hurst exponent

$$q \in \begin{bmatrix} -45:5:-5,-3.1:0.1:-0.1, \\ 0.1:0.1:3.1,5:5:45 \end{bmatrix}.$$

Figure 3 shows that as q increases from -45 to 45, the generalized Hurst exponent $h(q)$ decreases non-linearly, indicating that the six indices return series exhibits a multifractal nature.

The degree of multifractality in the returns series of the six indices can be measured by the difference between the smallest and largest values of $H(q)$, $\Delta H = H(q_{min}) - H(q_{max})$.

A monofractal series is characterized by $\Delta H = 0$. The larger ΔH is, the higher the degree of multifractality.

Table 3 presents the degree of multifractality for the six indices.

Table 3. Degrees of multifractality based on the generalized Hurst exponent

Index	ΔH
JSE	0,506
MASI	0,618
BSE	0,670
NGX	0,620
EGX	0,441
BRVM	0,435

Figure 4 compares the degrees of multifractality of the six indices.

Figure 4 shows that the Botswana index has the highest degree of multifractality, followed by the indices of Nigeria, Morocco, JSE, EGX, and BRVM. The higher degree of multifractality suggests that the market exhibits more complex volatility dynamics. It also indicates inefficiency, as it implies that market prices are not purely random and may be influenced by long-term correlations.

3.4.3. Rényi exponent $\tau(q)$

Figure 5 shows the plot of the Rényi Exponent $\tau(q)$ as a function of the variable

$$q \in \begin{bmatrix} -45:5:-5,-3.1:0.1:-0.1, \\ 0.1:0.1:3.1,5:5:45 \end{bmatrix}.$$

Figure 5 shows that as q increases from -45 to 45, the Rényi exponent $\tau(q)$ increases non-linearly, indicating that the six indices return series exhibits a multifractal nature, which suggests a weak form inefficiency in the six stock markets.

3.4.4. Hölder singularity spectrum $f(\alpha)$

Another interesting way to characterize the multifractality of time series is to use the singularity spectrum $f(\alpha)$ of the Hölder exponent α.

Figure 6 shows the plots of the singularity spectrum $f(\alpha)$ for the six indices.

From Figure 6 it can be observed that the curves of the singularity spectrum function $f(\alpha)$ have an inverted parabolic shape, indicating that the six indices return series exhibits multifractal nature, which suggests a weak form inefficiency in the six stock markets. Recall that for monofractal time series, the curve of the singularity spectrum theoretically reduces to a single point $\alpha = H$ with $f(\alpha) = 1$. The degree of multifractality can be measured by calculating the width of the spectrum $\Delta\alpha = \alpha_{max} - \alpha_{min}$.

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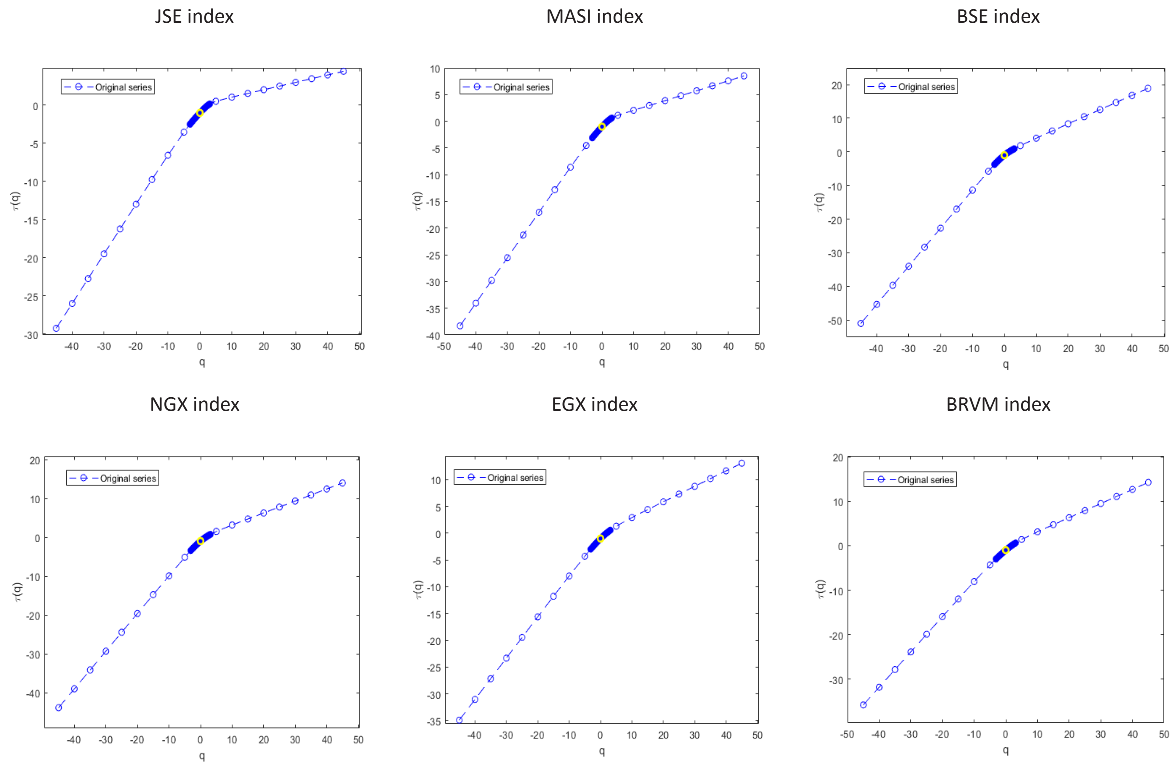

Figure 5. Plots of $\tau(q)$ vs. q for the six index returns

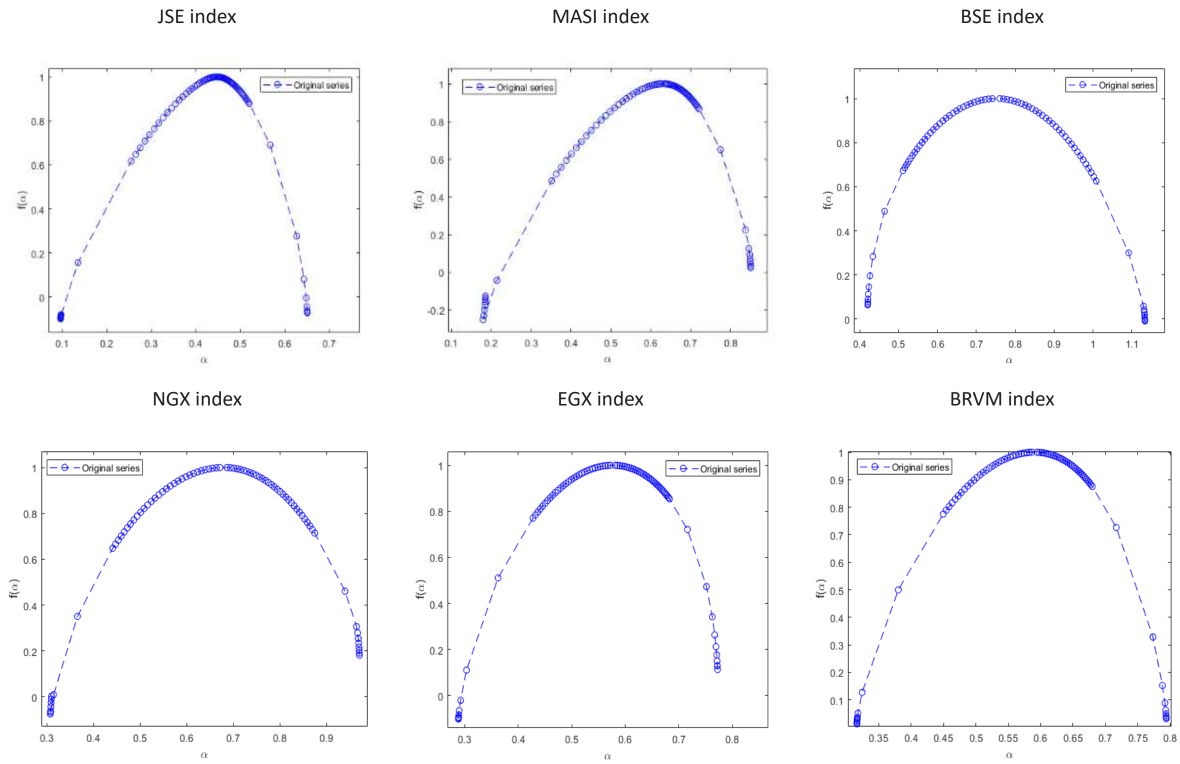

Figure 6. Plots of $f(\alpha)$ vs. α for the six index returns

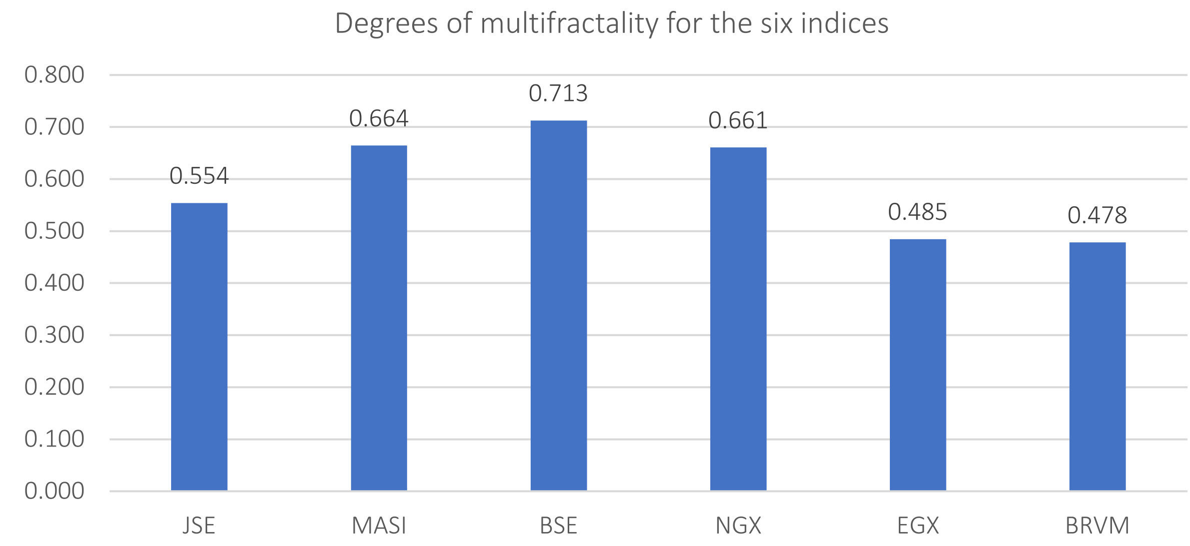

Figure 7. Degrees of multifractality based on the singularity spectrum

Table 4 presents the degree of multifractality for the six indices based on the singularity spectrum.

Table 4. Degrees of multifractality based on the singularity spectrum

Index	$\Delta\alpha$
JSE	0,554
MASI	0,664
BSE	0,713
NGX	0,661
EGX	0,485
BRVM	0,478

Figure 7 compares the degrees of multifractality of the six indices based on the singularity spectrum.

Figure 7 shows that the Botswana index has the highest degree of multifractality, followed by the indices of Morocco, Nigeria, JSE, EGX, and BRVM, which is nearly identical to the results obtained using the generalized Hurst exponent.

3.4.5. Source of multifractality for the six indices

As previously noted, there are two different sources of multifractality in a time series, long-term temporal correlations and the heavy tails distributions. To determine how each source contributes to the overall multifractality, two transformations on the original geometric return series will be used:

- Shuffling (Random permutation)
- Surrogate (Phase randomization).

This study used two shuffling techniques, "randperm" and "randi". For phase randomization, the Inverse Fast Fourier Transform (IFFT) method (Proakis and Dimitris, 1996) was applied.

Figure 8 compares the curves of the generalized Hurst exponent $H(q)$ for the original logarithmic return series of the six indices with those of the surrogate series and the shuffled series.

As shown in Figure 8, all the curves of the generalized Hurst exponent $H(q)$ for the original, surrogate, and shuffled series for the six indices, decrease non-linearly, indicating that these series exhibit multifractal behavior.

To compare the degrees of multifractality of the four series, the values of $\Delta H = H(q_{min}) - H(q_{max})$. are calculated.

the MF-DFA program was run 100 times for the six indices, and each time different values of ΔH were obtained for the surrogate series and the two shuffled series, the results for the original series remained consistent. This variability is due to the algorithms generating the surrogate and shuffled series as they use random permutations in their procedure.

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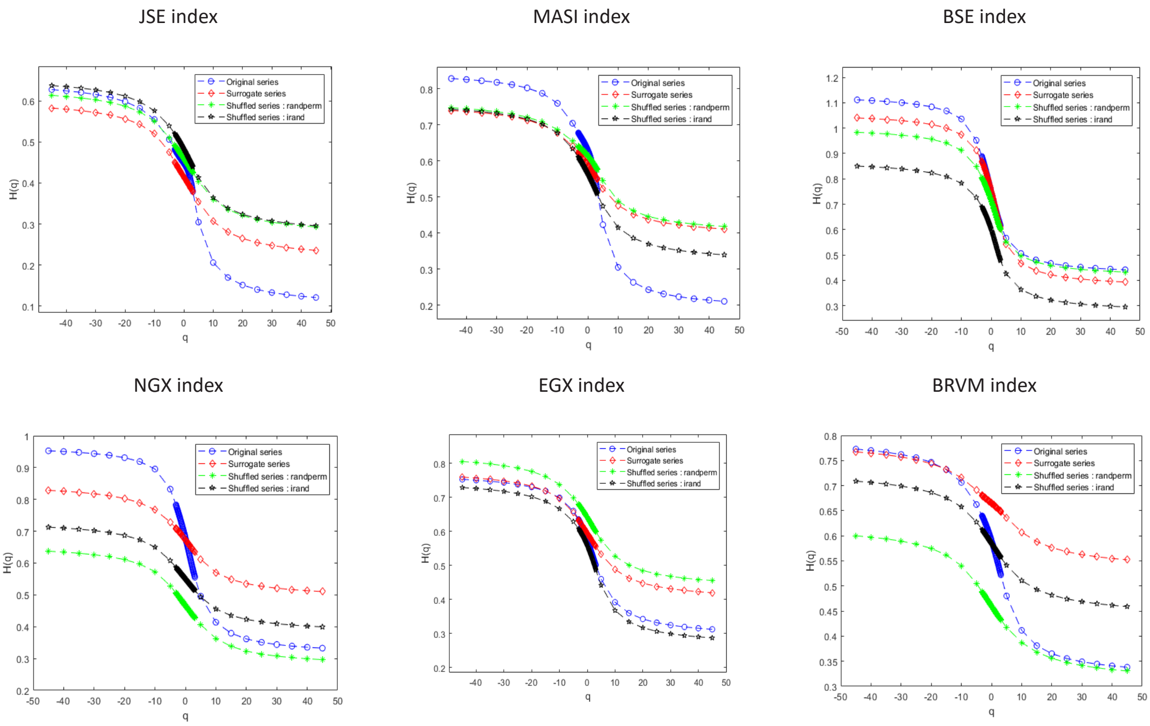

Figure 8. Plots of $H(q)$ vs. q for the original, surrogate, and shuffled series

However, in all 100 executions, the ΔH of the original series is consistently greater than the ΔH of the surrogate and the two shuffled series. Table 5 presents the results from one of the 100 executions.

The results indicate that $\Delta h_{orginale} > \Delta h_{Surrogate}$ and $\Delta h_{orginale} > \Delta h_{shuffled}$ for all six indices, as confirmed. This indicates that the multifractality of the six indices has been diminished by both the surrogate and shuffled series. It can be concluded that both long-term correlations and heavy-tailed distributions contribute to the multifractal behavior.

Figure 9 compares the curves of the singularity spectra $f(\alpha)$ for the original logarithmic return series of the six indices with those of the surrogate and the two shuffled series.

To compare the degrees of multifractality of the four series, the values of $\Delta\alpha = \alpha_{max} - \alpha_{min}$. are calculated.

The MF-DFA program was run 100 times for the six indices. Table 6 presents the results from one of these simulations.

Table 5. Degrees of multifractality of original, surrogate and shuffled series based on the generalized Hurst exponent

Index	$\Delta H = H(q_{min}) - H(q_{max})$			
	Original	**Surrogate**	**Shuffled-randperm**	**Shuffled-randi**
JSE	0,506	0,347	0,320	0,341
MASI	0,618	0,328	0,329	0,404
BSE	0,670	0,647	0,550	0,555
NGX	0,620	0,319	0,341	0,314
EGX	0,441	0,340	0,350	0,466
BRVM	0,435	0,215	0,269	0,250

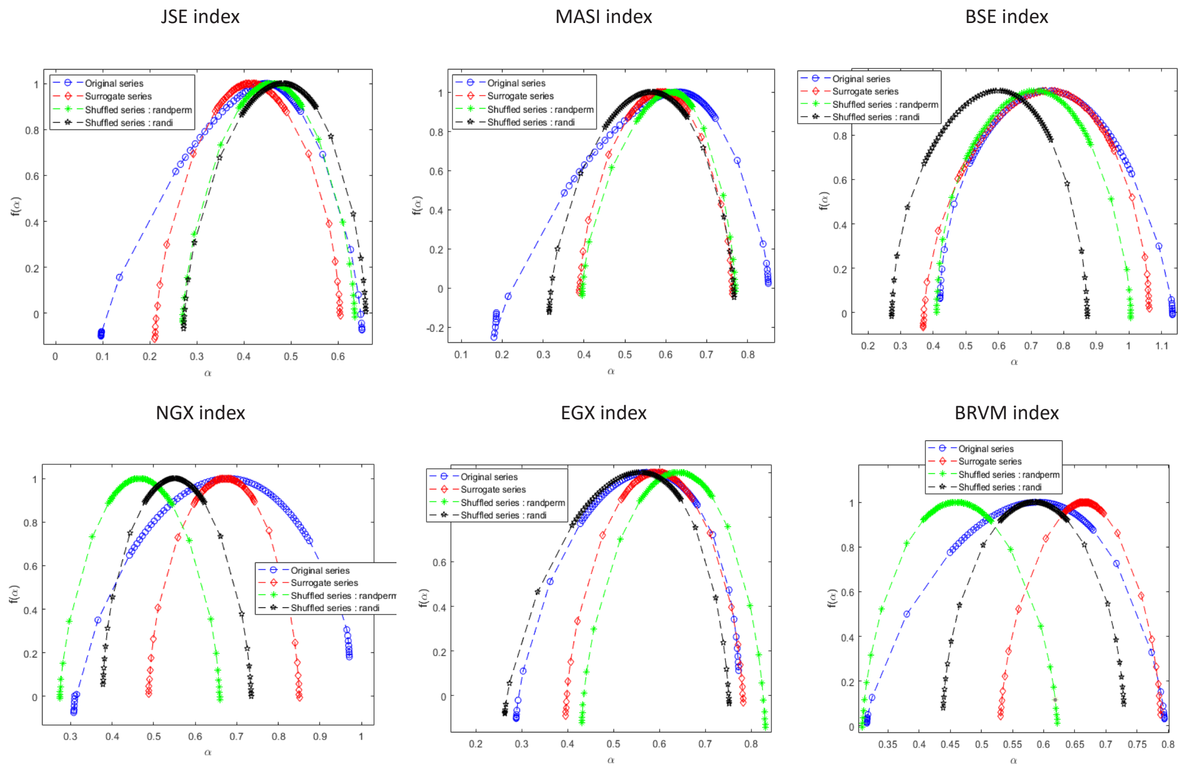

Figure 9. Plots of $f(\alpha)$ vs. α for the original, surrogate and shuffled series

Table 6. Degrees of multifractality of original, surrogate, and shuffled series based on the singularity spectrum

Index	$\Delta\alpha = \alpha_{max} - \alpha_{min}$			
	Original	**Surrogate**	**Shuffled-randperm**	**Shuffled-randi**
JSE	0,554	0,394	0,366	0,387
MASI	0,664	0,374	0,374	0,452
BSE	0,713	0,692	0,585	0,600
NGX	0,661	0,363	0,386	0,357
EGX	0,485	0,387	0,400	0,489
BRVM	0,478	0,257	0,314	0,290

The results indicate that $\Delta\alpha_{orginale} > \Delta\alpha_{Surrogate}$ and $\Delta\alpha_{orginale} > \Delta h_{shuffled}$ for all six indices, as confirmed in Table 6. This indicates that the multifractality has been reduced by both the surrogate series and the two shuffled series across the six indices. It is concluded that both long-term correlation and heavy-tailed distribution contribute to the multifractal behavior of these six indices.

4. DISCUSSION

This study applied Multifractal Detrended Fluctuation Analysis (MF-DFA) to examine the multifractal behavior of six stock markets, including the Johannesburg, Casablanca, Botswana, Nigerian, Egyptian, and Regional Stock Exchange. The analysis revealed significant multifractality for the six indices, driven by long-term correlations and heavy-tailed distributions. To contextualize the results, they were compared with previous studies on multifractality of African stock markets, highlighting both similarities and key differences.

Several studies have examined the Moroccan, South African, and Nigerian stock markets, with a focus on multifractality and market inefficiency. Benbachir and El Alaoui (2011a) identified multifractality in the Moroccan All Shares Index (MASI) and the

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Most Active Shares Index (MADEX), driven by long-term correlations and heavy-tailed distributions, a finding that aligns with the result on the MASI index. Similarly, Saâdaoui (2024) observed asymmetric multifractality and inefficiency in the MASI index, which is consistent with our results, although our study extends the analysis to other African markets. Mensi et al. (2022) used asymmetric MF-DFA during periods of financial instability to analyze the MASI index, revealing asymmetric multifractality, which also aligns with the findings of this study. Additionally, Faheem et al. (2021) observed multifractal behavior in the MASI index, which is consistent with the findings of this study. Lee and Choi (2023) also identified multifractality in the MASI index, suggesting inefficiency, which matches our findings as well.

In South Africa, Tilfani and El Boukfaoui (2019) found multifractality during the 2007–2008 subprime crisis using rolling window MF-DFA, which is consistent with this study. Tiwari et al. (2019) also identified multifractality in South Africa, matching the findings of this study. Similarly, Lee and Choi (2023) found multifractality and inefficiency in South Africa, which agrees with our observations.

Regarding Nigeria, Samuel (2023) focused on multifractality across different sectors of the Nigerian stock market and emphasized the impact of the 2007–2008 global financial crisis, finding that the crisis increased multifractality. While this study also identified multifractality in the Nigerian market, it provided a broader long-term analysis of market dynamics and multifractal characteristics, rather than focusing on a specific crisis period.

In summary, this study aligns with previous research on multifractality in African stock markets, revealing significant multifractality driven by long-term correlations and heavy-tailed distributions. Identifying long-term correlations as a critical factor in the multifractality of African stock markets underscores their non-linear dynamics. This suggests that past price movements can influence future price behavior over extended timeframes, which is essential for comprehending market trends and formulating effective trading strategies. The presence of heavy-tailed distributions in African stock markets indicates that extreme price movements – both gains and losses – are more probable than would be predicted by traditional models that assume normal distribution. This characteristic heightens the risk associated with investing in these markets, necessitating more robust risk management strategies. Investors must be prepared for sudden and significant fluctuations, which could substantially impact their portfolios.

CONCLUSION

The objective of this study was to explore the multifractal behavior of the six largest African stock markets (Johannesburg, Casablanca, Botswana, Nigerian, Egyptian, and Regional Stock Exchange), with a focus on assessing the degree of multifractality and identifying the underlying factors driving it. Using the MF-DFA method, the analysis employed generalized Hurst exponents, Rényi exponents, and the singularity spectrum, confirming that all six markets exhibit multifractal properties. The application of shuffling and phase randomization techniques further revealed that long-term correlations and heavy-tailed distributions are significant contributors to the observed multifractality.

These results highlight the complex dynamics of African stock markets, emphasizing their non-linear behavior and the presence of persistent long-range correlations. The findings suggest that past price movements have a lasting influence on future market behavior, challenging the assumption of market efficiency and indicating that price patterns can persist longer than expected in markets following a simple random walk. The detection of heavy-tailed distributions suggests that extreme price movements are more likely than traditional models would predict, underscoring the heightened risks involved in investing in these markets.

The findings of this study offer valuable practical implications for stakeholders in financial markets. For investors, incorporating advanced analytical techniques, particularly multifractal analysis, is crucial for improving market predictions and risk management. Additionally, the presence of multifractality and inefficiencies highlights the necessity for diversification within portfolios to mitigate risks linked to individual market behaviors. For policymakers, enhancing market transparency is vital for addressing inefficiencies and promoting greater market efficiency. Furthermore, regulations tailored to the unique characteristics of these markets are necessary to manage extreme events and oversee long-term dependencies. Strengthening market stability through measures such as stress testing can also ensure financial systems are resilient to extreme conditions and long-term trends.

AUTHOR CONTRIBUTIONS

Conceptualization: Benbachir Soufiane.
Data curation: Benbachir Soufiane.
Formal analysis: Benbachir Soufiane.
Investigation: Benbachir Soufiane.
Methodology: Benbachir Soufiane.
Project administration: Benbachir Soufiane.
Resources: Benbachir Soufiane.
Software: Benbachir Soufiane.
Supervision: Benbachir Soufiane.
Validation: Benbachir Soufiane.
Visualization: Benbachir Soufiane.
Writing – original draft: Benbachir Soufiane.
Writing – review & editing: Benbachir Soufiane.

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