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Modeling, risk assessment and portfolio optimization of energy futures

Abstract

This paper examines the portfolio optimization of energy futures by using the STARR ratio that can evaluate the risk and return relationship for skewed distributed returns. We model the price returns for energy futures by using the ARMA(1,1)-GARCH(1,1)-PCA model with stable distributed innovations that reflects the characteristics of energy: mean reversion, heteroskedasticity, seasonality, and spikes. Then, we propose the method for selecting the portfolio of energy futures by maximizing the STARR ratio, what we call "Winner portfolio". The empirical studies by using energy futures of WTI crude oil, heating oil, and natural gas traded on the NYMEX compare the price return models with stable distributed innovations to those with normal ones. We show that the models with stable distributed innovations are more appropriate for energy futures than those with normal ones. In addition, we discuss what characteristics of energy futures cause the stable distributed innovations in the returns. Then, we generate the price returns of energy futures using the ARMA(1,1)-GARCH(1,1)-PCA model with stable ones and choose the portfolio of energy futures employing the generated price returns. The results suggest that the selected portfolio of "Winner portfolio" performs better than the average weighted portfolio of "Loser portfolio". Finally, we examine the usefulness of the STARR ratio to select the winner portfolio of energy futures.

Keywords: energy futures markets, portfolio optimization, principal component analysis, α-stable distributed innovations, *t* copula.
JEL Classification: C51, G11, Q40.

Introduction

This paper examines the portfolio optimization of energy futures by using the STARR ratio that can evaluate the risk and return relationship for skewed distributed returns. Additionally, we conduct empirical studies by using the WTI crude oil, heating oil, and natural gas futures traded on the NYMEX.

Commodities such as energy, agriculture, and metal have been considered as the third investment assets, compared with the stocks and bonds. Financial institutions and hedge funds have recently recognized the commodities as the alternative investment objects and then tailored their own trading strategies in order to generate the cash. Down the line commodity trading is providing high returns as in Geman (2005) because of the diversification effects in the financial portfolio of the stocks and bonds. For example, Erb and Harvey (2006) and Miffre and Rallis (2007) recently examined and discussed the profitability of momentum strategies in commodity futures. In particular, energy trading, one of the commodities, has recently got roaring, because energy is traded not only by financial companies that provide the liquidity to the market but also by energy companies that have to be responsible for the demand for energy. Under the market circumstances, hedge funds, one of savvy financial institutions, often have a joy to dive into energy markets by employing their trading strategies basically tested in financial markets. One of their famous trading strategies applied to financial markets is a long and short trading strategy that makes zero cost portfolio of long and short positions and then generates the cash owing to the price convergence, which is categorized as the statistical arbitrage and convergence trading. In order to investigate the performance of the trading, Gatev, Goetzmann, and Rouwenhorst (2006) test the pairs trading, one of long and short trading strategies, by using historical stock prices. In addition, Jurek and Yang (2007) compare the performance of their optimal mean reversion strategy with that of Gatev, Goetzmann, and Rouwenhorst (2006) using the simulated data.

The pairs trading is very close to zero-investment strategy as in Rachev, Jăsić, Stoyanov, and Fabozzi (2007) and Rachev, Jăsić, Biglova, and Fabozzi (2006) in the sense that the zero cost portfolio by using the winner and loser ones historically produces the profit. However, the long and short trading strategies including pairs trading are not applied to energy markets as long as we know. In order to conduct the long and short trading strategies in energy markets, we have to know how to model the energy futures prices and how to construct the portfolio to be chosen as the winner or loser portfolio. Thus, this paper investigates the portfolio optimization

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of energy futures by using the STARR ratio that can evaluate the risk and return for skewed distributed returns often observed in energy futures markets.

In order to do this, we start with the modeling of energy price returns. Energy commodity prices such as crude oil and natural gas have four highlighted characteristics compared with those of financial assets such as stocks and bonds. In the beginning, it is well documented that energy prices have mean reversion as in Pilipovic (1998) and among others. Then, the volatility in energy prices is larger than that in stock prices and time varying. Because of the characteristics of the volatility in energy prices, energy prices present the inverse leverage effect such that the price volatility increases in the prices as in Eydeland and Wolyniec (2003), while stock prices have the leverage effect as in Black (1975). In addition, energy prices have stronger seasonality than the financial prices owing to the seasonality of supply and demand for energy. In order to demonstrate the seasonality in the price models, energy price returns are represented by using the Principal Component Analysis (PCA) as in Geman (2005). It is consistent with the energy market observation that the common factor with seasonality such as temperature makes different energy prices fluctuate in the same direction. Finally, the imbalance between supply and demand gives rise to the sudden price soaring: spikes often observed in deregulated electricity and natural gas markets as in e.g., Huisman and Mahieu (2001), Eydeland and Wolyniec (2003), Geman and Roncoroni (2006), and among others. As discussed in Kanamura (2006), energy prices are strongly affected by the supply and demand relationship and then the innovation terms of energy price returns are more skewed than those of stock price returns due to price spikes by way of the more upward sloping supply curve transformation of the mean-reverting demand process than the exponential. Like these since energy has four unique characteristics such as ARMA effect (mean reversion), GARCH effect (heteroskedasticity), PCA (seasonality), and skewed innovations (spikes), we should incorporate these characteristics into the model of the energy price returns. Down the line we model the price returns for energy by using the ARMA(1,1)-GARCH(1,1)-PCA model with stable distributed innovations by reflecting the characteristics of energy prices[1,2].Then, we propose the method for selecting the portfolio of energy futures by maximizing the STARR ratio as in e.g., Rachev, Menn, and Fabozzi (2005) that can evaluate the risk and return for skewed distributed returns often observed in energy futures markets.

The empirical studies by using energy futures prices of WTI crude oil, heating oil, and natural gas traded on the NYMEX compare the price return models for energy futures, especially focusing on the distributions of the innovations. We show that the models with stable ones are more appropriate for energy futures than those with normal ones. In addition, we offer some arguments that the stable innovations may come from price spikes in energy futures markets. We then generate the price returns by using the proposed ARMA(1,1)-GARCH(1,1)-PCA model with stable ones and choose the portfolio of energy futures by maximizing the STARR ratio. The results will illustrate that the selected portfolio, what we call "Winner portfolio", performs better than the average weighted portfolio, what we call "Loser portfolio", in energy markets. Finally, we examine the usefulness of the STARR ratio to select the winner portfolio of energy futures.

This paper is organized as follows. Section 1 explains the ARMA(1,1)-GARCH(1,1)-PCA model with stable distributed innovations for energy price returns and then proposes the method for choosing the winner portfolio in energy markets by using the STARR ratio. Section 2 empirically compares price return models for energy futures traded on the NYMEX, and conducts the portfolio optimization based on the procedure as in Section 1. Section 3 concludes and offers the directions for our future research.

1. The model

1.1. The price return model for energy. Energy price returns are well known to have mean reversion and heteroskedasticity. In addition, they often present the large outliers in the distributed noises partially due to price spikes. Thus, the return model requires the ARMA type model for the mean reversion, GARCH type model for the heteroskedasticity as in Bollerslev (1986), and the stable distributed innovations for the price spikes.

Furthermore, energy prices are correlated with each other and they are expected to have common principal components particularly due to seasonality. Thus, this paper models the price return of energy futures by using the ARMA(1,1)-GARCH(1,1)-Principal Component Analysis (PCA)[3] model with stable distributed innovations as in Appendix A as follows.

[1] The principal component such as temperature produces the seasonality in the prices. Thus, we do not incorporate the seasonality in the model directly.

[2] Spikes are generated partly by supply and demand relationship as in Kanamura (2006) and partly by time varying volatility. Thus, we do not employ jump diffusion model for our model.

[3] The principal component analysis is introduced by Hotelling (1933).

$$r_{i,t} = p_i + \sum_{j=1}^{S} q_{i,j} f_{j,t} + e_{i,t}\,, \tag{1}$$

where $i = 1,...,I$, $t = 1,...,T$,

$$f_{j,t} = a_{j0} + a_{j1} f_{j,t-1} + b_{j1}\varepsilon_{j,t-1} + \varepsilon_{j,t}\,, \tag{2}$$

$$\varepsilon_t = \sigma_t z_t\,, \tag{3}$$

$$\sigma_t^{\,2} = \alpha_0 + \alpha_1 \varepsilon_{t-1}^{\,2} + \beta_1 \sigma_{t-1}^{\,2}\,,$$

$$\sigma_t > 0\,, \tag{4}$$

where $e_{i,t}$ denotes stable innovations for each variable, and z_t has a multivariate distribution having a skewed Student's t copula with stable marginals. Note that I, T, and S represent the numbers of energy futures, observations, and principal components, respectively.

By using this model, we simulate the price returns of energy futures in order to choose two portfolios of energy futures: high and low performance portfolios.

1.2. Portfolio selection based on the price return model. In order to conduct a long and short trading, we have to construct high and low performance portfolios, what we called "Winner portfolio" and "Loser portfolio", respectively. This section proposes the method for selecting the winner portfolio of energy futures by using the STARR ratio that can evaluate risk and return of skewed distributed returns.

Denote by z_{pl} the weight of asset l in a portfolio of n assets, and denote by $r^{(p)}$ the total random return of the portfolio consisting of n assets:

$$r^{(p)} = \sum_{l=1}^{n} z_{pl} r_l\,, \tag{5}$$

where r_l is the random daily return of asset l. Denote by $R^{(p)}$ the total expected daily return of the portfolio of n assets:

$$R^{(p)} = E(r^{(p)}) = \sum_{l=1}^{n} z_{pl} R_l\,, \tag{6}$$

where R_l represents the expected return of asset l.

We define the objective function of the portfolio optimization by using the STARR ratio[1] as in e.g., Rachev, Menn, and Fabozzi (2005) and Biglova and Rachev (2007) as follows:

$$STARR_\delta(r^{(p)}) = \frac{R^{(p)}}{ETL_\delta(r^{(p)})}\,. \tag{7}$$

STARR ratio represents the ratio between the expected excess return and its Expected Tail Loss (ETL). Note that the ETL is a downside tail risk measure, also known as Total Value-at-Risk (TVaR), Expected Shortfall (ES), and Conditional Value-at-Risk (CVaR), and defined as

$$ETL_\delta(X) = \frac{1}{\delta}\int_0^\delta VaR_q(X)\,dq\,, \tag{8}$$

where

$VaR_\delta(X) = -F_X^{-1}(\delta) = -\inf\{x / P(X \le x) \ge \delta\}$

is the Value-at-Risk (VaR) of the random return X. If we assume a continuous distribution for the probability law of X, ETL can be interpreted as the average loss beyond VaR as in Rachev, Ortobelli, Stoyanov, Fabozzi, and Biglova (2007).

We choose the portfolio weights z_{pl} to maximize the STARR in Eq. (7).

$$\max_{z_{pl}} STARR_\delta(r^{(p)})\,, \tag{9}$$

$$\text{s.t. } z_p = \sum_{l=1}^{n} z_{pl} = 1\,, \tag{10}$$

$$z_{pl} > 0. \tag{11}$$

The selected portfolio is set to be the winner portfolio in energy futures markets. On the other hand, we define the loser portfolio as the equally weighted portfolio of energy futures in order to examine the performance of the winner portfolio comparing to the average return in energy futures markets.

2. Empirical studies for energy futures prices

2.1. Data. The studies use three series of daily closing prices of WTI crude oil, heating oil, and natural gas futures traded on the New York Mercantile Exchange (NYMEX). They include six different delivery months from one to six whose sources are obtained from Bloomberg and whose observations start from April 3, 2000 to July 10, 2003. The price quotes of the WTI crude oil, heating oil, and natural gas futures are US dollars per barrel, cents per gallon, and dollars per mmBtu, respectively.

2.2. Comparisons of price return models for energy prices. In this section, we compare the stable assumption for price returns of energy futures with the normal assumption by fitting the data with joint stable and normal distributions, respectively. We implicitly assume that returns are uniquely determined by the location parameter μ and the scale parameter σ as in Appendix A[2]. Assuming

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[1] The STARR ratio is also called as the CVaR ratio.

[2] The location parameter μ and the scale parameter σ are the mean and the standard deviation, respectively in the case of normal distribution hypothesis.

that the observations are i.i.d., we estimate two main parameters of the stable law as in Appendix A: the index of stability α and skewness parameter β, which characterize the heavy-tailedness and asymmetry of the price return distributions of energy futures, respectively. For the Gaussian fit, we compute the first moment and standard deviation. Finally, to test the normal and stable distribution hypotheses, we compute the Kolmogorov-Smirnov (*KS*) statistic according to

$$KS = \sup_{x \in R} | F_S(x) - \hat{F}(x) |, \qquad (12)$$

where $F_S(x)$ is the empirical sample distribution and $\hat{F}(x)$ is the standard normal cumulative distribution function evaluated at *x* for the Gaussian or stable fit, respectively. This statistic emphasizes deviations around the median of the fitted distribution. It is a robust measure in the sense that it focuses only on the maximum deviation between the sample and fitted distributions.

Our sample comprises returns of 18 risky assets of energy markets for the period from April 3, 2000 to July 10, 2003.

In the simple setting of the i.i.d. returns model, we have estimated the values for the four parameters of the stable Paretian distribution using the method of maximum likelihood. Figure 1 shows the scatter plots of the estimated pairs of α and β for all assets.

A comparison between the Gaussian and stable hypotheses clearly indicates that stable distributions approximate the returns' distribution much better than the Gaussian one. With *KS* test we can compare the empirical cumulative distribution of several assets returns with either a simulated Gaussian or a simulated stable distribution.

Table 1 shows that we can generally reject the hypothesis of normality of returns' distribution at different levels of confidence considered. Analogously, we cannot generally reject the stable distribution hypothesis for return distributions at different levels of confidence considered.

Table 2 shows that the average *KS* statistic across different energy futures prices equals about 0.74, when $F_S(x)$ is the cumulative Gaussian distribution for the case of confidence level equal to 0.05. When $F_S(x)$ is the cumulative stable distribution, the average *KS* statistic among different assets is about 0.07. The *KS* statistic for the stable non-Gaussian test is almost 10 times smaller than the *KS* distance in the Gaussian case.

We notice from Figure 1 that all estimates of parameter α are less than 2. We see also from Table 2 that the third quartile for α is approximately 1.90. This implies that none of the asset returns is normally distributed.

The majority of energy futures prices have negative estimate β as it can be seen from Figure 1. The mean of β is equal to about -0.54 as in Table 2. This fact also confirms that the stable fit outperforms the Gaussian one.

Table 1. Normality and stable distribution hypotheses for i.i.d. model

Confidence level	95%	99%	99.9%	99.95%	99.99%
% of energy futures prices for which the normal distribution hypothesis is rejected	73.98	74.00	74.04	74.40	74.24
% of energy futures prices for which the stable distribution hypothesis is rejected	7.10	7.14	7.37	7.13	6.62

Table 2. Summary of statistics for sample of 18 assets on i.i.d. model

	α	β	KS distances (normal)	KS distances (stable)
Mean	1.8754	-0.5396	0.7398	0.0710
Median	1.8923	-0.5368	0.7407	0.0677
1 quartile (25%)	1.8653	-0.7606	0.7241	0.0431
3 quartile (75%)	1.9021	-0.4082	0.7524	0.0997

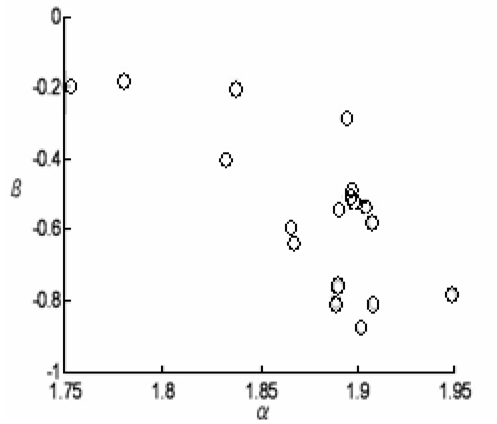

Fig. 1. Scatter plots between index of stability α and skewness parameter β for daily returns of 18 assets

As a next step, we estimate the normal and stable GARCH(1,1) models for energy futures price returns. The assumptions of i.i.d. returns and conditional homoskedasticity are often violated in energy data where we observe volatility clustering. Such behavior is captured by Autoregressive

Conditional Heteroskedastic (ARCH) models and their generalization (GARCH models, see Bollerslev (1986)). Accordingly, as the second test we consider the GARCH models with normal and stable distribution innovations. Recall that the GARCH model of the asset returns (y_t)'s can be represented by the expressions that assume that return process is given by

$$y_t = \sigma_t z_t, \quad (13)$$

where z_t's are i.i.d. mean zero and unit variance random variables representing the innovations of the return process and where the conditional variance in the GARCH(p,q) model is given by

$$\sigma_t^{\,2} = a_0 + \sum_{i=1}^{q} a_i y_{t-i}^{\,2} + \sum_{j=1}^{p} b_j \sigma_{t-j}^{\,2}. \quad (14)$$

In the most common form of the GARCH model, $z_t \sim N(0,1)$, so that the returns are conditionally normal. We observe that the GARCH model with a conditionally normal return distribution can lead to heavy tails in the unconditional return distribution.

If we assume that the distribution of the historical innovations z_{t-n}, ... , z_t is heavier-tailed than the normal, then the returns will not be conditionally normal any more so that the GARCH model will exhibit non-Gaussian conditional distribution. Note that in this model, σ_t given by Eq. (14) can be interpreted as a scale parameter and not necessarily volatility, since for some distributional choices for z_t, the variance may not exist. Specifically, in the case that z_t 's are realizations from a α-stable non-Gaussian distribution, the GARCH model is represented by the modified expression:

$$\sigma_t = a_0 + \sum_{i=1}^{q} a_i |y_{t-i}| + \sum_{j=1}^{p} b_j \sigma_{t-j}. \quad (15)$$

Note that the index of stability α for the stable distribution is constrained to be greater than one[1]. We call the representation Eq. (15) with its assumption "a stable-GARCH model".

Similar to common GARCH models that do not assume stable distributed innovation processes, the stable-GARCH model may prove beneficial to model the conditional distribution of asset returns by capturing the temporal dependencies of the return series appropriately. To test the goodness-of-fit of the models, the standard Kolmogorov distance statistic can be applied. We fit the GARCH(1,1) models in Eqs. (14) and (15) with the Gaussian innovations and α-stable distributions, respectively. The model parameters are estimated using the method of maximum likelihood assuming the normal distribution of innovations. In this the strong consistency property of estimators of the model under the stable Paretian hypothesis is preserved since the index of stability of the innovations is greater than 1 as in Rachev and Mittnik (2000). After estimating the GARCH(1,1) model parameters, we computed the model residuals and then verified which distributional assumption is more appropriate.

Table 3 shows the results of testing the normal and stable distribution hypotheses for stable-GARCH(1,1) models of energy futures prices with normal and stable innovations, respectively. The results show that at the 95% confidence level, the hypothesis of normality is rejected for 37% of assets residuals and the hypothesis of stable distribution is rejected only for 4% of assets with residuals. Comparing the results in Table 3 to those in Table 1, we observe that the Gaussian model is rejected in fewer cases in the GARCH(1,1) model than in the simple i.i.d. model.

Table 3. Normality and stable distribution hypotheses for GARCH (1,1) model

Confidence level	95%	99%	99.9%	99.95%	99.99%
% of energy futures prices for which the normal distribution hypothesis is rejected	36.86	36.78	37.37	37.41	37.95
% of energy futures prices for which the stable distribution hypothesis is rejected	4.37	4.72	5.06	4.65	4.69

Table 4. Summary of statistics for sample of 18 assets on GARCH(1,1) model

	α	β	KS distances (normal)	KS distances (stable)
Mean	1.9087	-0.6633	0.3686	0.0437
Median	1.9113	-0.7610	0.3700	0.0437
1 quartile (25%)	1.8910	-0.8663	0.3559	0.0344
3 quartile (75%)	1.9279	-0.4855	0.3768	0.0517

A summary of the computed statistics for the residuals of the GARCH(1,1) model is reported in Table 4. The results in Table 4 show that the average *KS* statistic across different innovation series equals about 0.37, when $F_S(x)$ is the cumulative Gaussian distribution for the case of confidence level equal to 0.05. When $F_S(x)$ is the cumulative stable distribution, the average *KS* statistic among different innovation series is about 0.04. The *KS* statistic for the stable non-Gaussian

[1] Note that term y_{t-i}^{2} in assuming stable innovation process z_t can become infinite rendering the whole expression meaningless. The condition of $\alpha > 1$ means that we impose a finite mean condition.

test is almost 10 times smaller than the *KS* distance in the Gaussian case. Generally, the results imply that the stable Paretian distribution is more adequate as a probabilistic model for the innovations compared to the Gaussian assumption. A plausible model for asset returns is then a model that exhibits the properties of volatility clustering captured by the GARCH process and heavy-tails captured with stable non-Gaussian innovations in the GARCH model.

Figure 2 shows the scatter plots of the estimated pairs of α and β for all assets' innovation series. We notice from Figure 2 that all estimates of parameter α are less than 2. We see also from Table 4 that the third quartile for α is approximately 1.93. This implies that none of the asset returns is normally distributed.

The majority of energy futures prices have negative estimate β as it can be seen from Figure 2. The mean of β is equal to about -0.66 as in Table 4. This fact also confirms that the stable fit outperforms the Gaussian one.

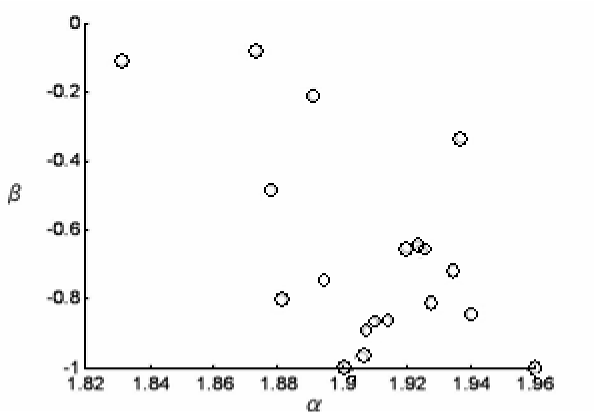

Fig. 2. Scatter plots between index of stability α and skewness parameter β for innovations of GARCH(1,1) fit of daily returns of 18 assets

Comparing Tables 3 and 1, the stable distribution model is rejected in fewer cases in the GARCH(1,1) model than in the simple i.i.d. model. It implies that by removing the heteroskedasticity from price returns that may cause the transformation of normally distributed innovations, i.e., stable ones, the stable distribution innovations are highlighted. It may be helpful to increase the possibility of justifying our assumption that the stable distributed innovations in price returns stem from the price spikes.

As a next step, we estimate ARMA(1,1)-GARCH(1,1) model. The sequence $h = (h_n)$ is described by the ARMA model, if

$$h_n = \mu_n + \sigma\varepsilon_n , \tag{16}$$

where

$$\mu_n = (a_0 + a_1 h_{n-1} + \ldots + a_p h_{n-p}) + (b_1\varepsilon_{n-1} + b_2\varepsilon_{n-2} + \ldots + b_q\varepsilon_{n-q}). \tag{17}$$

From Eqs. (16) and (17) we find that

$$h_n - (a_1 h_{n-1} + \ldots + a_p h_{n-p}) = a_0 + [\varepsilon_n + b_1\varepsilon_{n-1} + b_2\varepsilon_{n-2} + \ldots + b_q\varepsilon_{n-q}]. \tag{18}$$

Notice that in the case of $q = 0$, the model is reduced to the case of AR(p) model. In the case of $p = 0$, the model is reduced to the case of MA(q) model. The special case of ARMA(p,q) is ARMA(1,1), which is the combination of AR(1) and MA(1) models:

$$h_n = a_0 + a_1 h_{n-1} + b_1\varepsilon_{n-1} + \sigma\varepsilon_n . \tag{19}$$

For the case of $|a_1| < 1$, the time series, described by the model, is stationary. Figure 3 presents the computer realization of the sequence, for which the ARMA(1,1) model holds.

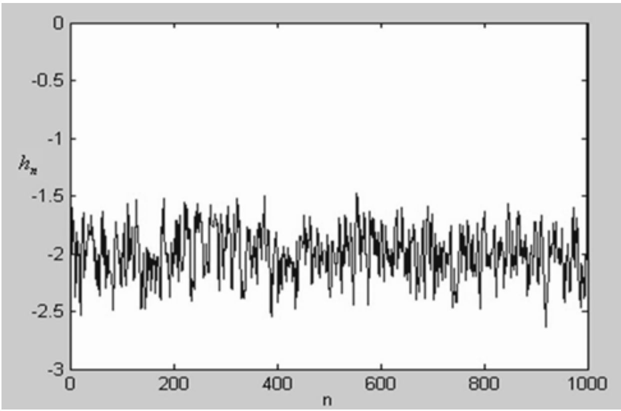

Fig. 3. Simulation of ARMA(1,1) model of $h_n = a_0 + a_1 h_{n-1} + b_1\varepsilon_{n-1} + \sigma\varepsilon_n$, where $a_0 = -1$, $a_1 = 0.5$, $b_1 = 0.1$, and $\sigma = 0.1$

The algorithm is offered in Appendix B.

We test the hypotheses about the stable and normal distributions of innovations applying the *KS* statistics. We have estimated the values for the four parameters of the stable Paretian distribution using the method of maximum likelihood for the sequences of innovations. Figure 4 shows the scatter plots of the estimated pairs of α and β for the innovations of GARCH(1,1) fit of residuals, obtained from ARMA(1,1) fit of daily returns of 18 assets. A comparison between the Gaussian and stable hypotheses clearly indicates that stable distributions approximate the innovations' distribution much better than the Gaussian one. With *KS* test we can compare the empirical cumulative distribution of innovations with either a simulated Gaussian or a simulated stable distribution. Table 5 shows that we can generally reject the hypothesis of normality of innovations' distribution at different levels of confidence considered.

Analogously, we cannot generally reject the stable distribution hypothesis for innovations' distributions at different levels of confidence considered.

Table 5. Normality and stable distribution hypotheses for ARMA (1,1)-GARCH (1,1) model

Confidence level	95%	99%	99.9%	99.95%	99.99%
% of energy futures prices for which the normal distribution hypothesis is rejected	37.60	36.26	36.85	36.99	36.52
% of energy futures prices for which the stable distribution hypothesis is rejected	4.63	4.51	4.37	4.47	4.33

Table 6 shows that the average *KS* statistic across different energy futures equals about 0.38, when $F_S(x)$ is the cumulative Gaussian distribution for the case of confidence level equal to 0.05.

Table 6. Summary of statistics for sample of 18 assets on ARMA(1,1)-GARCH(1,1) model

	α	β	KS distances (normal)	KS distances (stable)
Mean	1.9096	-0.6730	0.3760	0.0463
Median	1.9074	-0.8505	0.3750	0.0480
1 quartile (25%)	1.8939	-0.9562	0.3645	0.0357
3 quartile (75%)	1.9289	-0.4451	0.3903	0.0529

When $F_S(x)$ is the cumulative stable distribution, the average *KS* statistic among different assets is about 0.05. The *KS* statistic for the stable non-Gaussian test is almost 8 times smaller than the *KS* distance in the Gaussian case.

We notice from Figure 4 that all estimates of parameter α are less than 2. We see also from Table 6 that the third quartile for α is approximately 1.91. This implies that none of the innovation sequences is normally distributed.

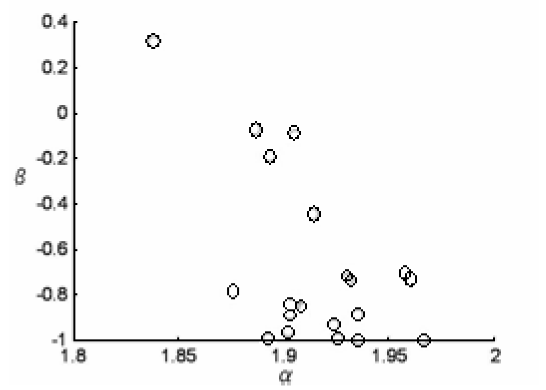

Fig. 4. Scatter plots between index of stability α and skewness parameter β for innovations of ARMA(1,1)-GARCH(1,1) fit of 18 assets

The majority of innovations have negative estimate β as it can be seen from Figure 4. The mean of β is equal to -0.67 as in Table 6. This fact also confirms that the stable fit outperforms the Gaussian one.

2.3. Principal component models for energy futures price returns. We modeled price returns of energy futures so far and then found that the price return models with stable distributed innovations are more appropriate for energy futures than those with normal ones. However, they have dependent structure not only for the different maturity futures prices of the same energy but also for the different energy futures prices. For example, temperature often affects demand for energy. Accordingly, it is possible that the corresponding prices move together. In order to capture the whole dependency structure for energy, we test the asset return models based on the Principal Component Analysis (PCA) as in Rachev, Mittnik, Fabozzi, Focardi, and Jăsić (2007).

In the beginning, we perform the PCA for the analyzed asset returns in an effort to examine how many factors influence them. We consider the influence of factors, which are combinations of analyzed assets. We replace the original n ($n = 18$ for our case) correlated time series X_i with n uncorrelated time series P_i, supposing that each X_i is a linear combination of the P_i. Supposing that only p of the portfolios P_i have a significant variance, while the remaining n-p have very small variances, we implement a dimensionality reduction by choosing only those portfolios whose variance is significantly different from zero. We call these portfolios factors F. We can then approximately represent each series X_i as a linear combination of the factors plus a small uncorrelated noise (e):

$$X_i = \sum_{i=1}^{p} \alpha_i F_i + \sum_{i=p+1}^{n} \alpha_i P_i = \sum_{i=1}^{p} \alpha_i F_i + e \,. \qquad (20)$$

The PCA works either on the variance-covariance matrix or on the correlation matrix. Although the technique is the same, the results are generally different. The PCA applied to the variance-covariance matrix is sensitive to the units of measurement, which determine variances and covariances. This observation does not apply to returns, which are dimensionless quantities. Therefore, we apply the PCA to the correlation matrix, as the returns of our analyzed assets are heavy tailed.

Having performed the PCA by using the correlation matrix of analyzed asset returns, we obtained 18 principal components, which are linear combinations of the original series, $X = (X_1,..., X_n)'$, i.e., they are obtained by multiplying X by the matrix of the eigenvectors.

Table 7 shows the total variance explained by a growing number of components. Thus, the first component explains about 66.82% of the total variance, the first two components explain about

90.81% of the total variance and so on. Obviously 18 components explain 100% of the total variance. From Table 7 it follows that 7 of the portfolios P_i explain 99% of total variance. Therefore, we implement a dimensionality reduction by choosing only the first 7 factors for further analysis of their distributions' properties. We consider that each series X_i of asset returns can be represented as a linear combination of these 7 factors plus a small uncorrelated noise.

Table 7. % of total variance by growing # of components on covariance matrix

Principal component	% of variance explained	% of total variance explained
1	66.8151	66.8151
2	23.9985	90.8136
3	3.5411	94.3548
4	1.8570	96.2118
5	1.3153	97.5272
6	0.9210	98.4482
7	0.5769	99.0251
8	0.3256	99.3508
9	0.2547	99.6056
10	0.1323	99.7380
11	0.1084	99.8465
12	0.0698	99.9164
13	0.0339	99.9503
14	0.0271	99.9774
15	0.0117	99.9892
16	0.0067	99.9960
17	0.0030	99.9990
18	0.0009	100

In the simple setting of the i.i.d. returns model, we have estimated the values for the four parameters of the stable Paretian distribution using the method of maximum likelihood. Figure 5 shows the scatter plots of the estimated pairs of α and β for all analyzed factors.

A comparison between the Gaussian and stable hypotheses clearly indicates that stable distributions approximate the factors' distribution much better than the Gaussian one. With *KS* test we can compare the empirical cumulative distribution of several assets returns with either a simulated Gaussian or a simulated stable distribution.

As Table 8 shows we can generally reject the hypothesis of normality of factors' distributions at different levels of confidence considered. Analogously, we cannot generally reject the stable distribution hypothesis for factors' distributions at different levels of confidence considered.

Comparing the results in Table 8 to those in Table 1, we observe that the Gaussian model is rejected in fewer cases in the i.i.d. model of factors than in the simple i.i.d. model of asset returns.

The results in Table 9 show that the average *KS* statistic across different energy futures equals about 0.75, when $F_S(x)$ is the cumulative Gaussian distribution for the case of confidence level equal to 0.05. When $F_S(x)$ is the cumulative stable distribution, the average *KS* statistic among different factors is about 0.20. The *KS* statistic for the stable non-Gaussian test is almost 5 times smaller than the *KS* distance in the Gaussian case.

We notice from Figure 5 that all estimates of parameter α are less than 2. We see also from Table 9 that the third quartile for α is approximately 1.87. This implies that none of the factors is normally distributed.

Some factors have negative estimate β as it can be seen from Figure 5. The mean of β is equal to -0.18 as in Table 9. This fact also confirms that the stable fit outperforms the Gaussian one.

Table 8. Normality and stable distribution hypotheses for i.i.d. PCA model

Confidence level	95%	99%	99.9%	99.95%	99.99%
% of energy futures prices for which the normal distribution hypothesis is rejected	75.07	73.20	73.78	72.97	72.51
% of energy futures prices for which the stable distribution hypothesis is rejected	19.98	21.09	20.70	21.60	20.40

Table 9. Summary of statistics for sample of 18 assets on i.i.d. PCA model

	α	β	KS distances (normal)	KS distances (stable)
Mean	1.5749	-0.1786	0.7507	0.1988
Median	1.5245	0.0130	0.7573	0.1428
1 quartile (25%)	1.3897	-0.4025	0.7361	0.1092
3 quartile (75%)	1.8660	0.0713	0.7613	0.3303

In addition to the comparison between Tables 8 and 1, the stable distribution model is rejected in more cases in the i.i.d. PCA model than in the simple i.i.d. model.

It implies that the stable distributed innovations are accommodated in each energy price returns. It may partially support our assumption that the stable distributed innovations in price returns stem from the price spikes in that the price spikes occur due to the imbalance of supply and demand for each energy.

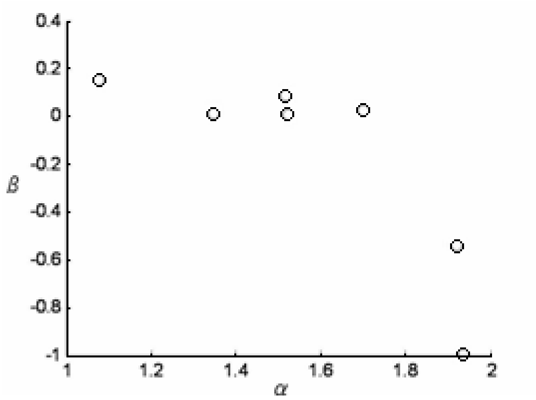

Fig. 5. Scatter plots between index of stability α and skewness parameter β for 7 factors' series

As a next step, we estimate the normal and stable GARCH(1,1) models for factors, obtained from the PCA. Table 10 shows the results of testing the normal and stable distribution hypotheses for stable-GARCH(1,1) models of factors with normal and stable innovations, respectively. The results show that at the 95% confidence level, the hypothesis of normality is rejected for about 43% of assets residuals and the hypothesis of stable distribution is rejected only for about 8% of assets' residuals.

Comparing the results in Table 10 to those in Table 3, we observe that the Gaussian model is rejected in more cases in the GARCH(1,1) model for factor series than in the GARCH(1,1) model for asset returns.

Table 10. Normality and stable distribution hypotheses for GARCH(1,1)-PCA model

Confidence level	95%	99%	99.9%	99.95%	99.99%
% of energy futures prices for which the normal distribution hypothesis is rejected	42.75	41.15	40.71	41.00	41.52
% of energy futures prices for which the stable distribution hypothesis is rejected	7.79	9.42	8.58	8.18	9.04

A summary of the computed statistics for the residuals of the GARCH(1,1) model is reported in Table 11. The table shows that the average *KS* statistic across different factors equals about 0.43, when $F_S(x)$ is the cumulative Gaussian distribution for the case of confidence level equal to 0.05. When $F_S(x)$ is the cumulative stable distribution, the average *KS* statistic among different factors is about 0.08. The *KS* statistic for the stable non-Gaussian test is almost 5 times smaller than the *KS* distance in the Gaussian case. Generally, the results imply that the stable Paretian distribution is more adequate as a probabilistic model for the innovations compared to the Gaussian assumption. A plausible model for factors' returns is then a model that exhibits the properties of volatility clustering captured by GARCH process and heavy-tails captured with stable non-Gaussian innovations in the GARCH model.

Table 11. Summary of statistics for sample of 18 assets on GARCH(1,1)-PCA model

	α	β	KS distances (normal)	KS distances (stable)
Mean	1.6614	-0.2458	0.4275	0.0779
Median	1.6514	0.0213	0.4248	0.0554
1 quartile (25%)	1.5755	-0.7642	0.3860	0.0535
3 quartile (75%)	1.9001	0.0827	0.4424	0.0603

Figure 6 shows the scatter plots of the estimated pairs of α and β for all factors' innovation series. We notice from Figure 6 that all estimates of parameter α are less than 2. We see also from Table 11 that the third quartile for α is approximately 1.90. This implies that none of the innovations is normally distributed.

The majority of innovations have negative estimate β as it can be seen from Figure 6. The mean of β is equal to about -0.24 as in Table 11. This fact also confirms that the stable fit outperforms the Gaussian one.

In contrast to the comparison between Tables 10 and 3, the stable distribution model is rejected in more cases in the GARCH(1,1)-PCA model than in the GARCH(1,1) model. It implies that the stable distributed innovations may be accommodated not only in principal component returns but also in each energy price returns. It may partially support our assumption that the stable distributed innovations in price returns stem from the price spikes in that the price spikes occur not only due to the events influencing whole energy markets such as wars and cold waves but also due to the imbalance of supply and demand for each energy such as shortage of natural gas storage.

Comparing Tables 10 to 8, the stable distribution model is rejected in fewer cases in the GARCH(1,1)-PCA model than in the i.i.d. PCA model. It implies that by removing heteroskedasticity from price returns, the stable distribution innovations are highlighted. It may support our assumption that the stable distributed innovations in price returns stem from the price spikes[1].

As the last step, we have estimated ARMA(1,1)-GARCH(1,1) model for factors as in Tables 12 and 13.

In the simple setting of the i.i.d. returns model, we have estimated the values for the four parameters of the stable Paretian distribution using the method of maximum likelihood. Figure 7 shows the scatter plots of the estimated pairs of α and β for the innovations of GARCH(1,1) fit of residuals, obtained from ARMA(1,1) fit of factor series.

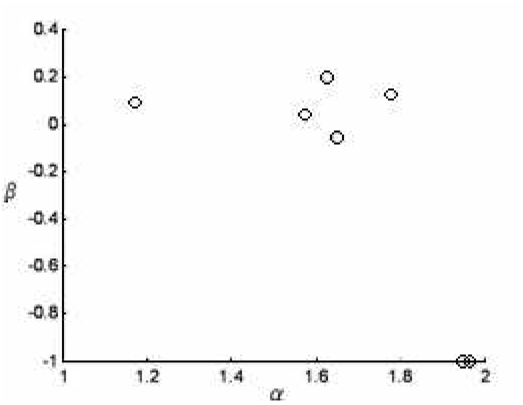

Fig. 6. Scatter plots between index of stability α and skewness parameter β for innovations of GARCH(1,1) fit of 7 factors' series

Table 12. ARMA(1,1)-GARCH(1,1) model for principal components

r_i	p_i	$q_{i,1}$	$q_{i,2}$	$q_{i,3}$	$q_{i,4}$	$q_{i,5}$	$q_{i,6}$	$q_{i,7}$
i = 1	0.007	0.190	0.085	0.170	0.126	0.154	0.026	-0.554
i = 2	0.007	0.173	0.078	0.143	0.078	0.077	0.014	-0.094
i = 3	0.006	0.157	0.072	0.135	0.071	0.033	0.001	0.032
i = 4	0.006	0.144	0.066	0.130	0.065	0.009	0.000	0.103
i = 5	0.006	0.134	0.062	0.124	0.061	-0.007	-0.003	0.154
i = 6	0.005	0.126	0.058	0.119	0.055	-0.020	-0.005	0.188
i = 7	0.020	0.549	0.103	-0.925	-0.197	1.395	0.408	0.528
i = 8	0.022	0.515	0.142	-0.658	-0.132	0.408	0.119	-0.091
i = 9	0.023	0.464	0.156	-0.502	-0.192	-0.059	-0.016	-0.183
i = 10	0.024	0.426	0.160	-0.355	-0.232	-0.383	-0.094	-0.147
i = 11	0.023	0.397	0.156	-0.251	-0.255	-0.535	-0.121	-0.069
i = 12	0.023	0.373	0.144	-0.172	-0.252	-0.626	-0.134	0.003
i = 13	0.003	0.030	-0.081	-0.067	0.144	0.003	-0.118	0.018
i = 14	0.003	0.028	-0.073	-0.020	0.061	-0.035	0.026	-0.021
i = 15	0.003	0.025	-0.065	0.000	0.017	-0.029	0.110	0.004
i = 16	0.003	0.021	-0.051	0.017	-0.029	0.001	0.028	0.006
i = 17	0.004	0.018	-0.044	0.021	-0.041	0.015	-0.028	-0.001
i = 18	0.004	0.017	-0.041	0.021	-0.038	0.016	-0.040	-0.004

Table 13. ARMA(1,1)-GARCH(1,1) model for principal components (cont'd)•

f_j	$a_{j,0}$	$a_{j,1}$	$b_{j,1}$	α_0	α_1	β_1
j=1	0.0042	0.5373	-0.6056	0.07030	0.9059	0.0202
j=2	-0.0073	0.7114	-0.7787	0.0371	0.8619	0.1047
j=3	0.0070	0.3378	-0.3000	0.0513	0.9078	0.0421
j=4	0.0468	-0.4911	0.3571	0.0356	0.7329	0.2425
j=5	-0.0463	0.5192	-0.5566	0.4708	0.1682	0.5576
j=6	0.1039	-0.7754	0.2478	0.1421	0	0.9999
j=7	-0.0128	-0.2500	0.2048	0.5862	0	0.5913

A comparison between the Gaussian and stable hypotheses clearly indicates that stable distributions approximate the innovations' distribution much better than the Gaussian one. With *KS* test we can compare the empirical cumulative distribution of innovations with either a simulated Gaussian or a simulated stable distribution.

The results in Table 14 show that we can generally reject the hypothesis of normality of innovations' distribution at different levels of confidence considered. Analogously, we cannot generally reject the stable distribution hypothesis for innovations' distributions at different levels of confidence considered.

Comparing the results in Table 14 to those in Table 5, we observe that the Gaussian model is rejected in more cases in the ARMA(1,1)-GARCH(1,1) model

[1] In this case, the price spikes consist in the principal components.

for factor series than in the ARMA(1,1)-GARCH(1,1) model for asset returns.

Table 15 shows that the average *KS* statistic across different innovation series equals about 0.41, when $F_S(x)$ is the cumulative Gaussian distribution for the case of confidence level equal to 0.05. When $F_S(x)$ is the cumulative stable distribution, the average *KS* statistic among different innovation series is about 0.10. The *KS* statistic for the stable non-Gaussian test is almost 4 times smaller than the *KS* distance in the Gaussian case.

We notice from Figure 7 that all estimates of parameter α are less than 2. We see also from Table 15 that the third quartile for α is approximately 1.91. This implies that none of the innovation sequences is normally distributed.

The majority of innovations have negative estimate β as it can be seen from Figure 7. The mean of β is equal to about -0.23 as in Table 15. This fact also confirms that the stable fit outperforms the Gaussian one.

In contrast to the comparison between Tables 14 and 5, the stable distribution model is rejected in more cases in the ARMA(1,1)-GARCH(1,1)-PCA model than in the ARMA(1,1)-GARCH(1,1) model. It implies that the stable distributed innovations may be accommodated not only in principal component returns but also in each energy price returns. It may partially support our assumption, especially in the proposed model, that the stable ones stem from the price spikes in that the price spikes occur not only due to the events influencing whole energy markets but also due to the imbalance of supply and demand for each energy.

Comparing Tables 14 to 10, the stable distribution model is rejected almost in the same way between in the ARMA(1,1)-GARCH(1,1)-PCA model and in the GARCH(1,1)-PCA model. It implies that the mean reversion as in ARMA effect does not affect the stable distributed innovations of principal components.

Table 14. Normality and stable distribution hypotheses for ARMA(1,1)-GARCH(1,1)-PCA model

Confidence level	95%	99%	99.9%	99.95%	99.99%
% of energy futures prices for which the normal distribution hypothesis is rejected	40.97	41.83	40.39	41.85	41.46
% of energy futures prices for which the stable distribution hypothesis is rejected	9.88	8.39	9.76	8.40	8.72

Table 15. Summary of statistics for sample of 18 assets on ARMA(1,1)-GARCH(1,1)-PCA model

	α	β	KS distances (normal)	KS distances (stable)
Mean	1.6735	-0.2298	0.4097	0.0988
Median	1.6514	0.0400	0.3928	0.0812
1 quartile (25%)	1.5883	-0.7650	0.3820	0.0751
3 quartile (75%)	1.9058	0.1143	0.4224	0.0603

Our results show that the ARMA(1,1)-GARCH(1,1)-PCA model with stable distributed innovations is more appropriate forecasting model for the PCA factors' series of energy futures prices than that with normal ones. In addition, the ARMA(1,1)-GARCH(1,1)-PCA model with stable distributed innovations is likely to be more desirable to model for energy futures than the ARMA(1,1)-GARCH(1,1) model with stable distributed innovations in that the PCA model can incorporate not only the whole market price behavior of stable distributed innovations but also each energy one.

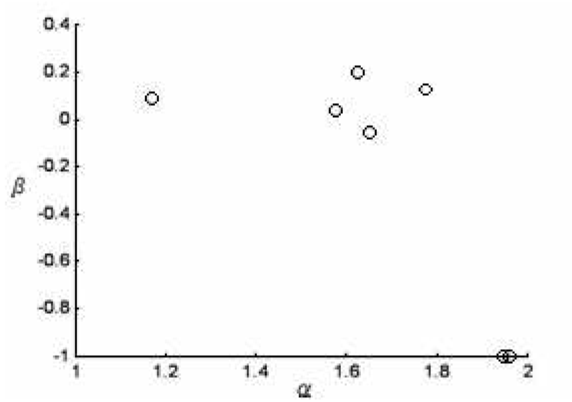

Fig. 7. Scatter plots between index of stability α and skewness parameter β for innovations of ARMA(1,1)-GARCH(1,1) fit of 7 factors' series

2.4. Scenarios generation based on the asset price return model. By using the PCA, we determined that each series (i = 1,..., 18) of asset returns can be represented as a linear combination of 7 factors plus a small uncorrelated noise. Now, we have to generate the series of i = 1,..., 18 using the formula of factor models as in Eq. (1). In our model, I = 18, T = 812, and S = 7 are employed as the numbers of energy futures, observations, and principal components, respectively.

The algorithm is as follows. We use the window of 250 observations to estimate parameters and residuals of the factor model as in Eq. (2) and then to generate the factor returns' values and residuals' values for the next day as in Eq. (1). Here in the following we describe the method of return's generation for 251 day.

We consider the first 250 observations of returns of the each asset and the matrix of factor returns as independent variables[1] to estimate parameters p_i, $q_{i,j}$ and the sequence of residuals $e_{i,t}$. We also have the factor return series $f_{j,t}$ for the corresponding 250 days. For each j = 1,..., 7 we fit the ARMA(1,1)-GARCH(1,1) model with stable innovations $u_{j,t}$ to the factor, respectively[2]. We fitted stable distribution into the sample innovations obtained from the ARMA(1,1)-GARCH (1,1)-fit: $\hat{u}_{j,t}$. In the sample innovations of $\hat{u}_{j,t}$ for the corresponding 250 days, we fit a skewed t copula so as to capture their dependence. The results are illustrated as in Table 16. Note that γ_i's denote the asymmetry parameters for each component, μ_i's denote the location parameters for each component, ν represents the degree of freedom, and σ shows the modified covariance matrix.

Table 16. Estimates of skewed t copula parameters

	f_1	f_2	f_3	f_4	f_5	f_6	f_7
γ_i	-0.0542	-0.0671	-0.0116	0.0335	0.0236	-0.0157	-0.1251
μ_i	0.1108	0.1709	0.0245	-0.0785	0.0505	-0.0440	0.2471
ν	5	5	5	5	5	5	5

Here the covariance matrix is estimated as follows.

$$\hat{\Sigma}_\nu = \begin{pmatrix} 0.591275 & 0.155343 & 0.119897 & 0.244183 & -0.168410 & 0.065424 & 0.239456 \\ 0.155343 & 0.575963 & 0.037945 & 0.240159 & 0.116046 & 0.034315 & -0.085000 \\ 0.119897 & 0.037945 & 0.595065 & 0.041207 & -0.039040 & 0.044453 & -0.079760 \\ 0.244183 & 0.240159 & 0.041207 & 0.587961 & 0.043619 & 0.094310 & 0.090417 \\ 0.168410 & 0.116046 & -0.039040 & 0.043619 & 0.595625 & -0.053620 & -0.102840 \\ 0.065424 & 0.034315 & 0.044453 & 0.094310 & -0.053620 & 0.911118 & -0.015520 \\ 0.239456 & -0.085000 & -0.079760 & 0.090417 & -0.102840 & -0.015520 & 0.549806 \end{pmatrix}.$$

We have already estimated the stable marginal distribution of any innovation: $u_{j,t}$.

Then having estimated the stable distribution for each factor's innovations $u_{j,t}$, we employ it in order to transform it into the uniform scenarios generated by the skewed t copula by taking the inverse of the fitted one-dimensional stable distribution functions. So, as a result, we have generated innovations z_t at the date 251 having skewed t copula as dependence and stable distributions for the marginals of the innovations ($u_{j,t}$).

Now we can generate scenarios for the factors $f_{j,t}$ at date 251 using the estimated ARMA(1,1)-GARCH(1,1) model for every $f_{j,t}$ with known coefficients and with known and generated innovations. So, having generated innovations of GARCH(1,1), i.e., values of z_t, from skewed t copula and the stable distribution for the marginal distributions and then using the estimated coefficients of the GARCH(1,1) model denoted by α_0, α_1, and β_1 as in Eq. (3), we'll obtain the returns of GARCH(1,1) denoted by ε_t in Eq. (3) for the next day. In addition, we know the coefficients of the ARMA(1,1) model as in Eq. (2). So, we have generated the factor returns of $f_{j,t}$ at date 251 fitting skewed t copula in the factor innovations and using stable marginal distributions for those innovations.

Table 17. ARMA(1,1)-GARCH(1,1) stable distributed innovations

e_i	$a_{i,0}$	$a_{i,1}$	$b_{i,1}$	α_0	α_1	β_1
i = 1	7.93E-06	0.113	-0.149	2.00E-07	0.871	0.062
i = 2	-1.23E-05	-0.249	0.131	2.78E-07	0.892	0.036
i = 3	-3.84E-06	0.556	-0.490	1.76E-07	0.835	0.066
i = 4	3.37E-06	-0.528	0.623	4.15E-08	0.840	0.061
i = 5	4.75E-05	-0.696	0.678	4.40E-07	0.000	0.318
i = 6	5.44E-05	-0.659	0.609	2.00E-07	0.834	0.071
i = 7	3.23E-05	0.529	-0.530	4.03E-06	0.000	1.000
i = 8	1.60E-04	-0.874	0.821	3.86E-06	0.019	0.935
i = 9	4.35E-05	-0.917	0.890	8.19E-06	0.099	0.000
i = 10	2.65E-05	-0.941	0.900	2.00E-07	0.954	0.000
i = 11	-8.66E-06	-0.868	0.837	2.48E-06	0.233	0.000
i = 12	2.77E-06	0.251	-0.317	8.70E-06	0.356	0.000
i = 13	-4.71E-04	-0.992	1.000	5.04E-07	0.466	0.534
i = 14	2.17E-04	0.150	-0.466	1.17E-06	0.493	0.507
i = 15	-1.24E-06	0.766	-0.943	6.36E-07	0.735	0.265
i = 16	6.85E-04	-0.881	0.898	5.21E-06	0.000	1.000
i = 17	-2.82E-07	0.749	-0.726	1.29E-06	0.602	0.000
i = 18	4.63E-07	0.593	-0.514	4.36E-06	0.469	0.000

[1] The matrix of size is 250×7.

[2] We distinguish the notation of $u_{j,t}$ from z_t, because u represents the stable innovation for each factor.

Going back to our PCA model, using Eq. (1), we have 250 sample residuals $e_{i,t}$ for every i = 1,..., 18 based on the first 250 observations. We fit in each individual series $e_{i,t}$ the stable innovations and generate scenarios for $e_{i,251}$. This scenario generation is done independently for every i and also independently of the factor-innovations generations not by applying skewed t copula but by fitting the α-stable distribution on individual series of e.

We finally calculate the value of the return $r_{i,t}$ as in Eq. (1) for every asset on the next day, i.e., 251 day, by using the estimated parameters of p_i and $q_{i,j}$, the generated values of factor returns $f_{j,t}$, and the generated value of the small uncorrelated noise $e_{i,t}$.

In this way, we can generate the price returns of each energy futures with 3 types of energy and 6 different maturities.

2.5. Portfolio selection based on the price return model. By using the simulations of energy futures price returns, we obtain the winner and loser portfolios for energy futures. As the winner portfolio, we introduce the maximization of the STARR ratio for energy futures portfolio. The details are illustrated in Appendix C. In contrast, as the loser portfolio we employ the average weighted portfolio of energy futures prices. The realized portfolio wealth and total return of winner and loser portfolios are illustrated in Figures 8 and 9, respectively. Note that the wealth is defined as $\Pi_{i=1}^{n}(1+r_i)$.

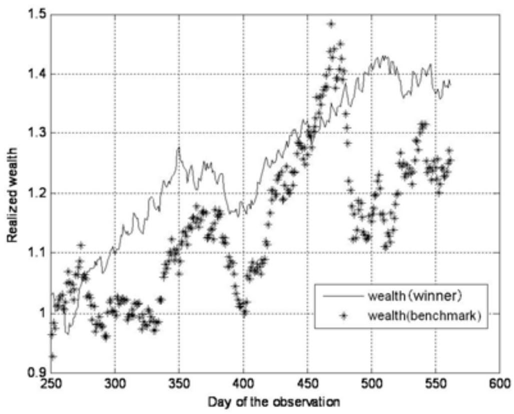

Fig. 8. Realized wealth of winner and loser portfolios

Figure 8 illustrates that the realized wealth of the winner portfolio almost exceeds that of the loser one except time around 270 and 470. In addition, Figure 9 illustrates that the realized total returns of the winner portfolio are more than those of the loser one except the same periods. Figures 8 and 9 imply that the STARR based portfolio, i.e., the winner portfolio, can generate more profit than the equally weighted benchmark portfolio, i.e., the loser portfolio, as long as judging from the simulation results.

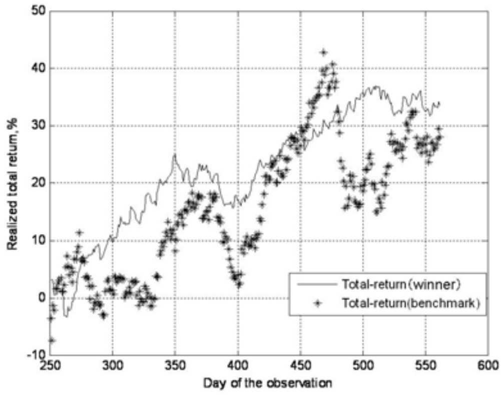

Fig. 9. Realized total return of winner and loser portfolios

Additionally, we also conduct another simulation. The results of two simulations are tabulated in Table 18.

Table 18. The results of simulations

Measures	Simulation №1		Simulation №2		Benchmark
	Values (daily, %)	Relative difference	Values (daily, %)	Relative difference	
Average mean return	0.107	0.191	0.106	0.179	0.090
Estimated ETL (99%)	3.471	-0.418	3.493	-0.414	5.961
STARR (99%)	3.085	1.046	3.033	1.011	1.508
Estimated standard deviation	0.862	-0.535	0.861	-0.535	1.852
Sharpe Ratio	12.424	1.559	12.312	1.536	4.855

Note that the relative difference is defined by the ratio of the value of simulation minus that of benchmark over the value of benchmark.

Table 18 suggests that STARR ratios of the two are 3.085 and 3.033, respectively and they are greater than that of benchmark of 1.508. Thus, the performance of the winner portfolios in energy futures markets by using the STARR ratio is likely to be better than that of loser portfolio by using the average return, as long as we employ the data in this paper.

Judging from Sharpe ratios as in Table 18, two simulations perform better than the benchmark with the relative differences of 1.559 and 1.536, respectively. On the other hand, from STARR ratios as in Table 18, two simulations perform better than the benchmark only with the differences of 1.046 and 1.011, respectively. The differences come from the risk measures: the standard deviation for the Sharpe ratio on one hand, and the ETL for the STARR ratio on the other hand. The standard deviation captures the risk of the portfolio by assuming that the price returns follow normal distributions, while the ETL does it by assuming that they do not necessarily follow normal ones. Thus, the estimated standard deviations evaluate the portfolio risk less than the estimated ETLs. Taking into account that price returns of energy futures have the stable distributed innovations, the STARR ratio may be more appropriate than the Sharpe ratio. It leads to the usefulness of the STARR ratio so as to obtain higher performance portfolio than the average in energy markets appropriately.

Conclusions and directions for future research

This paper has examined the portfolio optimization of energy futures by using the STARR ratio that can evaluate the risk and return relationship for skewed distributed returns. We have modeled the price return for energy by using the ARMA(1,1)-GARCH(1,1)-PCA model with stable distributed innovations that reflects the characteristics of energy: mean reversion, heteroskedasticity, seasonality, and spikes. Then, we have proposed the method for selecting the portfolio of energy futures by maximizing the STARR ratio. The empirical studies by using energy futures prices of WTI crude oil, heating oil, and natural gas traded on the NYMEX have compared the price return models with stable distributed innovations to those with normal ones for energy futures. We have show that the models with stable distributed innovations are more appropriate for energy futures than those with normal ones. In addition, we have offered some arguments that the stable innovations may come from price spikes in energy futures markets. Then, we generate the price returns by using the proposed ARMA(1,1)-GARCH(1,1)-PCA model with stable ones and choose the portfolio of energy futures. The results have illustrated that the selected portfolio performs better than the average weighted portfolio. It implies that the STARR ratio may work well in selecting the winner portfolio of energy futures.

This paper did not examine the performance of the long and short trading strategy in order to focus on the method for selecting the winner portfolio in energy futures markets. We leave it to the direction for our future research.

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Appendix A. α -stable distribution

The log-returns of energy prices are well known for having high skewness and kurtosis. So it is difficult to model such time series appropriately by using the normal distribution. α -stable distribution is often introduced as a tool to model such high skewness and kurtosis. Unfortunately, it does not have distribution function and density in closed form. Stable distributions are introduced by their characteristic function as follows,

$$\log F(t) = \begin{cases} -\sigma^{\alpha} |t|^{\alpha} (1 - i\beta \operatorname{sgn}(t) \tan(\frac{\pi\alpha}{2})) + i\mu t, & \alpha \neq 1 \\ -\sigma |t| (1 + i\beta \frac{2}{\pi} \operatorname{sgn}(t) \log |t|) + i\mu t, & \alpha = 1, \end{cases} \tag{A1}$$

where $F(t)$ denotes the characteristic function of the stable law:

$$F(t) = \int e^{itx} \frac{1}{\sigma} f\left(\frac{x-\mu}{\sigma}\right) dx . \tag{A2}$$

The parameter α describes the kurtosis of the distribution with $0 < \alpha \leq 2$. The smaller α is, the heavier is the tail of the distribution. The parameter β describes the skewness of the distribution, $-1 \leq \beta \leq 1$. If β is positive (negative), then the distribution is skewed to the right (left). μ and σ are the shift and scale parameters, respectively. If α and β equal 2 and 0, respectively, then the α -stable distribution reduces to the normal one.

Appendix B. The algorithm

In the beginning, knowing the number of observations N (N = 812) and return series R_t^i, we determine coefficients a and b of the ARMA(1,1) model: $R_t = a_0 + a_1 R_{t-1} + b_1 \varepsilon_{t-1} + \varepsilon_t$. Then, for that purpose we have to solve the system of equations:

$$\begin{cases} E[\varepsilon_i] = 0, \\ E[\varepsilon_i \varepsilon_{i-1}] = 0, \\ E[\varepsilon_i \varepsilon_{i-2}] = 0. \end{cases} \tag{B3}$$

If parameters a and b are found, we perform the next steps. We, then, restore empirical values of residuals $\varepsilon = (\varepsilon_t)$ from the ARMA(1,1) model: $R_t = a_0 + a_1 R_{t-1} + b_1 \varepsilon_{t-1} + \varepsilon_t$ based on found coefficients a and b:

$$\varepsilon_t = R_t - a_0 - a_1 R_{t-1} - b_1 \varepsilon_{t-1} . \tag{B4}$$

After we have found the residuals from ARMA(1,1), we finally apply the GARCH(1,1) model for them, obtain innovations and check the hypotheses for normality and stability for the innovations of the GARCH(1,1) model of the residuals from the ARMA(1,1) model. We determine parameters of stable distribution for the sequence of innovations: α is the index of stability ($\alpha \in (0,2]$), β is the skewness parameter ($\beta \in [-1,1]$), σ is the scale parameter ($\sigma \in R_+$), and μ is the shift parameter ($\mu \in R$).

Appendix C. Optimization problem solving for energy futures

$$\max_{z_{pl}} STARR_{\delta}(r^{(p)}) , \tag{C5}$$

$$STARR_{\delta}(r^{(p)}) = \frac{R^{(p)}}{ETL_{\delta}(r^{(p)})} , \tag{C6}$$

where z_{pl} is the weight of asset l in the portfolio of n assets, $r^{(p)}$ is the total random return of the portfolio consisting of n assets: $r^{(p)} = \Sigma_{l=1}^{n} z_{pl} r_l$, where r_l is the random daily return of asset l, $R^{(p)} = E(r^{(p)}) = \Sigma_{l=1}^{n} z_{pl} R_l$ is the total expected (daily) return of the portfolio of n assets, where R_l represents mean return (expected value of r_l-vector of dimension equal to 250 working days), and n is set to 18 such that $z_{pl} > 0$ where z_{pl} is the weight of individual asset l in the portfolio of n assets:

$$z_p = \sum_{l=1}^{n} z_{pl} = 1 , \tag{C7}$$

$$n = 18. \tag{C8}$$