RISK LIMIT SYSTEMS AND CAPITAL ALLOCATION IN FINANCIAL INSTITUTIONS

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Abstract

Financial institutions aim to cap the market risk taken by their trading divisions through limited providing of risk capital or setting risk limits, respectively. Risk limits must be based on the market risk model internally used. The paper addresses the questions of how to efficiently allocate risk capital and how to consistently construct hierarchical systems of risk limits. We develop a very concrete model to allocate risk capital and to control market risk in trading divisions of financial institutions. Based on Value-at-Risk we show how to build up a consistent system of risk limits which guarantees both an optimal profitability and limiting market risk. An optimization approach is used to construct a system of risk limits for any hierarchical order of trading portfolios. Because of instable or simply unknown correlations between portfolio's loss variables there is in result a trade-off between exploiting risk capital and strictly keeping to risk limits.

Key words: Market risk, Risk capital allocation, Risk limits, Value-at-Risk.
JEL Classification: G21, G31, C10.

I. Introduction

The quantification of risks has been known and mastered by the majority of financial institutions – for example by applying the Value-at-Risk concept – and risk models used lead to sufficiently exact results in most cases[1]. The modern risk management is now focusing more and more onto the effective control of risks. This is the consequent conclusion because risk controlling can exclusively relate to risks which are sufficiently objective, i.e. measurable. Apart from effect-related methods of risk controlling, such as hedging, causal-related ex ante restrictions of risky business activities arise due to risk limits and possibly their intertemporal adjustment being dependent on business success[2]. The related restriction and control of the assumption of risk are required by supervisory and economic reasons to ensure the stability and further existence of the financial institution[3]. Moreover, effective risk restrictions can minimise the costs of asymmetric information between the management and the shareholders and between the financial institution and customers[4].

Up to now, the main part of the scientific examination of the subject of research, that is the risk management in financial institutes, has focused on the adequate modelling of risk measurement. Questions about risk controlling such as the limitation of risks, the construction of hierarchic systems of risk limits and the allocation of risk capital in such systems seem to have been discussed insufficiently so far.

The management of risk capital in financial institutions was already discussed by Merton and Perold (1993), later by Matten (1996) and Kupiec (1999). Later on, the problem of allocation of risk capital was brought into discussion e.g. by Schierenbeck and Lister (1998) and Saita (1999), who describe some allocation processes, but concrete algorithms to solve the allocation problem are not provided. In part based on Froot and Stein (1998), Stoughton and Zechner (1999) analyse decisions of capital allocation onto decentralized autonomous business divisions. By using

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[1] See Berkowitz, O'Brien (2002) for a recent empirical investigation of the quality of internal Value-at-Risk-models in American banks.
[2] For example Beeck et al. (1999) tackle the problem of the intertemporal adjustment of Value-at-Risk limits.
[3] See for example Stulz (1996) and recently Danielsson et al. (2002) with further references for a discussion of risk management incentives.
[4] See Kinder et al. (2001), p. 282.

so called risk-adjusted profitability measures they develop shareholder value maximising allocation procedures[1]. Froot and Stein (1998) and Gründl and Schmeiser (2002) investigate the degree up to which capital allocations based on such risk-adjusted profitability measures are really compatible with a company management maximising shareholder value. Contrary to frequent statements they show that this is not necessarily the case. Denault (2001) compares the problem of allocation of risk capital with the cost allocation in coalitional games (whereby the cost function represents the risk measure) and shows that the optimum risk capital allocation onto the divisions is given by the gradient of the positive homogeneous and differentiable risk measure. In the Value-at-Risk context this corresponds to the vector of the marginal Value-at-Risk[2]. A game theory approach is also presented by Kinder et al. (2001) who develop allocation methods based on cost gap procedures.

The contributions just mentioned have in common that they discuss the problem of the allocation of risk capital among divisions on corporate level. In this paper, however, we constructively want to turn to the allocation of risk capital within one division, the trading division of a financial institute in particular. In doing so, we will investigate the construction of a risk limit system for a hierarchic pattern of trading portfolios and develop a model for the optimum allocation of risk capital in this hierarchy. For this purpose, we start from an externally given amount of risk capital for the trading division, so to speak from an upstream allocation on corporate divisions. Particularly for the trading division of banks, Burmester et al. (1999), Ridder (1999) and Eisele and Knobloch (2000) present first analytic model approaches. In the following, these approaches will be partly used, improved and extended. For the first time, in this paper a consistent solution is presented for the allocation problem in a portfolio hierarchy. A model is developed which is able to explain the empirical finding of not fully utilised risk limits on the highest aggregation level[3].

This contribution is structured as follows. Section II starts with a discussion of the objective function of the financial institute. For the further analysis we act on the assumption of maximising a risk adjusted profitability measure. Afterwards, a well-known analytic Value-at-Risk model is presented by the parametric delta-normal model, to which we return in the following to simplify the matter. Moreover, the term risk capital will be defined for the use in this paper. After clarifying the temporal constitution of the model, in Section III we will discuss the structure of a risk limit system and afterwards we will present an optimising approach for the selection of an optimum allocation of risk capital. Finally, the proposed procedure will be illustrated by a two-dimensional allocation problem. This contribution will end with a final consideration in Section IV.

II. Analysis Framework and Model Structure

1. Objective Function of the Financial Institution

For the analysis that follows, we assume the financial institution to aim at maximisation of the risk adjusted profitability of its trading division. It is further assumed that the available risk capital of the institute which is necessary to back risky positions is a short resource and does not have substitution relationships to other capital resources[4]. A management ratio out of the class of the so called Risk Adjusted Profitability Measures (RAPM) is used as objective function[5]. These measures do

[1] Risk-adjusted profitability measures are used in relation with the corporate division management and capital allocations in financial institutions in a series of papers, for example in James (1996), Uyemura et al. (1996), Zaik et al. (1996), Smithson et al. (1997) and Lehar et al. (1998).
[2] See Jorion (2000), p. 154 for the marginal Value-at-Risk.
[3] Perold (2001), for example, reports that the risk capital effectively used by trading divisions often just amounts to about 30% of the risk capital available on aggregated level.
[4] See for example Berger et al. (1995).
[5] For this widely-used proceeding see Matten (1996), Uyemura et al. (1996), Zaik et al. (1996), Schierenbeck, Lister (1998), Lehar et al. (1998), Stoughton, Zechner (1999) or Kinder et al. (2001) as examples of a lot of others. But from the theoretic point of view there are doubts about the compatibility of a corporate management based on these measures if the superior target to maximise the shareholder value is accepted. For this problem see Froot, Stein (1998) and Gründl, Schmeiser (2002).

not aim to determine the gained profit in relation to the invested capital but in relation to the capital being at risk. In literature, decisions about the allocation of risk capital are mentioned among others as a possible field of application of these ratios[1]. Such an approach is based on the idea that for two positions with the same invested capital a higher demand for risk capital will exist for this position which has the higher risk proportion – e.g. measured via the Value-at-Risk. The definition of risk capital will be made in Section II.2. It depends on the accepted risk measure.

In literature uncounted variants of these RAPM[2] exist. They all can, however, be reduced to one basic formula. So we define the risk-adjusted profitability measure for a portfolio i in its ex-post form as[3]:

$$\text{RAPM}_{i;t}(C_{i;t}) = \frac{G_{i;t}(C_{i;t}) - r_{i;t} \cdot C_{i;t}}{C_{i;t}} = \frac{G_{i;t}^{\text{ad}}(C_{i;t})}{C_{i;t}}. \tag{1}$$

Here, $G_{i;t}(C_{i;t})$ is used for the profit and $G_{i;t}^{\text{ad}}(C_{i;t})$ for the risk adjusted profit of the portfolio. Further on $r_{i;t}$ denotes the rate of risk capital costs and $C_{i;t}$ the risk capital (or risk limit). In the ex-ante consideration being of interest in this context we have to understand the incoming values principally as unknown and at least the profit as stochastic. Within the profitability determination, the latter is regularly defined as the profit of the portfolio minus operating and refinancing costs[4]. The variable being in question within the present allocation problem is the risk capital allocated or to be allocated to the portfolio.

The risk adjusted profit is modelled by a strictly monotonic increasing function with decreasing marginal profit[5]. So the RAPM is a monotonic increasing function:

$$\frac{dG^{\text{ad}}(C)}{dC} > 0, \quad \frac{d^2 G^{\text{ad}}(C)}{dC^2} < 0 \quad \Rightarrow \quad \frac{d\text{RAPM}(C)}{dC} > 0. \tag{2}$$

We interpret the rate of risk capital costs as the demanded target return of the risk capital (also Hurdle Rate). There are controversial opinions about the question, whether different target returns should be used for different portfolios or whether a uniform target return should be applied for different portfolios. The main argument for the use of a uniform target return are the "influence costs". This term describes both the struggles for power among the corresponding portfolio managers when coming to an agreement on the target returns and their efforts to agree upon the target returns as low as possible due to overestimation of the future risk[6]. It remains unclear why one should change to a risk adjustment of the numerator in (1), if the risk capital costs are uniform. The demand for different target returns for the distributed risk capital goes along with its often

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[1] See Merton, Perold (1993), p. 27, James (1996), p. 2 and Saita (1999), p. 97. Here, decisions about business policy, restructuring measures and managers' awards are mentioned as further fields of application of RAPM.

[2] The variants of RAPM and the terms mentioned in their context are not consistently used and clearly defined in literature. The differences are to be found mostly in the question, whether the numerator and/or the denominator of the measure will be risk-adjusted or not.

[3] See Matten (1996), p. 62, Punjabi (1998), p. 71, Lehar et al. (1998), p. 949, Stoughton, Zechner (1999), p. 14 and Smithson, Hayt (2001), p. 80 for identical or similar approaches. Here, the measure defined in this way is also designated as Risk Adjusted Return on Risk Adjusted Capital (RARoRAC). The index t used for the values refers to the time of consideration in this context. This index refers to the interval $[t-T, t]$ in ex-post considerations and to the interval $[t, t+T]$ in ex-ante considerations.

[4] See Matten (1996), p. 63, Punjabi (1998), p. 71, Lehar et al. (1998), p. 861 and Stoughton, Zechner (1999), p. 13 for similar approaches.

[5] Burmester et al. (1999) use a quadratic profit function.

[6] See Merton, Perold (1993), p. 25 and Lehar et al. (1998), p. 951. Saita (1999), p. 103, proposes the introduction of "punishment" for under-utilised risk limits. James (1996), p. 16, justifies uniform target interests with the uniform risk quantification.

suggested derivation from the Capital Asset Pricing Model (CAPM)[1], and the argument that trading portfolios can only be compared by RAPM if the systematic risk taken by them is taken into account. Apart from theoretic doubts about this procedure, the relation to the risk capital does not make the distinction between non-systematic and systematic risks unnecessary. In the same way as in the Value-at-Risk models non-systematic and systematic market price risks cannot be quantified separately, the risk capital will always cover non-systematic risks, too.

To determine the cost of risk capital, we use the approach suggested by Froot and Stein (1998) here. They propose a two-factor-model to derive the return claim for the risk capital and apply this model onto the decisions of financial institutions about capital allocations and capital structures. If this two-factor-model is transferred to the context given in this paper we get the following form[2]:

$$r_i = \frac{r_M - r_f}{\sigma_M^2} \cdot \mathrm{cov}_{i,M} + a \cdot \mathrm{cov}_{i,P} \,. \tag{3}$$

The first summand corresponds to the well-known market risk component with the expected market return r_M, the risk-free return r_f, the variance of the market return σ_M^2 and the covariance $\mathrm{cov}_{i,M}$ between the market return and return of the risk capital of the trading portfolio i. The second summand reflects – with the covariance $\mathrm{cov}_{i,P}$ between the risk capital return of the portfolio i and the aggregated overall portfolio P – the contribution of this portfolio to the total risk of the trading division. The standard price of the second component is given by the measure of the risk aversion a of the financial institution. This risk aversion depends, among others, convexly on the equity capitalisation of the institution, i.e. that the institution is less risk-averse with an increasing equity[3].

2. Quantification of Risk and Risk Capital

It is assumed that the risk measure and risk controlling criterion internally used by the financial institute is Value-at-Risk (VaR). For reasons which will not be discussed in this context, it has become generally accepted as the measure for the quantification of market price risks relevant for trading divisions. As generally known, the Value-at-Risk specifies the smallest (positive) loss barrier l, which will not be exceeded by the stochastic loss L at least with the given probability p at the end of a defined time interval T[4]:

$$\mathrm{VaR}_{t+T}(p) := \inf\{l : l \geq 0, \mathrm{prob}(L_{t+T} \leq l) \geq p\}. \tag{4}$$

We hereby define the loss of a portfolio as the negative change of its market price W_t:

$$L_{t+T} := W_t - W_{t+T} \,. \tag{5}$$

[1] See Matten (1996), p. 95, Uyemura et al. (1996), p. 105, Zaik et al. (1996), p. 88, Lehar et al. (1998), p. 950, Punjabi (1998), p. 71 and Kinder et al. (2001), p. 285. The CAPM is often seen as the theoretic starting point for the RAPM. In the result it is declared that the RAPM correspond ex post to the increase of a capital market line (Sharpe measure). A theoretic relation between the RAPM and the CAPM, however, can not be detected. It could possibly be constructed through the profit discounting model for the evaluation of the own capital. If the risk-adjusted interest rate used here is assumed as being derived from the CAPM, then the return expected according to CAPM corresponds to the sum made of the profit-course rate and rate of profit growth. The latter on its part depends on the withholding rate and the internal Return on Equity (RoE). $\mathrm{RoE} = G/C \cdot C/EK$ provides the relation of RoE and RoRAC (G/C).

[2] See Froot, Stein (1998), p. 66.

[3] See Froot, Stein (1998), p. 65 and Froot et al. (1993), p. 1639.

[4] See e.g. Huschens (1999), p. 39 for the definition and Uhlir, Aussenegg (1996), Duffie, Pan (1997), Linsmeier, Pearson (2000) and Jorion (2000) among a lot of others for an overview about the Value-at-Risk.

The Value-at-Risk does not generally define a so called coherent risk measure according to Artzner et al. (1999), and it is not generally compatible with the criterion of second order stochastic dominance according to Rothschild and Stiglitz (1970). Although the Value-at-Risk provides theoretic vulnerabilities it allows an acceptable approximation of market price risks and is established as the conventional industrial standard today[1]. Because of the normality assumption, in the delta-normal model used here it also overcomes the vulnerabilities mentioned[2] and presents a risk measure that is suitable in the context of market risk control. Moreover, we start from the assumption that we can clearly define the Value-at-Risk as p-fractile of the loss distribution $F_L(l)$ that is to be prognosticated:

$$\mathrm{VaR}_{t+T}(p) = F_{L_{t+T}}^{-1}(p)\,. \tag{6}$$

For this purpose we use the parametric delta-normal model (also covariance approach)[3]. This model operates with two central approximations[4]. First, the continuous returns $\boldsymbol{Y}_{t+T}$ of the M risk factors[5] $\boldsymbol{R}_t$ determining the market value of the portfolio are modelled independent and identically normally distributed with the expectation value vector $\boldsymbol{\mu}_Y = (\mu_1, \ldots, \mu_M)'$ and covariance matrix $\boldsymbol{\Sigma}_Y = (\sigma_{jk})$, $j, k = 1, \ldots, M$. Second, the map of these risk factor returns onto the portfolio's change in market value (or loss according to (5)) is described in a linear function $L_{t+T} = L(\boldsymbol{Y}_{t+T})$[6]. The restrictive approximations of the delta-normal model are not adequate in a lot of practical cases[7]. But only the use of this comparably simple risk model allows to obtain a complete analytic solution of the allocation problem in the following. Even for this simple frame we are able to explain empirical findings such as the often significantly under-utilised risk limits that can be observed in relation with risk capital allocations. We obtain the linear loss function of a portfolio:

$$L(\boldsymbol{Y}_{t+T}) = -\boldsymbol{\delta}_t{}' \boldsymbol{R}_t^{\mathrm{diag}} \boldsymbol{Y}_{t+T}\,. \tag{7}$$

In this equation $\boldsymbol{\delta}_t$ presents the gradient of the portfolio value with respect to the risk factors and $\boldsymbol{R}_t^{\mathrm{diag}} = \mathrm{diag}(\boldsymbol{R}_t)$ the diagonal matrix of the market values of the risk factors. Based on the model assumptions we can conclude that the loss is normally distributed with the expected value $\mu_{L;t} = -\boldsymbol{\delta}_t{}' \boldsymbol{R}_t^{\mathrm{diag}} \boldsymbol{\mu}_Y$ and variance $\sigma_{L;t}^2 = \boldsymbol{\delta}_t \boldsymbol{R}_t^{\mathrm{diag}} \boldsymbol{\Sigma}_Y \boldsymbol{R}_t^{\mathrm{diag}} \boldsymbol{\delta}_t{}'$. The normal distribution characteristics of the loss allows to estimate the Value-at-Risk by[8]:

[1] See e.g. Szegö (2002) for a critical consideration of Value-at-Risk.
[2] See Read (1998) for a proof.
[3] In parametric models the loss distribution to be prognosticated is drawn from the distribution of the risk factors determining the market value or from the distribution of their relative changes. For the latter distribution assumptions are made. In contrary to this, non-parametric models operate without distribution assumptions. See e.g. Locarek-Junge et al. (2002) for the application of parametric and non-parametric estimation models.
[4] See Huschens (2000) for the discussion of different methods of market risk estimation and its relevant assumptions.
[5] Such risk factors are for example interest rates, exchange rates, stock prices, prices for commodities or index states.
[6] See e.g. Duffie, Pan (1997), p. 23, Linsmeier, Pearson (2000), p. 53 and Jorion (2000), p. 219.
[7] The assumption of independent and identically normally distributed risk factors cannot be sustained in a lot of cases. Empirical investigations of return time series often indicate leptokurtic and asymmetric conditional probability distributions. Moreover, return time series are often heteroscedastic, that means the moments of the distributions are not constant over the time but have a changing variance over the time in particular. The assumption of a linear loss function is especially problematic for non-linear financial instruments such as options. In the cases mentioned serious doubts arise about the quality of the Value-at-Risk estimation if the delta-normal model is used. See e.g. Gaumert (2000) for a critical discussion about this topic.
[8] See Jorion (2000), p. 113, as an example.

$$\mathrm{VaR}_{t+T}(p) = T \cdot \mu_{L;t} + \sqrt{T} \cdot \sigma_{L;t} \cdot z(p). \tag{8}$$

Hereby, $z(p)$ describes the *p*-fractile of the standard normal distribution. Along with the comparably simple analytic determination of the Value-at-Risk, it is also easily possible to aggregate the Value-at-Risk of several portfolios in this model frame. Via the correlation matrix $\boldsymbol{P}_{L;t} = (\rho_{jk;t})$, $j,k = 1,\ldots,K$, of the portfolio losses the Value-at-Risk of K portfolios integrated in the vector $\mathbf{VaR}_{t+T}$ aggregate by applying the additional assumption $\boldsymbol{\mu}_Y \approx \mathbf{0}$[1]:

$$\mathrm{VaR}_{t+T}^{\mathrm{agg}} = \sqrt{\mathbf{VaR}_{t+T}^{'} \boldsymbol{P}_{L;t} \mathbf{VaR}_{t+T}}\,. \tag{9}$$

Now a strict distinction is to be made between the measured risk amount and the (risk) capital amount, which is used for risk controlling purposes. The understanding of risk capital depends on the accepted risk measure. By using the risk measure Value-at-Risk, the risk capital of the financial institution is defined as the capital amount which is at least required to cover the unexpected loss with the probability p[2]. But if the Conditional Value-at-Risk[3] was used as the risk measure the risk capital would be understood as the capital amount required to cover the unexpected loss with the probability p and additionally to cover the expected loss of all losses exceeding the Value-at-Risk with the probability $1-p$.

We further assume an exogenously given amount of risk capital for the allocation in the trading division. Depending on equity and the management's attitude toward risk, it is provided by the bank management and is to be interpreted as the total risk limit according to the above introduced definition. It presents the highest possible risk – quantified by the risk measure Value-at-Risk – which may be taken. Therefore, the allocation of the risk capital is realised via the risk limit system.

3. Temporal Constitution of the Model

In contrast to the quantification of taken risks at the present point of time given the present information, the construction of a system of risk limits at present time affects the limitation of risks to be taken in the future. Available information to quantify risks at that future points of time is unknown from the present perspective.

If, in addition to the previous modelling, the parameter or information vector $\boldsymbol{\theta}_t$ is introduced, which is still not defined in detail for the first instance, Value-at-Risk can be written as: $\mathrm{VaR}_{t+T}(p;\boldsymbol{\theta}_t)$ and $\mathrm{VaR}_{\tau+T}(p;\boldsymbol{\theta}_\tau)$. One could imagine e.g. market data, statistical parameters and portfolio structures as content of such an information vector. In $\tau > t$, the information $\boldsymbol{\theta}_\tau$ is unknown at time t. A risk limit is typically determined at time t with the aim to restrict the risk taking at time τ. Because the information $\boldsymbol{\theta}_\tau$ which is needed for Value-at-Risk estimation at time τ is unknown at time t the risk limit is parameterised by $\boldsymbol{\theta}_\tau$: $C(\boldsymbol{\theta}_\tau) \geq \mathrm{VaR}_{\tau+T}(p;\boldsymbol{\theta}_\tau)$. Figure 1 points up the temporal configuration which forms the basis for our further argumentation and modelling.

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[1] See e.g. Geyer, Pichler (1998), p. 946 and Uhlir, Aussenegg (1996), p. 833.
[2] See for similar discussion Kupiec (1999), p. 42, Wilson (1997), p. 194 and Matten (1996), p. 104.
[3] See for the Conditional Value-at-Risk e.g. Rockafellar, Uryasev (2002) and Acerbi, Tasche (2002).

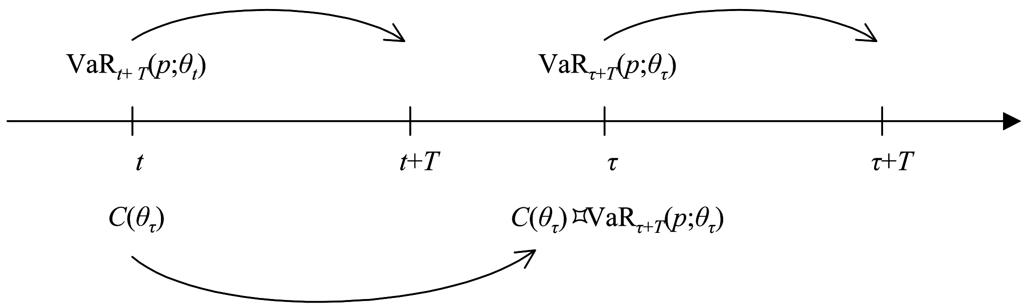

Fig. 1. Temporal Constitution of the Modelling

The described temporal modelling only affects the ex ante limitation of future risk taking which is established by the risk management. The portfolio managers decisions on investments which were delegated to them remain unaffected. They are not controlled directly but rather indirectly by means of risk amounts maximally allowed.

III. Risk Limit System and Risk Capital Allocation

1. Consistent Risk Limit System

We define a risk limit system as a multi-stage hierarchic system of risk limits to be used to simultaneously restrict the market price risks of all trading portfolios and aggregated portfolios within a trading division of a financial institution. It must be designed in such a way that apart from the desired limitation of the risk taking by the institution a target-optimum allocation of risk capital is generated at the same time.

In the following we consider a portfolio hierarchy which may exist in the trading division of the financial institution and which is simplified in Figure 2. Within this portfolio hierarchy consisting of a total number of K portfolios we consider $J < K$ basic portfolios which are aggregated via $K - J - 1$ intermediate portfolios to an overall portfolio.

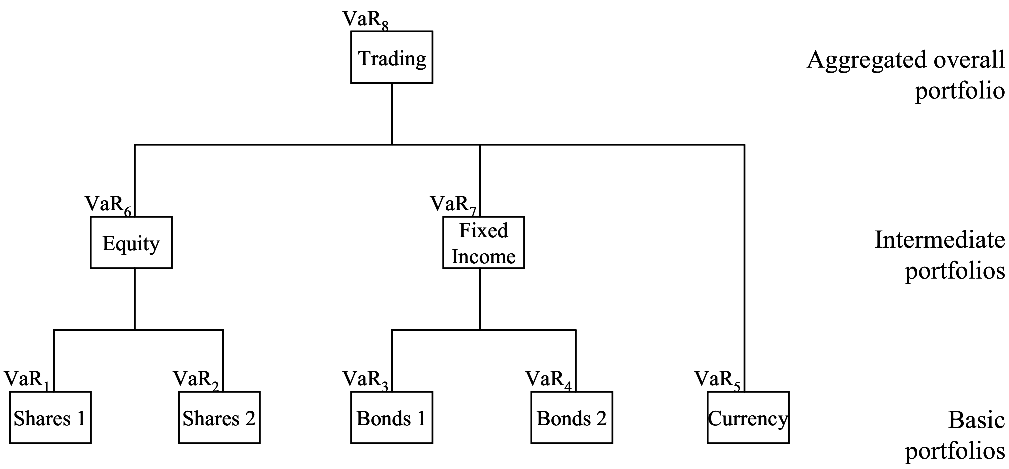

Fig. 2. Example of a portfolio and risk limit hierarchy

The goal is now to construct a consistent system of risk limits along the portfolio hierarchy which fulfils the following demands[1]:

- The aggregated risk limit of the trading division may never exceed the risk capital provided for the trading division.
- The maintenance of the risk limits in the decentralised basic portfolios must ensure the maintenance of the risk limits on all the aggregation levels located above.
- The risk limits of the basic portfolios are to be applicable clearly and independently from the risks taken by the other basic portfolios.
- The risk capital is to be distributed completely for the purpose of its efficient utilisation.

To solve this task we modify and expand the delta-normal model frame appropriately. Overall all basic portfolios a total number of N financial instruments $\boldsymbol{I}$ are traded. The market value $w_{i;t}$, $i = 1,\ldots,N$, of the i^{th} financial instrument develops as a function of the relevant risk factors $w_{i;t} = w(\boldsymbol{R}_t)$. $\boldsymbol{w}_t$ shall be the vector of the market values of all financial instruments. To reflect the concrete structure of the J basic portfolios in relation to the N financial instruments, we introduce the structure vectors $\boldsymbol{\theta}_{i;t}$, $i = 1,\ldots,J$. They include the numbers of the financial instruments in the basic portfolio. The structure vector contains the element zero for a financial instrument which is not traded or may not be traded in the portfolio (for example equity portfolios would not include bonds and vice versa). Therefore, the market value of the i^{th} basic portfolio results as:

$$W_{i;t} = \boldsymbol{\theta}_{i;t}'\boldsymbol{w}_t . \tag{10}$$

Amount-specific limitations of the investment universe $\boldsymbol{u}_i \leq \boldsymbol{\theta}_{i;t} \leq \boldsymbol{o}_i$ with minimum limits $\boldsymbol{u}_i$ and maximum limits $\boldsymbol{o}_i$ are possible for the structure vectors of the basic portfolios. Thus, it is additionally possible to integrate volume limitations common in actual trading into the model. In order to understand the overall structure of the trading division we generate first the matrix $\boldsymbol{\Theta}_{J;t} = (\boldsymbol{\theta}_{1;t},\ldots,\boldsymbol{\theta}_{J;t})$ from the structure vectors of the basic portfolios. The J columns of this matrix correspond to the basic portfolio structures. To illustrate this portfolio hierarchy we further introduce the matrix $\boldsymbol{H}$ which will contain the element one if a portfolio is to be aggregated to the next superior level, otherwise the element zero. Therefore, the structure of the complete portfolio hierarchy results in the following equation[2]:

$$\boldsymbol{\Theta}_t = \boldsymbol{\Theta}_{J;t}\boldsymbol{H} = (\boldsymbol{\theta}_{1;t},\ldots,\boldsymbol{\theta}_{J;t},\ldots\boldsymbol{\theta}_{K;t}) . \tag{11}$$

In each case, we obtain the structure of the next higher aggregation level from the sum of the structures of the portfolios which are positioned below and are to be aggregated; $\boldsymbol{\theta}_{K;t}$ finally represents the structure of the aggregated overall portfolio of the trading division.

By considering the assumption (being inherent to the Value-at-Risk estimation) of a portfolio structure $\boldsymbol{\theta}_{i;t}$ being constant over the time interval T, we obtain the loss of the i^{th} basic portfolio from[3]:

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[1] See Denault (2001), p. 5, Saita (1999), p. 100 and Ridder (1999), p. 7.

[2] Due to the linear dependencies of the column vectors the matrix $\boldsymbol{\Theta}$ will not possess the full column rank. See e.g. Bosch, Jensen (1994), p. 335.

[3] The addition and multiplication in $\boldsymbol{R}_t(1+\boldsymbol{Y}_{t+T})$ are to be understood as per element in each case.

$$L_{i;t+T} = W_{i;t} - W_{i;t+T} = \boldsymbol{\theta}_{i;t}'[\boldsymbol{w}_t - \boldsymbol{w}_{t+T}] = \boldsymbol{\theta}_{i;t}'[\boldsymbol{w}(\boldsymbol{R}_t) - \boldsymbol{w}(\boldsymbol{R}_t(1+\boldsymbol{Y}_{t+T}))]. \tag{12}$$

We can also summarise the loss variables of all K portfolios of the given portfolio hierarchy as:

$$\boldsymbol{L}_{t+T}(\boldsymbol{Y}_{t+T},\boldsymbol{\Theta}_t) = \boldsymbol{\Theta}_t'[\boldsymbol{w}(\boldsymbol{R}_t) - \boldsymbol{w}(\boldsymbol{R}_t(1+\boldsymbol{Y}_{t+T}))]$$
$$= (L_{1;t+T} \ldots L_{J;t+T} \ldots L_{K;t+T})'. \tag{13}$$

Unknown determination factors of the losses of the portfolios according to (12) are not only the returns of the risk factors as in (7) any longer. Ex ante unknown are also the concrete (future) portfolio structures. The vector of portfolio losses according to (13) is ex ante additionally parameterised by the structure matrix $\boldsymbol{\Theta}_t$.

The starting point of all further considerations is the covariance matrix $\boldsymbol{\Sigma}_I$ of the returns of all N financial instruments in question[1]. The returns of the instruments are independent and identically normally distributed. From this covariance matrix we can derive the covariance matrix of the loss variables of the portfolio hierarchy by means of the diagonal matrix $\boldsymbol{w}_t^{\text{diag}} = \text{diag}(\boldsymbol{w}_t)$ of the market values of all financial instruments and of the structure matrix $\boldsymbol{\Theta}_t$ as:

$$\boldsymbol{\Sigma}_{L;t} = \boldsymbol{\Theta}_t'\boldsymbol{w}_t^{\text{diag}}\boldsymbol{\Sigma}_I\boldsymbol{w}_t^{\text{diag}}\boldsymbol{\Theta}_t. \tag{14}$$

We also determine the expected value vector of all loss variables as:

$$\boldsymbol{\mu}_{L;t} = \boldsymbol{\Theta}_t'\boldsymbol{w}_t^{\text{diag}}\boldsymbol{\mu}_I. \tag{15}$$

Finally, we get the vector for the Value-at-Risk structure of the complete portfolio hierarchy directly on the main diagonal of the loss-covariance-matrix[2]:

$$\mathbf{VaR}_{t+T}(p,\boldsymbol{\Theta}_t) = \text{diag}(\sqrt{T \cdot \boldsymbol{\Sigma}_{L;t}} \cdot z(p)) + T \cdot \boldsymbol{\mu}_{L;t}$$
$$= (\text{VaR}_{1;t+T} \ldots \text{VaR}_{J;t+T} \ldots \text{VaR}_{K;t+T})'. \tag{16}$$

Like the vector of portfolio losses according to (13) the Value-at-Risk vector according to (16) is ex ante additionally parameterised by the structure matrix $\boldsymbol{\Theta}_t$. From the present point of view t, the risks measured in future points of time τ also depend on the unknown portfolio structures (see Section II.3). So the precise form of the risk limit structure, too, significantly depends on the covariance matrix of the portfolio losses. The ex-ante limitation of risks has to face the additional difficulty that the future covariances (and correlations) between the portfolio losses will not only be determined by possibly predictable market developments. They also considerably depend on the delegated (future) decisions about the structure of the basic portfolios made by the portfolio managers independently in the frame of the defined investment universe.

The Value-at-Risk vector includes the risk measures of all basic portfolios and aggregated portfolios. Its last element is the aggregated Value-at-Risk of the overall portfolio of the trading division.

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[1] See also Ridder (1999), p. 10. The number of the financial instruments does not comply with the number of the risk factors ($N \neq M$). Thus, the market value of an option is determined by several risk factors. A portfolio of N bonds, however, can show $M < N$ risk factors. Only in the case in which $N = M$, being for example applicable for domestic shares, the covariance matrix of the risk factor returns $\boldsymbol{\Sigma}_Y$ is used at this point.

[2] The root operation $\sqrt{\boldsymbol{\Sigma}_L}$ is to be understood as per element.

Due to the exogenous default of this aggregated Value-at-Risk in form of the risk capital provided for the trading division ($\mathrm{VaR}_K = \mathrm{C}$), we can numerically determine a set of Value-at-Risk combinations by recursion and by varying the structures of the sub-portfolios. The last element of these vectors corresponds to the given amount of the maximally allowed risk in each case. This procedure will be explained in more detail in Section III.3. The result is a permissible set V of Value-at-Risk vectors whose last element always corresponds to the given risk capital:

$$V = \left\{\mathbf{VaR}_{t+T} : \mathrm{VaR}_K = \mathrm{C}\right\}.$$

If we interpret the Value-at-Risk vector as the risk limit structure, that leads to a set of risk limit structures which (ceteris paribus) all maintain the risk limit on the highest aggregation level for the complete distribution of risk capital. Therefore, we designate V as the set of permissible iso-risk limit structures.

2. Optimal Risk Capital Allocation

Which risk limit structure is to be selected out of the set of permissible iso-risk limit structures? To answer this question we use an optimisation calculus. The optimum risk capital allocation is achieved as soon as the overall aggregated RAPM of the trading division maximises for the given risk capital C (and well-known, unchangeable profit functions $G_i^{\mathrm{ad}}(C_i)$ and target returns r_i). It is assumed that the allocated amounts of risk capital or the risk limits, respectively, are always completely exploited by the portfolio managers of the basic portfolios. The optimisation problem is[1]:

$$\mathrm{RAPM}_K^{\mathrm{agg}}(\boldsymbol{C}) = \frac{\sum_{i=1}^{K} G_i^{\mathrm{ad}}(C_i)}{\sqrt{\boldsymbol{C}'\boldsymbol{P}_L\boldsymbol{C}}} \rightarrow \max!$$

subject to $\sqrt{\boldsymbol{C}'\boldsymbol{P}_L\boldsymbol{C}} \leq \mathrm{C}$. (17)

Hereby, $\boldsymbol{C}$ is the vector of the risk capital amounts of the portfolio hierarchy and represents the risk limit structure. By applying the Lagrange approach we get (by the Lagrange multiplier λ) the Lagrange function:

$$L = \frac{\sum_{i=1}^{K} G_i^{\mathrm{ad}}(C_i)}{\sqrt{\boldsymbol{C}'\boldsymbol{P}_L\boldsymbol{C}}} + \lambda\left(\mathrm{C} - \sqrt{\boldsymbol{C}'\boldsymbol{P}_L\boldsymbol{C}}\right). \tag{18}$$

The real solution of the equation system determined by

$$\frac{\partial L}{\partial \lambda} = 0, \quad \frac{\partial L}{\partial C_i} = 0, \quad i = 1,\ldots,K, \tag{19}$$

always clearly leads to a global maximum[2]. To find solutions we can apply well-known numeric optimisation algorithms such as the gradient decline method.

We find the following property concerning the aggregated RAPM. Whereas the risk-adjusted profits in the numerator are additive, the risk capital in the denominator is sub-additive for not per-

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[1] See Burmester et al. (1999), p. 402 for similar approaches. The time indication is suppressed in the following.
[2] See Bosch, Jensen (1994), p. 246.

fectly correlated changes in portfolio values ($\boldsymbol{P}_L \neq \mathbf{1}$)[1]. This leads to the result that the aggregated profitability will normally be larger than the sum of the profitabilities of the incoming portfolios:

$$\sqrt{\boldsymbol{C}'\boldsymbol{P}_L\boldsymbol{C}} \leq \sum C_i \Rightarrow \text{RAPM}_K^{\text{agg}} \geq \sum \text{RAPM}_i \ .$$

The formulation of the equation system to be solved makes immediately clear that the optimum risk capital allocation $\boldsymbol{C}^*$ (or the optimum risk limit structure) – as elaborated already above – considerably depends on the correlation matrix $\boldsymbol{P}_L$ of the portfolio losses. The higher the correlations between the value changes of the portfolios are, the lower the risk capital contributions to be allocated to the portfolios. The problem for the allocation of the risk capital (or for the risk limit system) is that the correlations are ex ante unknown and can be changed due to the delegated, independent decisions made by the portfolio managers about the future concrete structure of the basic portfolios. But even in case of an unchanged composition of the portfolios the correlation matrix is unstable over the time due to changes in the market behaviour. The risk capital allocation has to consider these effects properly, because changes of the correlation matrix alone may cause the risk of exceeding the risk limit on the aggregated level, although the risk limits in the basic portfolios are maintained.

3. Example of the Two-dimensional Allocation Problem

To make the proposed procedure clear and to discuss the results to be observed we use a simple two-dimensional example of a risk capital allocation. We consider $J = 2$ basic portfolios which are aggregated to an overall portfolio ($K = 3$). In each basic portfolio $N = M = 2$ financial instruments are traded. To simplify the matter we assume $\boldsymbol{\mu}_I = \mathbf{0}$ for them. We get the following structure vectors:

$$\boldsymbol{\theta}_{1;t} = \begin{pmatrix} \theta_{11} \\ \theta_{12} \end{pmatrix}, \quad \boldsymbol{\theta}_{2;t} = \begin{pmatrix} \theta_{21} \\ \theta_{22} \end{pmatrix}. \tag{20}$$

We express the portfolio hierarchy via the matrix:

$$\boldsymbol{H} = \begin{pmatrix} 1\,0\,1 \\ 0\,1\,1 \end{pmatrix}. \tag{21}$$

Thus, we obtain

$$\boldsymbol{\Theta}_t = \boldsymbol{\Theta}_{J;t}\boldsymbol{H} = \begin{pmatrix} \theta_{11}\ \theta_{21}\ (\theta_{11} + \theta_{21}) \\ \theta_{12}\ \theta_{22}\ (\theta_{12} + \theta_{22}) \end{pmatrix} \quad \text{with} \quad \boldsymbol{\Theta}_{J;t} = \begin{pmatrix} \theta_{11}\ \theta_{21} \\ \theta_{12}\ \theta_{22} \end{pmatrix} \tag{22}$$

for the structure matrix of the complete portfolio hierarchy. From the covariance matrix of the financial instruments

$$\boldsymbol{\Sigma}_I = \begin{pmatrix} \sigma_1^2 & \text{cov}_{2,1} \\ \text{cov}_{1,2} & \sigma_2^2 \end{pmatrix} \tag{23}$$

and the diagonal matrix of the market values of the financial instruments

[1] For a proof of the sub-additivity property see Read (1998), p. 26.

$$\boldsymbol{w}_t^{\text{diag}} = \begin{pmatrix} w_1 & 0 \\ 0 & w_2 \end{pmatrix} \tag{24}$$

we obtain the covariance matrix of the portfolio losses as:

$$\boldsymbol{\Sigma}_{L;t} = \boldsymbol{\Theta}_t{}'\boldsymbol{w}_t^{\text{diag}}\boldsymbol{\Sigma}_l\boldsymbol{w}_t^{\text{diag}}\boldsymbol{\Theta}_t = \begin{pmatrix} \sigma_{L_1}^2 & \text{cov}_{L_1,L_2} & \sigma_{L_1}^2 + \text{cov}_{L_1,L_2} \\ \text{cov}_{L_1,L_2} & \sigma_{L_2}^2 & \sigma_{L_2}^2 + \text{cov}_{L_1,L_2} \\ \sigma_{L_1}^2 + \text{cov}_{L_1,L_2} & \sigma_{L_2}^2 + \text{cov}_{L_1,L_2} & \sigma_{L_1}^2 + \sigma_{L_2}^2 + 2\,\text{cov}_{L_1,L_2} \end{pmatrix}. \tag{25}$$

We finally get the Value-at-Risk vector by:

$$\mathbf{VaR}_{t+T} = \text{diag}(\sqrt{T \cdot \boldsymbol{\Sigma}_{L;t}} \cdot z(p)) = \begin{pmatrix} \text{VaR}_{1;t+T} \\ \text{VaR}_{2;t+T} \\ \sqrt{\text{VaR}_{1;t+T}^2 + \text{VaR}_{2;t+T}^2 + 2\rho_{L_1,L_2}\text{VaR}_{1;t+T}\text{VaR}_{2;t+T}} \end{pmatrix}. \tag{26}$$

If the risk capital amount C is given and we equate it with the Value-at-Risk for the aggregated portfolio, we can derive conclusively the set of permissible iso-risk limit structures:

$$V = \{(\text{VaR}_{1;t+T}\ \ \text{VaR}_{2;t+T}\ \ \text{VaR}_{3;t+T})' : \text{VaR}_{3;t+T} = C\}$$

$$= \{(C_1\ \ C_2\ \ C_3)' : C_3 = \text{C}\}.$$

The solution of

$$C_1^2 + 2\rho_{L_1,L_2}C_1C_2 + C_2^2 - \text{C}^2 = 0$$

leads to

$$C_{1(1,2)} = -\rho_{L_1,L_2}C_2 \pm \sqrt{\rho_{L_1,L_2}^2 C_2^2 - C_2^2 + C^2}\,. \tag{27}$$

$$\rho_{L_1,L_2} = 1: \qquad C_1 = \text{C} - C_2$$

$$\rho_{L_1,L_2} = 0: \qquad C_1 = \sqrt{\text{C}^2 - C_2^2}$$

By the same procedure in principle all higher dimensional allocation problems can be solved. For the n-dimensional case the solution is in each case identified for one C_i, $i = 1,\dots,n$, with all other $C_{j\neq i}$, $j = 1,\dots,n$, remain unchanged. So it is possible in our model to construct risk limit systems and to allocate risk capital for any portfolio hierarchy.

For the presentation of the permissible risk limit combinations on the C_1-C_2 level we get an intercept of an ellipse concave to the origin. The less the changes in value of the basic portfolios correlate, the more this curve becomes more intensively arched (see Figure 3). For perfectly positive correlated changes in portfolio value we obtain a straight line, i.e. that the risk capital amounts (or risk limits) sum up to the total risk capital amount that is available for the trading division.

Finally, we select the allocation of risk capital which maximises the risk-adjusted profitability of the trading division out of a set of permissible iso-risk limit structures via the proposed optimisation approach:

$$\mathrm{RAPM}_3^{\mathrm{agg}}(\boldsymbol{C}) = \frac{G_1^{\mathrm{ad}}(C_1) + G_2^{\mathrm{ad}}(C_2)}{\sqrt{C_1^2 + 2\rho_{L_1,L_2} C_1 C_2 + C_2^2}} \to \max!$$

subject to $\sqrt{C_1^2 + 2\rho_{L_1,L_2} C_1 C_2 + C_2^2} \le \mathrm{C}$. (28)

The assumption of concave functions for the risk adjusted profits (see (2)) and the super-additivity property of the RAPM (see Section III.2) imply the concavity of the aggregated RAPM for the overall portfolio[1]. On the C_1-C_2 level this results in RAPM indifference levels being convex to the origin (see Figure 3). One question remains to be answered: Which set of permissible iso-risk limit structures resulting for different correlations should be used to settle the allocation problem?

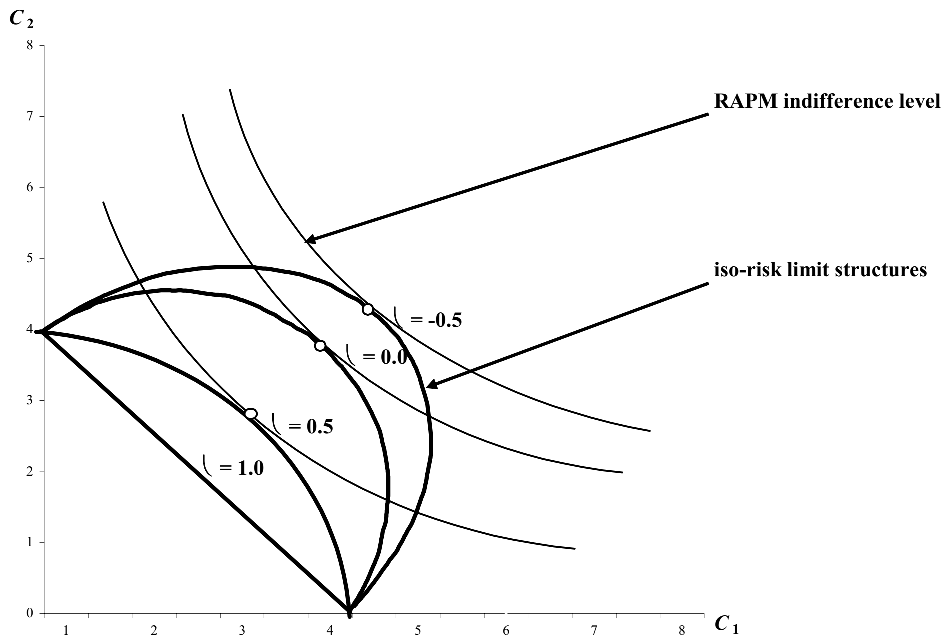

Fig. 3. Two-dimensional allocation problem

The attained results allow for the following conclusions. The lower the losses of the basic portfolios correlate, the higher the risk limits can be assigned to these portfolios without exceeding the default risk capital on the aggregated level by the actual risk taking, and vice versa (see (27) and Figure 3). Particularly this aspect explains the consequences which are caused for the allocation of the risk capital by the ex ante unknown and timely unstable correlation of the portfolio losses and the ex ante also unknown basic portfolio structures influencing them. If the (optimum) allocation is performed on the basis of an assumed correlation and if the portfolio losses correlate higher in the future, the risk capital amount of the trading division will be exceeded inevitably, provided that the risk limits of the basic portfolios are exhausted. But if the portfolio losses correlate less in the future, the risk capital amount of the trading division will be not utilised fully, even if the risk limits

[1] See Straßberger (2002), p. 213.

of the basic portfolios are exhausted[1]. The demands, which have been placed on the risk limit system at the beginning, for:

- the maintenance of the risk limit on the aggregated level at any time (non-exceeding of the risk capital given), and
- the complete allocation of the risk capital

can obviously not be made compatible with each other. There is a target conflict between the two demands. For the management of the financial institution these findings mean that it faces a trade-off between the permanent maintenance of the risk limit of the trading division and the complete utilisation of the risk capital provided.

IV. Final Discussion

The simple model developed above for constructing a system of risk limits and for allocating the risk capital in trading divisions of financial institutions offers a complete analytic solution of the allocation problem. We have an instrument at our disposal which allows us, among other options, to explain the phenomenon of not-exhausted risk limits on aggregated levels of the trading division which can be observed in actual business. The cause for this observation can be found in the fact that the correlation structure of the losses of the trading portfolios required for the allocation of the risk capital is ex ante unknown. It is not only unknown for non-reliable market developments and its resulting instability over the time but – even if this aspect is not taken into account – it is also unknown for the ex ante unknown structures of the future trading portfolios.

Due to their independent decisions on the composition of their portfolios the portfolio managers generate correlation structures which principally cannot be predicted. After all, the necessity of risk limitation is just caused by this decentralised and independent decision competence in the basic portfolios. Due to these findings we obtain the result of a trade-off between the efficient utilisation of the risk capital and the required permanent maintenance of the risk limit.

Which recommendation could be derived for the further proceeding from these findings for the management board of the financial institution? If we assume maximum possible correlations of the portfolio losses it is in fact ensured that the risk capital will never be exceeded by the actual risk taken. But the risk capital of the trading division will not be utilised completely in most cases. The introduction of a “super trader” might be possible here. It establishes market risk positions for the non-utilised portion of the risk capital on the division level[2]. Such an institution could also be used in the contrary situation. If the allocation of the risk capital was purposefully based on lower loss correlations, the “super trader” might establish appropriate opposite positions if the risk arises that the risk capital amount provided will be exceeded. The questionable aspect of these recommendations is the fact that the decisions made by such a “super trader” immediately change the correlation structure.

An interesting alternative could be given in the organisation of an internal market for the risk capital[3]. Risk capital that has not been utilised would be traded by the portfolio managers on a market, on which the target return is set as the market price. From the agency-theoretic point of view, however, doubts arise about the efficiency of such a solution for the allocation problem[4]. In addition to this, it will also be difficult to consider the unknown correlation structure properly.

The decision whether the present risk limits given are tried to be maintained and the risk capital provided is not completely utilised therefore, or whether the risk capital is fully exhausted and the

[1] Bühler, Birn (2001) come to similar findings when they identify instable correlation patterns to be responsible for increasing demands of risk capital in financial institutions.
[2] See Dresel et al. (2002) for such an approach.
[3] See Stein (1997) above all and also Saita (1999), p. 99 for the first attempt.
[4] Such a market is characterized by information asymmetries of the participants. Distribution fights among the portfolio managers can cause inefficient allocations. See Harris, Raviv (1998) and Scharfstein, Stein (2000) above all for this.

risk limit may be exceeded, is to be made by the management of the financial institution based on its individual attitude toward risk.

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