“The Relationship Between Size, Value, and Market Risk: Some Evidence”

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THE RELATIONSHIP BETWEEN SIZE, VALUE, AND MARKET RISK: SOME EVIDENCE
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Abstract
This study examines the risk-return characteristics of common stock in the Indian stock market. We propose a vector autoregressive model, Granger-causality tests and variance decomposition analysis to find the nature of relationship that exists among the three factors (market risk, size, and value) proposed by Fama-French (1992). The empirical findings of our study indicate that market risk proxy has persistent effects on stock returns in the Indian market. Moreover, a causal relationship is found to exist between market risk factor and non-market based measures. The results of the study are expected to provide better insights to investors in understanding the risk return characteristics that exist among the factors that affect stock prices.

Key words: Book-to-market Equity; Firm size; Vector autoregression; Granger-causality; Variance decomposition analysis.

JEL classification: G11, G12, C01.

1. Introduction
The transition of Indian markets from an underdeveloped state to a fast developing one has attracted the attention of many researchers. This transition has reached a stage where it is expected to be in tune with the direction of major developed markets like the NYSE and NASDAQ. In this setting, we have attempted to empirically test the three-factor model of Fama-French in an intertemporal setting in the Indian stock markets.

The Fama-French three-factor model, viz. the market risk, size, and book to market ratio, is used on Indian stock data. The study is carried out using an intertemporal approach with the help of a vector autoregressive model (VAR). We have tested the general equilibrium asset pricing model to endogenously determine the stochastic process followed by the equilibrium price of the stock, and to show how this process depends on the underlying real variables in an intertemporal setting. In order to capture this time varying effect, we have employed the vector autoregressive (VAR) method. With the help of VAR, we have tried to see the extent of share price movements that can be explained by its own variance, and the extent of these movements caused by the variance of other factors relating to market, size, and book to market ratios. Although the significance of the factors relating to size and book to market ratios were established in earlier studies (Banz, 1986; Bhandari, 1988; Basu, 1983; Rosenberg, Reid, and Lanstein, 1985), there is little evidence regarding the nature of relationship. In our study, we have tried to address this issue by studying the effect of changes in factors relating to market, size and book to market on the changes in returns. In addition, we have also attempted to verify the nature of impact – either positive or negative, of market, size, and book to market ratios on stock price returns.

Several studies have investigated CAPM effects, and tests of Fama-French model in the Indian context and the results have shown mixed evidence (Varma, 1988; Yalwar, 1988; Gupta, 1981). Manjunatha, Mallikarjunappa and Mustiary (2007) conducted a study on CAPM to test intercept, beta and a number of risk factors. Kumar & Shegal (2004) examined the relationship between selected company characteristics and common stock returns and recommended an alternative stock classification system based on company size and relative distress. Connor and Sehgal (2001) attempted to test the one-factor linear pricing relationship implied by CAPM and the three factor linear pricing model of Fama-French. Shegal (2006) evaluated for momentum patterns in

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Indian stock returns and reported strong momentum profits for individual stocks as well as a wide range of characteristic-sorted portfolios. Bahl (2006) empirically studied the Fama-French three-factor model for stock returns along with its variants. The methodology and findings of all these studies are discussed in the literature review section.

The rest of the paper is organized into four sections: The second section covers the literature on CAPM, and Fama-French model as established in literature. The third section describes the rationale for this study as well as the methodological differences. The fourth section comprises data and methodology, details of VAR, Granger causality, Impulse response functions, variance decomposition analysis, and various econometric techniques used. The last section is devoted to results and conclusions.

2. Literature Review

2.1. Traditional CAPM model

The capital asset pricing model CAPM developed by Sharpe (1964), Treynor (1961), Lintner (1965) and Mossin (1966) measures the risk of an asset by covariance of assets return with return on all invested wealth i.e. market return. Nevertheless, weak empirical performance and undue assumptions have made CAPM a target of attack from critiques. According to CAPM, the only relevant risk measure is beta. It is based on the assumption that all market participants have identical expectations regarding expected returns and variance of expected returns. Empirical evidence however shows time varying return (Engle, 1982; Bollerslev, 1986). The commonly used proxy for market return, the value weighted index or well-diversified portfolio was also questioned.

According to Breeden (1979), risk of an asset should be measured by its sensitivity to changes in investor’s consumption. In an equilibrium version of the model estimated by Blume (1975), price of risk is determined by the coefficient of relative risk aversion of an investor. Further, Merton (1973) opines that assets risk should be measured in terms of co-variance with marginal utility of investors. The main proposition that stock market return can be passably proxied by a stock index return was challenged by Roll (1977). This led to development of multifactor models in which risks are measured by co-variances with several common factors. Roll and Ross (1980) stated that the single period expected return on any risky asset is approximately linearly related to its associated factor loadings. Conventional factor extraction techniques like maximum likelihood factor analysis and principal component approach were used to measure these common factors (Chen, 1983; Roll and Ross, 1980; Reinganum, 1981; Lehmann and Modest, 1988). A major problem in empirically testing this was that pervasive factors affecting asset returns are unobservable.

2.2. Fama-French three-factor model

Empirical studies have shown that factors other than market beta can explain cross section of expected returns. Basu (1977) finds that price earnings ratios and risk-adjusted returns are related. Banz (1981) showed that during the period of 1936-1975 in NYSE, common stock of small firms had, on average, higher risk-adjusted returns than the common stock of large firms. Fama and French (1995) studied the joint roles of the above factors in the cross-section of average stock returns. The results of the study revealed that a three-factor asset-pricing model that includes a market factor and risk factors related to size and BE/ME seem to capture the cross-section of average returns on U.S. stocks.

In Fama-French three factor model, the expected return on each stock depends on its exposure to three factors; namely, 1) market factor (Rm-Rf) measured in terms of return on market index minus risk free interest rate, 2) size factor (SMB) i.e. return on small firm stocks minus return on large firm stocks, and 3) Book- to-market-factor or value factor (HML) which is measured in terms of return on high book to market ratios stocks minus return on low book to market ratio stocks.

A classic test by Fama and Macbeth (1973) combined time series and cross sectional steps to investigate the relationship between average return and risk for NYSE common stocks. Using time series methodology, beta was estimated with a set of twenty portfolios of assets. Sub-
sequently, cross-sectional regression for each month over a period ranging from 1935-1968 was conducted in the second pass regression. Their results showed that residual risks have little effect on security returns. Insignificant beta coefficient and the intercept much greater than risk free rate indicated that CAPM does not hold good. It showed that pricing of common stock reflects the attempt of risk-averse investors to hold portfolios that are efficient in terms of expected value and dispersion of return.

Fama and French (1992) find contradicting evidence for Sharpe (1964), Lintner (1965), and Black (1972) model, commonly known as Sharpe, Linter & Black (SLB) model. The positive linear relation of market beta and expected returns are violated. Although in univariate tests the relationship between average return and size, leverage, earnings price ratio (E/P), and book-to-market equity showed significant results, multivariate tests showed a negative relation between size and average returns. Further, there existed a positive relation between book-to-market equity and average return. The results of the study revealed the combined effect of size and book-to-market equity which captured the roles of leverage and E/P in average stock returns, during the period of 1963-1990. Since leverage and book to market are largely driven by market value of equity they proxy the risk factors. In other words risks are multidimensional, proxied by Book equity to Market equity (BE/ME) ratio and factors relating to size. Book to market ratio captures the risk of relative distress factor (Chan and Chen, 1991). The results confirm that factors related to market size and value explain returns on a well diversified portfolio.

The three-factor model is related to earnings by Fama and French (1995) for a deeper economic foundation. They study the relationship of risk factors relating to market, size, and value in terms of earnings behavior. Their results conform to the rational theory that high BE/ME signals poor earnings and low BE/ME is persistent with strong earning. But with regard to size factors they do not find a common association, i.e. they find little evidence for market and size factors in earnings which can explain market and size factors related to returns, a reason they owe to ‘noisy’ measures of shocks to expected return.

The present study uses a different methodology to capture this issue. As predicted by rational pricing models, factors relating to market, size, and value are related to its persistent properties in earnings. The prevailing methodology, while capturing the significance of common factors persistent in risk and returns fail to predict the extent of variation caused by these factors. We study the relationship of risk related factors related to market, size, and value and its reflection in earnings. This involves constructing a vector autoregression (VAR) model suitable for our analysis and applying relevant tests to investigate inter-linkages and causality.

There exists considerable evidence in support of three-factor model in developed markets. Chan, Hanao and Lakonishok (1991) studied the cross sectional variation on Japanese stocks to the behavior of earnings yield, size, value and cash flow yield with the help of seemingly unrelated regression method. The results of the study revealed a significant relationship between these variables and expected returns in Japanese markets during the period of 1971-1988. Book to market ratio and cash flow yield owed to the most significant positive impact on expected stock returns. Returns of growth and value stocks of six countries were analyzed by Capasul, Rowley and Sharpe (1993) and it showed that value stocks outperformed growth stocks in each country during the period of 1971-1988. The relationship between expected stock returns, market beta, book to market equity and size in five Pacific-Basin emerging markets by Chui and Wei (1998) showed weak relationship between market beta and stock return. The study finds mixed results in explaining cross sectional returns. Size effect showed significant effect in all markets except Taiwan. In emerging markets such as India the tests of Fama-French model have shown mixed results.

2.3. Studies on Indian Capital Markets

A recent empirical study conducted in India for an adjusted sample of 364 companies that form a part of CRISIL-500 index was carried out by Kumar and Sehgal (2004). Using both market-based as well as non-market based measures of company size, their results suggested that there is a strong size effect in the Indian stock market. They also detected a weak value effect in stock returns, especially when E/P ratio is employed as a relative distress proxy. The study further found that the present stock classification system in India fails to differentiate in returns on different
categories of stocks. In another study, Shegal (2006) evaluated for momentum patterns in Indian stock returns and reported strong momentum profits for individual stocks as well as a wide range of characteristic-sorted portfolios. However, the Fama-French three-factor model explained momentum, as the characteristic-sorted momentum portfolios tend to load on size and book equity to market equity factors. Their results suggested that there are rational sources of momentum profits in the Indian environment. The findings are in contrast with those of US where the momentum explanation is tilted towards delayed price reaction to firm-specific information.

Bahl (2006) studied the Fama-French three-factor model of stock returns along with its variants, including the one-factor Capital Asset Pricing Model for 79 stocks listed on the BSE-100 stock market index for India. The sample stocks were sorted into six portfolios on size and book-to-market equity ratio. The factor portfolios that explained the returns are the market factor, size factor (SMB) and value factor (HML). There was strong evidence for the market factor in all the portfolios, having highest explanatory power. The study confirmed that the three-factor model captures better the common variation in the stock returns than the CAPM.

Obaidullah’s (1994) study over a period of sixteen years (1976-1991) found mixed results for CAPM effect. Connor and Sehgal (2001) attempted to test the one-factor linear pricing relationship implied by the CAPM and the three factor linear pricing model of Fama-French. Their study analyzed whether the market, size and value factors are pervasive in the cross-section of random stock returns. In addition, they also investigated whether there are market, size and value factors in corporate earnings similar to those in returns and whether the common risk factors in earnings translate into common risk factors in returns. Their results showed significant effects of market, size and book to market factors for returns on Indian stock. Conversely, mixed evidence was found for the above factors in earnings. Further to this, they failed to establish a link between factors common in earnings and stock returns. Reasons for this may be due to the inefficiency prevalent in Indian market due to serial correlation, nonlinear dependence, day-of-the week effects, parameter instability, conditional heteroscedasticity (GARCH), daily-level seasonality in volatility, short term interest rate (in some sub-periods of some indices) and some dynamics in the higher order moments (Sarkar and Mukhopadhyay, 2002)

Thus, there exists a need for a comprehensive approach, which can capture the true effects of the market-related risk factors and its reflection on earnings related factors.

3. Rationale for the Study

Earlier researchers have used several estimation procedures to capture the presence of time varying moments in return distribution which form clustering large shocks of dependent variable, thereby exhibiting a large positive or negative value of error term (Mandelbrot, 1963; Fama, 1965). Engle (1982) suggested the autoregressive conditionally heteroscedastic or ARCH model as an alternative to time series treatments, where conditional variance of a zero mean normally distributed random variable \(u_t\) is equal to the conditional expected value of the square of \(u_t\). The autocorrelation in volatility is modeled by allowing conditional variance of error term \(\sigma_t^2\) to depend on the immediately previous value of the error term (Brooks, 2002). Bollerslev (1986) demonstrated a generalization of ARCH model identified as GARCH which allows conditional variance to be dependent upon previous own lags so that with a small number of terms it appears to perform as well as or better than ARCH model. The assumption in ARCH and GARCH is that return distribution is characterized with time variation only in its variance. However, Domowitz and Hawkins (1985) identified the presence of time variation in both mean and variance of return distribution. Engle, Lilien and Robins (1987) suggested an ARCH-M specification where conditional variance of asset returns enters into the conditional mean equation. Bollerslev (1988) has formulated a GARCH-M model to account for time varying moments more efficiently. Campbell (1991) used vector autoregressive system in combination with log linear asset pricing to calculate the impact of an innovation on the stock price. The rational expectations of future dividends and changes in rational expectations of future returns are examined in the perspective of VAR.

Empirical studies have found that the role of other variables in explaining cross-sectional return is significant. Choice of econometric technique is important in this regard. Roll & Ross

(1994) have demonstrated that one can arrive at diverse results with the same data. Replicating Fama-French (1992) model with GLS as an alternative to OLS, Amihud, Christensen and Mendelson (1992) found contrasting results. Conversely, GLS estimates pose a problem of unknown parameters which makes true covariance matrix of returns unknown.

Given the different conclusions on applications of Fama-French model, we have taken the factors relating to market, size, and value into the VAR model to examine the expected return, as shown by Kendall and Stambaugh (1988) who investigate expected stock returns over short and long horizons using both equilibrium asset pricing model and vector autoregression (VAR). The results of the study reveal that VAR can account for several long-run characteristics of the data like autocorrelations, R-squared values, and conditional expected returns that are close to those computed with actual long-horizon return.

Our approach is different in that we allow current and past values of common risk related factors to affect the earnings counterpart and also allow for feedback between the current and past values of common factors in risk and earnings. The dynamic response of each variable to various economic shocks is obtained. Further, we check the causal relationship of the factors related to the market, size and value i.e. the variables in the model that have statistically significant impact on future values of the variables in the system. We also explain how long these effects take place.

4. Methodology

4.1. Vector Auto Regressive (VAR) model

To derive the testable implications of factors relating to market, size and value we employ a vector autoregressive (VAR) model popularized by Sims (1980). In this model, all variables are often treated as being \textit{a priori} endogenous, and allowances are made for rich dynamics. In simplest case of a bivariate VAR we can let time path of \( \{ y_t \} \) be affected by current and past realizations of \( \{ z_t \} \) sequence and let time path of the \( \{ z_t \} \) sequence be affected by current and past realizations of the \( \{ y_t \} \) sequence. A simple bivariate VAR can be expressed as follows.

\[
y_t = b_{10} - b_{12} z_t + \gamma_{11} y_{t-1} + \gamma_{12} z_{t-1} + \varepsilon_{yt},
\]

\[
z_t = b_{20} - b_{21} y_t + \gamma_{21} y_{t-1} + \gamma_{22} z_{t-1} + \varepsilon_{zt},
\]

where both \( y_t \) and \( z_t \) are assumed to be stationary; \( \varepsilon_{yt} \) and \( \varepsilon_{zt} \) are white-noise disturbances with standard deviations of \( \sigma_y \) and \( \sigma_z \) respectively; and \( \{ \varepsilon_{yt} \} \) and \( \{ \varepsilon_{zt} \} \) are uncorrelated white noise disturbances. A set of reduced form of equations is obtained by transforming the set of above equations into matrix algebra as follows (Walter Enders, 2004)

\[
\begin{bmatrix}
1 & b_{12} \\
b_{21} & 1
\end{bmatrix}
\begin{bmatrix}
y_t \\
z_t
\end{bmatrix}
= \begin{bmatrix}
b_{10} \\
b_{20}
\end{bmatrix} + \begin{bmatrix}
\gamma_{11} & \gamma_{12} \\
\gamma_{21} & \gamma_{22}
\end{bmatrix}
\begin{bmatrix}
y_{t-1} \\
z_{t-1}
\end{bmatrix} + \begin{bmatrix}
\varepsilon_{yt} \\
\varepsilon_{zt}
\end{bmatrix}
\]

\[
B_{sr} = \Gamma_0 + \Gamma_1 x_{t-1} + \varepsilon_t
\]

\[
B = \begin{bmatrix}
1 & b_{12} \\
b_{21} & 1
\end{bmatrix}, \quad x_t = \begin{bmatrix}
y_t \\
z_t
\end{bmatrix}, \quad \Gamma_0 = \begin{bmatrix}
b_{10} \\
b_{20}
\end{bmatrix}
\]

\[
\Gamma_1 = \begin{bmatrix}
\gamma_{11} & \gamma_{12} \\
\gamma_{21} & \gamma_{22}
\end{bmatrix}, \quad \varepsilon_t = \begin{bmatrix}
\varepsilon_{yt} \\
\varepsilon_{zt}
\end{bmatrix}
\]

Premultipication by \( B^{-1} \) allows us to obtain the VAR model in standard form

\[
x_t = A_0 + A_1 x_{t-1} + \varepsilon_t,
\]
By defining \( a_{i0} \) as element \( i \) of the vector \( A_0 \), \( a_{ij} \) as the element in row \( i \) and column \( j \) of the matrix \( A_1 \), and \( e_{it} \) as the element \( i \) of the vector \( e_t \), equation can be rewritten in the equivalent form:

\[
\begin{align*}
    y_t &= a_{10} + a_{11} y_{t-1} + a_{12} z_{t-1} + e_{1t}, \\
    z_t &= a_{20} + a_{21} y_{t-1} + a_{22} z_{t-1} + e_{2t}.
\end{align*}
\]

(4)

Since \( e_{yt} \) and \( e_{zt} \) are white noise processes it shows that \( e_{1t} \) and \( e_{1t-1} \) have zero means, constant variances, and are individually serially uncorrelated.

\[
E e_{1t} = E(e_{yt} - b_{12}e_{zt})/(1-b_{12}b_{21}) = 0.
\]

The variance of \( e_{1t} \) is given by

\[
E e_{1t}^2 = E[(e_{yt} - b_{12}e_{zt})(e_{yt-1} - b_{12}e_{zt-1})]/(1-b_{12}b_{21})
\]

\[
= \sigma_y^2 + b_{12}^2 \sigma_z^2 / (1-b_{12}b_{21})^2
\]

(5)

This makes the variance \( e_{1t} \) time-independent. Further the autocorrelations of \( e_{1t} \) and \( e_{1t-1} \) are

\[
E e_{1t} e_{1t-1} = E[(e_{yt} - b_{12}e_{zt})(e_{yt-1} - b_{12}e_{zt-1})]/(1-b_{12}b_{21})^2 = 0.
\]

(6)

A higher order VAR which we use to capture the common variation in factors relating to market, size, and value can be expressed as follows:

\[
\begin{align*}
    [x_{1t}] &= [a_{10}] + [a_{11}(L)] [A_{11}(L) A_{12}(L) \cdots A_{1n}(L)] [x_{1t-1}] + [e_{1t}] \\
    [x_{2t}] &= [a_{20}] + [a_{21}(L)] [A_{21}(L) A_{22}(L) \cdots A_{2n}(L)] [x_{2t-1}] + [e_{2t}] \\
    \vdots \\
    [x_{nt}] &= [a_{n0}] + [a_{n1}(L)] [A_{n1}(L) A_{n2}(L) \cdots A_{nn}(L)] [x_{nt-1}] + [e_{nt}]
\end{align*}
\]

(7)

where \( A_{0} \) = the parameters representing intercept terms, \( A_{i}(L) \) = the polynomials in the lag operator \( L \).

4.2. Granger Causality/Block Exogeneity Wald Tests

We use Granger test (Granger, 1969) to test whether factors relating to market size and value cause significant effects. Granger causality is used to check the significance of a set of variables on each dependent variable by restricting lags of a particular variable to zero. A multivariate generalization of granger causality, known as block-causality, checks whether the lags of one variable \( W \) granger cause any of the variables in the system. In the three-variable case with \( W, y, \) and \( z \), we test whether lags of \( W \) granger cause \( y \) or \( z \). This cross equation is tested by likelihood ratio test (Walter Enders, 2004)

\[
(T-c)(\log \sum_{y} - \log \sum_{w})
\]

(8)
4.3. Impulse response function and variance decomposition

Although causality explained above identifies significant impact on the future values of each variables in the model, it will not explain the sign of the relationship or the duration of the effects. Impulse responses mark out the responsiveness of the endogenous variables in VAR to shocks of each of the variable. One unit of shock is applied to the error term of each variable and its effect upon the VAR system over time is noted. If the system is stable, the shock would gradually die away. This can be expressed as follows:

In a bivariate VAR

\[ y_t = A_1 y_{t-1} + u_t \]

where

Expressed in terms of the elements of matrices and vectors as

\[
\begin{bmatrix}
  y_{1t} \\
  y_{2t}
\end{bmatrix} = \begin{bmatrix}
  0.5 & 0.3 \\
  0.0 & 0.2
\end{bmatrix} \begin{bmatrix}
  y_{1t-1} \\
  y_{2t-1}
\end{bmatrix} + \begin{bmatrix}
  u_{1t} \\
  u_{2t}
\end{bmatrix}
\]

The effect of time \( t = 0, 1, \ldots \), of a unit shock to \( y_{1t} \) at time \( t = 0 \)

\[
y_0 = \begin{bmatrix}
  u_{10} \\
  u_{20}
\end{bmatrix} = \begin{bmatrix}
  1 \\
  0
\end{bmatrix}
\]

\[ y_1 = A_1 y_0 = \begin{bmatrix}
  0.5 & 0.3 \\
  0.0 & 0.2
\end{bmatrix} \begin{bmatrix}
  1 \\
  0
\end{bmatrix} = \begin{bmatrix}
  0.5 \\
  0
\end{bmatrix},
\]

\[ y_2 = A_1 y_1 = \begin{bmatrix}
  0.5 & 0.3 \\
  0.0 & 0.2
\end{bmatrix} \begin{bmatrix}
  0.5 \\
  0
\end{bmatrix} = \begin{bmatrix}
  0.25 \\
  0
\end{bmatrix}.
\]

The effect of a unit shock to \( y_{2t} \) at time \( t = 0 \)

\[
y_0 = \begin{bmatrix}
  u_{10} \\
  u_{20}
\end{bmatrix} = \begin{bmatrix}
  0 \\
  1
\end{bmatrix},
\]

\[ y_1 = A_1 y_0 = \begin{bmatrix}
  0.5 & 0.3 \\
  0.0 & 0.2
\end{bmatrix} \begin{bmatrix}
  0 \\
  1
\end{bmatrix} = \begin{bmatrix}
  0.3 \\
  0.2
\end{bmatrix},
\]

\[ y_2 = A_1 y_1 = \begin{bmatrix}
  0.5 & 0.3 \\
  0.0 & 0.2
\end{bmatrix} \begin{bmatrix}
  0.3 \\
  0.2
\end{bmatrix} = \begin{bmatrix}
  0.21 \\
  0.04
\end{bmatrix}.
\]

Further, we use variance decomposition analysis in uncovering the interrelationships among variables used to mimic factors relating to market size and value. It helps to determine how much of the s-step-ahead forecast error variance of a given variable is explained by innovations to each explanatory variables for \( s = 1, 2, \ldots, n \)

5. Data and Variables

Data for this study comprise monthly stock prices of S&P CNX 500 for a period of ten years ranging from March 1996 to March 2006. S&P CNX 500 is considered to be India’s first broad based benchmark which helps in comparison of portfolio returns with market returns. With a market capitalization of 92.66% S&P CNX 500 accounts for 86.44% of the total turnover of the national stock exchange in India. It covers a wide range of 72 industries and reflects industry
weightings prevalent in Indian market. Data pertaining to the companies are taken from prowess database maintained by Center for Monitoring Indian Economy (CMIE).

We use the same six portfolios of Fama-French (1992) to mimic the factors relating to size and book to market equity. All stocks which form a part of S&P CNX 500 from 1996 March to 2006 March are ranked on size ME (price times shares). S&P CNX 500 shares are then split into two groups: small and big (S and B). Further, we split S&P CNX 500 stocks into three groups: bottom 30% (Low), middle 40% (Medium), and top 30% (High) based on BE/ME. Based on the intersection of two size portfolios and three BE/ME portfolios, six portfolios S/L, S/M, S/H, B/L, B/M, B/H were formed. S/L consists of firm with small size and low BE/ME ratios. Likewise B/H consists of big sized firms with high BE/ME ratios.

Value weighted returns are calculated for each month as proposed by Fama and French (1992) and simple average of returns of three small sized portfolios (S/L, S/M and S/H) and big sized (B/L, B/M, B/H) portfolios were calculated. The differences between these two sets of portfolios were calculated to form SMB (Small Minus Big) which proxies the risk factor in returns. Similarly, in order to capture factors in returns related to value, HML (High Minus Low) is constructed. HML is the difference between the simple average of returns of two high BE/ME portfolios (S/H and B/H) and average returns of two low BE/ME portfolios (S/L and B/L). This helps to get two sets of portfolios which are free of value and size effects.

We use S&P CNX NIFTY as a proxy for aggregate economic wealth. S&P NIFTY, introduced in the year 1995, is based on 50 of the largest and highly liquid stocks. It accounts for 85% of the total market capitalization as at the end of March 2006 and is the professionally maintained index. The data on monthly averages of S&P CNX NIFTY are taken from Reserve Bank of India database.

Further, as a surrogate for risk free proxy we use 91-day treasury bills which are also taken from Reserve Bank of India database. The market factor proposed by the traditional CAPM is calculated as the difference between S&P CNX NIFTY index and 91-day treasury bills ($R_m - R_f$).

6. Results of The Study

6.1. VAR and Stationarity

Sequentially, to examine the relationship established by Fama-French in a VAR context, all the variables need to be stationary. The stationarity of the variables are tested with Augmented Dickey-Fuller (1979) and Phillips-Perron (1988) tests. The results of the test show unit root for the variables ($R_m - R_f$), B/L, B/M, B/H, S/L S/M and S/H at conventional significance levels. Conversely, the variables mimicking size (SMB), value (HML) and aggregate wealth confirm no signs of unit root. To estimate vector autoregressive model, all variables in the system are required to be stationary. Therefore we obtain the first difference of the variables showing signs of nonstationarity, thus making all variables in the system stationary and amenable for a VAR analysis.

6.2. Selection of Lag length

Selecting appropriate lag length is an imperative criterion in VAR. If there are ‘g’ equations, one for each of ‘g’ variables and with ‘k’ lags of each of the variables in each equation, (g + kg) parameters will have to be estimated. Longer lag length consumes more degrees of freedom implying large standard errors: however, shorter lag length leads to model misspecification. Based on sequential modified LR test statistic (LR), Akaike information criterion (AIC), and Final prediction error (FPE) at 5% significance level we choose ‘one’ as optimum lag length (Table 1).

<table>
<thead>
<tr>
<th>Lag</th>
<th>LogL</th>
<th>LR</th>
<th>FPE</th>
<th>AIC</th>
<th>SC</th>
<th>HQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-6639.578</td>
<td>NA</td>
<td>7.04e+37</td>
<td>112.6878</td>
<td>112.8991*</td>
<td>112.7736*</td>
</tr>
<tr>
<td>1</td>
<td>-6538.491</td>
<td>185.0408*</td>
<td>5.03e+37*</td>
<td>112.3473*</td>
<td>114.4605</td>
<td>113.2053</td>
</tr>
</tbody>
</table>
Stability of the VAR model was tested by AR roots graph. AR roots graph shows the inverse roots of the characteristic AR polynomial (Lütkepohl, 1991). The estimated VAR is considered stable (stationary) if all roots have modulus less than one and lie inside the unit circle. If the VAR is not stable, certain results (such as impulse response standard errors) are not valid. Figure 1 shown below substantiates the stability of the VAR model.

![Inverse Roots of AR Characteristic Polynomial](image)

The parameters of VAR can be estimated by applying OLS estimates. This gives consistent and efficient estimates. Following the principle of parsimony, only the significant variables of OLS are taken and Seemingly Unrelated Regression Estimation (SURE) technique was applied. The SURE method, also known as the multivariate regression, or Zellner's method, estimates the parameters of the system, accounting for heteroskedasticity, and contemporaneous correlation in the errors across equations. The estimates of the cross-equation covariance matrix are based upon parameter estimates of the unweighted system. After estimating VAR-SURE, granger causality was calculated to estimate the causal relationship among the variables.

6.3. Granger causality tests

The granger causality tests results (Table 2) test the null hypothesis that there exists no causal relationship among the variables. The results of the tests signify that big companies with high book to market ratios (BH) show granger-cause relationship with the market risk factor and small companies with low book to market ratios. The results are significant at 1% and 5% levels respectively. Companies with high market capitalization and low book to market ratios are not affected by any of the factors. Moreover, big companies with medium book to market ratios (BM) are affected by companies with low book to market ratios and high market capitalization at 10% significance level. BM companies also affect SL and SH companies at 5% significance level. However, value factor (HML) is not affected by any of the factors at conventional significance levels. Market risk factor ($R_m - R_f$) shows significant influence on big companies with high, low and medium book to market ratios (BH, BL, BM) at 10%, 5% and 1% significance levels respectively. Small companies with high book to market ratios (SH) are affected by BM companies and SL companies at 1% and 5% significance level. Small companies with low book to market ratios (SL) are affected by companies with high market capitalization and book to market ratios (BH) at 1% significance level. SL companies also show a causal relationship with companies with low book to market ratios in the same category. Companies that have medium book to market ratios, and are small in relation to size (SM) show signs of value effects at 10% significance level. Con-
tradiicting earlier studies, we do not find any influence of size factor (SMB) on any of the factors at conventional significance levels.

The results of our study reveal that there exists a causal relationship between BH companies and risk factor RMRF. BM companies also show signs of causal relationship with BL, SH and SL companies. Further, risk factor RMRF shows a causal relationship with all the three categories of high market capitalization companies. SL companies reveal a causal relationship with BH and SM companies. However SM companies show some signs of value effect. Moreover, we find evidence in support of bidirectional causality between BH and SL companies, and BH and RMRF.

### Table 2

<table>
<thead>
<tr>
<th>Variable</th>
<th>BH</th>
<th>BM</th>
<th>BL</th>
<th>SH</th>
<th>SM</th>
<th>SL</th>
<th>SMB</th>
<th>HML</th>
<th>RMRF</th>
<th>ALL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent variable: BH</td>
<td>0.21</td>
<td>1.71</td>
<td>0.09</td>
<td>0.77</td>
<td>3.79**</td>
<td>0.36</td>
<td>2.21</td>
<td>5.50***</td>
<td>11.18</td>
<td></td>
</tr>
<tr>
<td>Dependent variable: BM</td>
<td>0.07</td>
<td>3.06*</td>
<td>5.35**</td>
<td>0.01</td>
<td>4.74**</td>
<td>0.01</td>
<td>0.17</td>
<td>0.05</td>
<td>14.87*</td>
<td></td>
</tr>
<tr>
<td>Dependent variable: BL</td>
<td>0.06</td>
<td>0.98</td>
<td>0.42</td>
<td>0.07</td>
<td>2.29</td>
<td>0.06</td>
<td>0.74</td>
<td>0.61</td>
<td>5.06</td>
<td></td>
</tr>
<tr>
<td>Dependent variable: SH</td>
<td>0.165</td>
<td>8.29***</td>
<td>0.63</td>
<td>0.59</td>
<td>4.18**</td>
<td>0.85</td>
<td>0.06</td>
<td>0.08</td>
<td>28.71***</td>
<td></td>
</tr>
<tr>
<td>Dependent variable: SM</td>
<td>0.84</td>
<td>0.67</td>
<td>0.15</td>
<td>0.98</td>
<td>0.95</td>
<td>0.65</td>
<td>3.03*</td>
<td>0.37</td>
<td>9.77</td>
<td></td>
</tr>
<tr>
<td>Dependent variable: SL</td>
<td>3.61**</td>
<td>0.23</td>
<td>0.45</td>
<td>0.11</td>
<td>5.01**</td>
<td>0.99</td>
<td>0.28</td>
<td>0.01</td>
<td>12.14</td>
<td></td>
</tr>
<tr>
<td>Dependent variable: SMB</td>
<td>0.01</td>
<td>0.55</td>
<td>0.08</td>
<td>0.04</td>
<td>0.83</td>
<td>0.56</td>
<td>0.49</td>
<td>0.06</td>
<td>2.172</td>
<td></td>
</tr>
<tr>
<td>Dependent variable: HML</td>
<td>0.26</td>
<td>0.19</td>
<td>0.01</td>
<td>0.31</td>
<td>0.18</td>
<td>0.34</td>
<td>0.13</td>
<td>0.21</td>
<td>1.171</td>
<td></td>
</tr>
<tr>
<td>Dependent variable: RMRF</td>
<td>2.79*</td>
<td>16.68***</td>
<td>4.37**</td>
<td>1.48</td>
<td>0.03</td>
<td>2.22</td>
<td>0.06</td>
<td>0.18</td>
<td>34.61***</td>
<td></td>
</tr>
</tbody>
</table>

*, **, and *** indicate significance at 10%, 5% and 1% level, respectively.

### 6.4. Impulse response functions

Through impulse response, we introduce a shock to each of the variables in the Fama-French model and analyze the impact on the variables affecting size, value and market risk. We use generalized impulse responses where the ordering of the variables are not relevant. The results of the impulse responses (Figures 2.1-2.9) reveal the nature and sign of the relationship and the duration of the effect.

Among the factors, market risk proxy (RMRF) seems to have the most significant effect on the variables proposed by Fama-French model (Figure 2.1). In response to the shocks of BH companies, market risk factor shows a positive relation which is felt over the first three months. It also responds to the shock of BL and BM with a significant positive effect, which is felt over a period of four months. Comparing with the response of other shocks market risk proxy seems to show more variation. Converse to the impact of Value factor (HML) it responds with a negative trend that is felt over a period of four months. Risk factor proxy also seems to show a significant effect on companies with small market capitalization. In response to SH companies, risk factor shows a positive response over a period of three months. With regard to SL companies even though the shocks are not high as in the case of SH still it shows a positive response over a period of four months. Risk factor also responds to the shocks of SM companies over a period of three months with a positive effect. Nevertheless, it does not show any significant response to size factors.
Fig. 2.1. Impulse Responses RMRF
Response of size factor (SMB) to other variables denotes that size is affected by SL and SM companies (Figure 2.2). The variation is reduced with increase in book to market ratios. In other words the variation is more with regard to SL companies than SM companies and evens out when it reaches SH companies with no significant effect. Size factor also shows a significant negative effect on the value factor (HML) over the initial two months but size shows no effect on the market risk factor.

Value factor (HML) shows a positive trend in the initial month to the shocks of BH companies. However it responds negatively to the shocks of BM companies over a one-month period of time. It also shows signs of negative trend to the shocks of SL companies. Size and value factors also seem to be related, in which value factors show a significant effect to size related shocks.
with a negative effect. However BL, SH, SM and market risk factor (RMRF) show little influence on the shocks of value factors.

Response to Generalized One S.D. Innovations

Small companies with low BE/ME ratios show some positive effect on BM companies during the initial month. It shows a negative response to the Value Factor (HML), which turns out to be positive at the beginning of the second month and eventually settles down towards the end of the month. SL companies also show signs of Value effect, which is positive initially, and some signs of negative effect before it settles down at the beginning of the third month. However, SL companies do not show any significant impact with respect to the shocks of BH, BL, SH, SM and market risk factor of the companies.
Further, SM companies show signs of BL and BM effect which is positive. It negatively responds to the shocks of Value factor over the initial couple of months. SM companies also seem to be affected by the market risk during the initial month which is positive and show some signs of mixed effect with respect to SH companies. SM companies show some signs of size effect during the initial month. However, the shocks of BH and SL seem to be insignificant.
Small cap companies with high BE/ME ratio show a significant positive effect to the shocks of BL companies over a period of three months. With regard to BM companies, the positive effect comes down to a negative effect at the end of second month and settles down at the end of the third month. However it shows a negative effect to the Value shocks that are roughly felt over the next four months. SH companies show a significant positive effect to the market risk proxy during the initial month. SH companies also show signs of positive effect on SM, SL, and size factors (SMB) of the companies.
Response to Generalized One S.D. Innovations

Response of SH to BH

Response of SH to BL

Response of SH to BM

Response of SH to HML

Response of SH to RMRF

Response of SH to SL

Response of SH to SMB

Response of SH to SMB

Fig. 2.6. Impulse Responses SH companies

Response of BL to the shocks of BM companies shows a persistent positive trend which is felt over the subsequent three months’ BM companies also respond to the market risk factor (RMRF) with a significant positive trend felt over the next three months. Further, BL companies show a significant positive trend to the shocks of SH companies, which evades out by the end of the first month. BL companies show some signs of positive influence over the one-month period. Moreover, BL companies show little significance to the shocks of BH, SL, size (SMB) and value (HML) factors.
Response of BM companies to the shocks of BL companies is felt over the subsequent three months with a positive effect; however, it shows a negative trend to value related factor (HML). BM companies also show a significant positive trend to the shocks of market risk factor (RMRF) and SH companies which is felt over the subsequent month. Response of BM companies to the shocks of SM companies shows a persistent positive trend in the initial month and further comes down to a negative trend and settles down by the end of the third month. BM companies also show signs of positive size effect over the next couple of months.
Impulse responses of BH companies to shocks of BM companies show a negative relation initially and a positive trend at the start of the second month and settle down by the end of the third month. BH companies respond to the shock of Value factor (HML) with a positive trend which comes down to a negative shock at the beginning of the second month and evades out at the end of the third month. Response of BH companies to market risk factor (RMRF) shows a positive trend which is felt over the subsequent three months and settles down by the end of the third month. Response of BH to SL companies shows little effect which is negative and evades off by the end of the first month. BH companies respond to the shocks of size (SMB), which is negative in the initial month and grows up and settles down by the end of the second month. However, BH companies show no significant effect to the shocks of BL, SM and SH companies.
6.5. Variance decomposition Analysis

The variance decomposition results at the end of 10 periods are shown in Table 3. Size factor (SMB) explains nearly 15% of its own forecast error variance while value factor explains (43%) BM (12%), SM (11%) and SH (5%). Other factors BL (0.98), RMRF (0.08), SL (0.42) do not explain the variation in the size factor at the end of time period 10. Decomposition of variance of small companies with medium book to market ratios (SM) explains over 72% of its own variance. While factors BM (10%), BL (8%), HML (2%), SH (3%), and SL at (1%) show some signs of influence. However the influence of size factor SMB (0.45) is not significant. Correspondingly SL companies account for 22% of its own variance but value factor (HML) explains nearly 37% of the variance of SL companies. Factors BM (25%), BH (7%), BL (3%), SH (2%), and SM (3%) also explain some amount of forecast error variance. However the influence of market risk (0.81) and size factor (0.63) seems to be trivial. Among the significant factors in variance decomposition analysis of SH companies, other than its own shock which is at 51% are BM (22%), BL (19%),
RMRF (3%), and SL at (2%). Remaining factors relating to value (0.92), size (0.50), and SM (0.44) show little influence. Market risk proxy explains 40% of its own variance and remaining significant factors are BL (33%), BM (16%), BH (5%), HML (2%), SH (2%), and SL (1%). However, SM (0.23) companies, and proxy for size SMB (0.03) show little significance at the end of the 10th time period. Value factor owes for 65% of its own variance and, BH and BM explain 17% each. Other factors RMRF (0.03), SH (0.27), SL (0.22), SM (0.10) and SMB (0.09) show little influence. BM companies explain 73% of its own variance while BL explains 15%, BH – 3%, HML – 2%, SH – 3% and SL at 3%. Factors relating to market risk (0.31), size (0.01), and SM show modest significance. BL companies explain 94% of its own forecast error variance along with other factors: BH (1%), SH (1%), and SL (2%). Conversely factors relating to size (0.11), value (0.18), market risk (0.08), SM (0.39), and BM (0.29) are trivial. Finally the BH companies explain 91% of its own shocks along with BM and SL at 1% and 2% respectively. Other factors which proxy size (0.34), value (0.34) SM (0.49) SH (0.84) and BL (0.24) show little significance in explaining the variation of BH.

Table 3

Variance Decomposition Analysis

<table>
<thead>
<tr>
<th>Variables</th>
<th>time period</th>
<th>Variance Decomposition of BH</th>
<th>Variance Decomposition of BL</th>
<th>Variance Decomposition of BM</th>
<th>Variance Decomposition of HML</th>
<th>Variance Decomposition of RMRF</th>
<th>Variance Decomposition of SH</th>
<th>Variance Decomposition of SL</th>
<th>Variance Decomposition of SMB</th>
</tr>
</thead>
<tbody>
<tr>
<td>BL</td>
<td>0.242574</td>
<td>94.99744</td>
<td>15.32257</td>
<td>0.118154</td>
<td>33.10071</td>
<td>19.16501</td>
<td>2.464631</td>
<td>8.942439</td>
<td>9.81924</td>
</tr>
<tr>
<td>HML</td>
<td>0.593844</td>
<td>0.179070</td>
<td>65.4218</td>
<td>1.874941</td>
<td>0.926585</td>
<td>36.99183</td>
<td>2.483426</td>
<td>42.71757</td>
<td></td>
</tr>
<tr>
<td>RMRF</td>
<td>2.702653</td>
<td>0.080399</td>
<td>0.312261</td>
<td>0.036820</td>
<td>40.47217</td>
<td>3.139466</td>
<td>0.811860</td>
<td>1.543193</td>
<td>0.087588</td>
</tr>
<tr>
<td>SH</td>
<td>0.846012</td>
<td>1.067380</td>
<td>2.578946</td>
<td>0.278420</td>
<td>1.641176</td>
<td>50.53138</td>
<td>1.713897</td>
<td>2.799392</td>
<td>5.208688</td>
</tr>
<tr>
<td>SL</td>
<td>2.264454</td>
<td>1.55864</td>
<td>3.257366</td>
<td>0.228656</td>
<td>1.062425</td>
<td>2.321412</td>
<td>22.31099</td>
<td>1.052841</td>
<td>0.429160</td>
</tr>
<tr>
<td>SM</td>
<td>0.493735</td>
<td>0.399582</td>
<td>0.306390</td>
<td>0.104054</td>
<td>0.232973</td>
<td>0.449205</td>
<td>3.268647</td>
<td>71.60745</td>
<td>10.6048</td>
</tr>
<tr>
<td>SMB</td>
<td>0.345078</td>
<td>0.104792</td>
<td>0.019390</td>
<td>0.096029</td>
<td>0.039432</td>
<td>0.506598</td>
<td>0.639193</td>
<td>0.451213</td>
<td>14.54772</td>
</tr>
<tr>
<td>SE</td>
<td>3356.671</td>
<td>4888.790</td>
<td>3621.441</td>
<td>0.346232</td>
<td>90.26149</td>
<td>1677.310</td>
<td>172.0349</td>
<td>923.1691</td>
<td>0.264066</td>
</tr>
</tbody>
</table>

Finally, we conduct Hasbrouck (1995) tests to find the information share of various factors. The average information share of a factor is computed by taking the average of the proportional contribution of the factors to the innovation variance. Our results reveal that the factors BL (12%), HML (11.05%), BL (10.61%), and BH (8.75%) share the maximum information criteria. The factors can be ranked accordingly (Table 4) based on the Hasbrouck information criteria.

Table 4

<table>
<thead>
<tr>
<th>Hasbrouck information criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>BM</td>
</tr>
<tr>
<td>12%</td>
</tr>
</tbody>
</table>

To summarize, market risk appears to be prevalent in the Indian stock market. There exists some amount of size and value effect with certain categories of companies. We find evidence of bidirectional causality between BH and SL companies, and BH and market risk factor. There also exists a causal relationship of BM companies with BL, SH and SL companies. Through impulse response and variance decompositions we find some evidence of size and value effect. Hasbrouck tests show that according to the information criteria provided by the variables it can be ranked as in Table 4.
7. Conclusion

In this study, we explore the relationship between the variables as given in Fama-French model between size, value and market risk. We try to find out whether there exists some causality in the variables when they are subjected to time varying treatment. Findings of our study reveal that among the factors of Fama-French model, market risk proxy seems to be the most persistent factor. Other factors also show some amount of significance. We further find evidence of causal relationship between the companies with high market cap and book to market ratios, with market risk. There exists bidirectional causality between BH and SL companies, and BH and market risk factor. The dynamic linkages of variables studied through impulse response function show mixed results with direction and duration of these effects. Finally the variance decompositions substantiate the influence of these effects. According to the relevance of the information criteria the variables can be ranked as follows: BM, HML, BL, BH, SH, SM, SL, RMRF, SMB.

This study recognizes the nature of relationship among the variables of Fama-French model. The results of the study will provide better insights to investors in understanding the risk return characteristics that exist among the factors which affect stock prices. Similar line of researches in different markets will give more economic foundations to the existing model. Future researches can be aimed at proxies that can capture the effect of size and value in a better manner.

References


