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A CAPITAL STRUCTURE MODEL

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Abstract

This paper develops a capital structure model (CSM) that offers perpetuity gain to leverage (G_L) equations for debt-for-equity and equity-for-debt exchanges. The CSM equations offer potential application for financial managers as these equations have two components in contrast to the one-component equations of Modigliani and Miller (1963) and Miller (1977). One CSM component embodies both the traditional tax shield and also what this paper calls an "agency shield." The other CSM component represents a financial distress effect. Each CSM component shows how changes in borrowing costs influence firm value. This "change in borrowing costs" factor is missing from the MM and Miller equations and explains why these equations cannot account for the leverage-related effects predicted by agency and financial distress theories. A new outcome of this paper's CSM equations involves comparing a tax shield effect with an agency shield effect. This paper analyzes the variables that can cause the positive agency shield effect to dominate the positive tax shield effect. Even in countries where a corporate tax shield is absent, an agency shield effect can still cause the issuance of debt to increase firm value. The CSM equations can incorporate the predictions of existing capital structure theories and clarify points of controversy including the rate at which the tax shield should be discounted. Thus, this paper contends to have solved the controversial tax shield discount problem. Given this paper's extensions of the MM-Miller perpetuity G_L framework, the corporate finance world of MM-Miller long suspected as "flat" can be made "round."

Key words: Capital Structure Model, Gain to Leverage, Costs of Borrowing, Tax Shield, Agency Shield, Financial Distress.

JEL Classification: G32; C00.

1. Introduction

Modigliani and Miller (1963), MM, derived a gain to leverage (G_L) equation when an unleveraged firm issues perpetual riskless debt to replace risky equity[1]. By focusing on a pure capital structure change (in their case, a debt-for-equity transaction), the MM analysis detached itself from wealth effects that occur when a security is issued with the purpose of increasing operating assets. For MM, G_L is simply a tax shield effect given by multiplying the corporate tax rate times the value of perpetual debt. Writing for the Commission on Money and Credit at the time as his famed article with Modigliani, Miller (1963) warned of the economic costs of excessive debt. This suggests that G_L should include more than a positive corporate tax shield component. Post-MM researchers considered a variety of wealth effects linked to leveraged (and thus linked to financial risk) including bankruptcy and agency effects but disagreed about the strength of these effects and related details such as the rate at which the tax shield is discounted[2].

While the capital structure research is abundant and multifaceted, it largely leaves unanswered the measurement of leverage-related wealth effects through a succinct G_L equation that managers might be more likely to understand and apply. Respected researchers (Leland, 1998; Graham and Harvey, 2001) have acknowledged that capital structure theory is distressingly imprecise and provides relatively little specific guidance. In light of this lack of guidance and the dis-

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[1] Throughout this paper, MM refers to their 1963 article.

[2] Miller (1977) and Warner (1977) argue that leverage-related effects have no real impact on firm value, while Altman (1984), Fischer, Heinkel and Zechner (1989) and Kayhan and Titman (2007) are among those who provide contrary evidence. Pinegar and Wilbrecht (1989) and Graham and Harvey (2001) indicate firms are more concerned with debt flexibility. Ehrhardt and Daves (2002) review the literature concerning the disagreement over the rate at which the tax shield should be discounted.

agreements surrounding the strength and even the existence of leverage-related effects, what is needed is a G_L equation that accomplishes the following goals:

(1) does not presuppose the existence of any specific leverage-related effects but is derived from definitions for variables that exist before and after a firm undergoes a leverage change;

(2) lends itself to measurability through a compact mathematical expression that is reminiscent of the to-the-point equations of MM and Miller (1977) but addresses more than just a tax shield;

(3) sheds light on controversies surrounding the details of leverage-related wealth effects such as the relative strength of hypothesized tax, agency and financial distress effects as well as the rate at which a tax shield is discounted;

(4) offers managers a tool to help choose a wealth-maximizing debt level; and

(5) sets in motion a re-analysis of the maligned MM-Miller based equations where the re-analysis can render improved equations for primary variables rooted in MM-Miller (such as betas, costs of borrowing and WACC) thus causing far-reaching consequences on capital budgeting computations (such as for NPV, IRR and APV)[1].

This paper pursues the above goals through a perpetuity G_L framework while trying to lay the foundation to address some of the more obvious shortcomings of the MM-Miller equations. In particular, there is no in depth description as to how changes in costs of borrowing (and accompanying risk changes) influence G_L. Thus, there is no direct link to measure hypothesized effects related to debt such as bankruptcy and agency effects. In addition, the MM-Miller equations do not address the leveraged situation and the multiple potential wealth effects that might occur if a leveraged firm undergoes an equity-for-debt transaction. Also, their equations are silent on the role of growth and make no attempt to determine how changes in tax rates might impact firm value when a leverage change causes new investors to enter and old investors to exit. Besides laying the foundation to deal with the above shortcomings, the perpetuity G_L framework offered in this paper can also address other areas of concern through the cash flow, tax rate and discount rate variables included in each equation[2].

The remainder of the paper is organized as follows. After surveying the extant research, this paper develops a capital structure model by deriving perpetuity G_L equations with two components. After analyzing the components of its equations, this paper discusses extensions, offers numerical analyses, and summarizes the features and contributions of its G_L model.

2. Literature Review and Critique

This section will review the G_L research and critique its weaknesses including the inability to capture financial risk (and its related effects) through changes in discount rates[3]. The critique lays the groundwork for the development of this paper's perpetuity capital structure model. Henceforth, this paper refers to its capital structure model (that contains its G_L perpetuity equations) as simply CSM.

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[1] For example, consider the beta formula of Hamada (1972) based on the MM framework. Because of its linear relationship with debt that treats incremental increases in debt as equally risky (regardless of how much debt is outstanding at the time the new debt is issued), the Hamada beta equation clearly cannot accommodate any expected rapidly increasing levels of financial risk as a firm reaches higher and higher debt levels. As argued by Booth (2007), Hamada's formula is incorrect if the WACC has an interior minimum. With an erroneous WACC, problems are created when using it in capital budgeting methods such as NPV, IRR and APV.

[2] For example, concerns about flotation costs, noncash expenses, tax credits, alternative minimum tax, tax carryforward and carryback, and similar items can be inputted when setting either the cash flow or effective tax rate variables used in this paper's G_L equations. Concerns that influence the riskiness of cash flows (such as financial flexibility or managerial autonomy) can be included through adjustments in the costs of borrowing used to discount cash flows. Signaling concerns about earnings or risk can be adjusted with the cash flow variable and discount rate variable. Later, we will supply numerical analyses to illustrate how this paper's equations can handle some of these concerns as voiced my existing capital structure theories.

[3] While financial risk is seen as creating an overall net negative effect through increasing the probability of default, a broader definition might also consider debt as having a reducing effect on the probability of default for some situations. For example, overall firm value (including the long-term welfare of equity) can benefit if the increased debt allows for better decision-making that avoids risky projects with negative NPVs.

2.1. The MM Gain to Leverage (G_L) Equation and What It Leaves Out

The MM analysis focused on an unleveraged firm with risky equity that issues riskless debt. Other noteworthy conditions included corporate taxes, level perpetuities, and no growth. Given these suppositions, MM contended that G_L is the exogenous corporate tax rate (T_C) times the value of perpetual riskless debt (D):

$$G_L = T_C\left(\frac{I}{r_f}\right) = T_C D, \tag{1}$$

where I is the before-personal tax perpetual interest payment, and r_f is the exogenous cost of capital on riskless debt[1]. As D increases, MM showed that the cost of borrowing on risky equity increases. However, no detailed analysis was made of any negative leverage-related impact on G_L that might result from the increase in equity's cost of borrowing. Other central analyses not addressed in detail include influences from personal taxes, growth and the firm being leveraged when the new debt is issued. The latter leveraged condition can cause wealth transfers among equity and debt owners and allow for an analysis of a return to an unleveraged situation if relevant firm, industry, political or economic factors dictate such a return.

2.2. Early Post-MM G_L Research

Early post-MM researchers (Baxter, 1967; Kraus and Litzenberger, 1973) extended the G_L equation of MM by examining in more detail the negative effects of debt. They advocated a trade-off model where increasing levels of risky debt lead to increasing bankruptcy costs such that an optimal debt level exists where the bankruptcy costs offset the tax shield effect. While this line of research conceded risky debt so as to introduce bankruptcy as a leverage-related cost, the bankruptcy costs variable was extraneous to the derivational process. By extraneous to the derivation, it is meant that $G_L = T_C D + EV$, where EV is the chosen extraneous variable that is simply attached and not derived from definitions of firm value before and after the leverage change. As a result, such G_L equations do not derive self-contained discounted cash flows within a concise formula that captures leverage-related effects through changes in borrowing costs.

Although initial post-MM research focused on bankruptcy costs, it is possible for debt to be risky due to other leverage-related costs. Jensen and Meckling (1976) posited that there is more than just bankruptcy costs by examining a broader range of leverage-related costs referred to as agency costs. Agency proponents argued that net agency effects impact G_L notwithstanding any corporate tax shield and bankruptcy possibilities. Whereas increasing debt can initially cause net gains due to limiting the cash flows that managers can squander, too much debt can eventually lead to net losses caused by restricting manager's ability to make wealth-enhancing decisions.

2.3. Miller's G_L Extension of MM with Personal Taxes

Building on the research of Farrar and Selwyn (1967), Miller (1977) included personal taxes and expanded equation (1) so that:

$$G_L = [1 - \alpha]D, \tag{2}$$

where $\alpha = \dfrac{(1-T_E)(1-T_C)}{(1-T_D)}$, T_E and T_D are the respective personal tax rates applicable to income from equity and debt, and D now includes personal taxes. With personal taxes, the value of debt is now given as:

$$D = \frac{(1-T_D)I}{r_d}, \tag{3}$$

where r_d is the cost of debt and equals r_f if debt is riskless.

Miller (1977) considered the costs related to the increase in debt as negligible such that the effect of personal taxes alone offsets the effect of corporate taxes at the firm level. Thus, for

[1] MM acknowledged that their G_L value given in equation (1) is a maximum that falls in value when underlying assumptions are relaxed.

Miller, incorporating personal taxes restored an earlier finding by Modigliani and Miller (1958) that a firm's value lies solely in its operating assets.

2.4. Post-Miller G_L extensions and related capital structure literature

Post-Miller trade-off theorists (DeAngelo and Masulis, 1980; Kim, 1982; Modigliani, 1982; Ross, 1985) considered a variety of leverage-related costs and showed that an optimal debt level exists even with personal taxes. A pecking order theory of capital structure (Myers, 1984; Myers and Majluf, 1984) argued for a hierarchy of financing preferences with internal equity being the first choice as it preserves financial slack and avoids negative signaling and flotation costs. Jensen (1986) claimed debt reduces the agency cost of free cash flow implying it is valuable beyond its tax shield benefits. Even for firms with high leverage ratios, adding debt can be useful in monitoring managers and can lead to a profitable restructuring. Leland (1994) considered a perpetuity-like framework with firm value following a diffusion process with constant volatility and linked optimal leverage to firm risk, taxes, bankruptcy costs, riskless interest rates, payout rates and bond covenants[1]. Graham (2000) offered evidence for the existence of a corporate and personal tax benefit of debt that is at least 4.3% of firm value. Most recently, Fama and French (2005) have found that security issuance decisions are inconsistent with existing theories[2].

2.5. Assessment of Extant Research

Even though plentiful and multifaceted, the extant G_L research digresses from the simple perpetuity equations of MM (1963) and Miller (1977) and can be largely characterized by the inability to make explicit how changes in costs of borrowing (and thus changes in financial risk) influence firm value within a compact valuation model. The next section addresses this shortcoming by incorporating discount rates within perpetuity G_L equations. The inclusion of discount rates is crucial to providing G_L equations where leverage-related costs change as debt changes. To the extent changes in discount rates (and other relevant variables such as cash flows and tax rates) can be adequately estimated, managers are provided with practical G_L equations where the tax shield advantage is negated by leverage-related costs as debt increases.

Table 1

Variables and Equations: Summarized with Notations and Definitions

Panel A: Cash Flow Variables (before corporate and personal tax considerations)	
C = the uncertain perpetual before-tax cash flow (from operating assets) for unleveraged equity owners	
I = the perpetual before-tax interest payment chosen for debt owners	
$C - I$ = the perpetual before-tax cash flow (from operating assets) available to leveraged equity owners	
Panel B: Corporate and Personal Tax Rate Variables	
T_C = corporate tax rate	T_E = personal tax rate on equity income
T_D = personal tax rate on debt income	$\alpha = \dfrac{(1-T_E)(1-T_C)}{(1-T_D)}$

[1] While more modest in design, the CSM model can give results consistent with some of Leland's findings. For example, Leland notes that he observes a "curious aspect", namely, that an increase in the risk-free rate (r_f) increases the optimal leverage. This has been shown by Hull and McNulty (2007) in a pedagogical application using the CSM framework given in this paper. They decrease r_f from 5% to 3% in their application and G_L rises about 6% while the optimal debt-to-firm value ratio (DV) drops slightly over 20%. Thus, a drop in the federal funds rate (or other factors that lower r_f) will cause a decrease in the optimal DV implying the firm is now overleveraged. To achieve the 6% increase the firm would have to lower its debt.

[2] Many other articles could be cited as the literature is abundant beginning with Berle and Means (1932) and Williams (1938) and continuing more recently with Mahrt-Smith (2005) and Hennessy and Whited (2005).

Table 1 (continued)

Panel C: Cost of Borrowing (Discount Rate) Variables	
r_f = the exogenous cost of riskless debt	r_d = the endogenous cost risky debt
r_u = the exogenous cost of unleveraged equity	r_l = the endogenous cost of leveraged equity
Panel D: Ownership Variables (personal and corporate taxes considered)	
V_U (E_U) = unleveraged firm value = $\frac{(1-T_E)(1-T_C)C}{r_u}$	D = debt value = $\frac{(1-T_D)I}{r_d}$
E_L = leveraged equity value = $\frac{(1-T_E)(1-T_C)(C-I)}{r_l}$	$V_L = E_L + D$
Panel E: Two Capital Structure Model (CSM) Gain to Leverage Equations	
G_L (debt-for-equity) = $V_L - V_U = POS + NEG$	$POS = \left[1-\frac{\alpha r_d}{r_l}\right] D$ and $NEG = -\left[1-\frac{r_u}{r_l}\right] E_U$
G_L (equity-for-debt) = $V_U - V_L = POS_1 + NEG_1$	$POS_1 = \left[1-\frac{r_u}{r_l}\right] E_U$ and $NEG_1 = -\left[1-\frac{\alpha r_d}{r_l}\right] D$

3. CSM's G_L Equation for an Unleveraged Firm Issuing Debt to Retire Equity

This section begins developing the capital structure model (CSM) by deriving a G_L equation for an unleveraged firm issuing debt to retire equity. The equation captures the valuation impact of a debt-for-equity exchange within a perpetuity framework through changes in financial risk as captured by changes in equity's cost of borrowing. Table 1 summarizes the variables and equations (along with notations and definitions) used in this section and subsequent sections.

Why extend the MM-Miller perpetuity framework? First, there exists a perpetuity that is practically equivalent to the present value of any stream of a firm's expected cash flows. While there is an unlimited number of perpetuities with the same value, there is only one that can best capture both the amount and riskiness of an individual firm's cash flows. In essence, whatever assumptions one wants to make (including those about the probability distribution of cash flows), there are perpetuities that will equal it and one of these perpetuities will best represent the firm's cash flows[1]. Second, by definition, equity provides perpetual cash flows. While debt has no infinite horizon, it is often refinanced (or restored in the long-run) causing an indefinite horizon. Given these considerations and the simplicity of working with perpetuities, it makes a perpetuity equation an ideal means for modeling the valuation impact of a managerial decision involving a pure leverage change (such as a debt-for-equity or equity-for-debt transaction).

3.1. G_L for an Unleveraged Firm with No Growth and Tax Rates Independent of Leverage

This paper's first G_L derivation uses the MM-Miller framework of an unleveraged firm with level perpetuities, two security types, and no growth. The latter condition implies that all real depreciation equals investment to keep the same amount of capital in place and no funds are retained internally or added externally to increase future payouts. The derivation allows for risky debt (with concomitant agency and bankruptcy effects) and tax rates that are independent of the leverage change. Given these conditions, a perpetuity G_L equation including discount rates (borrowing costs) can be derived from the definition that G_L is leveraged firm value (V_L) minus unleveraged firm value (V_U):

$$G_L = V_L - V_U. \tag{4}$$

[1] Consider a company with a market value of $10 billion, which also represents management's best assessment of its true value given all assumptions and considerations about expected cash flows and risk. The problem concerns which perpetuity to choose from those that could render a value of $10 billion. Although there could be a vast number of perpetuities equal to $10 billion, let us consider (for illustration purposes) just two possibilities: a perpetual cash flow of $1 billion with a discount rate of 0.1 or a perpetual cash flow of $1.1 billion with a discount rate of 0.11. Both equal $10 billion, but only one would best represent the firm's cash flow and risk situation.

Noting that V_U is the same as unleveraged equity value (E_U) since there is no debt, V_U is defined as:

$$V_U = E_U = \frac{(1-T_E)(1-T_C)C}{r_u}, \tag{5}$$

where tax rates can be viewed as effective rates that include actual taxes paid at all levels (municipal, state, federal and international); C is the uncertain perpetual before-tax cash flow (from operating assets) available to owners of unleveraged equity and can be viewed as the accounting number given by EBIT if all expenses are cash expenses; and r_u is the exogenous unleveraged equity discount rate with $r_u > r_d$.

V_L is leveraged equity value (E_L) plus debt value (D) where D was defined earlier in (3). E_L is defined as:

$$E_L = \frac{(1-T_E)(1-T_C)(C-I)}{r_l}, \tag{6}$$

where $(C – I)$ is the uncertain perpetual before-tax cash flow to owners of leveraged equity with $C > I$ and r_l is the leveraged equity discount rate that increases with debt such that $r_l > r_u$ holds. Inserting equations (6) and (3) into the definition $V_L = E_L + D$ gives:

$$V_L = E_L + D = \frac{(1-T_E)(1-T_C)(C-I)}{r_l} + \frac{(1-T_D)I}{r_d}. \tag{7}$$

A perpetuity G_L equation for an unleveraged firm issuing debt that incorporates tax rates and discount rates can now be derived from the definitions in equations (3) through (7). Appendix A shows:

$$G_L = \left[1-\frac{\alpha r_d}{r_l}\right]D - \left[1-\frac{r_u}{r_l}\right]E_U, \tag{8}$$

where $\alpha = \dfrac{(1-T_E)(1-T_C)}{(1-T_D)}$ with $\alpha < 1$ expected to hold unless there is some unusual and unexpected tax rate situation, $r_d < r_u < r_l$, and $D < E_U$. Relaxing the latter restriction such that $D = E_U$, debt owners become (by legal decree) the new unleveraged equity owners and there is no G_L (or $G_L = 0$).

As shown and explained in Appendix A, equation (8) can be expressed as:

$$G_L = POS + NEG, \tag{9}$$

where $POS = \left[1-\frac{\alpha r_d}{r_l}\right]D > 0$ and $NEG = -\left[1-\frac{r_u}{r_l}\right]E_U < 0$. Equation (9) emphasizes that G_L can be positive or negative depending on whether $|POS|$ or $|NEG|$ is larger. Thus, an unleveraged firm with no growth attempting to increase its value would issue debt as long as $|POS| > |NEG|$ holds. If restricting the decision-making process to one choice from a set of finite possible debt-for-equity choices, managers would estimate G_L values for all choices and choose the one that gives the largest positive G_L value and do nothing if all G_L values are negative[1].

[1] Equation (8) yields two endpoints where $G_L = 0$: $D = 0$ and $D = E_U$. Using Rolle's Theorem and a qualitative argument that managerial behavior is wealth maximizing, it is a rudimentary exercise to show G_L has at least one maximum value. Under the simplifying conditions assumed in the derivation (and adding in very low risk debt), one can use differential calculus to show the optimal debt-to-equity ratio (DE) $\approx \dfrac{r_u}{\alpha r_d}$. Thus, given the Miller beliefs that $\alpha = 1$ and the negative effects of debt are trivial, the optimal is simply a function of the costs of borrowing existing at the time of the debt decision. Although not shown in this paper, the optimal DE is overestimated if one considers other factors like growth, a wealth transfer effect, and tax rates that change with leverage. In particular, growth and wealth transfers lower the optimal DE.

Table 2

Reduction of Equation (8) to the MM and Miller Equations

Panel A: Reduction of the Miller Equation to the MM Equation
If personal taxes are ignored ($T_E = T_D = 0$), and debt is riskless ($r_d = r_f$), then Equation (2) reduces to Equation (1): $G_L = [1 - \alpha]D = \left[1 - \frac{(1-T_E)(1-T_C)}{(1-T_D)}\right]\frac{(1-T_D)I}{r_d} = \left[1 - \frac{(1-0)(1-T_C)}{(1-0)}\right]\frac{(1-0)I}{r_f} \Rightarrow G_L = T_C D$ with $D = \frac{I}{r_f}$.
Panel B: Reduction of the CSM Equation to the Miller Equation
If differences in costs of borrowing are ignored ($r_d = r_u = r_l$), then the CSM Equation given by Equation (8) reduces to the Miller Equation given by Equation (2): $G_L = \left[1 - \frac{\alpha r_d}{r_l}\right]D - \left[1 - \frac{r_u}{r_l}\right]E_U = \left[1 - \frac{\alpha \cancel{r_d}}{\cancel{r_l}}\right]D - \left[1 - \frac{\cancel{r_u}}{\cancel{r_l}}\right]E_U = [1 - \alpha]D - [1 - 1]E_U \Rightarrow G_L = [1 - \alpha]D$.
Panel C: Reduction of the CSM Equation to the MM Equation
If differences in costs of borrowing are ignored ($r_d = r_u = r_l$), personal taxes are ignored ($T_E = T_D = 0$), and debt is riskless ($r_d = r_f$), then the CSM Equation given by Equation (8) reduces to the MM Equation given by Equation (1): $G_L = \left[1 - \frac{\alpha r_d}{r_l}\right]D - \left[1 - \frac{r_u}{r_l}\right]E_U = \left[1 - \frac{\alpha \cancel{r_d}}{\cancel{r_l}}\right]D - \left[1 - \frac{\cancel{r_u}}{\cancel{r_l}}\right]E_U = [1 - \alpha]D - [1 - 1]E_U = [1 - \alpha]D \Rightarrow$ $G_L = \left[1 - \frac{(1-T_E)(1-T_C)}{(1-T_D)}\right]\frac{(1-T_D)I}{r_d} = \left[1 - \frac{(1-0)(1-T_C)}{(1-0)}\right]\frac{(1-0)I}{r_f} \Rightarrow G_L = T_C D$ with $D = \frac{I}{r_f}$.

3.2. Why the MM and Miller G_L equations fail to capture agency and bankruptcy effects

Panel A of Table 2 illustrates how the Miller G_L equation given by (2) reduces to the MM G_L equation given by (1) when personal tax rates are ignored ($T_E = T_D = 0$) and debt is riskless ($r_f = r_d$). Panel B shows how equation (8) reduces to the Miller equation when differences in borrowing rates are ignored ($r_u = r_l = r_d$), while Panel C illustrates how equation (8) reduces to the MM equation when differences in costs of borrowing are ignored ($r_d = r_u = r_l$), personal taxes are ignored ($T_E = T_D = 0$), and debt is riskless ($r_d = r_f$).

The reduction of equation (8) to equation (2) when differences in costs of borrowing are ignored, and to equation (1) when differences in costs of borrowing and personal tax rates are both ignored, reflects the respective procedures implied by MM and Miller. While these procedures have their own rationale (and this paper will not debate their logic), it can be pointed out that both procedures can be viewed as implying, from an algebraic viewpoint, that perpetual cash flows can be factored without denominators[1]. This implies that discount rates are equal[2]. The implied treatment of discount rates as equal explains why the resulting MM-Miller equations are disabled from capturing negative leverage-related effects predicted by optimal capital structure models that focus on the trade-off between the benefits and costs of debt.

In conclusion, this paper offers the following two results.

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[1] One need only look at the MM (1963) argument (pp. 434-435) to see that their perpetuity cash flows are rearranged without regard to discount rates. This is done before their famous arbitrage argument is invoked. A similar disregard can be found later when they derive their costs of capital.

[2] For the record, this paper is not claiming that MM "require" discount rates to be equal for their purposes since their procedure does not involve factoring cash flows with different discount rates. By choosing an algebraic procedure with factoring, the CSM must explicitly treat the inequality of discount rates in the derivation.

(1) The MM (1963) G_L equation, with riskless debt and only corporate taxes for an unleveraged firm with no growth, can be viewed as a specific application of the more inclusive formula given in equation (8) with the MM equation resulting when debt is riskless, discount rates are equal, and personal tax rates are zero.
(2) The Miller (1977) G_L equation, with personal and corporate taxes for an unleveraged firm with no growth, can be viewed as a specific application of the more inclusive formula given in equation (8) with the Miller equation resulting when discount rates are equal.

4. The Missing Effects from the MM-Miller Equations

This section breaks down the two components of equation (8). In scrutinizing the 1st component of (8), this section compares a tax shield effect with what is identified as an "agency shield" effect. Given these two effects, this 1st component is dubbed the "tax-agency shield" component. This section also investigates the 2nd component of (8) and characterizes it as the "financial distress" component due to its negative effect that increases in magnitude as debt increases. In the process, the effects missing from the MM-Miller equations are analyzed.

4.1. Miller's tax shield component modified to include a positive "agency shield" effect

An analysis of equation (8) reveals that its 1st component of $\left[1-\frac{\alpha r_d}{r_l}\right]D$ resembles the Miller tax shield equation given in (2) where $G_L = [1-\alpha]D$, except α is multiplied by a fraction that is less than one (i.e., $\frac{r_d}{r_l} < 1$) thus increasing this component's positive value. Ignoring differences in discount rates so as to allow $r_d = r_l$ to hold, the 1st component of (8) equals (2). Thus, due to the 1st component's similarity to (2), it appears this component should be named the tax shield component. However, suppose tax rates are ignored (i.e., $T_C = T_E = T_D = 0$) causing $\alpha = 1$ to hold[1]. Doing this causes the 1st component to now equal $\left[1-\frac{r_d}{r_l}\right]D$ (where D equals $\frac{I}{r_d}$ with $T_D = 0$). One would even suspect that $\frac{r_d}{r_l} < \alpha$ will hold causing a positive effect in the 1st component that would (in itself) be greater than the tax shield effect. Thus, instead of being just a tax shield component, the 1st component takes on an additional category that can be identified with an "agency shield" effect because it is consistent with agency theory that hypothesizes debt can be positive for reasons other than a tax shield[2]. A positive agency shield effect can be viewed as stemming from a synergistic impact due simply to how ownership claims are packaged and sold (with regard to risk) to "shield" the firm from costs associated with agency behavior.

Besides agency theory, the positive effect (with tax rates ignored) is also consistent with any less established theories that hypothesize a positive relationship between debt and firm value. For example, it is consistent with Boot and Thakor (2005) who posit (when the value of assets in place is high) a positive relationship between debt and firm value due to managerial autonomy even in the absence of tax shields, agency costs and signaling.

To sum up, the 1st component of equation (8) embodies both a tax shield component and an "agency shield" component where the latter stems from the choice of ownership claims that "shields" the firm from agency costs that might otherwise exist or exist in a greater degree. Henceforth, this paper refers to this component as the "tax-agency shield" component. The end result is

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[1] Miller (1977) believes $\alpha = 1$ but for a situation where all tax rates are not zero. Later, this paper will give a numerical analysis of a situation where $\alpha = 1$ and show that G_L can still be positive.

[2] This paper's derivations using standard definitions do not claim to have any particular model of financial distress or agency costs in mind. However, in interpreting the implications of the CSM model, this paper carefully tries to find consistency with prior research so that the CSM is not perceived as "totally disparaging standard capital structure theory" (as has been claimed by one reputed scholar).

that the 1st component expands the Miller tax shield representation to one that also makes explicit a wealth effect consistent with agency theory. Thus, even for countries without a tax shield advantage of debt over equity, an optimal leverage ratio can still exist.

4.2. The MM-Miller missing "financial distress" component

If one only examines the 1st component of (8), one would have just another equation, like the MM equation, that suggests firms issue unlimited amounts of debt. This holds even if the debt tax shield is zero since the agency shield is positive. This leads us to the key missing component from the MM-Miller equations, which is the 2nd component of (8) given by $-\left[1-\frac{r_u}{r_l}\right]E_U$. It is this component that captures a negative effect. As the gap between r_u and r_l increases with more debt, financial risk increases leading to a negative financial distress factor capable of offsetting the positive "tax-agency shield" effect of the 1st component.

Given that the negativity of the 2nd component of equation (8) can increase with debt, this component is referred to as the "financial distress" component. This negativity for a leverage increase can be associated with both bankruptcy costs as well as agency costs arising when the firm's creditworthiness is in doubt. While this component is missing from the MM-Miller G_L equations, intuitive discernment reveals that this component's effect should occur in a G_L formula in a world of risky securities. Otherwise, the capacity to capture financial distress with increasing levels of debt is incongruously absent.

With the integration of financial distress and tax-agency effects, it can be concluded that:

Managers of an unleveraged firm (with negligible growth and tax rates independent of leverage) can use equation (8) as a feasible G_L formula capable of quantifying tax, agency and bankruptcy considerations with key measurable dimensions involving the difference between r_d and r_l and the change from r_u to r_l.

5. Resolution of the Controversy on the Discount Rate for the Tax Shield

Equation (8) can provide insight on the controversy surrounding the rate at which the interest tax shield should be discounted. MM's equation given by $\frac{T_C I}{r_d}$ implies that the tax shield's cash flow of $T_C I$ should be discounted by the cost of debt. Likewise, Miller's equation given by $\frac{[1-\alpha](1-T_D)I}{r_d}$ implies that the tax shield's after-tax cash flow of $[1-\alpha](1-T_D)I$ should also be discounted by the cost of debt. However, others disagree and argue that the tax shield should not be discounted by r_d but rates higher than r_d. Ehrhardt and Daves (2002) review the literature and argue that the tax shield should be discounted by the unleveraged equity rate (r_u). According to equation (8), the dispute concerning the appropriate discount rate is muted given that multiple discount rates can be shown to be present in each component's numerator and denominator when discounting a cash flow representing the gain to leverage. To illustrate, consider the 1st component of (8) that includes the tax shield effect (one of the two effects just identified). This component can be expressed with two discount rates in the denominator: $\left[1-\frac{\alpha r_d}{r_l}\right]D = \left[1-\frac{\alpha r_d}{r_l}\right]\frac{(1-T_d)I}{r_d} = \left[\frac{r_l}{r_l}-\frac{\alpha r_d}{r_l}\right]\frac{(1-T_d)I}{r_d} = \left[\frac{r_l-\alpha r_d}{r_l}\right]\frac{(1-T_d)I}{r_d} = \frac{[r_l-\alpha r_d](1-T_D)I}{r_l r_d}$. The latter expression indicates there is no clear-cut rate in the denominator at which the interest payment can be discounted. Only if one divides numerator and denominator by r_l can one get an expression with r_d in the denominator. But this procedure can also be used to get r_l in the denominator. In fact, any value in the denominator could be obtained by simply manipulating the coefficient that multiplies the cash flow of I. Similar deductions can be made about the 2nd component of equation (8) that can be expressed as $\frac{[r_l - r_u](1-T_C)(1-T_E)C}{r_l r_u}$.

It can be argued that the controversy about how to discount the tax shield has its origins in the lack of an algebraic procedure to factor cash flows with discount rates. As noted previously, from an algebraic standpoint, this is tantamount to treating rates as equal. As was seen in Table 2, treating discount rates as equal causes the 1st component in equation (8) to reduce to Miller's equation with a discount rate of r_d and the 2nd component in (8) to become zero. The end product of a non-algebraic procedure is that a discount rate of r_d results. But, this rate of r_d (suggested by the MM and Miller G_L equations) does not make sense to investigators who embark on a search to find the "Holy Grail" of discount rates. Simply put, if a G_L equation is not derived from the definitions for V_L and V_U with discount rates in tact, one is left to assign discount rates just as one is left to extraneously assign bankruptcy and agency variables (whose effects could be captured by discount rates if these rates are not lost in the derivational process)[1].

6. An Equity-for-Debt Exchange and Other Extensions to CSM

This paper now derives G_L for an equity-for-debt exchange and also looks at extensions of CSM given in equation (8). These other equations can form a series of G_L formulations capable of expanding the CSM to include growth, wealth transfers among security holders, and changes in corporate and personal tax rates.

6.1. Suppose a leveraged firm becomes unleveraged

Suppose a firm can increase its value through an equity-for-debt exchange where it retires all of its debt and becomes unleveraged. For this scenario, G_L is referred to as $G_L^{\text{Equity-for-Debt}}$ and defined as:

$$G_L^{\text{Equity-for-Debt}} = V_U - V_L, \tag{10}$$

where managers now believe $V_U > V_L$ holds. Using equation (10) and definitions given previously for D, E_U, E_L and V_L, Appendix B shows:

$$G_L^{\text{Equity-for-Debt}} = \left[1 - \frac{r_u}{r_l}\right] E_U - \left[1 - \frac{\alpha r_d}{r_l}\right] D, \tag{11}$$

where $\left[1 - \frac{r_u}{r_l}\right] E_U > 0$ and $-\left[1 - \frac{\alpha r_d}{r_l}\right] D < 0$ and the components and signs found in (8) are reversed. Given the assumption about signs for components, equation (11) can be expressed in a fashion akin to the POS and NEG components in equation (9). The expression is:

$$G_L^{\text{Equity-for-Debt}} = POS_1 - NEG_1, \tag{12}$$

where $POS_1 = \left[1 - \frac{r_u}{r_l}\right] E_U > 0$ and $NEG_1 = -\left[1 - \frac{\alpha r_d}{r_l}\right] D < 0$. Using equation (12), a firm would become unleveraged as long as $|POS_1| > |NEG_1|$ holds.

For a firm that becomes unleveraged through an equity-for-debt exchange, the 1st component of (11) indicates a positive effect through reduction in financial distress as captured by the fall in equity's discount rate. In fact, the 1st component can be shown to equal the percentage change in r_l times E_U. The 2nd component of (11) indicates that a leverage decrease has a negative effect from reducing the positive tax-agency shield. Equation (11) is consistent with the logic of trade-off theory, which suggests that a firm will undergo a leverage decrease when the positive effect from

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[1] Although not covered in this paper due to length concerns, incorporating growth also does not produce one distinct discount rate because any discount rate that occurs can be shown to depend on the growth rate. For the 1st component of a G_L equation with growth, only by coincidence (which is when the growth adjusted rate of leveraged equity equals r_d) can the discount rate be r_d. Similarly, the discount rate can only be r_u if the growth adjusted rate of leveraged equity happens by chance to equal r_u. Incorporating a wealth transfer effect rising from a leveraged situation adds a third component with multiple discount rates (albeit the two discount rates are two costs of debt).

reducing financial distress costs dominates the negative effect from reducing the positive tax-agency shield.

6.2. Extensions with growth, wealth transfers, and changes in tax rates

Equations incorporating growth and wealth transfer effects can be derived in a similar fashion to the CSM equations found in (8) and (11). Due to length concerns, this paper only briefly comments on these extensions.

First, let us consider growth within a perpetuity situation[1]. The only change in a resulting equation is that equity discount rates are adjusted for growth making them smaller. While the derivation is similar, interpretations can differ. For example, with unleveraged and leveraged equity discount rates adjusted for growth, the denominators in the CSM equations become smaller. This can lead to sign reversals for both the 1st and 2nd components if the firm chooses (or achieves for whatever reason) an extremely high leveraged situation. The sign reversal for the 1st component behaves as predicted by Jensen and Meckling (1976) by changing from a positive effect to a negative effect as debt increases. The sign reversal for the 2nd component is consistent with positive restructurings and free cash flow theory that predict the possibility of a positive value with more debt even if the firm already has high debt levels.

Expanding on equation (8) so as to consider a leveraged situation, one can allow for a wealth transfer from outstanding debt to equity. This adds a 3rd component to the resulting CSM equation that captures the transfer of wealth among security holders including that among prior debtholders and new debtholders. This "wealth transfer" component is consistent with Galai and Masulis (1976) who introduced an agency model dealing with risk shifting causing a transfer of wealth among owners when a firm's managers changes its mix of debt and equity. The shift in risk can be explained in terms of changes in the costs of borrowing among debt and equity owners.

Finally, the author has also derived equations with changes in corporate and personal tax rates and examined their effect on the resulting G_L equations (albeit one might not expect much of a clientele change in terms of taxes paid). These derivations introduce taxes into the financial distress component and the wealth transfer component in a manner similar to that found in the tax-agency shield component. This tax rate change situation caused by newer owners replacing prior owners along with the growth and wealth transfer situations are all reserved for future research where all three situations can be more properly addressed without space limitations.

Table 3

Linkages between Existing Theories and Capital Structure Model (CSM)

Existing Theory	Numerical Analysis Using CSM
Trade-Off Theory: This theory asserts that the optimal leverage ratio occurs when the last dollar of debt issued has benefits that are offset by its costs. A numerical analysis using equation (8) shows, in the adjacent cell, that CSM is consistent with this assertion as G_L first increases and then decreases with debt. The decrease is caused by the negative 2nd component of (8) increasing rapidly with debt.	CSM supports trade-off theory as optimal DV can exist: Assume these values: T_C = 0.26, T_D = 0.13, T_E = 0.07, α = 0.79103, r_f = 0.055, $r_d = r_f + 0.07\left(\frac{D}{E_U}\right)^2$, r_u = 0.10, $r_l = r_u + 0.095\left(\frac{D}{E_U}\right)^2$, and E_U = \$10 billion. The values for r_u and E_U imply that the after-tax cash flows available to equity are \$1 billion. Choosing after-tax values for D in increments of \$1 billion for nine choices from D = \$1 billion to D = \$9 billion, we get r_d values ranging from 0.0557 to 0.1117 and r_l values from 0.10095 to 0.17695. G_L values (in billions of \$) as debt increases are 0.47, 0.75, 0.87, 0.86, 0.76, 0.62, 0.45, 0.29 and 0.16. Thus, G_L increases until we get to D = \$3 billion where the maximum G_L is \$0.87 billion. The optimal debt to firm value ratio (DV) is 0.28 for the \$3B debt choice.

[1] See Hull (2005) for more details on the consequences of incorporating growth within a perpetuity G_L equation (as applied to a case study of an individual firm).

Table 3 (continued)

Existing Theory	Numerical Analysis Using CSM
Agency Theory: This theory says that a capital structure change can cause a change in firm value even without taxes. The numerical analysis in the adjacent cell shows that when there is no tax shield issuing debt still adds about \$0.52B.	CSM shows how tax shield not needed for debt to increase firm value: Assume the numbers just given in the 1st row except let us change tax rates so that $\alpha = 1.0$ by assuming $T_C = 0.213$, $T_D = 0.2407$ and $T_E = 0.0351$. Doing this renders G_L values (in billions of dollars) of about 0.35, 0.520, 0.518, 0.38, 0.15, -0.13, -0.44, -0.75 and -1.03. The firm issues \$2B in debt and the optimal DV is 0.19. However, issuing \$3B in debt only drops firm value about \$0.002 billion or 0.39%.
Signaling Theory: One aspect of this theory says that issuing equity signals negative news because the market fears the equity is overvalued. On the other hand, retiring equity can signal that the equity is undervalued (else the firm would not have reason to repurchase its shares). This undervaluation is illustrated in the adjacent cell where issuing debt to retire undervalued equity causes an increase in G_L and allows a firm to issue more debt causing a higher leverage ratio. The numerical analysis shows that the CSM can incorporate this positive signaling effect.	CSM shows how signaling can affect G_L and the chosen debt level: Let us assume the numbers in the 1st row do not properly account for G_L because retiring equity signals that the equity is undervalued (a positive effect that increases the value of equity). The undervaluation by the market could be based on its underestimation of the future expected cash flows and/or its discounting these cash flows by a rate that reflects too high of a risk premium. For simplicity, let us just assume the latter. To reflect the reduced risk, let us drop the coefficient in the r_d equation from 0.07 to 0.06 and the coefficient in the r_l equation from 0.095 to 0.085; and also let us decrease r_u from 0.10 to 0.095. The latter causes V_U to increase from \$10B to \$10.53B because the after-tax perpetual cash flow of \$1 billion is now divided by a r_u of 0.095 instead of 0.10. With these changes, we get G_L values (in billions of dollars) of about 0.47, 0.77, 0.90, 0.91, 0.83, 0.70, 0.56, 0.42 and 0.31. The optimal DV is now 0.37 instead of 0.28. Keeping V_U at \$10B, but allowing r_d and r_l to still increase with debt, has no visible affect on the optimal DV and only causes G_L to fall from 0.91 to 0.87.
Pecking Order Theory (POT): This theory ties into signaling theory because it says firms prefer securities with lower costs including signaling costs. This implies that the choice of security considers its signaling impact. Thus, POT can be linked to CSM through the above numerical analysis. In regards to flotation costs, POT suggests these costs are a factor in the security choice even leading to a range of DV values. The numerical analysis in the next cell captures this aspect of POT showing how flotation costs relate to the optimal DV.	CSM shows how flotation costs can have large impact on DV range: Assume the numbers in the 1st row. Also assume that flotation costs are composed of a fixed fee of \$0.1B plus 1% of every dollar of debt raised to retire a dollar of debt. This means that the flotation costs for \$3 billion of debt is \$0.13B (about 4.33% per dollar raised). If the firm had issued \$4 billion of debt it would have paid \$0.14B (about 3.50% per dollar raised). The difference in flotation costs between the \$3B and \$4B choices is \$0.01B. This difference is greater than the difference in G_L between the \$3B and \$4B choice which is \$0.8722B – \$0.8623B = \$0.0099B. Suppose the transaction costs had been fixed at \$0.14B for all debt choices. If so, the firm would have chosen \$0.4B in debt and achieved an increase in G_L of about \$0.0001B and the optimal DV would have been 0.37 instead of 0.28. Thus, a slight change in flotation costs can have major ramifications on the optimal DV value. Even without considering any need for financial slack, firms could have a range of DV values depending on flotation costs.

6.3. Incorporating existing capital structure theories

There may be other expected costs (OEC) that this paper may appear to not explicitly consider in its CSM equations. For example, consider the OEC suggested by the pecking order theory such as the costs of floating a new issue and any costs that might occur from a firm not having sufficient financial slack. If these costs are not somehow factored into the variables in equation (8) then, in terms of equation (9), a manager would issue debt only if (POS + NEG) > $\lvert$OEC$\rvert$ holds. However, if flotation costs are deemed the primary domain of residual owners, then these expenses can be inputted into the cash flow variable to equity owners. Similarly, if additional risk for equity owners results from lack of financial slack then this can be captured by a greater discount rate on equity thereby creating a net negative impact on G_L.

Table 3 provides linkages between the CSM equations and existing capital structure theories demonstrating (via numerical analysis) how these theories can be supported within the CSM framework. The 1st column describes four existing capital structure theories and their predictions. The numerical analysis in the 2nd column assumes nine choices for a firm and the firm maximizes the G_L value for the best choice among these nine choices. This column shows that the CSM framework can quantity the predictions of these theories showing how they impact G_L and the optimal debt to firm value (DV) ratio. The table reveals the potential of the CSM framework to account for and shed light on major competing hypotheses.

In conclusion, this paper's CSM equations are able to incorporate the effects predicted by existing capital structure theories by making them workable within its framework. These effects could sway managers in their capital structure choice when using equations (8) and (11) or extensions of these equations that consider factors such as growth, wealth transfers or tax rate changes. Future research can provide a more exhaustive analysis.

7. Summary and Conclusions

The perpetuity framework of this paper's capital structure model (CSM) equations shares a kinship with the perpetuity-based structure of the Dividend Valuation Model (DVM). Like the DVM, the CSM has the advantage over a finite period model by agreeing with the notion that the present value of all cash flows contains the real value of any financial claim. As such, CSM equations are consistent with the belief that rational investors will appraise an asset (such as a tax shield) based on the discounted value of all of its cash flows. The CSM's critical impact is not just computing G_L but lies more with breaking down its positive and negative components so managers can better understand the nature of the wealth effects. By analyzing all relevant leverage-related wealth effects, managers are less likely to error in their decision-making as occurs when they use an imprecise equation that does not include all valuation effects.

Table 4

Summary of the Salient Features and Contributions of This Paper's Capital Structure Model (CSM)

(1) The CSM contains G_L equations derived from definitions embodying germane variables needed to explain the effects of a pure capital structure change. As a result, the equations have no need to extraneously insert variables to capture a hypothesized effect and thereby run the risk of crafting a variable that, if it exists, can be hard to measure.
(2) The CSM supplies G_L equations that make explicit how changes in equity and debt discount rates (i.e., costs of borrowing) impact firm value. These rate changes (and accompanying risk changes) are missing from the MM-Miller G_L equations explaining why these equations do not have the capacity to capture the leverage-related effects hypothesized by mainstream capital structure theories.
(3) The CSM offers perpetuity G_L equations that are more inclusive than the MM (1963) and Miller (1977) equations revealing the limitations of these latter two models including applications derived from their framework.
(4) The CSM offers equations with two components that are dubbed as the "tax-agency shield" component and the "financial distress" component.
(5) The tax-agency shield component can be broken down into a tax shield effect and an "agency shield" effect where the latter can be even greater than the former.
(6) The financial distress component can capture leverage-related costs as equity's cost of borrowing increases with debt.
(7) The CSM equations provide an answer to the controversial area of research that disagrees about the rate at which the tax shield should be discounted. This paper shows that cash flows given in its G_L equations do not have a single discount rate.
(8) While the primary concern is to extend perpetuity G_L research for an unleveraged framework for firms undergoing debt-for-equity transactions, this paper also offers a G_L equation for equity-for-debt transactions. This latter equation exists only for a leveraged situation and is a mirror image of its counterpart equation derived from a debt-for-equity exchange.
(9) Although epitomizing trade-off theories, the CSM equations can show how other major capital structure theories influence a manager's capital structure choice.
(10) To the extent cash flows, tax rates and costs of borrowing can be accurately estimated, the CSM gives G_L equations usable by managers to measure the dollar impact of an anticipated leverage change. Because the impact can be either positive or negative, it is possible for managers to identify the optimal debt level where G_L is maximized.

Table 4 summarizes the salient features and contributions of the CSM to the capital structure literature. As the table notes, the CSM equations are derived from definitions for firm values before and after pure leverage change transactions so that they include discount rates for equity and debt. The inclusion of these rates (i.e., costs of borrowing) makes it possible for G_L values to eventually decrease with increasing debt levels. By including discount rates, the CSM offers G_L equations with more practical potential than prior equations, which either are unrealistic by disregarding the role of discount rates or consist of variables (often extraneously added) that are more difficult to measure. As such, financial managers are hard pressed to find utility in their application. This paper's G_L equations can overcome previous measurability problems to the extent changes in the costs of borrowing for equity and debt are easier to estimate than the unenviable task of trying to gather data to compute the indirect and direct bankruptcy costs and the multitude of (real or unreal) hypothesized agency effects.

In conclusion, the groundwork laid in this paper offers potential to generate a set of G_L equations that can form a coherence sequence of equations applicable for a variety of real world conditions and adaptable enough to account for competing capital structure theories. Collectively, these equations can synthesize and broaden prior G_L research, while discovering a new perspective from which to understand and measure the theoretical dynamics that can guide the capital structure decision-making process. Future research is now needed to realize the full implications of the theoretical considerations and practical applications inherent in the CSM.

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Appendix A. Gain to Leverage for an Unleveraged Firm Issuing Debt to Retire Equity*

Proof of equation (8) for an unleveraged firm undergoing a debt-for-equity increment with no growth and with tax rates independent of leverage. Using equation (4) for G_L while noting $V_L = E_L + D$ and V_U is the same as E_U:

$G_L = V_L - V_U = E_L + D - E_U$.

Inserting for E_L using equation (6):

$$G_L = \frac{(1-T_E)(1-T_C)(C-I)}{r_l} + D - E_U.$$

Multiplying out the 1st component and rearranging:

$$G_L = D - \frac{(1-T_E)(1-T_C)I}{r_l} - E_U + \frac{(1-T_E)(1-T_C)C}{r_l}.$$

Multiplying the 2nd component by $\frac{(1-T_D)r_d}{(1-T_D)r_d} = 1$ to get $-\left[\frac{(1-T_E)(1-T_C)r_d}{(1-T_D)r_l}\right]\frac{(1-T_D)I}{r_d}$ which is $-\left[\frac{(1-T_E)(1-T_C)r_d}{(1-T_D)r_l}\right]D$, factoring out D, and setting $\alpha = \frac{(1-T_E)(1-T_C)}{(1-T_D)}$:

$$G_L = \left[1-\frac{\alpha r_d}{r_l}\right]D - E_U + \frac{(1-T_E)(1-T_C)C}{r_l}.$$

Multiplying the last component by $\frac{r_u}{r_u} = 1$ to get $\left(\frac{r_u}{r_l}\right)\frac{(1-T_E)(1-T_C)C}{r_u}$ which is $\left(\frac{r_u}{r_l}\right)E_U$, and factoring out E_U:

$$G_L = \left[1-\frac{\alpha r_d}{r_l}\right]D - \left[1-\frac{r_u}{r_l}\right]E_U. \tag{8}$$

Q.E.D.

Note on expressing (8) as a positive component and a negative component.

The 1st component of (8), $\left[1-\frac{\alpha r_d}{r_l}\right]D$, is positive if $D > 0$. This is because the firm would not knowingly issue debt unless $\frac{\alpha r_d}{r_l} < 1$ holds. If $D = 0$, then this component is zero. The 2nd component of (8), $-\left[1-\frac{r_u}{r_l}\right]E_U$, is negative if $D > 0$. This is because $E_U > 0$ and $\frac{r_u}{r_l} < 1$ must both hold when $D > 0$. If $D = 0$, then r_u is the same as r_l and the 2nd component is zero. Thus, if $D = 0$ then (8) implies that $G_L = 0$. But if $D > 0$ then (8) can be expressed as:

$$G_L = POS + NEG \tag{9}$$

where $POS = \left[1-\frac{\alpha r_d}{r_l}\right]D > 0$ and $NEG = -\left[1-\frac{r_u}{r_l}\right]E_U < 0$.

* Although this proof has not been shown, two similar proofs (with no personal taxes and no growth, and with taxes and growth) have been shown by Hull (2005) in a case study publication for a real firm application.

Appendix B. Gain to Leverage for a Leveraged Firm Issuing Equity to Retire All Debt

Proof of equation (11) for a firm becoming unleveraged by undergoing an equity-for-debt increment with no growth and with tax rates independent of leverage. Using equation (10) for $G_L^{\text{Equity-for-Debt}}$ while noting V_U is the same as E_U and $-V_L = -E_L - D$:

$$G_L^{\text{Equity-for-Debt}} = V_U - V_L = E_U - E_L - D.$$

Inserting for E_L using equation (6):

$$G_L^{\text{Equity-for-Debt}} = E_U - \frac{(1-T_E)(1-T_C)(C-I)}{r_l} - D.$$

Multiplying out the 2nd component and rearranging:

$$G_L^{\text{Equity-for-Debt}} = E_U - \frac{(1-T_E)(1-T_C)C}{r_l} - D + \frac{(1-T_E)(1-T_C)I}{r_l}.$$

Multiplying the 2nd component by $\frac{r_u}{r_u} = 1$ to get $\left(\frac{r_u}{r_l}\right)\frac{(1-T_E)(1-T_C)C}{r_u}$ which is $\left(\frac{r_u}{r_l}\right)E_U$, and factoring out E_U:

$$G_L^{\text{Equity-for-Debt}} = \left[1-\frac{r_u}{r_l}\right]E_U - D + \frac{(1-T_E)(1-T_C)I}{r_l}.$$

Multiplying the last component by $\frac{(1-T_D)r_d}{(1-T_D)r_d} = 1$ to get $-\left[\frac{(1-T_E)(1-T_C)r_d}{(1-T_D)r_l}\right]\frac{(1-T_D)I}{r_d}$ which is $-\left[\frac{(1-T_E)(1-T_C)r_d}{(1-T_D)r_l}\right]D$, factoring out D, and setting $\alpha = \frac{(1-T_E)(1-T_C)}{(1-T_D)}$:

$$G_L^{\text{Equity-for-Debt}} = \left[1-\frac{r_u}{r_l}\right]E_U - \left[1-\frac{\alpha r_d}{r_l}\right]D \qquad (11)$$

where $\left[1-\frac{r_u}{r_l}\right]E_U > 0$ and $-\left[1-\frac{\alpha r_d}{r_l}\right]D < 0$.

Q.E.D.