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Structural Models and Default Probability: Application to the Spanish Stock Market

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Abstract

The Basle Committee on Banking Supervision has recently published a new Capital Agreement (Basle II) that replaces the agreement of 1988 currently in force. The theoretical inspiration for this new Agreement, which is expected to come into force at the end of 2006, is the search for convergence between economic capital and regulatory capital. Thus, one of the most encouraging aspects of the new Agreement is that it decidedly favors new and better systems of measurement of credit risk, giving financial entities the freedom and incentive to develop their own models and to employ them in the determination of bank capital.

In this article we analyze the so-called structural models, for which the model of Merton (1974) constitutes the theoretical inspiration; according to this model, the default in the obligations of a company is an endogenous variable related to the capital structure of the company, and such default occurs when the market value of the firm's assets falls below a certain critical level, related to the book value of the debt outstanding. With this approach, the default probabilities, as at December 31, 2003, are estimated for the companies that comprise the IBEX-35 index of the Spanish stock market at that date, by applying the model of Merton (1974).

The study concludes that, although the option pricing theory provides a very interesting alternative framework for estimating the risk of default of a firm, its application to countries with relatively few quoted companies presents serious limitations.

Key words: Credit risk, Structural models, Option pricing theory, Default probability, Financial entities.

JEL Classification: G13, G21, G28.

1. Introduction

In recent years a growth of interest can be observed in new and better methodologies for the measurement of credit risk. This is principally the consequence of the various proposals of the Basle Committee on Banking Supervision for a new Capital Agreement, known as Basle II, which is expected to come into force at the end of 2006[1]. One should add to this the increasing interest of researchers in analyzing the relationship between the differential of corporate bond yields and credit risk, or the spectacular growth being experienced by new types of credit derivatives.

Estimating the probability of default is a key factor in the management of credit risk, and this has made it necessary to seek different alternatives for measuring the rate of insolvency of a possible borrower.

Making such estimations is not, however, a simple task: on the contrary, it presents great obstacles. There are three possibilities that, in principle, are available to us for determining the default probability (Trujillo, 2002): utilizing the historical experience of defaults derived from internal systems of credit rating, based on the financial entity's own or shared client portfolio; associating the system of rating of the bank with the probability of default derived from the historical experience of some of the credit rating agencies; or lastly, employing some statistical or financial model to derive, from knowledge of a data series easily accessible to the analyst, the probability of default individually for each asset, without the need to link it to discrete categories of risk.

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[1] In June 2004 the Basle Committee on Banking Supervision published the document "*International Convergence of Capital Measurement and Capital Standards: a Revised Framework*", commonly termed the Basle Agreement II. The theoretical inspiration of this new Capital Agreement, which would replace that of 1988 currently into force, is the search of the convergence between economic capital and regulatory capital. In this respect, one of the most encouraging aspects of the new Agreement is that it favors better and more sophisticated systems of measurement of credit risk, giving financial entities the freedom and incentive to develop their own models and employ them in the determination of bank capital.

It is the last alternative that we explore in this study, in which we analyze a series of models for determining the probability of default; the theoretical inspiration for these models is that of Merton (1974), according to which the default is an endogenous variable related to the capital structure of the company, and a default occurs when the value of the firm's assets falls below a certain critical level, related to the debt outstanding. Consequently they are known by the name of structural models.

Merton proposes that the position of the shareholders can be considered as similar to purchasing a call option on the assets of the company, and the price at which they will exercise this option to purchase is equal to the book value of company's debt due for payment in the defined time horizon. In this way, Merton was the first to demonstrate that a firm's option of defaulting can be modeled in accordance with the assumptions of Black and Scholes (1973).

Thus, if the company is quoted on an organized market, we can utilize the option pricing theory to derive both the market value and the volatility of the asset, from knowledge of the value of the shares that comprise the equity of the company analyzed, and their volatility. This process can be considered similar to that utilized by investors in determining the implied volatility of an option from the observed option price.

Once the market value of the company and the debt due for payment in a defined time horizon are known, it should be easy to obtain the probability of a company going bankrupt at any given moment of time.

The most important restriction of Merton's model is that it assumes that the liability of a company consists of a single issue of bonds and that its insolvency can occur only when such obligation becomes due. In principle, this would prevent the probability of default being determined for a time horizon shorter than the period until the debt falls due. This hypothesis is relaxed in later studies, such as that of Black and Cox (1976) or the more recent study of Zhou (1997). In both studies the default can be considered before the debt falls due, for example, in the event that the value of the assets falls to certain lower limit. These approaches are known by the name of first-passage models.

Geske (1977) proposes a generalization of Merton's model using the idea that if a share is an option on the assets of the company, then an option on a share is one option on another, that is, a compound derived asset. In this way, several types of debt with different terms of payment can be included.

Leland (1994, 1998), Anderson et al. (1996) and Mella-Barral and Perraudin (1997) extend the models of Merton and Geske to take into account the possibility of renegotiating the debt, and the presence of agency and bankruptcy costs. In a recent article, Forte and Peña (2002) introduce the concept of a refinancing contract that would permit the repayment of the debt by the issue of new obligations.

Finally, an unrealistic hypothesis, common to several of the models, is that the risk-free rates of interest are deterministic. This is a clear limitation because unanticipated changes in interest rates can affect the value of the debt in two ways. First, an increase in interest rates reduces the actual value of the debt repayments and, therefore, their market value. Second, an increase in interest rates will tend to reduce the value of the company due to the empirical evidence of negative correlation between rates and returns from the shares. This effect will also reduce the value of the debt. The generalization of Merton's model to the case of stochastic interest rates that follow the model of Vasicek (1977), is presented in Shimko et al. (1993). In Longstaff and Schwartz (1995) we find this extension for the model of Black and Cox (1976).

Another approach being discussed in the financial literature for determining the probability of default is that whose analytical basis essentially consists of a series of ratios extracted from the financial statements of the customer.

In this approach, the principal analytical techniques utilized by the various authors in the prediction of corporate solvency are multiple regression analysis, discriminant analysis, qualitative regression (probit and logit) models and, most recently, neuronal network models.

Since Altman (1968) proposed the well-known Zeta model, there have been many researchers who have applied some of the previously-listed techniques in an attempt to determine the

default probability for a possible borrower. The majority of authors have opted to utilize the conditional probability models, logit and probit. Both statistical techniques have the objective of giving the probability that a particular observation belongs to a certain group, once the values of the independent variables for that observation are known. They are based on a cumulative probability function and require neither normality in the distribution of the independent variables nor equality of the matrices of variances-covariances.

These models constitute a valuable alternative in situations where it becomes necessary to derive the probabilities of default of companies that are not quoted on organized markets. For example, Moody´s Investors Service (2002) utilizes a logit model (RiskCalc®) to derive the default probabilities for unquoted companies. Similarly, Carey and Hrycay (2001), Westgaard and van der Wijst (2001) and Martín and Trujillo (2004) opt for a logit model to obtain rates of insolvency, through the knowledge of a series of financial ratios.

For Vassalou and Xing (2004) the most serious disadvantages of this type of "accounting" model stem, on the one hand, from the information taken as input being historical in character, in contrast to structural models that extract data from the capital market, which in theory should give them an initial advantage for determining the default probability in the future since these data incorporate the expectations of investors regarding the future development of the company. On the other hand, such models do not take into account the volatility of the company's assets in their estimation of the risk of default, which would mean that two companies with similar financial ratios, but different asset volatilities, would have similar probabilities of default. This variable, however, should play an important role in determining the probability of default and, in this respect, it constitutes a key factor in Merton's model and its later revisions.

Lastly, a more modern approach to determining default probability encompasses the so-called models of reduced form. Under this approach, the probability of default is extracted from the credit risk premium, which is determined by the market prices of the bonds traded in the financial markets. The studies of Litterman and Iben (1991), Jarrow and Turnbull (1995) and Duffie and Singleton (1999), in particular, are notable among several studies of this type.

This approach, however, does encounter a series of problems. In the first place, it is difficult to differentiate, without additional hypotheses, which part of the credit risk premium corresponds to the probability of default and which part to the rate of recovery. Added to this should be the finding of authors such as Elton et al. (2001) and Delianedis and Geske (2001) that the components associated with the risk of default explain only a very small proportion of the premium, and that the greater part of this can be attributed to factors associated with fiscal and systematic risk effects. In any case, the number of companies whose bonds are traded in organized markets is appreciably lower than the number of companies whose shares are quoted in such markets.

The article presented here aims to analyze the goodness of the so-called structural models and their possibilities for application to the Spanish market. To this end, the next part gives a brief exposition of the theoretical basis of the model of Merton (1974). In continuation, the operation of a structural model developed by the company KMV, recently acquired by Moody´s, is described. In the fourth part of the paper, we present the probabilities of default one year ahead, as at December 31, 2003, of the companies that comprise the IBEX-35 index of the Spanish stock market at this date, by the application of the option pricing theory. In the fifth part, a series of conclusions is offered.

2. Merton's model and default probability

Merton (1974) considers the equity of a firm as a European-type call option on its assets, and the price for exercising that option is the accounting value of the outstanding debt due for repayment in the defined time horizon.

If the value of the assets exceeds that of the debt on the date it falls due, whatever that value may be, the shareholders will repay the credit. The total value of the company is sufficient to do this. If the company does not have sufficient money in cash, the owners can sell part of the assets at their market value. Moreover, the shareholders will wish to repay the loan, since not to do so could force the bankruptcy of the company.

In the case that the market value of the company is less than the amount due to be amortized, the shareholders will not repay the debt, but will limit their loss to the amount of capital initially provided.

In the following we shall give a brief exposition of the bases of the model. Let us suppose, for this, a leveraged company that has made only one issue of debt, consisting of zero coupon bonds that fall due at time *T*. This company does not pay dividends. We also assume that the markets are perfect and that there are no frictions, such as taxes or costs of bankruptcy.

In this case, the market value of the firm´s equity, *E*, at time *T* when the debt falls due is:

$$E_T = \max (V_T - D, 0), \tag{1}$$

where V_T is the market value of the company's assets and *D* (the price of exercising the option or strike price) is the book value of the debt due at *T*. It should be noted that (1) represents the payment of a call option of the European type whose underlying security is the value of the company. Therefore, we can use the formulation of Black and Scholes (1973) to obtain the probability that the company may become bankrupt at any given moment of time.

If we assume the customary hypotheses of the Black-Scholes-Merton model (lognormality of the underlying security, volatility and constant rates of interest, continuous contracting, and perfect markets), we can relate the market value of the firm´s equity today, E_0, with the market value of the assets, V_0, and the volatility of the return on these assets, σ_V, using the known expressions of the model:

$$E_0 = V_0 N(d_1) - De^{-rT} N(d_2)$$

$$d_1 = \frac{\ln(V_0 / D) + (r + \sigma_V{}^2 / 2)T}{\sigma_V \sqrt{T}}$$

$$d_2 = \frac{\ln(V_0 / D) + (r - \sigma_V{}^2 / 2)T}{\sigma_V \sqrt{T}} = d_1 - \sigma_V \sqrt{T}, \tag{2}$$

where *N* is the cumulative density function of the standard normal distribution, *r* is the risk-free rate of interest in continuous terms, and the rest of the variables are as already defined.

It can be observed that the model has two unknowns, V_0 and σ_V. To estimate these parameters, we need an additional equation that relates the volatility of the option to that of the underlying security[1]:

$$\sigma_E = \frac{V_0}{E_0} \frac{\partial E}{\partial V} \sigma_V . \tag{3}$$

This equation, together with the preceding ones, shown in (2), enables us to determine V_0 and σ_V by means of a numerical algorithm using the values of E_0 and σ_E; these are variables that are easy to quantify in quoted companies.

In this model, the neutral-risk probability[2] of which the value of the company is greater than the value of the debt on the date *T*, that is $V_T \geq D$, is $N(d_2)$. Therefore, the risk-neutral probability that the company may default on the debt at time *T* determined at any time *t* is:

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[1] We can express the volatility of the firm´s equity, in function of the volatility of the assets, as $\sigma_E = \eta_{E,V} \cdot \sigma_V$, where $\eta_{E,V}$ denotes the elasticity of the firm´s equity against the value of the assets, that is, $\eta_{E,V} = (V / E)(\partial E / \partial V)$. This last partial derivative, ∂E/∂V is, simply, the delta of a call *option*, $\Delta = N(d_1)$, that has the assets of the company as the underlying security.

[2] A very important principle in the valuation of derivative financial assets is that known as the "risk-neutral valuation". This principle does not state that the investors are neutral to the risk, but rather that the derivative financial assets, like options, can be valued under the assumption that such investors are neutral to the risk. This means that the preferences of the investors in respect of the risk do not influence the price of the option when expressed as function of the price of the underlying assets. Risk-neutral valuation is a powerful tool because it represents two particularly simple results: 1) The expected yield of all the financial assets is the risk-free rate of interest, and 2) The risk-free rate of interest is the appropriate discount rate to apply to any expected cash flow.

$$q_t(T) = 1 - N(d_2) = N(-d_2) \cdot \tag{4}$$

This risk-neutral probability of default is that "foreseen" by the market and can be considered as the expected frequency of default conditional on the actual value of the company, on its leverage, volatility, structure of debt and risk-free interest rate. If the company's debt structure is more complex than a simple issue of zero-coupon bonds, one particular issue, whose duration is the weighted mean of the durations of all the outstanding liabilities due, could be utilized as an approximation.

The "natural" default probability can also be calculated, but the expected rate of growth of the company, μ, must be available for this. In this case, the probability that we seek is:

$$p_t(T) = N\left[-\frac{\ln(V_t / D) + \left(\mu - \frac{\sigma_V^2}{2}\right)(T-t)}{\sigma_V \sqrt{T-t}}\right] \tag{5}$$

the result of substituting the risk-free rate by the expected rate of growth of the assets, in equation (4).

3. Option models in practice: the KMV model

The empirical application of these types of model is relatively recent, since they have only begun to achieve a certain popularity in the last few years. The model developed by the KMV Corporation, a company recently acquired by Moody´s, is notable among those that are currently being developed in this field of analysis.

Insolvency is defined, as previously explained, as the situation in which the market value of the company's assets is insufficient for the repayment of its contracted debt.

There are four main variables utilized in the calculation of the *Expected Default Frequency*, *EDF* or default probability:

1. The assets of the company at market values.
2. The volatility of the future values of the assets.
3. The "shape" of the distribution curve of future values of the assets.
4. The amount of the outstanding debt falling due in the defined time horizon, an accounting value deduced from the financial statements of the company.

The expected rate of growth of the assets and the time horizon of the study, which must be defined by the analyst, should be added to these variables. The relationship between the variables is represented in Figure 1.

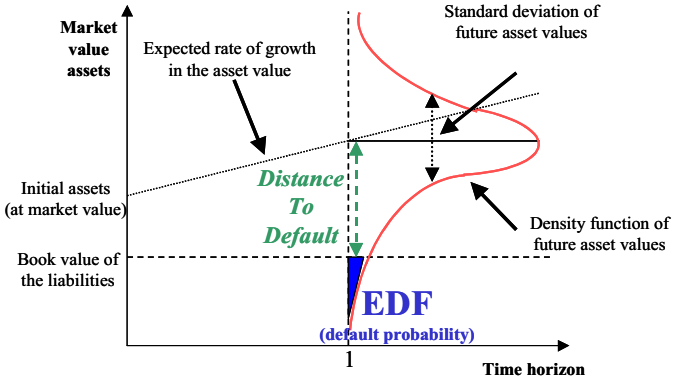

Fig. 1. Calculation of the *Expected Default Frequency*

In this figure, the value of the company, at some future date, is configured by a Normal probability distribution, defined by its corresponding mean and standard deviation. The shaded area of the density function, below the broken line representing the outstanding debt due for repayment, is the probability of default that we are seeking.

We can summarize the mechanics of the KMV model simply, in three stages.

First stage: Estimation of the market value and of the volatility of the company assets.

The market value of the assets is obtained by summing the amount of the various resources, own and external, that comprise the total liabilities of the company, also at market values. Unfortunately, few companies trade their debt, which makes it necessary to adopt an alternative approach.

This problem is resolved in the model by considering the relationship existing, on the one hand, between the market value of the assets and that of the shares that comprise the firm´s equity and, on the other, between their volatilities, the result of regarding the position of the shareholder in an indebted company as equivalent to that of the purchaser of a call option on the firm´s assets with a strike price equal to the book value of the firm´s liabilities, as originally suggested by Merton. Thus, if the company is quoted on an organized market, we can utilize the option pricing theory to derive both the market value and the volatility of the assets, the second variable of the model, from knowledge of the value of the shares that comprise the equity of the company analyzed, and their volatility. This process can be considered similar to that utilized by investors in determining the implied volatility of an option from the observed option price.

The values sought, that of the asset (V_0) and its volatility (σ_V), are obtained by resolving the following series of equations for the company analyzed:

$$E_0 = f(V_0, \sigma_V, L, c, r), \tag{6}$$

$$\sigma_E = g(V_0, \sigma_V, L, c, r), \tag{7}$$

where f is the function that determines the price of an option and g is the function that defines its volatility[1],

E_0 – Market value (actual) of the firm´s equity.

σ_E – Volatility of the equity.

L – Structure of liabilities of the company, defined by its leverage ratio.

c – Coupon paid by the debt in the long term.

r – risk-free rate of interest.

The analyst, therefore, can utilize the preceding expressions to determine both the value of the assets and its volatility, once the value of the shares comprising the own capital of the company studied, its volatility, the structure of liabilities of the company, and the risk-free rate of interest are known.

Second stage: Calculation of the “distance-to-default”.

Once the parameters that will define the probability function are known, the model determines the "default point", i.e. the book value of outstanding debt due for repayment in the time horizon considered[2].

When the time horizon defined by the user is one year, the debt to be taken into account for determining the default point is that which comprises the company's current liabilities[3]. In the

[1] Neither of these functions have been given as known by KMV, although both can be considered to be derived from the Black-Scholes-Merton model. This new model, utilized by KMV, is known by the surnames of its authors, Vasicek-Kealhofer (VK).

[2] As we have previously commented, the most important restriction of the model of Merton (1974) is that it assumes that the liabilities of a company consist of only one bond issue and that the company's insolvency may occur only when that obligation falls due. In principle, this would prevent the probability of default being determined for a time horizon shorter than the period until the debt falls due. To overcome this limitation, in practice it is assumed that there can be a default on the liability at the end of any given time horizon. That is, to estimate the probability of insolvency one year ahead, it is assumed that, without exception, the company may only default on the bond at the end of that year; to assess the rate of insolvency for five years ahead, it is assumed that the debt can only result in a default at the end of the fifth year, etc.

[3] In this case, KMV has observed empirically that the default usually occurs before that the value of the company falls below the current liabilities, perhaps because it is possible that some of the long term debt is covered by guarantees that demand advance repayment in defined situations of crisis. Therefore, the model usually takes as default point an amount situated between the short-term debt and the total liabilities, for example, D = Current liabilities + ½ of the long-term liabilities.

event that this amount may fall due more than one year ahead, a process of accumulation of the outstanding debt due for repayment takes place.

The model developed by KMV currently allows five different classes of liability to be included: short and long term debt, convertible bonds and shares, both ordinary and preferred stock; this compares with the more limited character of the initial models of Merton or Geske.

After having established the level of default and fixed the expected rate of growth of the assets, which will enable us to estimate their future value, V_T, both variables are used to calculate the parameter that KMV designate the "distance-to-default", this being the amount by which the value of a company must be reduced for insolvency to be reached, in the way that we have defined, divided by the standard deviation or volatility of the assets.

In dividing by this factor, we are placing the companies on a comparable scale: by this means, a company with low volatility, for which the difference between the market value of the assets and the outstanding debt due for repayment, in whatever time horizon we have defined, may be less than that of another more volatile company, could have the same distance-to-default as the other, or even a greater distance-to-default.

$$DD = \frac{V_T - D}{\sigma_V \cdot V_T}. \tag{8}$$

In this case, the standard deviation is measured in monetary units, $\sigma_V(\%) \times V_T$. A distance-to-default with a value of 2 tells us that a decrease in the value of the assets of two times its standard deviation would be necessary for that company to be in default. The greater the distance-to-default is, the lower the probability of default will be, and vice versa.

Third stage: Determination of the probability of default (*EDF*).

Once the distance-to-default of a particular company is known, and assuming that the future asset values follow a Normal probability distribution, it is easy to determine the "theoretical" default probability. Thus, continuing with the previous example, a company whose distance-to-default is found to be equal to 2, has a 2.5% probability of being in default or, what is the same, there is a probability of 2.5% that the market value of that company's assets may fall by more than twice its standard deviation (2σ). We should remember that, in a normal density function, a 95% probability exists that the random variable takes a value within the range of $\mu \pm 2, \sigma$ and, therefore, there is a 2.5% probability that this value may be higher or lower.

Therefore, if the shape of the probability function is known, the probability of default or *Expected Default Frequency* would simply be the likelihood that the company's asset value may be below the "default point", as a result of which the company would be insolvent. This probability is shown represented by the shaded zone in Figure 1. However, in practice, the distribution of the value of the assets is difficult to measure, which leads KMV to reject the normal or normal logarithmic density functions.

To determine the probability of insolvency, KMV study the relationship between the distance-to-default and the probability of default over a historical series of bankruptcies; in particular, they analyzed data on close to 250,000 companies, of which nearly 4,700 resulted in insolvency or problems of default. From these data, a table of frequencies was generated to relate probability of default to different levels of distance-to-default, which is easy to obtain when the market value of the company, its volatility and the point of insolvency are known. We thus obtain an "empirical" probability, the value of which may differ significantly from the previously calculated theoretical value.

In this way, if we want to calculate the probability of default one year ahead for a company whose distance-to-default (*DD*) is seven times the variability of its asset value, we refer to the historical series of companies in the same situation that became insolvent in the following year; then we divide the number of those insolvent companies by the total population of companies with the same DD and this gives us a probability of 0.05%, equivalent to the AA grading of Standard & Poor´s.

4. Empirical application to the companies comprising the IBEX-35

In the fourth part of this paper, we shall estimate the default probabilities for a one year time period, as at December 31, 2003, of the companies that comprised the IBEX-35 index of the Spanish stock market at that date, by applying the option pricing theory. The procedure that we shall follow will be similar to that employed by the company KMV, whose theoretical basis is the model of Merton (1974).

The total number of companies analyzed is 29, since we have eliminated from the study the companies of the financial services sector, because their peculiarities make it very difficult to apply the approach that we have been dealing with. In these cases, it is not clear that the situation of default corresponds to the point in time when the asset value is below that of the liabilities (Crosbie and Bohn, 2003).

The fact that these 29 companies are quoted facilitates access to the data that are needed, the data indicated in the previous expression (5) that we shall reproduce again here:

$$p_t(T) = N\left[-\frac{\ln(V_t / D) + \left(\mu - \frac{\sigma_V^2}{2}\right)(T-t)}{\sigma_V\sqrt{T-t}}\right].$$

In our case $t = 0 = 31/12/03$ and $T = 1 = 31/12/04$[1].

It should be observed that the equation that we utilize incorporates the rate of growth of the company, μ, in place of the risk-free rate of interest, r. With this, we obtain a "natural" probability of default that is different from the risk-neutral probability indicated in (4).

The determination of the rate of growth of the assets is not a simple task. Du and Suo (2003) utilize the mean variation of the asset values during the twelve months prior to the time when the default probability is to be estimated. In our case, we have chosen to use as a proxy the rate of growth of the Spanish GDP; for the year 2004, this is estimated at around 3%. In any case, this variable appears to have little discriminant power in the default of a firm (Crosbie and Bohn, 2003).

Since our objective is to determine the probability of default one year ahead, we will assume that the amount of debt falling due within a year or default point is equal to the book value of the current liabilities plus half of the long term debt[2]. These data have been extracted from the financial statements that the various companies quoted on the Spanish continuous market periodically have to provide. The information we have used refers to 31 December 2003.

Finally, with respect to the calculation of the asset values, V_0, and their volatilities, σ_V, these have been estimated by the expressions (2) and (3), by means of a numerical algorithm, using the market value of the equity, E_0, during the month of December 2003[3] and their daily volatilities annualized during this year, σ_E.

Once the default probability for the company has been estimated by means of equation (5), it is easy to quantify the company's distance-to-default (DD) by the expression (Vassalou and Xing, 2004):

$$DD_t = \frac{\ln(V_t / D) + \left(\mu - \frac{\sigma_V^2}{2}\right)(T-t)}{\sigma_V\sqrt{T-t}}. \quad (9)$$

[1] Although the model of Merton assumes that default can only occur when the debt falls due, in practice this limitation is usually overcome by assuming that there can be a default on the liability at the end of any given time horizon. In our case, this is one year ahead.

[2] Although we are determining the probability of default one year ahead, the inclusion of part of the long-term debt is customary in the studies on the subject. KMV argues that empirically it is observed that default usually occurs before that the market asset value of the company falls below the book value of the current liabilities, as we have already stated in note 6. Du and Sou (2003) and Vassalou and Xing (2004) comment on this same point. All of these authors add 50% of the long term to the short term debt.

[3] Although we wish to determine the default probability for the company at 31/12/2003, it seemed to us more appropriate to take the mean value of its shares during the month of December, instead of taking the value on the last dealing day of the year, with the object of trying to avoid possible market anomalies.

The preceding equation tells us how by many standard deviations the logarithm of the ratio of the asset value/debt must be decreased with respect to its mean for a default situation to occur[1]. The greater the distance-to-default of a company is, the less its probability of default is and vice versa, both variables being related by equation (10):

$$p_t(T) = N(-DD) = N\left[-\frac{\ln(V_t / D) + \left(\mu - \frac{\sigma_V^2}{2} \right)(T-t)}{\sigma_V \sqrt{T-t}} \right]. \quad (10)$$

Table 1 gives the default probability one year ahead and its corresponding distance-to-default, at 31/12/2003, of each of the companies that comprised the IBEX-35 at that date, excepting the companies of the financial sector.

We should make the following observations on the results obtained:

- The credit rating of the companies comprising the IBEX-35 is very high, and in all the cases analyzed, the probability of default one year ahead is close to zero. This finding makes sense, considering that most of the companies comprising this Index are of proven solvency and are kept within the investment grade in the classifications awarded by the principal international credit rating agencies.
- Among these companies, the lowest default probabilities correspond to ABERTIS, with a distance-to-default of 11.46, IBERDROLA, with a distance of 11.33 and TELEFÓNICA MOVILES, with 11.12. At the other extreme, the lowest distance-to-default values are found in the services sector of the market, where IBERIA and METROVACESA present distance values of 3.93 and 4.27, respectively; and in the sector of communications and information services, where SOGECABLE is notable with a distance-to-default of 3.35 standard deviations.
- We have assumed that the future asset values follow a distribution function of the Normal type. However, in practice, this probability function is difficult to test. A possible solution would consist of analyzing the relationship between the distance-to-default and the default probability in a historical series of insolvencies, generating a table of frequencies to relate the two variables, as done by KMV. Such an analysis would thus produce an "empirical" probability, the value of which may differ significantly from the previously calculated theoretical value, although the two would be expected to be strongly correlated.

5. Conclusions

The option pricing theory provides us with a very interesting alternative framework for estimating the risk of default of a firm. This estimation is an objective assessment, based on market data, which also allows it to be frequently updated. In this respect, the model presented constitutes a valuable alternative and should not be ignored.

However, in spite of it being evidently robust in theory, the approach is found to have a serious obstacle in countries like Spain, where the number of companies whose shares are submitted to stock market quotation, which is the principal input of the model, continues to be relatively low.

One should add to this that the probability distribution which the theoretical model makes use of is difficult to maintain in practice, which has led companies such as KMV to underrate it. In this case, it becomes necessary to map the distance-to-default and the real probability of bankruptcy, obtained from historical experience. However, this last alternative is not very viable in such restricted markets as the Spanish. Furthermore, the application of the results gathered in the North American market by the company KMV to our case would mean assuming that the characteristics of the companies of the two countries are similar or, put another way, that they present similar levels of risk, which brings in other disadvantages.

[1] Expression (9) is derived from the Black-Scholes model and is different from that included in (8), utilized by KMV.

Table 1

Probability of default for a one year time period of the companies of the IBEX-35 (at 31/12/03), determined by applying Merton's model

Company	Market value of equity (E_0)*	Volatility of equity (σ_E)	Default point (book liabilities) (D)*	Market asset value (V_0)*	Volatility of asset value (σ_V)	Default Probability (p_t)	Distance-to-default (DD)
ABERTIS	6.204.307,14	17,55%	1.580.832,00	7.751.204,47	14,05%	0,0000E+00	11,46127194
ACCIONA	3.004.961,75	17,66%	1.273.646,50	4.251.268,08	12,48%	0,0000E+00	9,833889064
ACERINOX	2.492.284,67	26,50%	412.174,50	2.895.611,66	22,81%	0,0000E+00	8,564628614
ACS	4.424.047,22	20,73%	2.051.363,00	6.431.374,93	14,26%	2,2204E-16	8,152371831
ALTADIS	6.477.664,67	20,17%	2.178.138,50	8.609.047,04	15,21%	0,0000E+00	9,159580659
AMADEUS	14.359.880,12	36,39%	502.788,50	14.851.875,52	35,18%	0,0000E+00	9,532074892
ARCELOR	7.161.403,09	39,52%	864.615,50	8.007.458,45	35,34%	2,7331E-10	6,205746332
ENAGAS	1.990.378,41	20,17%	1.052.412,50	3.020.199,15	13,29%	3,3307E-16	8,090313063
ENDESA	15.304.848,36	26,96%	8.634.228,00	23.753.731,51	17,37%	1,6978E-09	5,91178746
FCC	3.802.981,49	19,88%	1.026.583,50	4.807.527,60	15,73%	0,0000E+00	10,05709853
FERROVIAL	3.743.984,55	22,45%	1.841.974,50	5.546.419,22	15,15%	2,5979E-14	7,528074008
GAMESA	691.467,61	29,74%	79.108,50	768.878,34	26,75%	0,0000E+00	8,481136274
GAS NATURAL	7.968.920,21	22,68%	1.630.434,50	9.564.355,19	18,90%	0,0000E+00	9,532557128
IBERDROLA	13.467.138,62	15,30%	6.121.051,50	19.456.794,47	10,59%	0,0000E+00	11,3395317
IBERIA	2.089.231,98	37,44%	1.703.758,00	3.756.416,71	20,82%	4,1992E-05	3,932837185
INDITEX	10.541.549,18	38,72%	723.239,50	11.249.263,26	36,28%	2,7645E-14	7,519808846
INDRA	1.556.130,05	28,21%	717.365,50	2.258.096,35	19,44%	6,8965E-10	6,058475227
METROVACESA	1.525.643,66	33,01%	1.595.516,50	3.086.789,25	16,31%	9,7006E-06	4,271826685
NH HOTELES	1.109.067,02	29,34%	430.744,50	1.530.565,03	21,26%	5,5787E-10	6,092516253
PRISA	2.470.520,33	34,06%	397.990,00	2.859.966,65	29,42%	8,7823E-12	6,725770021
R.E.E.	1.683.510,30	15,90%	1.300.990,50	2.956.567,27	9,05%	0,0000E+00	9,574156767
REPSOL YPF	18.190.180,44	21,68%	4.866.445,00	22.952.160,68	17,18%	0,0000E+00	9,232284205
SACYR VALLE.	2.921.600,30	23,06%	735.275,50	3.641.091,89	18,50%	0,0000E+00	8,823711516
SOGECABLE	3.312.155,14	52,41%	1.190.531,00	4.477.082,99	38,77%	4,0238E-04	3,351193657
TELEFONICA	56.263.129,21	27,04%	21.084.337,00	76.894.864,37	19,78%	1,0941E-11	6,693683047
TELF.MOVILES	35.573.070,60	24,00%	3.454.965,00	38.953.870,86	22,00%	0,0000E+00	11,12893555
TPI	1.641.096,19	26,74%	242.875,50	1.878.757,87	23,36%	0,0000E+00	8,856029045
UNION FENOSA	4.351.662,45	25,11%	4.718.961,50	8.968.848,28	12,18%	9,4972E-09	5,621387533
ZELTIA	1.076.788,84	51,05%	36.481,00	1.163.815,88	47,23%	3,0143E-13	7,200790522

* Data in thousands of euros.

Despite this, the model analyzed is one of those that are being discussed by financial entities for the establishment of internal credit rating systems, which would allow them, in the last instance, to adopt an advanced approach to risks, in accordance with the new Capital Agreement recently approved by the Basle Committee on Banking Supervision. As this Agreement is configured, it would represent a substantial improvement in the capital requirement figures and, therefore, in the returns obtained, compared with systems based on ratings awarded by credit rating agencies.

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