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The Cyclical Structure of the British Stock Market Returns. An Approach Based on Long Memory Cycles

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Abstract

This paper deals with the presence of stochastic cycles in the British stock market returns. Stock market prices are usually characterised by the presence of a large value in the periodogram at the smallest frequency, which may suggest the need of first differences. Once this component is removed, we show in the paper that a cyclical component may still be present in the returns, and we model this property throughout new statistical techniques based on long memory cycles.

Key words: Financial time series; Long memory; Stochastic cycles.
JEL classification: C22.

1. Introduction

The adequacy of a mathematical model when describing the nonstationarity in macroeconomic and financial time series is a question that still remains open. Deterministic approaches were early discouraged in view of the fact that the series changed or evolved over time, and stochastic approaches (based on first differencing) became popular, especially after the seminal paper of Nelson and Plosser (1982). These authors used tests of Fuller (1976) and Dickey and Fuller (1979) and examined fourteen US macroeconomic variables. They were unable to reject the existence of unit roots in practically any of the series examined. Following that work, a battery of test statistics was developed for testing unit roots (Phillips, 1987; Phillips and Perron, 1988; Kwiatkowski et al., 1992; etc.), and they have been widely employed in the empirical work on finance.

Although there is still no consensus, most authors agree that stock market prices contain unit roots and the literature has been oriented towards the short-run dynamics of the returns. Thus, the key question is to examine the possible autocorrelated structure of the differenced series, either throughout the classical ARMA representations or using other more complex forms based on stochastic volatility models (see, e.g., Bollerslev, 1986; Taylor, 1986; and all subsequent work).

In this paper we examine the stochastic behaviour of the British stock market prices from a different time series perspective. First, we examine the order of integration of the series. However, instead of using classical procedures (e.g., Dickey and Fuller, 1979; Phillips and Perron, 1988; etc.), which are all based on autoregressive (AR) models, we nest the unit root in a fractional structure. The results show that the British stock market prices are difference-stationary. Then, the differenced series is examined in order to investigate a possible cyclical pattern in its behaviour. For the case of the long run or zero frequency, we use both parametric and semiparametric methods, whereas for the cyclical component we employ a version of a parametric testing procedure of Robinson (1994a). The outline of the paper is as follows: Section 2 describes the methods employed in the paper. In Section 3, we describe the historical dataset of the British stock market prices. Section 4 deals with the empirical work, while Section 5 concludes.

2. The statistical methods

For the purpose of the present paper, we define an integrated of order 0 (I(0)) process $\{u_t, t = 0, \pm 1, \ldots\}$ as a covariance stationary process, with spectral density function that is positive and finite at any frequency on the spectrum. In this context, we say that $\{x_t, t = 0, \pm 1, \ldots\}$ is integrated of order d (and denoted by $I(d)$) if:

$$(1 - L)^d x_t = u_t, \qquad t = 1, 2, \ldots, \tag{1}$$

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with $x_t = 0$ for $t \leq 0$, where d can be any real number, and where the unit root corresponds to $d = 1$. If $d \in [0, 0.5)$, x_t is covariance stationary and mean-reverting. If $d \in [0.5, 1)$, the process is no longer stationary but it is still mean reverting, with the effect of the shocks dying away in the long run, while $d \geq 1$ means nonstationarity and non-mean-reverting. Note that this specification is radically different from the traditional way of testing unit roots, usually based on models of form:

$$(1 - \rho L) x_t = u_t, \quad t = 1, 2, \ldots, \tag{2}$$

where the unit root is $\rho = 1$. Here, the series is stationary if $|\rho| < 1$. It is nonstationary but non-explosive for $\rho = 1$ and becomes explosive for $|\rho| > 1$. Thus, the different long-run behaviour around the unit root makes this specification untractable in terms of standard distributions, unlike what happens in case of the fractional model (1).

The use of fractional structures in stock return data has been empirically studied in numerous papers. A few examples are Cheung and Lai (1995), Jacobsen (1996) and Hiemstra and Jones (1997). Nowadays, there exist many well-known estimators for the fractional differencing parameter d (e.g., Fox and Taqqu, 1974; Dahlhaus, 1989; Geweke and Porter-Hudak, 1990; Sowell, 1992; etc.). Here, we present first a semiparametric method due to Robinson (1995a). It is a "Whittle" estimate in the frequency domain, based on a band of frequencies that degenerates to zero. The estimate is implicitly defined by:

$$\hat{d} = \arg\min_d \left(\log \overline{C(d)} - 2d \frac{1}{m} \sum_{j=1}^{m} \log \lambda_j \right), \tag{3}$$

$$\overline{C(d)} = \frac{1}{m} \sum_{j=1}^{m} I(\lambda_j) \lambda_j^{2d}, \qquad \lambda_j = \frac{2\pi j}{T}, \qquad \frac{m}{T} \to 0,$$

where m is a bandwidth parameter number, $I(\lambda_j)$ is the periodogram of the raw time series, and $d \in (-0.5, 0.5)$[1]. Under finiteness of the fourth moment and other mild conditions, Robinson (1995a) proved that:

$$\sqrt{m} (\hat{d} - d_o) \to_d N(0, 1/4) \quad \text{as } T \to \infty,$$

where d_o is the true value of d and with the only additional requirement that $m \to \infty$ slower than T[2]. Robinson (1995a) showed that m must be smaller than $T/2$ to avoid aliasing effects. A multivariate extension of this estimation procedure can be found in Lobato (1999). There also exist other semiparametric procedures for estimating the fractional differencing parameter, for example, the log-periodogram regression estimate (LPE), initially proposed by Geweke and Porter-Hudak (1983) and modified later by Künsch (1986) and Robinson (1995b) and the averaged periodogram estimate (APE) of Robinson (1994b). We have decided to use in this article the estimate of Robinson (1995a), firstly because of its computational simplicity. Note that using the "Whittle" estimate, we do not need to employ any additional user-chosen numbers in the estimation (as in the case with the LPE and the APE). Also, we do not have to assume Gaussianity in order to obtain an asymptotic normal distribution, Robinson's (1995a) estimate being more efficient than the LPE.

As mentioned in Section 1, in the context of financial time series, it is also important to examine the short-run dynamics underlying the series. Thus, we also perform a fully parametric procedure, due to Robinson (1994a), which is still based on model (1). It is a Lagrange Multiplier (LM) test of the null hypothesis:

$$H_o: d = d_o \tag{4}$$

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[1] Velasco (1999a, b) has recently showed that the fractionally differencing parameter can also be consistently semiparametrically estimated in nonstationary contexts by means of tapering.

[2] The exact requirement is that (1/m) + ((m1+2α(log m)2)/(T2α)) → 0 as T → ∞, where α is determined by the smoothness of the spectral density of the short run component. In the case of a stationary and invertible ARMA, α may be set equal to 2 and the condition is (1/m) + ((m5(log m)2)/(T4)) → 0 as T → ∞.

in model (1) for any real value d_o. The test statistics is given by:

$$\hat{R}_1 = \hat{r}_1^{\,2}, \quad \hat{r}_1 = \frac{T^{1/2}}{\hat{\sigma}^2} \hat{A}^{-1/2} \hat{a}, \tag{5}$$

where T is the sample size and

$$\hat{a} = \frac{-2\pi}{T} \sum_{j=1}^{T-1} \psi(\lambda_j) g(\lambda_j; \hat{\tau})^{-1} I(\lambda_j); \qquad \hat{\sigma}^2 = \sigma^2(\hat{\tau}) = \frac{2\pi}{T} \sum_{j=1}^{T-1} g(\lambda_j; \hat{\tau})^{-1} I(\lambda_j);$$

$$\hat{A} = \frac{2}{T} \left(\sum_{j=1}^{T-1} \psi(\lambda_j)^2 - \sum_{j=1}^{T-1} \psi(\lambda_j) \hat{\varepsilon}(\lambda_j)' \times \left(\sum_{j=1}^{T-1} \hat{\varepsilon}(\lambda_j) \hat{\varepsilon}(\lambda_j)' \right)^{-1} \times \sum_{j=1}^{T-1} \hat{\varepsilon}(\lambda_j) \psi(\lambda_j) \right)$$

$$\psi(\lambda_j) = \log \left| 2 \sin \frac{\lambda_j}{2} \right|; \qquad \hat{\varepsilon}(\lambda_j) = \frac{\partial}{\partial \tau} \log g(\lambda_j; \hat{\tau}); \qquad \lambda_j = \frac{2\pi j}{T}; \qquad \hat{\tau} = \arg\min \sigma^2(\tau).$$

$I(\lambda_j)$ is the periodogram of u_t evaluated under the null, and the function g is a known function related to the spectral density function of u_t,

$$f(\lambda; \sigma^2; \tau) = \frac{\sigma^2}{2\pi} g(\lambda; \tau), \qquad -\pi < \lambda \le \pi.$$

Note that this test is purely parametric and therefore, it requires specific modelling assumptions regarding the short memory specification of u_t. Thus, for example, if u_t is white noise, $g \equiv 1$, (and thus, $\hat{\varepsilon}(\lambda_j) = 0$), and if u_t is an AR process of form $\phi(L) u_t = \varepsilon_t$, then, $g = |\phi(e^{i\lambda})|^{-2}$, with $\sigma^2 = V(\varepsilon_t)$, so that the AR coefficients are a function of τ.

Based on the null hypothesis H_o (4), Robinson (1994a) established that under certain regularity conditions[1]:

$$\hat{R}_1 \to_d \chi_1^2 \quad as \quad T \to \infty, \tag{6}$$

and also the Pitman efficiency of the test against local departures from the null[2]. Thus, we are in a classical large sample-testing situation. Because $\hat{R}_1$ involves a ratio of quadratic forms, its exact null distribution can be calculated under Gaussianity via Imhof's algorithm. However, a simple test is approximately valid under much wider distributional assumptions. An approximate one-sided 100α% level test of H_o (4) against the alternative: H_a: $d > d_o$ ($d < d_o$) will be given by the rule: "Reject H_o if $\hat{r}_1 > z_\alpha$ ($\hat{r}_1 < - z_\alpha$)", where the probability that a standard normal variate exceeds z_α is α[3].

So far, we have presented methods that concentrate exclusively at the long run or zero frequency. However, we are also interested in the cyclical structure of the series. The existence of cycles is a well-known stylised fact, mainly in macroeconomics, but also in finance. With the development of the National Bureau of Economic Research (NBER)'s project of "Measurement without Theory", and the first extensive study of Burns and Mitchell (1946) on the American Economy, business cycles and their features have also constituted a direct object of empirical analysis. Numerous studies have tried to describe them and to consider their stability over time. Examples are Romer (1986, 1994), Diebold and Rudebusch (1992), Beaudry and Koop (1993), Watson (1994), etc. Hess and Iwata (1997) showed that complex linear or non-linear models (like Perron, 1989; SETAR, Markov Switching or Beaudry and Koop, 1993) do not better replicate business cycle features than a simple linear ARIMA model, though in another recent article, Candelon and Gil-Alana (2003) show that fractional models can do it even better. In the context of

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[1] These conditions are very mild, and concern technical assumptions, which are satisfied by ψ(λ).

[2] That is, if we direct the tests against local alternatives of form: Ha: d = do + δT-1/2, the limit distribution is $\chi_1^2(\nu)$, with a non-centrality parameter ν, that is optimal, under Gaussianity, compared with other rival regular statistics.

[3] This version of Robinson's (1994a) tests, with the singularity in the spectrum occurring exclusively at the long run or zero frequency, has been used in macroeconomic time series in Gil-Alana and Robinson (1997) and in financial contexts by Gil-Alana (2003).

financial data, the cycles have not received much attention. Andersen and Bollerslev (1997) found some evidence of strong intraday periodicity in return volatility in equity markets. They modelled periodicity by means of deterministic weights, though Robinson (2001) recommends the use of stochastic cyclical structures to explain the autocorrelations in financial series.

Another issue is to estimate the length of the cycles. A vast majority of researchers use the Hodrick-Prescott's (1997) filter or Baxter and King's (1999) band-pass filter, and most authors conclude that business cycles have a duration of about six years. Englund et al. (1992) and Hassler et al. (1994) use another filter in the frequency domain to extract cycles with duration between 3 and 8 years. Similar conclusions are obtained in Canova (1998), Burnside (1998), King and Rebelo (1999) and others.

In Section 4 we will use another version of Robinson's (1994a) tests, which is based on the model,

$$(1 - 2\cos w_r L + L^2)^d x_t = u_t \qquad t = 1, 2, \ldots. , \tag{7}$$

where $w_r = 2\pi r/T$, $r = T/j$, j indicating the number of periods per cycle. Here, if $d > 0$, the process is said to be long memory with respect to the cyclical part. These processes were examined by Gray et al. (1989, 1994), and they show that the series is stationary if $|\cos w_r| < 1$ and $d < 0.50$ or if $|\cos w_r| = 1$ and $d < 0.25$. They also showed that the second polynomial in (7) can be expressed in terms of the Gegenbauer polynomial C_{j,d_2}, such that, calling $\mu = \cos w_r$, for all $d \neq 0$,

$$(1 - 2\mu L + L^2)^{-d} = \sum_{j=0}^{\infty} C_{j,d}(\mu) L^j, \tag{8}$$

$$C_{j,d}(\mu) = \sum_{k=0}^{[j/2]} \frac{(-1)^k (d)_{j-k} (2\mu)^{j-2k}}{k!(j-2k)!}; \qquad (d)_j = \frac{\Gamma(d+j)}{\Gamma(d)},$$

where $\Gamma(x)$ represents the Gamma function and a truncation will be required in (8) to make the polynomial operational. Thus, the process in (7) becomes

$$x_t = \sum_{j=0}^{t-1} C_{j,d2}(\mu) u_{t-j}, \quad t = 1, 2, \ldots$$

and when $d = 1$, we have

$$x_t = 2\mu x_{t-1} - x_{t-2} + u_t, \qquad t = 1, 2, \ldots ,$$

which is a cyclical $I(1)$ process with the periodicity determined by μ. Its performance in the context of macroeconomic time series was examined, for example, by Bierens (2001)[1]. The nice property of a Gegenbauer polynomial is that it spectral density has a peak at frequency $\lambda \in [0, \pi]$. As we have rule out in this setting the long run frequency (as well as the seasonal frequencies), this peak typically deals with business cycle frequencies. Some papers (Gray et al., 1989; Ferrara and Guegan, 2001) have already used the Gegenbauer polynomial as a convenient way to analyse fluctuation at a particular frequency. Ferrara and Guegan (2001) consider a more general k-factor Gegenbauer polynomial to model all the peaks in the spectrum density (it exists indeed k peaks in $[0, \pi]$ for a k-factor Gegenbauer polynomial). Nevertheless, it is regrettable that no asymptotic distribution of the Whittle-form is available for a k-Gegenbauer polynomial. We prefer then to induce from the theory the presence of a simple one-factor Gegenbauer polynomial to model the cyclical behavior. The persistence of such a movement however has to be tested.

Similarly to the other versions of his tests, Robinson (1994a) proposes a test of H_o (4) in model (7) for real values d_o. The test statistics is:

[1] Unit roots cycles were also examined by Ahtola and Tiao (1987), Chan and Wei (1988) and Gregoir (1999a, b).

$$\hat{R}_2 = \hat{r}_2^{\,2}; \qquad \hat{r}_2 = \frac{T^{1/2}}{\hat{\sigma}_2^2}\hat{A}_2^{-1/2}\,\hat{a}_2, \tag{9}$$

where $\hat{A}_2$, $\hat{a}_2$ and $\hat{\sigma}_2^2$ are given as in (5) but for the new $\hat{u}_t$ just defined as $(1-2\cos w_r L + L^2)^{d_o} x_t$, and

$$\psi(\lambda_j) = \log\left|2\left(\cos\lambda_j - \cos w_r\right)\right|,$$

and the summation is now over $\lambda \in M$, where $M = \{\lambda: -\pi < \lambda < \pi, \lambda \notin (\rho_k - \lambda_1, \rho_k + \lambda_1)$, such that ρ_k is the pole in the spectrum.

Similarly to $\hat{R}_1$,

$$\hat{R}_2 \rightarrow_d \chi_1^2, \qquad as \quad T \rightarrow \infty. \tag{10}$$

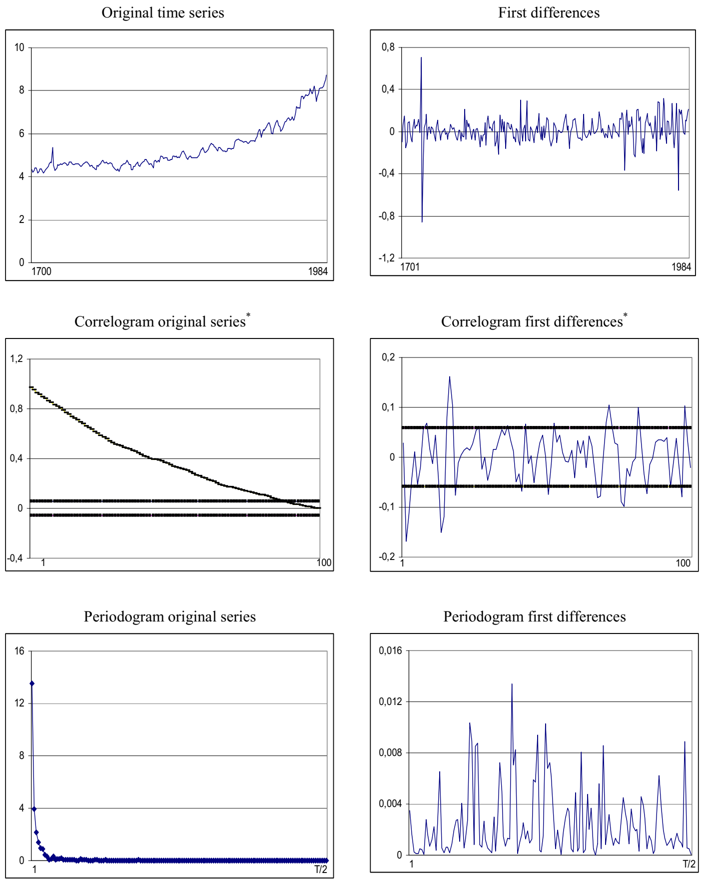
This version of the tests of Robinson (1994a) was used in an empirical application in Gil-Alana (2001) and its statistical properties in finite samples were also compared in that paper with other unit root cyclical tests (Ahtola and Tiao, 1987), showing that Robinson's (1994a) tests outperform Ahtola and Tiao (1987) in a number of cases. In Section 4 all the above procedures will be applied to the annual data of the British stock market prices.

3. The British stock market

The time series data analysed in this section correspond to the annual series of the British stock market prices, for the period of 1700-1984, and it is the longest available time-span record in finance across the world. The resources and explanation of the data can be found in http:/fisher.su.edu/resources_data/data/britann.txt.

The data for 1700-1790 are from P. Mirowski. For most of this period there were not many firms in the index (4-15). Most of the firms were quasi-public, i.e. Bank of England, East India Co. These data are not averaged over the year. But which month they are chosen from was not specified. For the period of 1790-1933 the data are from W. Hoffman. The series are based on several historical studies and include Hayek's stock price index. It is not clear if these data are averaged for the whole year. The data for 1934-1949 are from the L.C.E.S. index for 92 industrial companies excluding finance and property companies. The data are an arithmetic average over the year of mid-month prices. The data for 1950-1962 are from Moodies index of 60 representative equities excluding mines and plantations. This series is a geometric mean of weekly quotes. For 1963-1984 The Financial Times actuaries index was used. This is a value-weighted index of 500 shares. The index is an arithmetic average of the share prices from each day. After 1967 the data are from C.S.O. Economic Trends. For 1934-1966 they are from L.C.E.S., The British Economy, Key Statistics, 1900-1966.

As we can see in the above paragraph, the dataset comprises several sources, and the number of firms in the index range from as little as 4 to 500 firms. Also, in certain portion of the data, there is no averaging; some other parts use arithmetic averaging, while other parts are based on geometric averaging. However, despite all these inhomogeneities, it is worth its study in the sense that it is the longer available time-span record in finance across the world. On the other hand, long run dependence requires a sufficiently large ammount of observations in order to make inference about the degree of integration of the series. In that respect, the present dataset can be considered as a valuable information set about stock market prices and it should be taken into account for the analysis of long run dependence in financial time series.



* The large sample standard error under the null hypothesis of no autocorrelation is $1/\sqrt{T}$ or roughly 0.038.

Fig.1. Historical data of the British stock market prices and its first differences, with their corresponding correlograms and periodograms

Figure 1 displays plots of the original series, (log of the stock market prices) and its first differences (stock market returns), along with their corresponding correlograms and periodograms. We see that the series is nonstationary and this is substantained by the correlogram, with values decaying very slowly, and the periodogram, with a large value around the smallest frequency. The returns have a stationary appearance, though we still observe in the correlogram significant values even at some lags relatively far away from zero, along with some kind of cyclical oscillation. Also, in the periodogram, we observe some peaks at some frequencies different from zero. Thus, it may be of interest to deeper examine its structure in terms of a cyclical model.

4. The empirical application

First, we concentrate on the original series and examine the behaviour of the time series at the long run or zero frequency. Figure 2 displays the estimates of d based on the Whittle estimator of Robinson (1995a), for the whole range of values of the bandwidth parameter m[1]. Since the series is clearly nonstationary, we compute $\hat{d}$ in (3) based on the first differenced data, adding then 1 to the estimated values to obtain the proper order of integration. We also include in the figure the

[1] Some attempts to calculate the optimal bandwidth numbers have been examined in Delgado and Robinson (1996) and Robinson and Henry (1996). However, in the case of the Whittle estimator (Robinson, 1995a), the use of optimal values has not been theoretically justified. Other authors, such as Lobato and Savin (1998) use an interval of values for m but we have preferred to report the results for the whole range of values of m.

95%- confidence interval corresponding to the unit root hypothesis. We observe that practically all values of $\hat{d}$ are within the unit root interval, implying that the returns are $I(0)$ stationary.

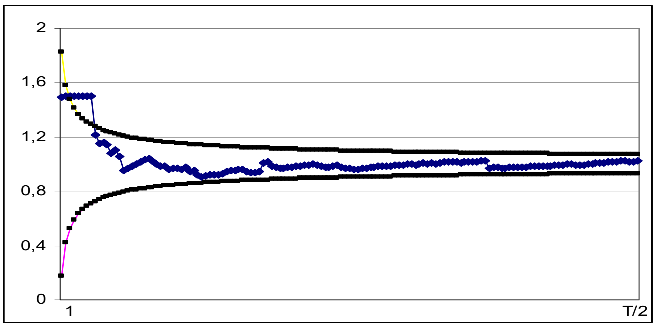


Fig. 2. Estimates of d based on the QMLE (Robinson, 1995a) for the whole range of values of *m*

Table 1

95%-confidence intervals of those values of d_o where H_o: $d = d_o$ cannot be rejected in model $(1 – L)^d x_t = u_t$	
ut - type	Confidence intervals
White noise	[0.94 - 1.10]
AR(1)	[0.83 - 1.16]
AR(2)	[0.88 - 0.99]
Bloomfield (1)	[0.83 - 1.15]
Bloomfield (2)	[0.82 - 1.17]

In order to confirm the existence of a unit root, we also perform the parametric procedure described in Section 2, testing H_o (4) in model (1) for values $d_o = 0, (0.01), 2$, using $\hat{R}_1$ in (5), with white noise and autocorrelated disturbances. However, instead of reporting the values for all statistics, we compute in Table 2 the confidence intervals corresponding to the non-rejection values of d_o at the 95% significance level[1]. We see that, similarly to the semi-parametric procedure, the unit root is included in all intervals. Thus, if u_t is white noise, the values range between 0.94 and 1.10, and, if we permit autocorrelated disturbances (either with AR or using the Bloomfield's (1973) exponential spectral model)[2], the intervals are slightly wider but still including the unit root. Thus, we can conclude by saying that the annual data of the British stock market prices contains a unit root. Additionally, other unit root tests based on AR alternatives (Dickey and Fuller, 1979; and Phillips and Perron, 1988) were also performed on the series, and we found in all cases strong evidence in favour of unit roots.

Once we have determined the existence of a unit root in the stock market prices, we concentrate on the returns. The classical approach here consists of testing for autocorrelation either in the original series or in some transformations of the returns. In this paper, however, we take a different approach and examine its cyclical structure throughout the model given by (7).

We computed the statistic $\hat{r}_2 = \sqrt{\hat{R}_2}$ given by (9) for values $d = 0, (0.25), 2$, and $r = 1, (1)$, T/2 (= 192)[3], assuming that u_t is white noise (Table 2) and AR(1) and AR(2) are processes (Tables 3 and 4 respectively). The first thing we observe is that H_o (4) is rejected for all values of d_o when r is

[1] The confidence intervals were built up according to the following strategy. First, we choose a value of d from a grid. Then, we form the test statistics testing the null for this value. If the null is rejected at the 5% level, we discard this value of d. Otherwise, we keep it. An interval is then obtained after considering all the values of d in the grid.

[2] The Bloomfield's (1973) model is a nonparametric approach of modelling the $I(0)$ disturbances, which produces autocorrelation in a similar way to an AR process, but that accommodates fairly well to the present version of the tests.

[3] Note that in case of $r = 1$, the model reduces to an $I(2)$ process, with the pole (or singularity) occurring exclusively at the long run or zero frequency.

smaller than 3 or higher than 10, which is consistent with the literature mentioned in Section 2 about business cycles length, that says that cycles have a duration constrained between 3 and 10 years.

Table 2

Testing H_o: d = d_o in the model: $(1 - 2\cos w_r L + L^2)^d x_t = u_t$; u_t is white noise

r / d	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
3	**-0.28**	-4.28	-6.55	-7.98	-8.96	-9.65	-10.15	-10.53	-10.82
4	2.99	**-1.27**	-2.98	-3.83	-4.35	-4.72	-4.99	-5.21	-5.39
5	**1.43**	-2.27	-5.01	-6.82	-8.06	-8.93	-9.57	-10.03	-10.83
6	**1.57**	-3.57	-6.57	-8.45	-9.66	-10.45	-10.97	-11.33	-11.59
7	**0.66**	-4.02	-7.07	-8.88	-9.94	-10.60	-11.03	-11.32	-11.53
8	**0.87**	-4.70	-7.52	-9.05	-9.95	-10.51	-10.89	-11.16	-11.35
9	**1.03**	-5.08	-7.65	-9.00	-9.80	-10.31	-10.67	-10.92	-11.11
10	**0.36**	-5.07	-7.62	-8.92	-9.67	-10.15	-10.49	-10.73	-10.92

In bold, the non-rejection values at the 5% significance level.

Table 3

Testing H_o: d = d_o in the model: $(1 - 2\cos w_r L + L^2)^d x_t = u_t$; u_t is AR(1)

r / d	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
3	**-0.39**	-5.93	-9.07	-11.06	-12.41	-13.37	-14.07	-14.57	-15.00
4	2.99	**-1.27**	-2.98	-3.83	-4.35	-4.77	-4.99	-5.21	-5.39
5	2.30	-2.58	-5.68	-7.72	-9.12	-10.11	-10.82	-11.34	-11.73
6	2.19	-6.10	-9.78	-11.92	-13.54	-14.54	-15.12	-18.79	-20.24
7	**1.10**	-6.75	-11.84	-14.86	-16.64	-17.76	-18.51	-19.06	-19.45
8	1.67	-9.03	-14.42	-17.34	-19.02	-20.09	-20.75	-21.10	-22.97
9	2.14	-10.60	-15.63	-18.72	-20.31	-21.25	-21.69	-22.52	-23.16
10	-1.03	-11.65	-16.58	-19.09	-20.37	-20.73	-21.09	-22.55	-24.30

Table 4

Testing H_o: d = d_o in the model: $(1 - 2\cos w_r L + L^2)^d x_t = u_t$; u_t is AR(2)

R / d	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
3	**1.25**	-5.96	-8.16	-9.52	-8.95	-7.98	-8.49	-8.92	-9.27
4	8.27	-4.80	-5.07	-5.27	-4.35	-2.61	-3.01	-3.36	-3.67
5	6.38	-6.16	-7.64	-8.65	-8.07	-6.71	-7.41	-8.12	-8.61
6	4.00	-7.35	-9.55	-10.44	-9.64	-8.54	-8.94	-9.54	-9.74
7	**1.46**	-5.15	-8.46	-10.20	-9.93	-9.10	-9.56	-9.92	-10.21
8	**0.63**	-5.34	-8.65	-10.28	-9.93	-9.11	-9.52	-9.85	-10.11
9	**0.12**	-5.50	-8.63	-10.17	-9.78	-8.99	-9.37	-9.68	-9.91
10	**-0.82**	-5.33	-8.20	-10.03	-9.65	-8.89	-9.25	-9.54	-9.74

Starting with the case of white noise disturbances, we see that there is a monotonic decrease in the value of the test statistics with respect to d_o. Such monotonicity is a characteristic of any reasonable statistics, given correct specification and adequate sample size. Note that the test statistic is one-sided. Thus, for example, if we reject H_o (4) with $d_o = 1$ against $d > 1$, an even more significant result in this direction should be expected when the null is tested with $d_o = 0.75$ or 0.50. We see that if $d_o = 0.5$, H_o (4) is always rejected against smaller degrees of integration, implying that the cycles are stationary. If $r = 4$, H_o (4) cannot be rejected at $d_o = 0.25$, while for the remaining values of r, $d_o = 0$ is the only non-rejection case. However, the significance of the above results may be in large part due to the un-accounting for I(0) autocorrelation in u_t. Thus, we also fit AR(1) and AR(2) processes, in Tables 3 and 4 respectively. The results are similar in both cases and most of the non-rejection values take place when $d_o = 0$ or slightly higher. Note that in many cases, the value of the test statistic changes its sign when we move from $d_o = 0$ to $d_o = 0.25$, implying that some non-rejections may take place between these two values.

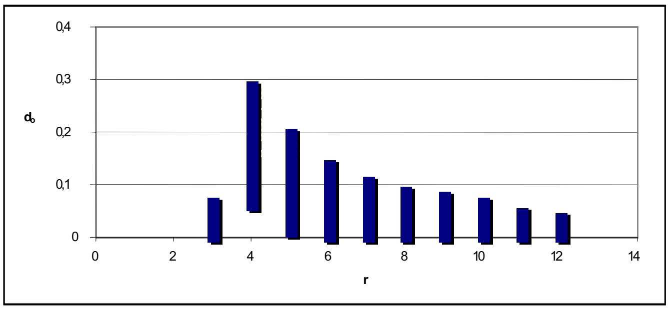


Fig. 3. (r / d_o) values where H_o: $d = d_o$ cannot be rejected in: $(1 - 2\cos w_r L + L^2)^d x_t = u_t$; u_t is white noise u_t

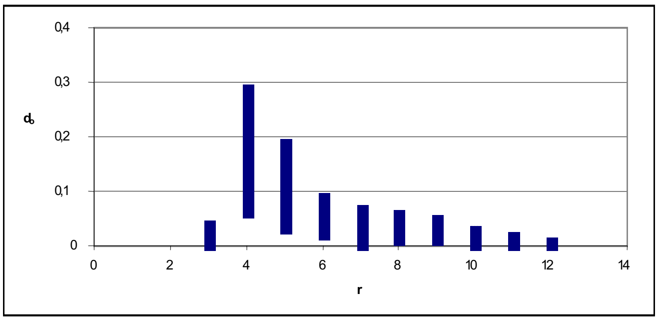


Fig. 4. (r / do) values where H_o: $d = d_o$ cannot be rejected in model: $(1 - 2\cos w_r L + L^2)^d x_t = u_t$; u_t is AR(1) u_t

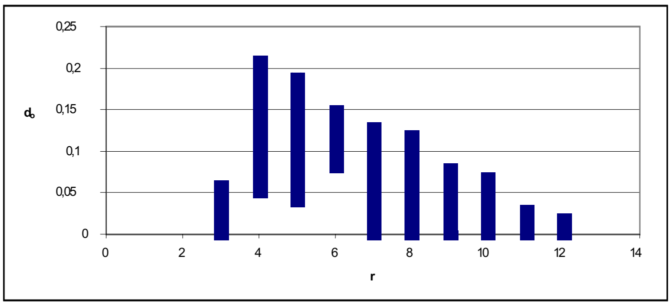


Fig. 5. (r / d_o) values where H_o: $d = d_o$ cannot be rejected in model: $(1 - 2\cos w_r L + L^2)^d x_t = u_t$; u_t is AR(2) u_t

In order to have a more precise view about the non-rejection values obtained across Tables 2-4, we recomputed the tests, but this time for values $d_o = 0, (0.01), 2$. Figures 3 – 5 display the (r, d_o) combinations of the non-rejection values of d_o at the 5% level, respectively for the three cases of white noise, AR(1) and AR(2) u_t. The results are again similar for the three cases: r is constrained between 3 and 12, and d_o oscillates between 0 and 0.3. If u_t is white noise (but also in some cases with AR disturbances), the null hypothesis of $d = 0$ cannot be rejected, though in all cases, it is "less clearly non-rejected"[1] than in the case of positive d_o. In the light of this, we can conclude by saying that there exists a component of long memory behaviour in the cyclical part of the British stock mar-

[1] By "less clearly non-rejected" we mean that the value of the test statistics is closer to the rejection critical values.

ket returns, with the order of integration ranging between 0 and 0.3 and thus, implying that cycles are stationary and mean reverting, with shocks affecting them, disappearing in the long run.

4. Concluding comments

In this paper we have examined the time series behaviour of the British stock market prices by means of fractionally integrated techniques. However, instead of looking exclusively at the case of roots occurring at the long run or zero frequency, we have also examined the possibility of long memory with respect to the cyclical part. Starting with the logged-transformed data, we show that the series contains a unit root. Here, we used both parametric and semiparametric procedures. In particular, we employed a Gaussian semiparametric method of Robinson (1995a) and a parametric testing procedure (Robinson, 1994a). We used these methods because of the distinguishing features that make them particularly relevant in the context of financial time series. Thus, they do not require Gaussianity, (which is an assumption rarely satisfied in most financial data), requiring a moment condition only of order 2. Additionally, they have standard null limit distributions, which is another unusual feature of the tests compared with other procedures for long memory. Using these and other methods, the evidence was strong in favour of unit roots.

Once it was confirmed the existence of a unit root, we concentrated on the first differenced data, and examined the returns by looking at the possibility of fractional cycles. Using another version of Robinson's (1994a) tests, we showed that long memory cycles are plausible alternatives when modelling the returns, with the periodicity of the cycles constrained between 3 and 10 years, (which is consistent with the literature on business cycle duration), and the order of integration ranging between 0 and 0.3. Thus, the cycles are stationary and mean reverting, with the effect of the shocks disappearing in the long run.

An argument that can be employed against this type of model is that cycles occurring in economics and finance are typically weak and irregular and are spread evenly over a range of frequencies rather than peaked at a specific value. A strong counter-argument is that, despite the fixed frequencies used in this specification, flexibility can be achieved through the first differenced polynomial, the ARMA components and the error term. In fact, Bierens (2001) also uses a model of this kind (with $d = 1$) to test for the presence of business cycles in the annual change of monthly unemployment in the UK. Our analysis also yields clear-cut results, which are consistent with earlier findings on the periodicity of cycles.

It would also be worthwhile proceeding to get point estimates of the fractional differencing parameter with respect to the cyclical frequency. Some attempts have been made by Arteche and Robinson (2000) and Arteche (2002). However, the goal of this paper is to show that a fractional cyclical model can be a credible alternative for the British returns to the conventional ARMA specifications. In fact, our approach leads us to some unambiguous conclusions, with the periodicity constrained to be between 3 and 10 years and the order of integration ranging between 0 and 0.3.

A potential drawback of the present work might be its univariate nature, with the limitation that it imposes in terms of theorising, policy-making or forecasting. Theoretical models and policy-making involve relationships between many variables, and forecast performance can be improved through the use of many variables (e.g., factor based forecasts based on data involving hundreds of time series beat univariate forecasts, as shown, e.g., in Stock and Watson, 2002). However, the univariate approach adopted in this paper is useful in enabling to decompose the series into a long run and a cyclical component. Moreover, theoretical econometric models for both long run and cyclical fractional structures in a multivariate framework are not yet available. In that respect, the present study can be viewed as a preliminary step in the analysis of financial data from a different time series perspective. Data mining is an additional relevant issue. Work in all these directions is now in progress.

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